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LIMIT STATE DESIGN—A UNIFIED PROCEDURE FOR THE DESIGN OF STRUCTURES

by
D. E. Allen

ANALYZED

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CALCUL À L'ÉTAT LIMITE - MÉTHODE UNIFIÉE POUR LE CALCUL DES STRUCTURES

SOMMAIRE

Cette communication décrit le calcul à l'état limite—une méthode unifiée pour le calcul de tous genres de structures du génie civil. Cette méthode est employée en U.R.S.S. depuis 1954 et son emploi pour le béton armé se répand en Europe. Les raisons de son usage de préférence à d'autres méthodes, telles celle des contraintes admissibles ou celle du comportement élastoplastique, sont présentées. Avant de décrire le calcul à l'état limite, on présente un bref aperçu des déficiences des règles actuelles de calcul et des critères du calcul optimal des structures.

Limit State Design — A Unified Procedure for the Design of Structures

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Paper presented at the 83rd Annual Meeting of the EIC, Vancouver, September 1969

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This paper describes limit state design—a unified procedure of structural calculations for the design of all types of civil engineering structures. It has been in use since 1954 in the U.S.S.R.[1] and it is being adopted in Europe[2, 3] for reinforced concrete construction. The reasons for its increasing popularity over existing procedures, such as allowable stress design and plastic design, are given in this paper. Before describing limit state design a brief review is given of shortcomings of existing design rules, criteria of structural design and optimum design.

1. INTRODUCTION

1.1 Shortcomings of Existing Design Rules

Structures are getting thinner but structural codes are getting very much thicker. The increasing complexity of design rules makes it more difficult for the designer to be aware of their intent and consequently he may lose sight of the principles of design. This in turn can lead to major design errors which usually occur because some aspect of structural behavior or loading is entirely neglected.

The increasing size of structural codes cannot be avoided, but it would help the designer considerably if they had a strong unifying basis.

Some shortcomings of existing design procedures follow:

(i) Design procedures, even if they are approximate, should directly reflect the conditions of collapse or unserviceability that are to be avoided. As shown later, allowable stress design and plastic design are limited to certain types of structures. Furthermore, serviceability criteria are sometimes expressed in a way which hides the actual conditions to be designed for (e.g. vibration uncomfortableness expressed as a member deflection limitation); this tends to promote incorrect or uneconomical design solutions.

(ii) Design procedures should be easily adaptable to new structures and different conditions of workmanship. This can only be done by stating the degree of safety required and by having a procedure which directly reflects the conditions to be designed for as stated in (i).

(iii) A uniform safety factor does not mean uniform safety. Sometimes, as shown later, the allowable stress (single safety factor on material strength) or the single load factor is a misleading indicator of the actual safety of the structure.

(iv) Not all members should have the same safety. It is very uneconomical to require wall cladding or a barn to have the same high safety as a roof truss in an arena.

1.2 Criteria of Structural Design[4]

Shortcomings such as these indicate a need to revise the way in which we carry out design calculations. Any new procedure must be based on what we ultimately require of the structure. This can be stated as follows:

The object is to design, construct and maintain economically a structure that will serve usefully (i.e. with little or no damage) under the expected loads, without disquiet to the user and no loss of life. Also, the likelihood of collapse from unusual loads such as earthquakes or extreme winds, which might endanger public safety, should be very small.

From this the criteria of structural design are *serviceability, public safety* and *economy*. Whereas it is in the interests of public safety and serviceability to increase the design safety factors, it is in the interest of economy to decrease them to a certain level. These influences lead to an optimum design solution[5] which is an extension of the most economic design solution.[6]

Figure 1A —Probability of Unserviceability

Figure 1B —Probability of collapse

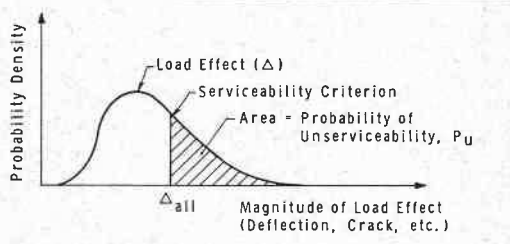
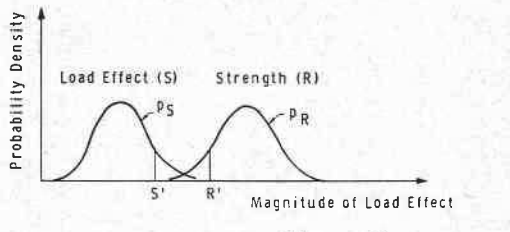


FIGURE 1A
PROBABILITY OF UNSERVICEABILITY



Probability of Collapse (7) $P_c = \int_0^\infty p_S(x) \cdot \int_0^x p_R(y)dy \cdot dx$

FIGURE 1B
PROBABILITY OF COLLAPSE

1.3 Optimum Design

The *most economical design* is the one that gives the minimum total cost per year of service. The total cost may be expressed as

$$TC = IC + C_c P_c + C_u (P_u - P_c) \qquad \ldots (1)$$

where: TC is the total cost; IC the initial cost; C_u the cost if the structure becomes unserviceable; C_c the cost if the structure collapses; P_u the probability that the structure becomes unserviceable; P_c the probability that the structure collapses. The probability of unserviceability, P_u, and the probability of collapse, P_c, for any particular design is calculated as shown in Fig. 1, although in general this is not easily done. The most economical design (i.e. the most economical dimensions, material properties and associated values of P_u and P_c) is the one that minimizes equation (1). The values of P_u and P_c so determined then give the most economic design safety factors.

This solution for the most economic design can be extended to include all the criteria—economy, serviceability and public safety. This is done by

representing the terms IC, C_u and C_c in equation (1) as *losses*, part of which can be represented directly in terms of money, and part of which cannot, especially certain unserviceabilities such as a crack. *The optimum design corresponds to the values of P_u and P_c for which the losses are a minimum.*

Direct optimum design as outlined above is not simple and it requires information not usually available—the relative losses (especially C_u in Eq. (1)) and the statistical information shown in Fig. 1 necessary for calculating P_u and P_c. There is, nevertheless, the possibility of estimating the missing information, in which case optimum design becomes an example of "decision theory in the face of uncertainty."[5]

The loss C_c must take into account human death and injury. Instead of assessing human life in terms of money which may be morally objectionable, the risk of death for any probability of collapse, P_c, can be compared with other risks we take during our lives.[7, 8] The risk of death, which at present is extremely low, must always be kept very much lower than for car travel, cigarette smoking, etc.

