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Molyneux, W. D.; Roddan, G.

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MODEL EXPERIMENTS ON A SIMPLIFIED PLANING HULL IN REGULAR AND IRREGULAR WAVES

by

David Molyneux,
Institute for Marine Dynamics,
&
George Roddan,
B. C. Research.

Abstract

A classical set of experiments was carried out, by Fridsma and extended by Zarnick and Turner, on a series of simplified planing hulls, in regular and irregular waves. The models used had constant deadrise angles between 10 and 30 degrees. Length to beam ratio was varied between 4 and 9, and each hull was tested at two loading conditions and three speeds. These results have been used extensively for preliminary predictions of the performance of planing hulls in waves, and also for comparison with both linear and non-linear numerical methods for motion calculation. Unfortunately the minimum length to beam ratio is greater than that typically required for many small boat applications.

It was thought to be highly desirable to extend the data to hull form proportions more appropriate for small boats. This paper describes experiments carried out on three models, with length to beam ratios of 4.0, 3.0 and 2.0 all with a constant deadrise angle of 20 degrees. The first phase of the project was to compare the results for one of the models with Fridsma's results, for regular and irregular waves. Once a satisfactory agreement had been obtained, then the two additional length to beam ratios were tested, at loading conditions and speeds selected in the spirit of the original series.

Introduction

Planing hulls have many interesting hydrodynamic properties because they rely on dynamic lift forces to improve their performance, relative to conventional

displacement hulls. The amount of dynamic lift varies with speed, but at planing and semi-planing speeds, it is a substantial portion of the total lift required to support the hull. Since the hull is no longer in static equilibrium about the zero speed water line, hydrodynamic forces must be included when calculating the motions induced by wave action. This feature, combined with the observed non-linear nature of the response with wave height makes numerical prediction of performance in waves more complex than for displacement hulls. Model experiments are therefore the most reliable performance prediction method for planing hulls.

However, the relatively simple hull shapes, and the low design budgets for this type of hull usually preclude extensive model experiment programs. An alternative approach is to consider a parametric series, representing the main features of a planing hull, and use these results to guide the designer in the selection of features such as length to beam ratio, deadrise angle and loading coefficient. Unfortunately there is no such data for designers of wide beam planing hulls, with length to beam ratios between 2.5 and 4.

This paper describes the first phase of a series of model experiments designed to fill this important gap, which includes the basic proportions of most small planing hulls. The experiments were carried out at the Ocean Engineering Centre (OEC) of the British Columbia Research Corporation. The work was carried out as part of an 'Agreement for Collaboration' between the National Research Council of Canada, the British Columbia Research Corporation and the University of British Columbia.

The Fridsma Series of Simplified Planing Hulls and Proposed OEC Extension

The most well known work to determine the factors influencing the behaviour of planing hulls in waves was carried out by Fridsma (references 1 and 2). In order to isolate the effect of beam and deadrise angle, which normally vary along the length of the hull, the design was simplified. The hull used by Fridsma was a prism, with a single chine and a simplified bow, identical for all the models with the same deadrise angle. Sections for the bow portion of the hull, which was the same length as the model beam, are shown in figure 1, for a deadrise angle of 20 degrees.

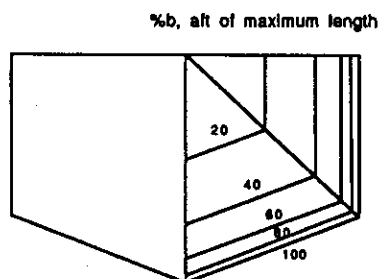


Figure 1, Bow Sections for Simplified Planing Hull

The design variables selected for Fridsma's study were speed-length ratio ($V/\sqrt{L} = 2, 4$ and 6), loading coefficient ($W/(b^3 \cdot \rho)$), running trim ($\tau = 4$ and 6 deg), deadrise angle, ($\beta = 10, 20$ and 30 degrees) and length to beam ratio ($L/b = 4, 5$ and 6). V is the model speed in knots, L is the overall length of the model in feet, W is the mass of the model (lb) and ρ the density of water (lb/ft³). Loading coefficient was varied with length to beam ratio, to reflect realistic relationships between overall length and displacement. The series was extended by Zarnick and Turner (reference 3) to include hull forms with length to beam ratios of 7 and 9. The speed coefficients were changed to V/\sqrt{L} of 2.0, 3.0 and 4.0. The running trim was also changed to the nominal values of 2, 3 and 4 degrees.

The models were originally tested in regular waves over a range of wave frequencies (reference 1). However, it was noticed that model responses were not linearly proportional to wave height, especially at planing speeds. In order to develop the

results into design predictions, additional model experiments (reference 2) were carried out in irregular head waves. Three representative sea states were selected, with significant heights which were constant proportions of the model beam (H_s/b of 0.22, 0.44 and 0.66). The sea states were derived from Pierson-Moskowitz spectra, and so modal period and significant height were not varied independently. The results of the irregular wave experiments on the slender hulls are given in reference 3.

