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SOMMAIRE

Ce mémoire décrit une méthode simple de calibrer le géophone Willmore sans utiliser la table vibrante. Il faut supposer l'un des paramètres physiques connus, soit, dans ce cas, la masse. Les résultats de cette méthode simplifiée sont comparés à ceux obtenus selon la méthode du pont à courant alternatif où l'on mesure une impédance électrique équivalente. Ils sont aussi comparés aux résultats obtenus selon une autre technique de pont décrite par Willmore où l'on mesure la sensibilité par une méthode de substitution.

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COMPARISON OF THREE METHODS FOR CALIBRATING A WILLMORE GEOPHONE

By R. J. Donato

ABSTRACT

This paper describes a simple method for calibrating a Willmore geophone without using a shake table. It is assumed that one of the mechanical parameters of the system is known—in this case, the mass. The results from this simplified method are compared with an a.c. bridge method in which the equivalent electrical impedance is measured and with another bridge method described by Willmore where the sensitivity is measured by a substitution method. The three methods give substantially the same sensitivity of 6.1 volts/cm/sec for the particular geophone, with an estimated accuracy of 2 per cent, and all agree to the manufacturer's approximate specification to within 7 per cent.

Introduction

The most direct way to calibrate an electromechanical system is to use a shake table. If the vibration properties of the shake table are known or can be measured, the measured electrical output may be correlated to the mechanical input. In some situations, this method has disadvantages. The Willmore geophone is a relatively heavy instrument and would require a correspondingly large shake table. The geophone may be used in either the vertical or horizontal position and would have to be mechanically driven by translational vibration in each direction. Finally, as the geophone uses an electromagnetic sensing device, one has to consider the stray magnetic fields that would be produced by the more commonly used types of tables.

The calibration methods described in this paper involve representing the geophone as an equivalent electrical circuit. The values of these components may be derived from the mechanical parameters, and it will be assumed that the vibrating system has a single-degree-of-freedom and that the vibrating mass is known.

Equivalent Circuit Representation

Figure 1*A* shows the equivalent circuit representation of the geophone. The mechanical arm is in mobility form Y_m and the electrical arm in impedance form Z_c. The term Z_c represents the clamped electrical impedance, which may be taken to be an inductance and a resistance. The turns ratio of the perfect coupling transformer is $S:1$. Thus, with the geophone being used to detect mechanical vibration of velocity, u, the open circuit voltage received would be Su, and S would be the sensitivity of the geophone. The mobility term Y_m may be split into the parallel circuit form of Figure 1*B*, where M is the vibrating mass, K the spring constant, and R' a dissipative term. Figure 1*C* gives the equivalent circuit form, where the effective impedances have all been transformed by the factor S^2. For ease of representation in the analyses below, the following apply: $C = M/S^2$, $L = S^2/K$, and $R = S^2/R'$.

Simple Calibration Method

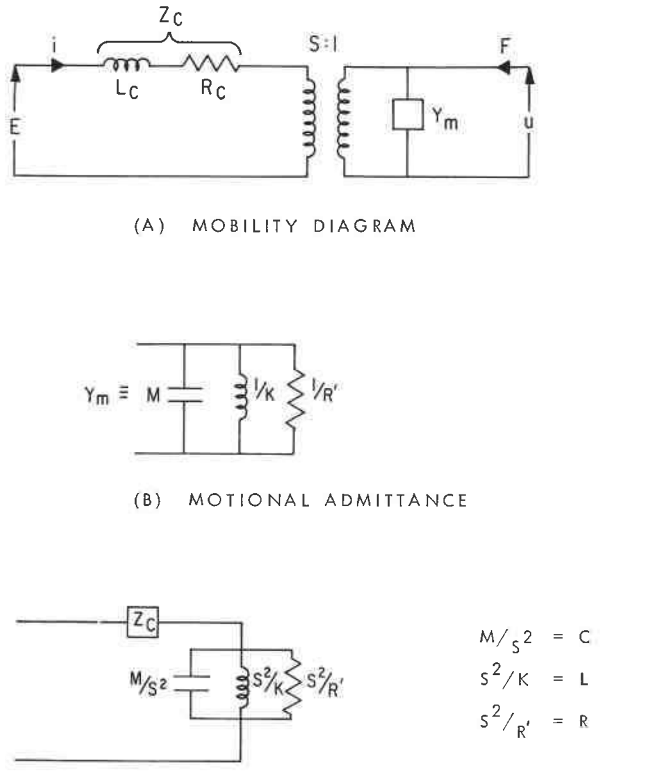
In this method, the damping at the natural frequency of vibration of the geophone is investigated. The natural frequencies of the system are given when the motional admittance becomes zero, i.e. when

