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## ***Building Vibrations from Human Activities***

by D.E. Allen

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### Abstract

Health clubs, gymnasia, stadia, dance floors and even office buildings have, in recent years, experienced vibrations due to rhythmic human activities which are annoying to the users and, in rare cases, unacceptable for safety. The vibration problems are reviewed and guidelines are presented based on the unifying principle of resonance, the main factor behind the problems. The guidelines are contained in Commentary A to Part 4 of the 1990 National Building Code of Canada. Examples are given of the application of the guidelines to the design or evaluation of typical concrete structures. Remedial measures to cure vibration problems with existing concrete buildings are also reviewed.

### Résumé

Les clubs de santé, les gymnases, les stades, les pistes de danse et même les immeubles à bureaux sont exposés depuis quelques années à des vibrations produites par des activités humaines rythmiques qui sont gênantes pour les occupants et, dans de rares cas, inacceptables du point de vue sécurité. L'auteur examine les problèmes de vibration et énonce des lignes directrices en se basant sur le phénomène de résonance, qui est en grande partie à l'origine de ces problèmes. Les lignes directrices sont contenues dans le Commentaire A portant sur la partie 4 du Code national du Canada 1990. L'auteur donne des exemples de l'application des lignes directrices à la conception et à l'évaluation de structures typiques en béton. Il fait aussi état d'un certain nombre de mesures correctives pour les problèmes de vibration dans les bâtiments existants.

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# **Building Vibrations from Human Activities**

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In recent years, vibrations due to human activities have occurred in health clubs, gymnasiums, stadiums, dance floors, and even office buildings. These vibrations are annoying to the building users and, in rare cases, are unacceptable for human safety. Three factors have given rise to these problems:

- increased human activities such as aerobics and audience participation
- decreased natural frequency because of increased floor spans
- decreased damping and mass in building construction

Table 1 summarizes a number of recent vibration problems in concrete building structures. To avoid such problems, Commentary A to Part 4 of the National Building Code of Canada (NBC)<sup>1</sup> introduced new design criteria for floor structures in 1985. Additional experience since then has required some revisions to the criteria.

The unifying principle is resonance, the main factor behind most serious vibration problems in buildings. Resonance occurs when the forcing frequency coincides with or is close to a natural frequency of the structure. With each cycle of loading, energy is fed into succeeding cycles of vibration and, as a result, the vibration grows to a large amplitude controlled only by the ability of the system to absorb this energy through damping.

## Acceleration limits

The reaction of people who feel the vibration depends very strongly on what they are doing. People in offices or residences do not like perceptible vibration (about 0.5 percent  $g$ ), whereas people taking part in an activity will accept much more (about 5 percent  $g$ ). People dining beside a dance floor or lifting weights beside an aerobics gym, will accept something in between (about 2 percent  $g$ ).

Because other factors besides occupancy affect the acceptability of vibration (such as remoteness of vibration source), the NBC recommends a range of acceleration limits for each occupancy (Table 2). These limits correspond reasonably well with recommendations of the International Standards Organization.<sup>2</sup>

## Dynamic response to sinusoidal load

A simplified model of building vibration due to rhythmic human activities is needed for design and evaluation. A basic model is a person exerting a sinusoidal force (for example, by flexing the knees) to a mass attached to the ground by a spring with viscous damping. The classical steady-state acceleration response can be expressed as follows (see box for a derivation)

$$\frac{a}{g} = \frac{\alpha W_p / W \cdot \sin 2\pi f t}{\sqrt{\left[\left(\frac{f_o}{f}\right)^2 - 1\right]^2 + \left[2\beta \frac{f_o}{f}\right]^2}} \quad [1]$$

where

- $a/g$  = acceleration as a fraction of the acceleration due to gravity  
 $f$  = forcing frequency  
 $f_o$  = natural frequency of the spring-mass system  
 $\beta$  = damping ratio  
 $W$  = mass weight  
 $W_p$  = weight of the person

and  $\alpha$  is the ratio of the dynamic component of loading to the person's weight, referred to as the dynamic load factor. The greater the person's exertion, the greater is the dynamic load factor  $\alpha$ .

To understand this basic model, Eq. (1), consider the following:

1. When the ratio  $f_o/f$  approaches zero, Eq. (1) corresponds to Newton's second law, i.e., the inertial acceleration of the mass unsupported.

2. When the ratio  $f_o/f$  becomes large, Eq. (1) corresponds to the acceleration due to the time variation of static deflection [load divided by the spring stiffness; see Eq. (A1) in box].