1.4 Practical Considerations

It appears, however, that direct optimum design might be practical only for certain types of structures such as transmission towers, precast joists, street lamps, etc., in which the economic return on extra design effort will pay off because of the large number produced.

From what has been said so far it is clear that existing structural design rules have many shortcomings, whereas direct optimum design is usually too complicated and lacks the necessary information. The rest of this paper is devoted to "limit state design" which might be better called "structural calculations for limit states." Limit state design is an improved version of existing design procedures.

2. LIMIT STATE DESIGN

2.1 Limit States

As already mentioned, the object is to design a structure which is serviceable under the expected (working) loads and unlikely to collapse due to unusual loads. We must, therefore, design the structure for the conditions which correspond to loss of serviceability and collapse. *Limit states* are just these conditions, that is, *those structural conditions for which the structure is no longer fit for the purpose for which it is intended.* The various limit states are as follows:[9]

Ultimate Limit States

—collapse due to fracture (fatigue), rupture, primary buckling, overturning, etc.
—excessive inelastic deformation (plastic mechanism).

Serviceability Limit States

—excessive deflection
—damage to non-structural elements due to structural deformation
—excessive local deformation—cracking, spalling
—excessive oscillation or acceleration.

Now in accordance with the criteria of structural design, *the aim of limit state design is to furnish satisfactory probabilities, P_u and P_c, that the limit states will not be reached during the service life of the structure.* Since limit state design is an approximate method, it must in general be conservative so that the failure probabilities,

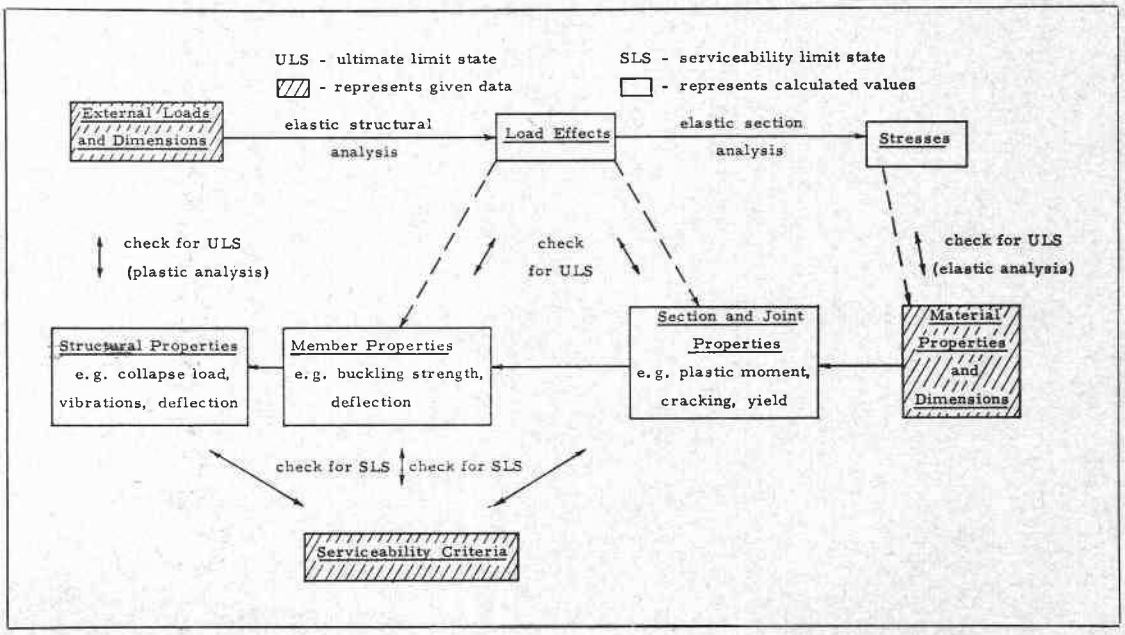


Figure 2—Scheme of structural calculations —limit state design

P_u and P_c, must be no greater than those which just satisfy optimum design. With this in mind, the procedure of limit state design will now be outlined.

2.2 Procedure of Limit State Design

Figure 2 illustrates the steps in calculating the adequacy of a particular structure for any limit state. The data in the calculation are the external loads, material properties, dimensions (part of which may be left unknown in direct design) and serviceability criteria (e.g. allowable crack width). The actual values of the external loads, material properties and dimensions as they occur in the structure during its life are not known to the designer beforehand.

In the procedure of limit state design, these parameters are represented by their specified values defined as follows.

2.3 Specified Values

In the development of limit state design[2] an attempt has been made to choose the values of loads and material properties, etc. given in codes on the basis of probability of occurrence. This has led to a new definition as follows: the *characteristic* value of the parameter (load, material property, etc.) is defined as the value of the parameter for which there is not more than a *given probability* that the *actual* parameter will be more unfavorable (see Fig. 1B—S' and R'). If the characteristic values are used, there is some positive control on the probabilities P_u and P_c. In this case limit state design is called a "semi-probabilistic" design procedure.

In actual practice, however, the statistical information needed for calculating the characteristic values, especially for certain loads, is not available or else the loading of the structure is liable to change due to change of use. Therefore, at present, limit state design is based partly on characteristic values (e.g. concrete cylinder strength, code wind speeds) and partly on specified values (e.g. live loads). The *specified* values are the values given in a code, given by the owner, or, in the case of dimensions, shown on the drawings.

In the remainder of this paper the characteristic values and the specified values will both be referred to as the *specified* values denoted by a prime, e.g. D' for dead load, f'_c for concrete compressive strength, and d' for depth to reinforcing in a reinforced concrete beam or slab.

2.4 Load, Material and Dimension Factors

To take account of unfavorable deviations in practice, the specified values of the loads, material properties and dimensions are multiplied by partial safety factors: in the case of loads by *load factors*† (α) and in the case of material properties and dimensions by *material* and *dimension factors* (β). Possible load, material and dimension factors are given in Table 1A for the ultimate limit states and in Table 1B for the serviceability limit states of local damage and deflection. Table 1 is based on existing codes, various proposals[9, 10, 11] and the author's assessment.

For any given limit state, these factors depend only on the type of load, material property or dimension. Since dead load can usually be predicted more accurately than other loads, the load factor for dead load is less severe than for other loads (Table 1A). Similarly, since steel strength can be predicted more accurately than concrete strength the material factor for steel is less severe than for concrete (Table 1A).