$$pC + (pL)^{-1} + (R)^{-1} = 0$$

or

$$p = -(2RC)^{-1} \pm \{(4R^2C^2)^{-1} - (LC)^{-1}\}^{1/2} \tag{1}$$

where $p = j\omega = j2\pi f$, f being the frequency, and where L, C, R are the parameters shown in Figure 1C.



(A) MOBILITY DIAGRAM

(B) MOTIONAL ADMITTANCE

(C) ELECTRICAL EQUIVALENT OF GEOPHONE

FIG. 1. Equivalent circuits for geophone.

If the system has damping less than critical, one would expect the open-circuit electrical signal of the damped system to be of the form

$$E(t) = A \exp(-t/2RC) \exp\{\pm j(1/LC - 1/4R^2C^2)^{1/2}\} \tag{2}$$

where A is a constant.

If this voltage is passed through a logarithmic amplifier and recorded, the resultant wave form $V(t)$ will be

$$V(t) = \ln[E(t)] = \text{constant} + (-t/2RC) + \text{oscillatory terms.} \tag{3}$$

With the display given by equation (3), the damping term is no longer a modulation of the wave form but a linearly-decaying superposition on the oscillation. Thus, a measure of the decay of the wave form peaks will give the time constant RC of the equivalent circuit. The method is similar to that used by Barr (1964). The major differences lie in the use of a logarithmic amplifier to determine the damping constants, which can then be calculated to the same order of accuracy as the frequency of oscillation, and in the use of the decay curves to determine S directly.

To obtain a reliable value for C (and hence S), the measurement of the time constant was made for different terminating resistances at the electrical output, i.e., with the tuned circuit damped by R and R_0 in parallel, where R_0 is the external resistance.

To R_0 must be added the clamped impedance of the geophone which is composed of resistance R_c and inductance L_c. These values were determined using the a.c. bridge circuit of Figure 2, and were found to be $L_c = 11$ H and $R_c = 3.3$ kΩ. The inductance is ignored because its impedance is less than 1 per cent of the lowest value of R_0. Allow for the input impedance of the amplifier by assuming 600 kΩ to be connected across R_0.

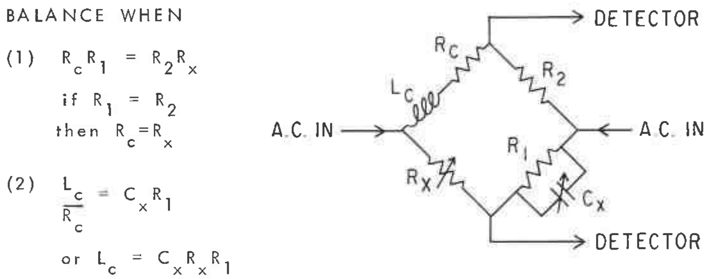


FIG. 2. a.c. bridge to determine clamped impedance.

If the measured slope of the amplitude decay is N in decibels per second rather than in nepers per second

$$1/(2R_TC) = N/8.68$$

where

$$1/R_T = 1/R + 1/R_x \qquad \text{and} \qquad R_x = R_0 + R_c.$$

Thus

$$\frac{1}{C}\left[\frac{1}{R} + \frac{1}{R_x}\right] = N/4.34. \tag{4}$$

If N is plotted against $1/R_x$, the slope m of the resulting line is $4.34/C$ and the vertical intercept is given by $4.34/CR$. It is known from Figure 1C that C is equal to M/S^2. Thus

$$m = \frac{4.34S^2}{M}. \tag{5}$$

Figure 3 shows the plot of slope in decibels per second against $1/R_x$. The calculated value of the capacitance C is 12.7 μF with an error of 0.5 μF. With a value of $M = 4.75$ kg, this gives a sensitivity S of 6.1 ± 0.1 volts/cm/sec.