3. At resonance when  $f_o = f$ , the inertial acceleration of the mass is increased in succeeding cycles of vibration until it reaches a steady-state maximum determined by the amplification factor  $1/2\beta$ .



Photo courtesy of Bally's Health & Tennis Corporation

A more useful basic model for group activities on floor structures, however, is a simply supported beam with mass weight  $wA$  and dynamic load,  $\alpha w_p A \cdot \sin 2\pi ft$  uniformly distributed over the tributary area  $A$ . In this case, the steady-state acceleration at midspan is very similar to Eq. (1)<sup>3</sup>

$$\frac{a}{g} = \frac{1.3 \alpha w_p / w \cdot \sin 2\pi ft}{\sqrt{\left[\left(\frac{f_o}{f}\right)^2 - 1\right]^2 + \left[2\beta \frac{f_o}{f}\right]^2}} \quad [2]$$

Although Eq. (2) is derived for a uniformly loaded, simply supported beam, it can be used for human activities on most floor systems.<sup>3</sup> Eq. (2) assumes that there is only one mode of vibration. In fact, there are many modes, but for practical problems where resonance is involved, this assumption is generally close enough.

Eq. (2) can be simplified for use in design as follows (see Fig. 1)

For resonance  $f_o = f$

$$\frac{a_{peak}}{g} = \frac{1.3}{2\beta} \cdot \frac{\alpha w_p}{w} \quad [2(a)]$$

For natural frequency greater than forcing frequency  $f_o > 1.2f$

$$\frac{a_{peak}}{g} = \frac{1.3}{\left(\frac{f_o}{f}\right)^2 - 1} \cdot \frac{\alpha w_p}{w} \quad [2(b)]$$

the latter being typical for most floor structures.

## Dancing

Consider dancers on top of a simply supported beam — a platform. The weight of the dancers can be approximated by a uniformly distributed load  $w_p$ . Dancing produces a nearly sinusoidal dynamic load of  $\alpha w_p \cdot \sin 2\pi ft$ , where  $\alpha$ , the dynamic load factor, ranges from 0.2 to 0.5 and  $f$ , the beat of music, ranges from 1.5 to

## Derivation of Eq. (1)

The standard solution for the steady-state response of an SDOF spring-mass system to sinusoidal load is given by<sup>7</sup>

$$y = \frac{P}{k} \frac{\sin(2\pi ft - \phi)}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left[\frac{2\beta f}{f_o}\right]^2}} \quad [A1]$$

where  $y$  is the deflection;  $P \sin 2\pi ft$  is the sinusoidal load at forcing frequency  $f$ ;  $M$  is the mass;  $f_o$  is the natural frequency of the spring-mass system;  $k$  is the stiffness of the spring;  $\beta$  is the damping ratio; and  $\phi$  is the phase angle between response and load. The acceleration is given by

$$\ddot{y} = a = \frac{P}{k} \frac{(2\pi f)^2 \sin(2\pi ft - \phi)}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left[\frac{2\beta f}{f_o}\right]^2}} \quad [A2]$$

It is more convenient to express Eq. (A2) in terms of mass rather than stiffness through the relationship<sup>7</sup>

$$\begin{aligned} f_o &= \frac{1}{2\pi} \sqrt{\frac{k}{M}} \\ a &= \frac{P}{M (2\pi f_o)^2} \cdot \frac{(2\pi f)^2 \sin(2\pi ft - \phi)}{\sqrt{\left[1 - \left(\frac{f}{f_o}\right)^2\right]^2 + \left[\frac{2\beta f}{f_o}\right]^2}} \\ &= \frac{P}{M} \cdot \frac{\sin(2\pi ft - \phi)}{\left[\left(\frac{f_o}{f}\right)^2 - 1\right]^2 + \left[\frac{2\beta f_o}{f}\right]^2} \quad [A3] \end{aligned}$$

Eq. (1) corresponds to Eq. (A3), with the symbols  $P$  and  $M$  replaced by  $\alpha W_p$  and  $W/g$  and, for simplification, the phase angle  $\phi$  is left out.