The values of the parameters obtained after multiplication by the load, material or dimension factors are called the *design* values, and are denoted by an asterisk. For example:

$$D^* = \alpha_D \cdot D'$$
$$f^*_c = \beta_{f_c} \cdot f'_c \qquad \ldots (2)$$
$$d^* = \beta_d \cdot d'$$

As already mentioned, the load, material and dimension factors take into account unfavorable deviations in practice from the specified values. In the case of loads, deviations are usually of a random nature but they may also take account of change of use or increase in future payload. In the case of material properties, these deviations include random deviations (variations in the field as measured by the standard test) as well as systematic differences (differences between the standard test value and the property in the structure). In the case of concrete, for example, systematic differences between the standard cylinder test and the strength in a beam arise due to differences in curing conditions, duration of load, etc. These systematic differences must be taken into account by the material factor, β_{f_c}.

Figure 3
Example 2

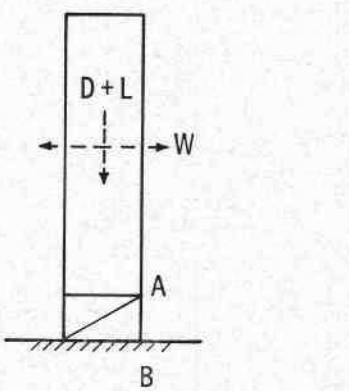


FIGURE 3 EXAMPLE 2

† Note that "load factor" is defined differently for limit state design than for plastic design.

Table 1A Possible Partial Safety Factors for Ultimate Limit State

Load Factors α		Overload[1]	Underload
Dead Load		1.20	0.80
Live Load	(i) uncontrolled	1.40	0
	(ii) controlled	1.30	0
Wind Load		1.40	0
Imposed Strain (Temp. etc.)		1.40[2]	0

Material and Dimension Factors β

Steel	(i) Yielding (f_y): 0.85
	(ii) Euler buckling: 0.80
Concrete	Crushing (f_c): 0.6
Depth to Reinforcing	$1 - 0.6/d'$ (d' in inches)
All Other Dimensions	1.00

Performance Factor[3, 4] γ

Steel Structures	(i) Yielding: 1.0
	(ii) Buckling or Fracture: 0.9
Reinforced Concrete Structures	(i) Tensile Failure: 1.0
	(ii) Compressive Failure: 0.85
Overturning	0.9
Sliding	1.0

Notes:
[1]For load combinations of dead load + two or more other loads, the overload factors may be reduced to 1.1 provided the combined load effect exceeds that for dead load + any other load.
[2]May be neglected if imposed strain does not affect the ultimate limit state.
[3]These values are for structures in which consequences of failure are serious. When consequences of failure are not serious (barns, cladding, etc.) add 0.15. When consequences of failure are very serious (hospitals, power stations, arena roof, etc.) subtract 0.15.
[4]These values refer to realistic or safe calculation assumptions and should be modified when necessary.

Table 1B Possible Partial Safety Factors for Serviceability Limit States

	Local Damage Limit State		*Deflection Limit State*	
Load Factors α	*Overload*	*Underload*	*Overload*	*Underload*
Dead Load, Prestress	1.00	1.00	1.00	1.00
Live Load	1.00	0	0.2-1.00[1]	0
Wind Load	1.00	0	0.80[2]	0
Imposed Strain (Temp. etc.)	1.00	0	1.00	0

Material and Dimension Factors β

	Local Damage Limit State	*Deflection Limit State*
Steel	0.85 (yield)	1.00 (modulus E_s)
Concrete	0.77	.3-1.0[1] (modulus E_c)
All Dimensions	1.00	1.00

Performance Factor γ

1.00

Notes:
[1]Depends on whether considering short term or creep deflection.
[2]May alternatively be given in the form of a reduced return period.

2.5 Structural Analysis

The next stage is the structural analysis (or, alternatively, testing). This can be depicted as a process of moving from left to right at the top of Fig. 2 and/or right to left at the middle of Fig. 2 in order to check that the structure and its parts will not exceed the limit states during its intended life. The type of structural analysis used depends on the limit state being considered and the behavior of the structure. For example, elastic analysis would be suitable for most serviceability limit states and for the ultimate limit state of fracture or fatigue, whereas plastic analysis might be suitable for the ultimate limit state of excessive inelastic deformation in continuous steel construction. In the case of overturning, a stability analysis is required.

Having done the analysis, the final step in the procedure of limit state design is to check that the limit states are not exceeded.

2.6 Design Checking Equations

The design checking equation for the ultimate limit state can be expressed in the form

$$f(D^*, L^*, W^*, \dots) \leq \gamma \cdot [g(f_y^*, f_c^*, d^*, \dots)] \quad \dots (3a)$$

when all the loads (D, L, W, etc.) *promote* failure (failure = exceeding the limit state) and

$$f(L^*, W^*, \dots) \leq \gamma \cdot [g(f_y^*, f_c^*, d^*, \dots; D^*)] \quad \dots (3b)$$

when the dead load *prevents* failure. Here f is the *most unfavorable* combination of design load effects which promote failure, and g is the "strength" due to the *most unfavorable* design material strengths, dimensions and stabilizing load effect. In the case where dead load helps prevent failure (e.g. overturning, stress reversal, masonry walls), underestimates in dead load are unfavorable, and so an underload factor (Table 1) is used to calculate D^*.

In equation (3), γ, the *performance factor*, takes into account the consequences of failure with regard to public safety and economy (i.e. type of failure—brittle or ductile, importance of member or structure) and approximations in structural analysis and computations. This factor should be increased when the consequences are not serious (e.g. wall cladding, barns) and decreased when they are very serious (hospitals, auditoria, power stations, etc.).

Thus there are three partial safety factors: α (loads), β (materials and dimensions) and γ (performance). If elastic analysis or plastic analysis is used, then the only difference between limit state design and allowable stress or plastic design is the use of partial safety factors.

The design checking equation for a serviceability limit state is of the form

$$\Delta(D^*, L^*, \dots) \leq \Delta_{allowable} \quad \dots (4)$$

where Δ is the effect (deflection, crack, acceleration, etc.) due to the most unfavorable combination of design loads, and $\Delta_{allowable}$ is the serviceability criterion (Fig. 1A). The consequences of exceeding the serviceability limit states are less severe than the consequences of exceeding the ultimate limit states and so the partial safety factors, usually 1.0, are also less severe.

3. EXAMPLES OF LIMIT STATE DESIGN

The best way to get an understanding of limit state design is by means of some simple examples. The partial safety factors used are those assumed in Table 1A.

Example 1: Simply-Supported Beam or Slab

A simply-supported beam or slab is to be designed to carry dead load plus live load. It is assumed that the design is governed by the ultimate limit state so that the beam or slab must have sufficient bending strength at mid-span to safely resist the bending moment due to dead load plus live load.