From the vertical intercept of the curve, one finds that the effective motional parallel resistance R is 320 ± 60 kΩ. The accuracy is poor and it is better to measure the undamped decay time directly, which gives $R = 400$ kΩ. The accuracy of this determination is better than 10 per cent, which is good enough for calculating the resistance for, say, 0.6 critical damping. Thus, if the calculated resistance for this damping is 9 kΩ, a 10 per cent error in R would give only a 0.25 per cent error in the calculated damping resistor.

Motional Impedance Bridge Method

A bridge of the type shown in Figure 4*A* may be used to balance the geophone at all frequencies. The geophone impedance Z_G is shown as the sum of an electrical clamped impedance Z_c and an equivalent electrical impedance Z_m to represent the

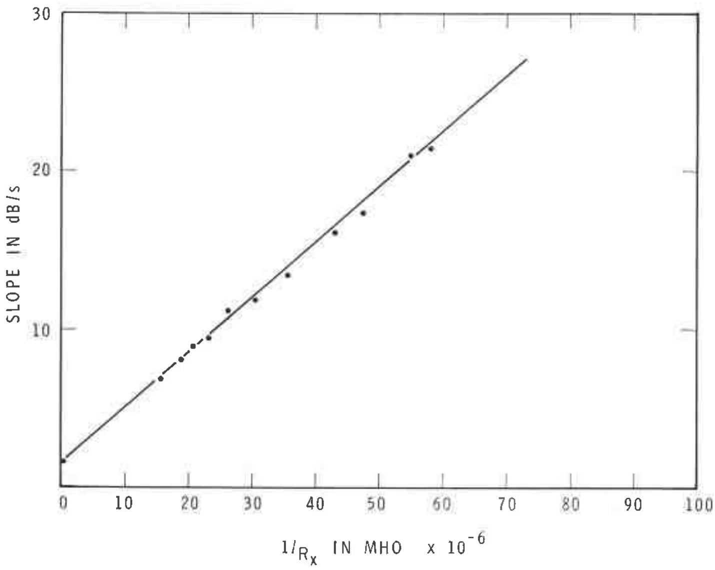


Fig. 3. Variation of decay rate with external resistance.

motional impedance. It is known that at the frequencies below resonance, the geophone impedance will be inductive. Taking the resonant frequency to be approximately 0.8 Hz, one may calculate the parallel inductance of Z_m to be around 3,000 H. The bridge will only balance if Z_G is capacitative; as a parallel capacitance would be too large to be readily available, a capacity must be put in series with the geophone. For frequencies just below resonance, the effective value of the parallel inductance of Z_m becomes very large. If it is specified that frequencies at 0.01 Hz around resonance are to be investigated, then there must be some means of counteracting inductive impedances of the order of 240 kΩ. At frequencies just above resonance, the capacitative impedance becomes very large and a capacity in parallel with the geophone is required. Finally, at frequencies higher than around 14 Hz, the geophone becomes inductive again and exhibits a series resonance at this frequency. A series capacity will raise this frequency out of the range of interest.

The circuit adopting all these features is shown in Figure 4*B*. For maximum sensitivity, the impedances of all the arms should be equal, and, to achieve this, the two fixed resistors were made equal to 39.2 kΩ. A further 39.2 kΩ was placed across the geophone.

Balance points were located on an oscilloscope. Steps were taken to isolate the geophone from both vibrating and stray electrical fields. After some calculation, the impedance of the geophone is obtained as the sum of a series resistance and a series reactance. Figure 5 shows a plot of the resistive and reactive terms about zero, and the analysis proceeds as follows.