**Table 1 — Cases of vibration problems with concrete buildings**

Type of building and location	Date	Type of construction	Cause	Complaints from	Natural frequency, Hz	Measured peak acceleration, percent g	Solution implemented
Health Clubs, Gymnasiums							
1. Alberta	1988	Reinforced concrete	Aerobics	Occupants above	5		Relocation
2. Switzerland	1977	Reinforced concrete	Aerobics	Occupants, safety	4.9	50	Stiffen with plates
3. New York	1988	Precast T	Aerobics	Office workers	4.5	2.5	Tuned dampers
Office Buildings							
4. Ontario	1985	Reinforced concrete - 18 Story	Aerobics	Office workers	4.4	0.4	Not known
5. Ontario	1986	Reinforced concrete - 26 story	Aerobics	Office workers	4.2	1	Stop activity
Concert Halls							
6. Switzerland	1982	Reinforced concrete cellular slab	Pop concert	Occupants, safety	4.5	30	Relocation
Dance Floors							
7. Great Britain	1974	Reinforced concrete	Rock dance	Occupants	2.9		Control music
8. Georgia	1983	Reinforced concrete?	Aerobics, Greek dance	Participants	—	—	Stiffen with cables
Stadiums							
9. Ontario	1980	Precast seats	Pop concert	Audience	2.4	34	No action
10. South Carolina	1986	Reinforced concrete cantilever	Football song	Fans	—		Alter activity

3 Hz.<sup>3</sup> The response of the platform is given by Eq. (2).

If resonance occurs, the peak acceleration given by Eq. [2(a)] will generally turn out to be very large, of the order of 10 percent *g* or more. Such large accelerations are unacceptable and usually the only practical way to avoid the problem at the design stage is to increase the stiffness. Increasing the stiffness increases the ratio  $f_o/f$ , which, according to Eq. [2(b)], results in a substantial decrease in peak acceleration. Eq. [2(b)], however, can be inverted to provide a simple design criterion for minimum natural frequency

$$f_o \geq f \sqrt{1 + \frac{1.3}{a_o/g} \cdot \frac{\alpha w_p}{w}} \quad [3]$$

where  $a_o/g$  is the acceleration limit.

## Aerobics

Jumping exercises produce periodic forces that are not sinusoidal and this results in harmonic overtones,<sup>3,4</sup> that is, sinusoidal loading components that involve not only the beat of music but also multiples or harmonics of the beat of music. Fig. 2 shows measured dynamic loading from a group of eight people jumping, which clearly indicates sinusoidal loading components for the first three harmonics. Sinusoidal loading components beyond the third harmonic can be ignored for practical purposes.<sup>5</sup> Dynamic load factors are recommended in Table 3 for design.<sup>1</sup>

The natural frequencies of floor structures are generally greater than 3 Hz and therefore the second and third harmonics are more likely to cause resonance.

Cases 1 through 6 in Table 1 are examples in which resonance occurred at the second harmonic. A case was encountered recently in which resonance, shown by the response in Fig. 3, occurred at the third harmonic.<sup>5</sup> Note the reversal in importance of the higher harmonics between loading in Fig. 2 and response in Fig. 3.

Jumping vibration therefore involves response to all three harmonics of the jumping frequency. Each sinusoidal loading component produces a steady-state sinusoidal vibration, the acceleration of which can be determined from Eq. (2). Because the three sinusoidal vibrations occur simultaneously, their peak accelerations must be combined. The combined effective peak acceleration to this motion can be determined from<sup>5</sup>

$$a_m = [a_1^{1.5} + a_2^{1.5} + a_3^{1.5}]^{1/1.5} \quad [4]$$

where  $a_i$  is the peak acceleration for each harmonic vibration determined from Eq. (2). Eq. (4) is based on human reaction criteria recommended by the International Standards Organization.<sup>3</sup>

Eq. (3) can also be used to estimate the minimum natural frequency for all three harmonics of aerobics, the highest governing. All three harmonics add together, however, and to account for this, the factor 1.3 in Eq. (3) should be increased to 2.0<sup>5</sup>

$$f_o \geq i f \sqrt{1 + \frac{2.0}{a_o/g} \cdot \frac{\alpha_i w_p}{w}} \quad [5]$$

where  $f$  is the jumping frequency and  $i$  is the harmonic.