The most unfavorable combination of design load effects (Eq. 3a) is given by:

$$f = M_D^* + M_L^* = 1.2M_D' + 1.4M_L'$$

where: M_D' is the bending moment due to the calculated dead load; M_L' the bending moment due to the specified live load; 1.2 and 1.4 are overload factors from Table 1A.

In the case of a *steel* beam, the design bending strength (i.e. g in Eq. 3a) is given by:

$$g = M_u^* = f_y^* Z_p^* = 0.85 f_y' Z_p' \quad \ldots (5)$$

where f_y' is the specified yield point, Z_p' the specified plastic section modulus, and 0.85 the material factor (β_{f_y}) from Table 1A.

In the case of a *reinforced concrete* beam or slab the actual bending strength is given closely by

$$M_u = f_y A_s d\left(1 - \frac{f_y A_s}{2 f_c b d}\right)$$

where A_s is the area of steel, d the depth to the centroid of the reinforcing, b the width of the section and f_c the strength of the concrete in the beam at failure. The design bending strength (β factors given in Table 1A), is then given by

$$M_u^* = 0.85 f_y' A_s' (d' - 0.6)\left[1 - \frac{0.85 f_y' A_s'}{2(0.6 f_c') b' (d' - 0.6)}\right] \quad \ldots (6)$$

In the case of a *wood* beam, the design bending strength might be given by

$$M_u^* = \beta_{f_w} f_w' Z_e' \quad \ldots (7)$$

where f_w' is the specified modulus of rupture, obtained from tests on small clear specimens ; β_{f_w} is the material factor for modulus of rupture which takes into account random and systematic deviations such as duration of load, flaws, moisture conditions; and Z_e' is the elastic net section modulus.

The design checking equation for the ultimate limit state is then given by (see Eq. 3a)

$$1.2M_D' + 1.4M_L' \leq \gamma \cdot (M_u^*) \quad \ldots (8)$$

where M_u^* is given by Eqs. (5), (6) and (7) for steel, reinforced concrete and wood, respectively; and γ, the performance factor, might be taken as 1.0 for steel and reinforced concrete (ductile failure) and 0.9 for wood (brittle failure).

Equation (8) therefore, determines the required dimensions.

Example 2: Column AB, Figure 3

A short column member (AB in Fig. 3) in a statically determinate structure carries axial forces due to dead load, live load and wind. This member must be designed to be strong enough both in compression and tension, where tension is caused by tensile wind effect overcoming compressive dead plus live load effect.

In the case of *compression*, the most unfavorable combination of design load effects promoting failure, according to Table 1A, is

$$D^* + L^* + W^* = \text{max. of } [1.2D' + 1.4L', \; 1.2D' + 1.4W', \; 1.1(D' + L' + W')] \quad \ldots (9)$$

where D, L and W represent axial loads due to dead, live and wind loads respectively.

For a reinforced concrete column, the design strength in compression is given by ††$0.85 f_y' A_s' + 0.60 f_c' A_c$, where A_s and A_c refer to steel and concrete areas respectively. The design checking equation for compression (Eq. 3a) is

$$D^* + L^* + W^* \leq 0.80[0.85 f_y' A_s' + 0.60 f_c'] \quad \ldots (10)$$

where $D^* + L^* + W^*$ is given by equation (9), and 0.80, the performance factor (γ), takes into account the type of failure (brittle) and the importance of the member.

For a steel column,

$$D^* + L^* + W^* \leq 0.90[0.85 f_y' A_s'] \quad \ldots (11)$$

where 0.90 (γ) takes into account the importance of the member.

In the case of *tension*, the most unfavorable combination of design load effects promoting failure is $-1.4W'$, whereas the most unfavorable design load effect preventing failure is $0.8D'$ (see Eq. 3b).

For a reinforced concrete or steel column, the tensile strength is given by $0.85 f_y' A_{st}$ where A_{st} is the area of steel capable of transmitting tension to the foundation. The design checking equation for tension is, therefore,

††Although there is always some eccentricity for reinforced concrete in compression, this will be neglected here.

$$1.4W' \leq 0.9[0.85f'_y A'_{st} + 0.8D'] \quad \ldots (12)$$

where 0.9 (γ) takes into account the importance of the member.

When there is no positive anchorage to the foundation or ground, Eq. (12) becomes

$$1.4W' \leq 0.9[0.8D'] \quad \ldots (13)$$

which says that the dead weight D' must be sufficient to prevent overturning.

Equations (9) to (13) then, determine the dimensions or dead weight required.

These two examples will now be used to demonstrate the advantages of limit state design, with the use of partial safety factors (α, β, γ), over existing allowable stress or plastic design which use the single safety factor.

4. ADVANTAGES OF LIMIT STATE DESIGN

4.1 Economy

When a single safety factor is used, many structures are overdesigned because the single factor must cover the worst cases.

In the case of our beam example above, the single factor does not vary for different ratios of live to dead load and so it must cover the worst case, namely, $\alpha = 1.4$. For heavy structures, such as bridges, reinforced concrete buildings and tall buildings, the adoption of separate load factors leads to greater economy without a decrease in minimum safety.

In the case of the axial strength of concrete (Eq. 10), the single safety factor is the same for all percentages of steel ($100\ A'_s/A'_c$) and so it must cover the worst case, namely when the percentage of steel is very small. This same reasoning also applies to the bending strength of reinforced concrete (Eq. 6) where the single safety factor must cover the worst case, that is, when the member is thin (d small). Thus, by using the single safety factor, we are often penalizing ourselves.

4.2 Stress Reversal, Overturning, Masonry Walls

One of the shortcomings of applying a single safety factor occurs in the case where dead load has a stabilizing effect, i.e. prevents failure. An error may occur because an overload factor is applied to the dead load when in fact an underload factor is required. This is accomplished in allowable stress design by calculating the wind effect minus the dead load effect and applying the usual safety factor to the difference.[12] It can also occur in plastic design if a single safety factor ("load factor") is applied to the loads in combination without considering more unfavorable effects when the safety factor is not applied to the dead load.

This fundamental error in present safety provisions for stress reversal contributed to the collapse of the Ferrybridge Cooling Towers.[13] Existing design rules based on allowable stress, plastic or ultimate strength design are being revised[14] to avoid this error; however, their empirical form does not promote clear thinking.

The same reasoning also applies to overturning, sliding and the stability of masonry walls. Allowable stress or plastic design has the further disadvantage in that what is required is a stability analysis, for which "allowable stress" has no meaning and for which "plastic strength" usually plays a minor role, the major role being the dead weight and geometry.