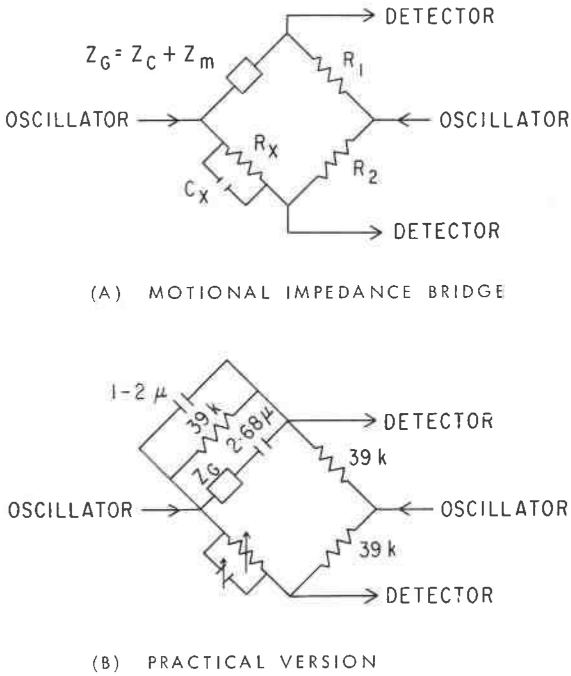


FIG. 4. Impedance bridge.

The resistive curve has a peak at 386 kΩ and the bandwidth at a resistance value of 272 kΩ is required—0.026 Hz. This is not a very accurate method for determining the Q, however, and the method is varied slightly. The admittance Y_m of the motional element is given by

$$Y_m = (R)^{-1} + j(\omega C - 1/\omega L)$$

$$Z_m = (Y_m)^{-1} = \frac{(R)^{-1} - j(\omega C - 1/\omega L)}{(R)^{-2} + (\omega C - 1/\omega L)^2}. \tag{6}$$

It can be shown by differentiating that the reactive term has extremes when $1/R = \omega C - 1/\omega L$. These are precisely those frequencies where the amplitude of the resistive term is 3 db down from its maximum. These extremes are easier to interpolate on Figure 5B than the corresponding points on the resistance curve. Taking the bandwidth to be 0.034 Hz, the value of the capacitor comes out to be 12.4 μF $\pm$ 0.4 μF. The agreement is close to that obtained using the simple method, 12.7 μF. The resonant frequency is around 0.765 Hz, which agrees with direct measurements on the resonant output from the geophone, and the value of S is calculated to be 6.2 $\pm$ 0.1 volts/cm/sec.

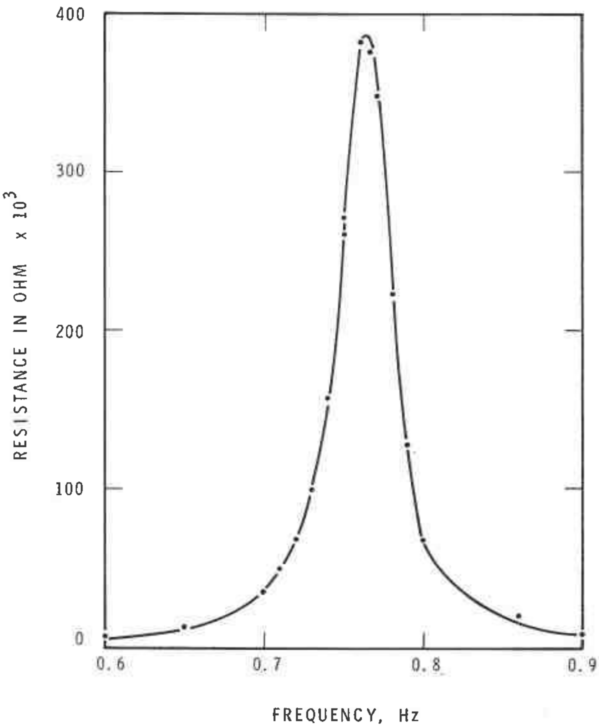


FIG. 5A. Motional resistance.