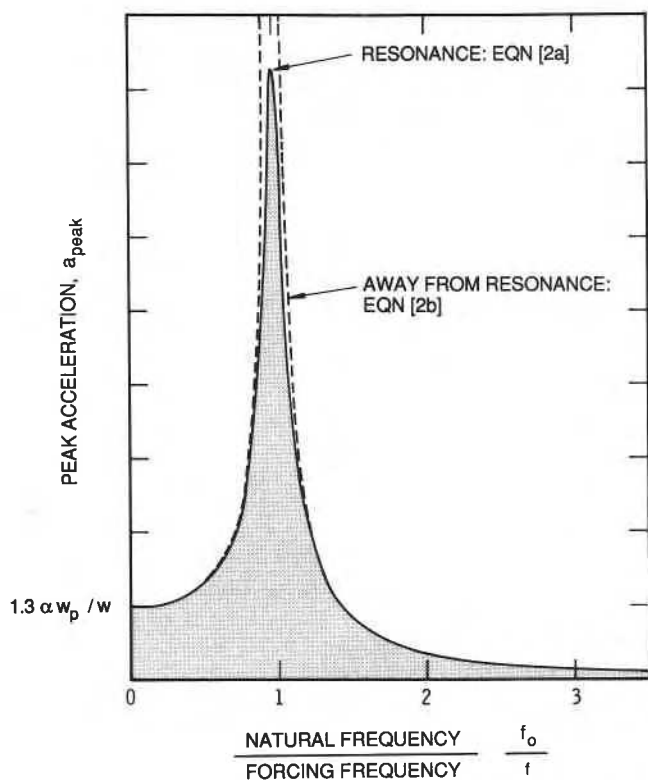


Fig. 1—SDOF response to sinusoidal load — Eq. (2)

Alternatively, Eq. (2) and (4) can be used to check the effective peak acceleration for any assumed natural frequency. This may be useful when third harmonic resonance vibration is sufficiently low, as occurs when damping times mass is much greater than dynamic loading (Eq. [2(a)]).

### Audience participation

Audience participation in stadiums, arenas, and concert halls involves a wide variety of human activities and a representative loading function is more difficult to define. The NBC recommends a loading function similar to dancing with the dynamic load factor reduced by half to account for the reduced overall effort and participation. Some people near the stage of rock concerts, however, jump to the music beat and the loading function for these individuals should be assumed similar to that for aerobics. Also, football fans will stamp to a beat and it is therefore recommended that a second harmonic be considered with a dynamic load factor of approximately 0.05.

### Estimation of natural frequency

Resonance is the most important factor affecting vibration from rhythmic activities; hence, natural frequency is the most important structural design parameter. Natural frequency of floor systems generally is estimated by means of the frequency formula based on beam flexure. Recent experience, however, has shown that shear deformation (important for deep beams, girders, and trusses) and flexibility of supports (girders, columns, etc.) can substantially increase the flexibility of the floor system and, consequently, reduce the natural frequency from that based on beam flexure. A

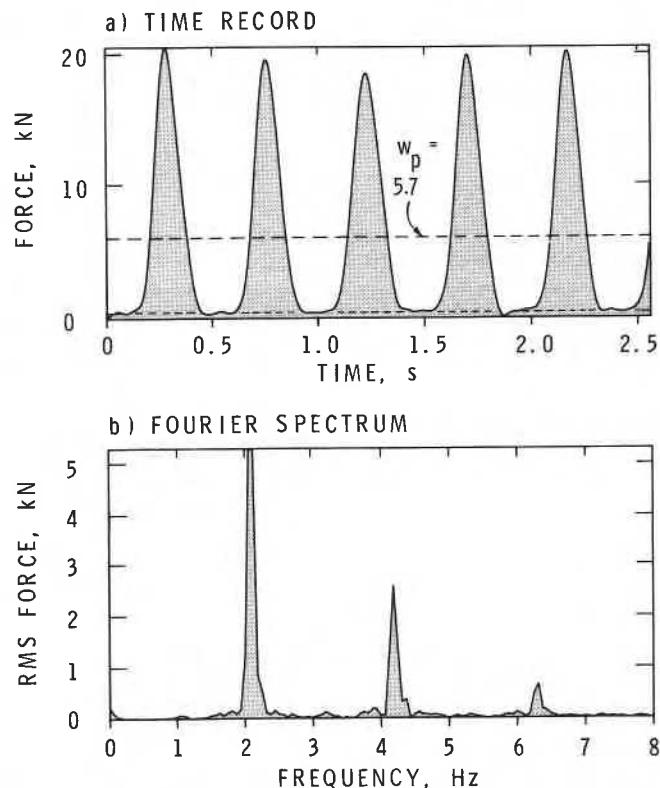


Fig. 2—Floor load from eight people jumping at 2.1 Hz.<sup>2</sup>

better approximation is to use the relationship for a simple spring-mass system

$$f_o = \frac{1}{2\pi} \sqrt{\frac{\text{stiffness}}{\text{mass}}} = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta}} \quad [6(a)]$$

where  $\Delta$  is the total deflection of the spring due to its mass weight. In the case of a beam and girder floor system, the equivalent "spring" deflection  $\Delta$  can be approximated by<sup>5</sup>

$$\Delta = (\Delta_B + \Delta_G)/1.3 + \Delta_s \quad [6(b)]$$

where

- $\Delta_B$  = the elastic deflection of the floor beam at mid-span due to bending and shear
- $\Delta_G$  = the elastic deflection of the girder at the beam support due to bending and shear
- $\Delta_s$  = the elastic deflection of the column or wall support (axial strain)

and where each deflection results from the total weight supported by the member, including the weight of people. Both supports are considered and the most flexible one is used in the calculation. The factor 1.3 in Eq. [6(b)] applies to most beam-floor systems. In the case of fixed cantilevers and two-way slabs, however, the factor should be increased to 1.5.