4.3 Composite Structures

For composite structures such as reinforced concrete, a clearer understanding of the actual safety is obtained by the use of separate β factors for the component materials. This is illustrated by Equation (6), the design bending strength of reinforced concrete. If the percentage of steel ($100\ A'_s/b'd'$) is low, then the bending strength is hardly affected by an unexpectedly low concrete strength. As the percentage of steel increases, however, an unexpectedly low concrete strength has an increasing effect on the lever arm, and hence also on the bending strength. Therefore, it makes more sense to apply the β factors to the material properties and dimensions rather than a φ factor to the ultimate moment, as is now done in the ACI Code.[15]

4.4 Workmanship

The use of partial safety factors easily accommodates differences in workmanship whereas the single safety factor does not.

It does this first of all by means of the clear separation into load factors, material and dimension factors, and performance factor. A change in workmanship is directly reflected by a change in the material or dimension factors, β; all other factors remain unchanged.

Secondly, it does this for composite structures by means of a clear separation of β factors for different material properties and dimensions. For example, in reinforced concrete structures, an improvement in workmanship is obtained by better control on concrete strength and better placing of the steel. Without too much trouble the value of $\beta_{f'_c}$ can be increased from 0.60 to 0.70 and β_d from $(1 - 0.6/d')$ to $(1 - 0.4/d')$. In this case the design axial strength becomes $0.85f'_y A'_s + 0.70f'_c A'_c$ and the design bending strength becomes

$$M^*_u = 0.85f'_y A'_s (d' - 0.4)\left[1 - \frac{0.85f'_y A'_s}{2(0.7f'_c)b'(d' - 0.4)}\right]$$

This improvement in workmanship gives direct material savings of from zero to 10% depending on the percentage of steel, slab thickness, etc.

In the same way, the effects of very poor

workmanship on structural safety can be immediately calculated.

4.5 Imposed Strains

Imposed strains (deformations imposées) due to temperature change, shrinkage and differential settlement are of a different nature than gravity or pressure loads (charges). Internal stresses from imposed strains arise because of statical indeterminacy, and these "constraint stresses" can be minimized by providing hinges, expansion joints, etc. Imposed strains should be considered in the design calculations for certain serviceability limit states and for ultimate limit states if, because of brittle behavior of the structure (e.g. fatigue failure), constraint stresses contribute to failure.

Allowable stress design takes the brittle attitude in that it considers imposed strains on an equal footing as gravity loads. Plastic design takes the purely ductile attitude in that it neglects imposed strains altogether. The advantage of limit state design is that it considers imposed strains in accordance with the behavior of the structure and the limit state being considered.

5. CRITICISM OF THE SYSTEM OF PARTIAL SAFETY FACTORS

Although the system of partial safety factors is a considerable improvement over the single safety factor, it has one important shortcoming[16]. This occurs when more than a few of the partial factors enter into the calculation, as a combination of various load types and/or a combination of various material properties and dimensions. An example is shown by Eqs. (8) and (6), the design equation for a reinforced concrete beam, in which the factors β_{f_c}, β_{f_y}, β_d, α_D, and α_L are combined together with their most unfavorable deviations; the probability of this actually happening is very small. The greater the number of parameters which enter the design checking equation, the smaller the probability of failure.

One way of getting around this shortcoming is illustrated by the reduction of load factors in Table 1A when dead load is combined with two or more other loads. Analogously the β factors might be increased when more than two factors enter the calculation. Such empirical adjustments, however, are somewhat unsatisfactory.

Another procedure which overcomes this shortcoming is a simplified statistical procedure of design[16, 17]. Essentially what is done is to consider the loads, material properties, dimensions, etc. as statistical quantities, represented by their average values and standard deviations. By means of the equations of structural analysis the probability of failure of a particular design can be calculated and compared to a specified value which, like the γ factor above, depends on the consequences of failure, etc. By using the simplest statistical assumptions the statistical procedure is fairly straightforward, involving only an extra calculation of the safety factor over existing design procedures.

This statistical method also has its shortcomings. For example, the statistical distributions of the parameters are represented by normal or log-normal curves and control of either the materials (e.g. steel) or loading are not taken into account very well. Also, the required statistical information is usually not available, although this is also a problem in the procedure of partial safety factors. Finally, some of the simplifying statistical assumptions are usually not satisfied.

6. CONCLUSIONS

(i) Limit state design is a unified procedure of design calculations for all kinds of structures: steel, reinforced concrete, wood, masonry, soil, etc.

(ii) The primary emphasis in limit state design is on the structural conditions for which we must design, i.e. the limit states. Allowable stress design and plastic design put too much emphasis on theory and not enough on the limit states.

(iii) The use of partial safety factors leads to more intelligent safety considerations by the designer and hence greater economy. This is because two of the factors, the load factors and the material or dimension factors, are applied directly to the parameters to which they belong.

(iv) Limit state design differentiates between dead load which promotes failure, which carries an overload factor, and dead load which prevents failure, which carries an underload factor. Thus, in regard to stability and stress reversal, safety is automatically assured without the need for special requirements.

Historical Note

The biggest contribution to limit state design was made in the U.S.S.R. in 1945-1954 by Streletskii, Gol'denblaat and others. As a result, the present U.S.S.R. structural building code[1] is in many ways the most advanced in the world. European countries are also adopting limit state design, especially for reinforced concrete structures[2, 3].

Partial safety factors, which can also be incorporated into traditional elastic design, were used as long ago as 1856 when separate safety factors for dead load and live load were used for bridge design[18]. In 1900, Moncrieff[19] adopted partial safety factors in deriving design formulas for columns. Perhaps the first proposal to use partial safety factors involving both load and material factors was made by Moe in 1936[20]. Many other contributions have been made in North America, notably by Boase and Morgan[21], Lin[10], and a safety subcommittee of the American Concrete Institute[11].

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Discussion by C. K. A. Stieda, Timber Engineering Section, Forest Products Laboratory, Vancouver

The design of structures which will perform adequately is a continuing heavy responsibility of structural design engineers. It is recognized generally that loading conditions assumed for design only bear a vague resemblance to actual loading conditions. Likewise, it is recognized that strength and deformation characteristics of all materials are more or less variable. The exact determination of the margin of safety preventing any particular structure from malfunctioning or collapsing is therefore usually a highly elusive goal. Nevertheless, structural engineers are obliged to make the best possible estimate on the basis of all available information of the safety of a structure.