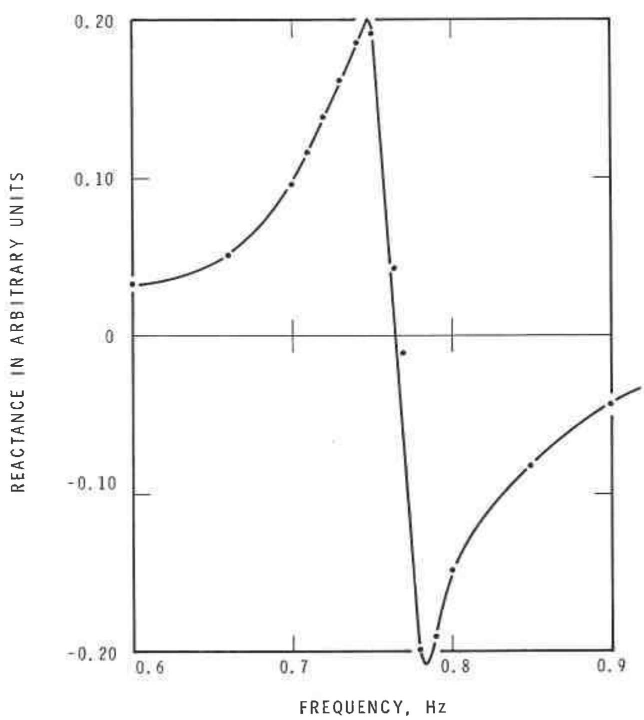


FIG. 5B. Motional reactance.

Willmore Method

Willmore's method (1959, 1960) uses a bridge similar to the one used in the former case to measure clamped impedances, but with an additional resistance to provide a facility for injecting an off-balance current into the geophone. This additional resistance R_s is placed at the junction of R_x and R_1 of Figure 3. The components are chosen so that $R_2 \gg R_c$, $R_x \ll R_c$ and $R_s \gg R_x$. Either R_1 or R_2 is made the balancing resistor—in this case R_1. The procedure is to balance the bridge clamped, unclamp and take the detector reading, and then insert the same driving voltage through R_s. This is repeated for each frequency. If the detector readings in the two cases are V_1 and V_2, then plotting $[V_2/V_1]^2$ against $(\omega - \omega_0^2/\omega)^2$, where ω_0 is the resonant frequency, yields a straight line of slope $[R_x R_2 C/R_s]^2$. In this way a value of $C = 12.9 \pm 0.3$ μF is obtained. This value is somewhat arbitrary because there is a strong dependence on the number of points chosen to fit the line. For example, if the two largest values are ignored, then C becomes 14 μF. The corresponding values of S are 6.05 ± 0.1 and 5.84 ± 0.1 volts/cm/sec.

Comparison of the Methods

All the methods agree in their value for C. It is principally the ease and speed of the determination that is important. The simple method is by far the fastest and the Willmore the slowest since it involves successively clamping and unclamping the geophone. The Willmore method also involves comparing two signals and depends a great deal on the accuracy to which ω_0 can be measured. The method of doing this may be either direct or by finding the minimum of the (V_2/V_1) versus ω curve. The impedance method is preferable to the Willmore in that it involves balancing rather than measuring, but the computation is cumbersome. One must be more careful in this method to isolate the geophone against mechanical and electrical disturbances. All of the methods fall within 7 per cent of the maker's specification of an approximate 5.75 volts/cm/sec, which is known to vary by at least 10 per cent between geophones.

Response to Ground Motion

For practical purposes, one may neglect the resistance $R = S^2/R'$ in the geophone and the inductive part of the clamped impedance. Then if R_d is the damping resistance across the output terminals, the received voltage V for an actuating force F will be

$$V = \{R_d/(R_c + R_d)\}\ (F/S)\ \{j(\omega M/S^2 - K/\omega S^2) + (R_c + R_d)^{-1}\}^{-1}. \quad (7)$$

For frequencies above resonance

$$V = \{R_d/(R_c + R_d)\}\ (F/S)(j\omega M/S^2)^{-1} \quad (8)$$

and by substituting $F = -\omega^2 Mx$, where x is the ground displacement

$$V = \{R_d/(R_c + R_d)\}\ Sj\omega x$$

$$= \{R_d/(R_c + R_d)\}\ S\dot{x}. \quad (9)$$

If the coefficient of damping is α

$$(R_c + R_d)^{-1} = 2M\alpha\omega_0/S^2.$$

Whence

$$V = \{R_d/(R_c + R_d)\}\, F/S\, \{M(j\omega + 2\alpha\omega_0 + \omega_0^2/j\omega)\}^{-1}$$

and

$$|\, V\,| = \{R_d/(R_c + R_d)\}\, S\dot{x}\, \{(1 - \omega_0^2/\omega^2)^2 + 4\alpha^2\omega_0^2/\omega^2\}^{-1/2}. \qquad (10)$$

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