In the case of beams or girders continuous over supports, the elastic deflection  $\Delta_B$  or  $\Delta_G$  should be determined using the complete mode shape, i.e., adjacent spans deflect in opposite directions with no change in



**Table 2 — Recommended acceleration limits for vibrations due to rhythmic activities**

Occupancies affected by the vibration	Acceleration limit, percent gravity
Office and residential	0.4 to 0.7
Dining and weightlifting	1.5 to 2.5
Rhythmic activity only	4 to 7

**Table 3 — Recommended dynamic load factors for aerobics<sup>5</sup>**

Harmonic	Forcing frequency, Hz	Dynamic load factor, $\alpha$
1	2 - 2.75	1.5
2	4 - 5.5	0.6
3	6 - 8.25	0.1

slope over the supports, and by assuming that the weight supported by each span always acts in the direction of deflection (up if the deflection is up, down if it is down). Application of this procedure shows, for example, that a continuous beam with equal spans on pin supports has the same natural frequency as any of its simply supported spans.

Most building vibration problems due to human activities concern natural modes of vibration of the floor structure. This is not always true, however. Cases 4 and 5 in Table 1 concern two high-rise office buildings in which the entire structure vibrated vertically as a result of aerobic exercises in the building. The mode of vibration in resonance with the second harmonic of jumping frequency consisted primarily of axial deformation of the tall column tiers. Estimation of natural frequency and response of this building mode to aerobics is described in Example 4.

## Determining distributed load and floor weight

For a simply supported beam or girder uniformly loaded by participants, the distributed values of  $w_p$  and  $w$  in Eq. (2), (3), and (5) are equal to their estimated values. In cases where participants occupy only part of the span, the value of  $w_p$  is correspondingly reduced. In cases where vibrating beams, girders, or columns support extra mass weights not included in the mass weight of the floor itself, these extra weights can be taken into account by increasing  $w$ . Reference 5 contains procedures for doing this.

It is reasonably conservative to include the participants as part of the floor weight  $w$ , especially for determining natural frequency. For some activities, such as rock concerts, the audience acts mostly as a rigid mass. For other activities, such as aerobics, the participants act mostly as an externally applied force, but

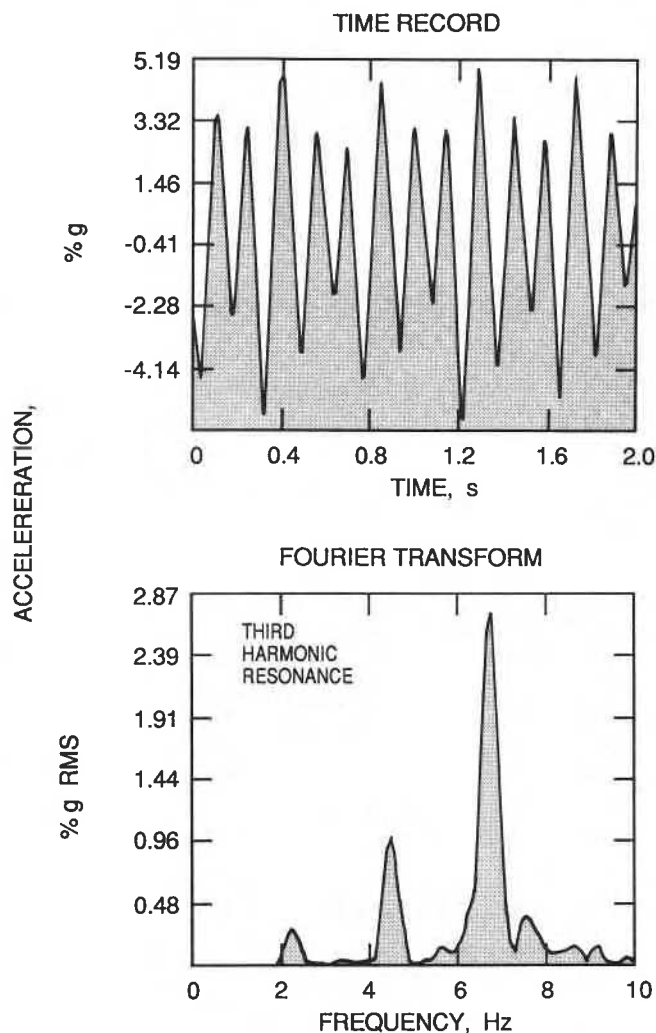


Fig. 3—Vibration of a 6.7 Hz floor due to aerobics at 2.25 Hz.<sup>5</sup>

their distributed weight is generally small compared to that of the floor.