Use of the concept of limit state design, as outlined by Dr. Allen, will be of considerable help to designers in their efforts to design safe and economical structures. Separate consideration of loads, materials and performance by means of partial safety factors will allow a more logical assessment of the safety of a structure than is possible at present through the use of one single factor of safety. By using partial safety factors it will be possible to focus attention on the various factors contributing to the overall safe performance of a structure. In Tables IA and IB Dr. Allen has presented a number of possible partial safety factors. It is to be hoped that structural engineers will consider these partial safety factors in detail and that the engineering profession will express a collective opinion regarding the adequacy or deficiency of these values.

Perhaps Dr. Allen would be kind enough to comment on Footnote No. 3 to Table IA, where he advocates an increase of 0.15 in the performance factor γ for structures where consequences of failure are not serious. This, in effect, means that for many minor structures the design strength of the structure would be less than the forces produced by the design loads. Even though the design loads, as defined here by Dr. Allen, are higher than conventional design loads, I wonder if it is a sound design philosophy to allow a structure to be weaker than required by some assumed load system.

The magnitude of the material and dimension factors β eventually to be adapted for the various materials will depend, of course, to a large extent on the variability of the strength properties of these materials. Here the usual practice is to assume a normal distribution of strength values and to calculate a lower exclusion limit by subtracting from the estimated average strength $\overline{(f)}$ a certain number (n) of estimated standard deviations (s). The specified strength values (f) are then obtained by dividing the lower exclusion limit by some constant (K).

$$f = \frac{\overline{f} - ns}{K}$$

If it is assumed that the frequency distribution of strength values follows a Gaussian distribution, then it can be demonstrated quite easily that the probability of failure at a specified strength value f is a function of the coefficient of variation $V\text{-}s\sqrt{f}$. The probability of failure at a stress level f will be determined by the number (n_1) of standard deviations lying between the average $\overline{(f)}$ and the specified value (f). This number of standard deviations is given by

$$n_1 = \frac{1 - 1/K}{V} + \frac{n}{K}$$

It is readily seen that for any given value of K an increase in the coefficient of variation V will result in a decrease in n_1 and therefore in an increase in the probability of failure. This dependence of the probability of failure on the magnitude of the variability of the material is usually not recognized by designers and ought to be taken into consideration in the determination of partial safety factors.

It also should be mentioned here that strength values do not necessarily follow a normal distribution, but that it is possible to determine lower exclusion limits without assuming any particular form of distribution. This has been done, for example, in a recent standard of the American Society for Testing and Materials (ASTM D245-T) for the establishing of structural grades of lumber.

In conclusion I would like to thank Dr. Allen for bringing the important concept of partial safety factors to our attention.

Discussion by
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The two design processes described by the author, the Optimum Design and the Limit State Design, are characterized by a number of parameters varying all the way in their reliability from those based on probabilistically interpreted physical measurements and observations to the ones, on the other side of the spectrum, simply assumed with seemingly no justification.

The difficulties facing the designer in following the recommended procedures may be observed in the basic relation of the optimum design, Eqn. (1), representing the total annual cost of a structure as consisting of the initial cost and the costs of replacement occasioned by the building collapse and by its unserviceability, the two latter components of the cost being expressed through the probabilities of the occurrences of the failure and the unserviceability. The writer presumes that the unmentioned maintenance cost is included with the last item.

It is the cost of collapse that presents something of a puzzle. As it appears to the writer, the designer is supposed to know the expected life span of the building, and is capable of calculating the probability of its collapse in this period. With this information he is able to arrive at the annual cost of clearing the wreck. But the question is, what sort of failure is it going to be: that of a single storey, a wing of the building, or the whole structure, and furthermore, will there be human casualties and of what kind? Different types of failure will result in different costs, and in addition to this, there is the question of how to balance human lives and suffering against the property loss? The writer can see no possibility of rational way of determining the cost of collapse. The author calls the optimum design "the decision theory in the face of uncertainty". This, the writer considers, is an illustration of the wonderful capacity of the English language for expressing fine shades of meaning, so valuable to diplomats.

The difficulties with the optimum design lead the author to the procedure named the Limit State Design. Here is what seems to be the basic idea of the method. The load effects, i.e., normally the load intensities of the building code, are overestimated by certain given coefficients, while the strength properties, attributed to the material of the structure, are described by some quantities similar to the allowable stresses, although different from them in magnitude. Furthermore, the structure under the specified loads must still be understressed by certain prescribed coefficients as compared with the given strength values. Some of the strength parameters are based on the actual properties of the material, while other parametric values are mere assumptions.

The writer does not see much point in making the live and the wind loads used in design, the products of two inseparable quantities (the given code intensities and the specified load factors) instead of the single ones. Furthermore, he questions the logic of overestimating the dead load by the coefficient 1.2. The dead load should be known to the designer. If he does not want to go to the trouble of computing it exactly, he should know enough whether to overestimate or to underestimate it, in order to be on the safe side. The only logical reason that occurs to the writer in explanation of the dead load factor 1.2, is that it represents an attempt to allow for possible numerical mistakes in design. If this conjecture is correct, are there similar provisions made in the live load and the wind load factors? Incidentally, the intensities and provisions for wind loads are widely different in different building codes. They cannot be all correct. How does the proposed procedure meet this situation?

The writer disagrees with the author's view that the limit state design is more economical than the conventional elastic design. The factor of safety of the elastic design is the factor of ignorance providing for all sorts of unfavourable contingencies in materials, loads, design, construction, fabrication and, to some extent, mistakes in the method and numerical work, as well as totally unexpected load conditions.[22] These exceptionable unfavourable circumstances do not occur all at once, and so the factor of ignorance can meet them one at a time or in small combinations. The limit state design on the other hand provides separately for each of the many contingencies, thus producing an uneconomical cumulative effect.

The two trival examples, the single simple beam and the single column, brought up for demonstration of the method are hardly convincing. Structures as a rule are not composed of separate single members. It is felt that the designer of a realistic structure, attempting to follow the outline of the limit state design in Fig. 2, will find serious difficulties in some steps of the procedure, when designing in accordance with the plastic theory.[23]

The writer's feeling of doubt, generated by the plastic state design, rises to disbelief when reading

the claim of the universal applicability of the method to all kinds of structures, even the new ones, and to different conditions of workmanship. With the view of this claim in particular the writer is at a loss to understand the total absence of any references in the paper to the earthquake effects or loads.

One factor important in design is left unmentioned and unprovided with quantitative parameters, –this is the knowledge and the experience of the designer. And yet there is hardly any question that an inexperienced designer can do more harm in a complex situation than all the cumulative effect of several incorrectly chosen parameters.