## Examples

These examples give an understanding of the principles involved in applying the recommended criteria. Other cases are described in Reference 6.

### Stadium with precast concrete seats

Vibration from audience participation is the concern in designing the prestressed precast seating (Fig. 4), with an acceleration limit of 4 to 7 percent  $g$  for this occupancy. The weight of people is estimated to be 1.4 kPa (29 psf) and the mass weight of the structure plus people is estimated to be 7.5 kPa (157 psf).

If, as recommended in the NBC, single harmonic loading is assumed at a maximum forcing frequency of 3 Hz with a dynamic load factor of 0.25, then applying Eq. (3) with an acceleration limit of 5 percent  $g$  results in a minimum natural frequency of 4.5 Hz.

Football fans stamping in unison could, however, create a second harmonic with a dynamic load factor of approximately 0.05 at a maximum frequency of 5.5 Hz. Applying Eq. (3) in this case results in a minimum nat-

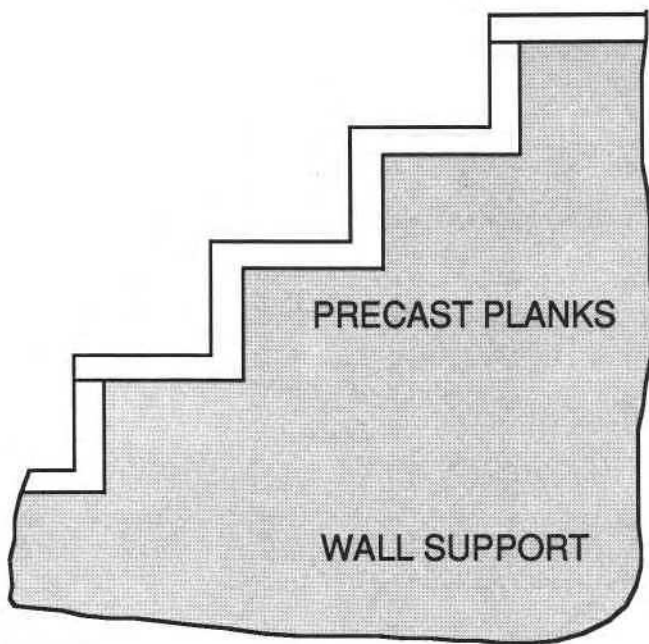


Fig. 4—Stadium with precast concrete seating.

ural frequency of 6.1 Hz. If the natural frequency of the seats is 5 Hz, then the peak acceleration for fans stamping at 2.5 Hz is, from Eq. [2(a)], equal to 10 percent  $g$  based on an assumed damping ratio of 0.06 for precast seating filled with people. This is definitely noticeable, and may cause concern to the participants.

### Gymnasium

An aerobics gym floor over a swimming pool consists of precast T-beams spanning 15 m (49 ft) supported on concrete block walls. Because there are no other occupancies involved, the acceleration limit for design can be taken as 5 percent  $g$ . It is estimated that the aerobicsists weigh 0.2 kPa (4 psf) maximum and that the floor weight is 5 kPa (104 psf). The damping ratio is estimated to be 0.03.

Applying Eq. (5) for aerobics at a maximum jumping frequency of 2.75 Hz results in minimum natural frequencies of 5.1, 7.7, and 8.9 Hz for the first, second, and third harmonics, respectively. Thus, a minimum natural frequency for the floor of 9 Hz is indicated.

If half the floor span were converted to weight lifting, the acceleration limit would be reduced to 2 percent  $g$  and the equivalent weight of aerobicsists reduced to 0.1 kPa (2 psf). This results in a small increase in minimum natural frequency.

If half the floor span were converted to offices with an acceleration limit of 0.5 percent  $g$ , the minimum natural frequency according to Eq. (5) would increase to 13.5 Hz, which would be very difficult to achieve for such a span. Therefore, this mix of occupancies is not recommended.

Assume that the natural frequency of the floor is 7 Hz. If the aerobics load for the whole span were kept below 0.1 kPa (2 psf) (by loading arrangement, for example), then for jumping at the critical frequency of 2.33 Hz the peak harmonic accelerations from Eq. (2)

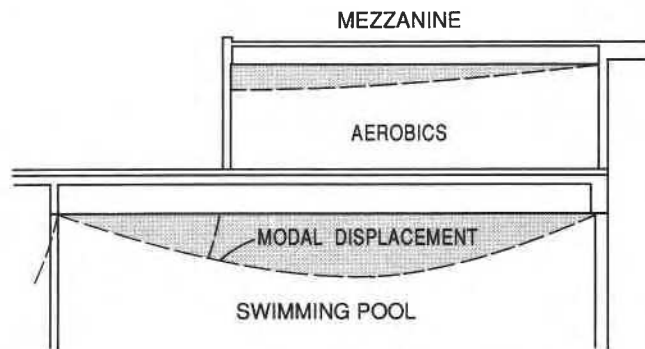


Fig. 5—Health club with reinforced concrete beams.

are 0.5, 1.2, and 4.3 percent  $g$ , respectively, with resonance occurring at the third harmonic. Substitution in Eq. (4) results in an effective peak acceleration of 4.8 percent  $g$ , which appears to be satisfactory for this occupancy.

### Health club — reinforced concrete

An aerobics area is supported by long-span, cast-in-place T-beams with a span/depth ratio of 17.5. At one-third span, alternate beams support columns from a mezzanine floor above (Case 1, Table 1; Fig. 5). Large vibrations due to aerobics associated with the second harmonic of the jumping frequency (5 Hz) are disturbing to people using the mezzanine.

This is a two-floor reinforced concrete structure in which the mezzanine floor provides additional mass to affect both frequency and distributed mass weight  $w$ . The effect on frequency is taken into account by including the column load in determining the deflection  $\Delta_b$  for Eq. (6), and the floor weight, 6.5 kPa (136 psf), is increased by approximately 1.5 kPa (31 psf) by using the procedure described in Reference 5. The peak acceleration felt by people using the mezzanine, however, is reduced by approximately 20 percent compared to that at midspan of the aerobics floor (based on the modal displacement sketched in Fig. 5).

The equivalent weight of aerobicsists over the floor span is estimated to be 0.10 kPa (2 psf). Applying Eq. (2) for jumping at 2.5 Hz to a 5 Hz floor, with an assumed damping ratio of 0.03, results in accelerations in the mezzanine floor of 12 percent  $g$ , almost entirely due to second harmonic resonance. This is obviously unacceptable. To obtain an acceptable floor with an acceleration limit of 2 percent for the mezzanine, the natural frequency would, according to Eq. (5), have to be increased to 9 Hz.

The solution to this problem was to relocate the aerobics to a stiffer floor area.

## 26-Story concrete office building

Aerobics takes place in the corner of the top floor of a reinforced concrete office building (Case 5, Table 1; Fig. 6) causing unacceptable vibration in the offices near the corners of the upper stories of the building. The vibration is associated with resonance at the second harmonic of the jumping frequency, 4.2 Hz. Initially it was thought that this was a natural frequency of vibration of the floor structure, but measurements showed that the lowest natural frequency of the flat slab was 10 Hz. After some confusion, it was determined that the outer part of the building corners were vibrating up and down, with the columns acting as simple springs.

The natural frequency of a typical column tier acting as a spring can be obtained from Eq. [6(a)] by determining the total deflection due to the dead weight supported by all columns in the tier. It is estimated that the columns carry, on average, a dead weight stress of 6 MPa (870 psi) on the concrete. For an assumed concrete modulus of 30,000 MPa (4350 ksi) and a total height of 80 m (262 ft), this results in a total shortening of 16 mm (0.63 in). Because the mass of the column tier is distributed along its height, the factor 1.3 in Eq. [6(b)] is applied to determine  $\Delta$  for Eq. [6(a)]. The resulting natural frequency, 4.4 Hz, is close to the measured one, 4.2 Hz.

The gym area is approximately 80 m<sup>2</sup> (861 ft<sup>2</sup>) with a total weight of aerobicists of  $0.2 \times 80 = 16$  kN (3.6 kips). All corners of the building participate in the vibration, with equivalent areas of approximately 100 m<sup>2</sup> (1076 ft<sup>2</sup>) per corner, or 400 m<sup>2</sup> (4306 ft<sup>2</sup>) per floor. Assuming an average building weight of 7 kPa (146 psf) of floor area, the overall participating building weight is estimated to be  $26 \times 400 \times 7 = 72,800$  kN (16,400 kips). Resonance response to the second harmonic can be estimated by Eq. (1) with  $f = f_o$  and the equivalent mass at the top of column springs equal to half the distributed building mass (determined according to the procedure described in Reference 5). The resulting peak acceleration for a damping ratio of 0.02 is 1.1 percent g, compared to a measured acceleration of 1 percent g. This level of vibration was completely unacceptable to office workers in the floors below.