The writer is in full accord with the author on the basic objectives of design, safety, serviceability and economy, but on reading this paper he still remains convinced of superiority of the conventional elastic design method over all others for the attainment of these ends.[24] The alleged use of the new methods in the Russian design practice need not be taken too seriously. The recognized Russian prowess in pure mathematics does not always extend to the applied areas. If the comfort of occupants and an attractive appearance of the building are the attributes of serviceability, the Russian designers seemingly do not live up to this requirement. The writer found this in person while attending an engineering conference in Leningrad in 1966, and staying in a big hotel constructed only in 1963.

Although disagreeing with the author in the estimate of the value of the new design procedures, the writer strongly commends his paper for the masterly exposition of the subject. Any new engineering development taken up seriously in Russia or elsewhere deserves close examination by Canadian engineers with the view of its subsequent incorporation in their practice or its considered rejection. The author must be given credit for presenting the basic principles of the optimum and the limit state designs before the Canadian engineering audience.

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Discussion by
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I should like to congratulate Mr. Allen on his excellent presentation on Limit State Design. As this subject has been almost neglected in Canadian publications, his paper is timely. However, with the 1970 National Building Code complete, it is a most appropriate time to consider seriously the adoption of limit state design for the NBC which will follow.

Fundamentally, there is nothing new in the concept of limit state design. It is a formal statement of what designers have supposed to have been doing heretofore. The adoption of limit state design will place the design process on a rational and logical basis, but will not make the determination of the individual limit states any easier. Currently, we find the determination of the collapse load of many structures extremely difficult and have to be satisfied with the failure load of individual members as criteria.

Our determination of deflection frequently is most unreliable as also are our current deflection limits. But it has been the absence in codes of a logical statement of the aims of the design process and the presentation of recommended partial load factors within which framework the designer could use his judgement and manoeuver, which has laid structural design open to attacks such as that made by Lind, Turkstra and Wright.[25] While this paper showed a lack of knowledge of overall design office operation, it contained some disturbing truths.

Perhaps I could clarify my argument by quoting a recent personal experience. I was designing a reinforced concrete chimney of moderate height (450 feet) but unusual shape, a tapering cruciform. On my firm's advice, the owner had obtained probably the best design data available in the world today by having a model tested in the Boundary Layer Wind Tunnel at the University of Western Ontario. The best reference design code available was obviously the new ACI Code for Reinforced Concrete Chimneys, ACI 307-69. But this code is one of the worst types available in that it does not even attempt to specify a single performance requirement and, therefore, offers little or no guidance to the designer of a structure which is not strictly a circular chimney. This code provides only stress limits in concrete and reinforcement under dead load plus wind, earthquake and temperature effects. Chimneys require only a small percentage of reinforcement, but no mention is made of the problem to which Mr. Allen refers, that in which a moderate stress level in reinforcement does not indicate an adequate reserve of strength for overload. No guidance is given in this code on crack width versus durability, no suggestion on what proportion of the maximum design load should just put the concrete into tension. Therefore, although extremely good statistical design loading data were available, no code committee appeared to have thought how it should be used.

The separation of the load factor into its partial elements is of great value when one is designing outside the scope of the code. To quote from the chimney example again, it was absurd that the same proportional allowance for displacement of reinforcement should have to be made in an element 40 feet deep as in one only a few inches in depth. Similarly, the effect of local understrength material or local lack of vibration in concrete would be of much less importance in a large element than a small one. The single load factor does not permit designers to recognize these factors. However, it is noteworthy that in the wood joist designs for the National Housing Standards, allowance has been made for the diminishing effect of knots, etc. on

increasing depths of joists.

I wondered why Mr. Allen introduced the subject of Optimum Design into his paper for it does not appear essential to his presentation. In fact, I think it is a distraction. Rowe, in presenting his paper on Limit State Design in London, suggested that designs be based on a given probability of failure and that insurance cover failures beyond that probability. This caused such a furor that a second meeting had to be held in London to continue the discussion and subsequent meetings were arranged in other cities throughout Britain. I feel that the cause of Limit State Design would be hindered by the introduction of any red herrings.

Why is there a 20% tolerance on the dead load factor α? Surely we can estimate dead load closer than that. Is an allowance included for use of the wrong material, such as normal weight aggregate rather than lightweight?

In the near future, the CSA-S16 Committee may start to formulate a new standard in which the working stress concept will be discarded. Load factors and yield strength will be the criteria. The 1963 ACI Code ultimate strength design provisions contained an emasculated form of limit state design which will doubtless be developed further in the next edition. Unless we take advantage now of the effective national bodies serving us, namely the CSA and NBC committees, a chaotic design state will result. It is imperative at this time that we agree upon partial safety factors for both the ultimate and serviceability limit states, such as the author has suggested. If limit state design is to be adopted, it must be followed consistently throughout the code for each material section. At present, the Structural Loads & Procedures Committee is the only unifying authority exercising control over the individual material sections. The Technical Committee of CSA could exercise control over the approach taken by the individual standard committees, but this does not appear to be their present function. I appeal to you to support my suggestion that a joint NBC-CSA Committee be formed in the near future to decide upon the detailed adoption of limit state design for all future CSA Standards and the 1975 NBC.

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Author's Closure
by D.E. Allen

The author wishes to thank the discussers for their informative and stimulating comments. These will be discussed by topic.

Optimum Design

The author agrees with Mr. Springfield that optimum design is not an essential part of limit state design. Limit state design is a practical calculation procedure, whereas, for most civil engineering structures, optimum design is primarily a principle or goal and a research tool in evaluating practical safety provisions. Optimum design has nevertheless been useful for the design of repetitive structures such as offshore drilling rigs.[26]

Regarding the cost of collapse, C_c, the extent of collapse and its consequences, discussed by Dr. Hrennikoff, is an important consideration in design safety provisions, but is explicitly neglected in existing building codes. This will be discussed later.

Specified or Characteristic Values

The comment raised by Dr. Hrennikoff and previously by Turkstra, Lind and Wright[25] that specified loads vary from code to code beyond the bounds of reason is well taken. Specified loads should be established on a consistent basis, namely probability of occurrence. The characteristic value promoted by limit state design provides such a basis.

In the procedure of limit state design, it is noted that the characteristic values are chosen independent of the limit state being considered. The partial load, material and dimension factors are then chosen for the various loads, material properties and dimensions on the basis of their expected deviations and the limit state being considered. Thus the use of specified loads and load factors, objected by Dr. Hrennikoff, is essential to the procedure of limit state design.

Load Factors

The comment by Mr. Springfield and Dr. Hrennikoff that the load factors, especially for dead load, are too high may be partly true. Minor design and construction errors not directly connected with the loads, material properties and dimensions should be taken into account by the performance factor. Accordingly, a possible alternative to Table 1 is suggested later.