There was no practical solution to this problem except to stop the activity.

## Remedial measures

Table 1 shows that a variety of solutions have been applied to cure problem vibrations, ranging from doing nothing to expensive stiffening. Remedial measures, and their advantages and disadvantages, include:

**Doing nothing** — If the problem is not severe, this is sometimes the best solution. It may be possible, however, to impose some control on the activity, for ex-

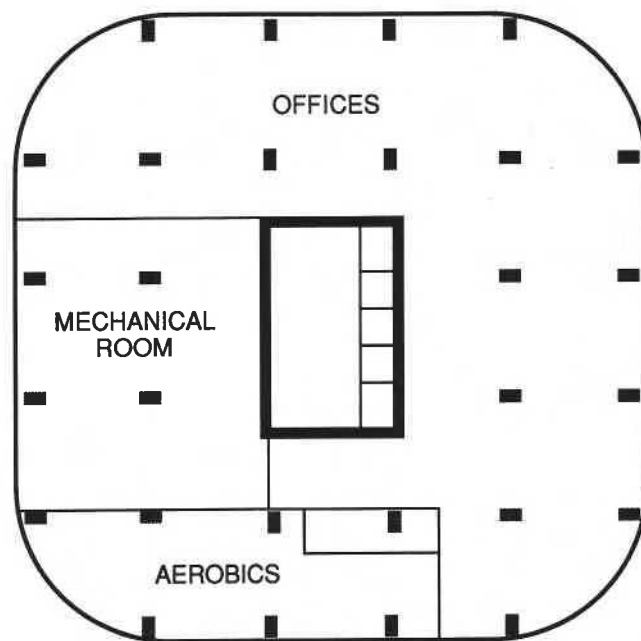


Fig. 6—Top floor of a 26-floor concrete office building.

ample, avoiding a particular music beat. In addition, objects that rattle or are tuned to the vibrating floor affect human reaction and should therefore be suitably altered.

**Relocation** — This is often the most economical solution. It is usually easier to separate a sensitive occupancy from an adjacent active one or to find a stiff floor elsewhere than to try to alter the annoying floor.

**Tuned Dampers** — Tuned dampers have so far not been very successful; reasons include:

- the dampers must be tuned to all modal frequencies likely to be in resonance with a harmonic
- the initial damping ratio must be low (about 0.02 or less) for the added damping to make a sufficient difference
- tuned dampers are not effective in altering off-resonance vibration, governed essentially by Eq. [2(b)]
- they must be maintained

Tuned dampers become viable when the ratio of mass weight  $w$  times damping  $\beta$  to dynamic load  $\alpha w_p$  is high, as occurred in Case 3 in Table 1.

**Partitions** — Partitions can be helpful in two ways. First they increase the damping. Second, if suitably arranged, they can prevent transmission of vibration down the floor (i.e., across the beams), effectively isolating the vibration to a portion of the floor. They can be harmful, however, if they transmit vibration from floor to floor. In Case 3 in Table 1, full-height fire partitions transmitted aerobics vibration to a sensitive office floor below.

**Stiffness** — If the span is very long and the requirements are severe (sensitive occupancy), then the cost becomes prohibitive. Cables and plates have been used successfully and incorporating story-height structural partitions is a possibility. Shear deflection, however, must be taken into account in the design of stiffening.

Curing the problem can sometimes be very difficult. In the future, it is recommended that any existing long-

span floor contemplated for rhythmic activities be evaluated by a performance test.

#### Notation

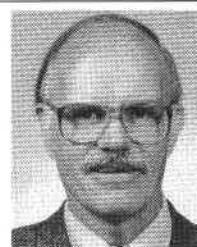
- $a$  = acceleration  
 $a_o$  = acceleration limit  
 $f$  = jumping frequency  
 $f_o$  = frequency of natural vibration  
 $g$  = acceleration due to gravity  
 $i$  = harmonic multiple of jumping frequency  
 $w$  = uniformly distributed weight of floor, with people  
 $w_p$  = uniformly distributed weight of people  
 $\alpha$  = dynamic load factor  
 $\Delta$  = deflection due to mass acting as a weight

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