It should be pointed out, however, that dead load is often more unpredictable than most people think. For example, 5-inch concrete slabs can turn out to be closer to 6 inches. Also, unit weights of concrete and wood can deviate substantially from assumed values. This suggests that there should be different dead load factors, α_D, for different materials.

Performance Factor

Dr. Stieda questions the philosophy of having the design load greater than the design strength for minor members, even though partial safety factors are already included in the design values. Accordingly, a possible alternative to Table 1 is suggested later. It is interesting to note, however, that the Canadian "Farm Building Standards"[27] allow a 25% increase in allowable stress for buildings of "low human occupancy," so that for mild steel subject to wind loads, an over-all safety factor of $\frac{33}{20 \times 1.33 \times 1.25}$ or 0.99 results. The most unfavorable case from Table 1 is represented by a *global* safety factor (specified strength/specified load effect) of $\frac{1.1}{0.85 \times 1.15}\left(\frac{\alpha}{\beta\gamma}\right)$ or 1.13.

The performance factor takes into account the consequences of failure, approximations in analysis and minor design or construction errors not directly connected to the partial load, material and dimension factors. At present most structural failures are due to gross errors in design or construction, for example those due to inexperience mentioned by Dr. Hrennikoff. Safety precautions against gross errors, however, must be provided by sound structural training, independent checking, inspection, suitable contract arrangements, etc., not by design safety factors. As approximations in analysis are almost always on the safe side, this leaves the consequences of failure as the major factor affecting the performance factor, γ.

Existing codes usually do not explicitly consider the consequences of failure, and even when they are considered, for example in the design of farm buildings or in earthquake design, this is not done consistently. The "brittleness" of failure which directly affects the consequences of failure is considered for some materials, e.g. reinforced concrete, but not always. This is in line with the philosophy expressed by Dr. Hrennikoff that the single safety factor takes care of everything and does not vary, e.g. 1.7 for yielding of steel. Another example of inconsistency is the use of the 33⅓% increase in allowable stress for members subject to wind loads. This rule is justified for cladding because the consequences of failure are not serious whereas it is justified for main structural members because of the decrease probability of simultaneous occurrence of live load and wind load. The point is, the present system can be made to work, but why not do it so designers know what is going on?

Revised Partial Safety Factors

From the discussion above it appears desirable to alter the design checking equation for the ultimate limit state to

$$\gamma \cdot f(D^*, L^*, \ldots) \leq g(f_y^*, f_c^*, d^*, \ldots)$$

where the performance factor, γ, is greater than 1.0 and increases with an increase in seriousness of collapse, with a decrease in accuracy of analysis (note that the new γ is related to the reciprocal of the former γ). At the same time, the load factors α and the performance factor γ might be readjusted from those in Table 1A to:

	Dead Load, α_D	Live Load, α_L	Wind Load, α_w
Overload	1.1	1.3	1.3
Underload	0.8	0	0

	Consequences of Failure		
	Not Serious	Serious	Very Serious
Performance Factor, γ	1.0*	1.1*	1.2*

*These values represent ductile or sliding type failures. Add 0.1 to 0.2 for buckling or brittle failures.

Economy

It was pointed out in the paper that the single safety factor must cover the worst case, namely when the dead load is small, when the percentage of steel in a reinforced concrete column is small, when the slab is thin, and so on. Now the system of partial safety factors must at first be partly derived on the basis of existing provisions, except where existing provisions are known to be unsafe, as for example, stress reversal. This "calibration" is done by making the global safety factors (defined above), when the consequences of failure are "serious," equal the existing ones for the worst cases cited above. This means that, except in a few isolated cases where existing procedures are not safe enough, the resulting material quantities will be less than or equal to those obtained by existing procedures. But it should be pointed out that the economics are not as great as directly indicated because deflection requirements would probably be the same at first.

This "calibration" procedure overcomes in part the criticism of partial safety factors given in the paper and mentioned by Dr. Hrennikoff, namely the uneconomical "cumulative effect," or small probability of simultaneous deviations. The problem of simultaneous deviations nevertheless remains, and perhaps the probabilistic methods suggested by Rzhanitzyn[16] and Cornell[17] could be used for evaluating corrective factors, especially for load combinations.

Limit State Design Universally Applicable?

As a procedure for structural design calculation in civil engineering, limit state design is universally applicable, not universally scientific or precise. Limit state design, like existing procedures, is based on a static representation. Thus, wind and earthquake dynamic effects are represented by equivalent static effects. In the case of earthquake design, the static representation is partly empirical, partly based on dynamic analysis, and partly based on ductility requirements. Improved knowledge of earthquake effects, complicated buckling situations, etc. can be incorporated into limit state design or existing procedures.

Allowable stress and plastic design are also applied universally, but it is a stretch of the imagination when they are applied in situations when the adjectives "allowable stress" and "plastic" are misleading, for example, stability of walls or deflection requirements. It is really a question of semantics, mentioned by Dr. Lash, because good designers – using "elastic," "plastic," "limit state" design or whatever – have known for a long time that they have to design for strength and serviceability. Nevertheless, semantics are important because they make codes more readable and reduce the probability of gross design errors. The terms "limit state calculations" or possibly just "structural calculations" are better than "limit state design" because other equally important aspects of design are not covered by structural calculations, e.g. structural concepts; detailing; fire, corrosion and sound protection; and interaction with architectural and mechanical requirements. Unfortunately, "limit state design" has gained international acceptance.

Building Code Application

Limit state design is used in the USSR as a basis for structural calculation for all kinds of structures and for all kinds of materials, including soil. Dr. Thadani pointed out, however, that the USSR code

Limit State Design – Author's Closure

has become too complicated for practical design. There are two reasons for this: (1) the procedures of calculation and the design requirements are excessively detailed in certain areas, a situation that is also beginning to occur in Canadian codes; (2) the performance factor, instead of being applied only at the final checking equation, has been introduced into various material factors resulting in an overly-complicated system of partial safety factors. If a system of partial safety factors is used for a code, it should not be complicated. A simple system, such as presented in this paper, is actually clearer and simpler than that in existing codes because safety factors in existing codes are often hidden in specified loads or allowable stresses. Furthermore, the system of partial safety factors is better suited to new forms of structures and differences of workmanship because only certain material and dimension factors will be affected.

Whether "limit state design" forms a basis to revision of future codes, as suggested by Mr. Springfield, depends on whether the majority of structural designers wants it. For this reason it is suggested that structural engineers send in their comments for publication in this journal.

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