The model experiments in irregular waves have been used for design prediction purposes. The original design charts, given by Fridsma, have been analyzed by Savitsky and Brown, (reference 4), and prediction formula have been developed. The results of experiments in regular waves have been used for checking the validity of numerical predictions. Examples are Martin (reference 5) and Zarnick (reference 6). The series has proven to be a useful tool for estimating performance of planing hulls in waves. It does however have a severe restriction, in that the minimum length to beam ratio is higher than that typically used for small craft. It was decided that a useful extension to the original series would be to include hull form parameters more typical of smaller boats.

A review of literature on small planing boats and the data base of the Ocean Engineering Centre, suggested that the expected range of length to beam ratio was approximately 2.5 to 4. The maximum speed in waves for hulls of these proportions was likely to be around a speed-length ratio of 4.0. This would correspond to 32 knots for a hull 65 feet (19.8 m) long. In order to establish the effect of speed on results, additional speed length ratios of 2.0 and 3.0 were chosen. The review also suggested that a logical extension to the original Fridsma series would be to add length to beam ratios of 2 and 3, using the same three deadrise angles for the hull with a length to beam ratio of 3. The full matrix of hull designs is given in table 1. It is recognized that a more modern approach for speed coefficient might have been Froude number based on beam, but it was felt that consistency with the original series was important. The speed coefficient chosen is directly related to Froude number based on length.

	L/b						
Deadrise angle, deg.	2	3	4	5	6	7	9
10		O		F		Z	
20	O	O	O/F	F	F*	Z	Z
30		O		F		Z	

F=Fridsma models

Z=Zarnick & Turner Models

O=OEC Extension

* Regular waves only

Table 1
Matrix of Hull Form Proportions for
Simplified Planing Hulls in Waves

The original model used by Fridsma, and later by Zarnick and Turner, had a beam of 9 inches (229 mm). A model of this beam would be very short with a length to beam ratio of 2, and so it was desirable to change the size of the models. A review of the technical specifications for the towing carriage and wavemaker at the OEC, as well as the dimensions of typical models tested for clients suggested that the beam could be increased to 14.4 inches (366 mm), and still meet the requirements for the proposed extension discussed above. If this was done however, it would be desirable to include an additional model with a length to beam ratio of 4. This would then provide a single data set covering the range of small planing boat parameters, as well as the opportunity to compare the results of experiments at the OEC with those carried out elsewhere.

Loading conditions for the series extension were estimated by extrapolating the values given by Fridsma and are given in table 2.

3. Comparison of OEC and Fridsma Model Experiments

3.1 Regular Head Waves

For the first phase of the experiment program a model with a length to beam ratio of 4 was constructed. Calm water experiments were carried out, repeating those described in reference 1, and it was found that there were no scale effects on the relationship between centre of gravity position and running trim angle.

Although the main objective was to develop an extension to the charts given by Fridsma, to predict responses in irregular waves, a secondary objective was to develop new experiment methods for the OEC, and calibrate them against published results. For this reason it was decided to repeat the experiments in regular waves. The loading condition reported by Fridsma for L/b of 4 was 0.631, with a radius of gyration of 0.238L. The model was tested at a single speed-length ratio of 4.0, for six wave lengths (corresponding to 1, 1.5, 2, 3, 4 and 6 model lengths). The waves were nominally a constant height of 0.11b, over the full frequency range. Measurements were made of resistance in waves, pitch and heave at the model's centre of gravity, acceleration at the centre of gravity and acceleration 10% aft of the bow. Phase angle was measured for pitch and heave, relative to a wave crest at the centre of gravity, with phase lag positive for heave and phase lead positive for pitch.

The OEC model was dynamically balanced to the values given above, and it was found that the resulting static trim of 1.3 degrees (bow down) compared well with the value given by Fridsma of 1.4 degrees. The OEC model was towed from the same location as the Fridsma model, 0.294b above the keel at the centre of gravity. The model was tested in regular waves repeating, as closely as possible, the experiment program described in reference 1. Data was recorded digitally, at a frequency of 100 Hz, with 50 Hz low pass filters on each channel.

L/b	W/(b ³ *ρ)	
	light displ.	deep displ.
2.00	0.126	0.158
3.00	0.253	0.316
4.00	0.384	0.480

Table 2
Loading Coefficients for Fridsma Series Extension

There were some differences between the OEC experiment techniques and those described in reference 1. The OEC model was towed at a constant speed, whereas Fridsma's model was towed free to surge. However, in subsequent experiments (reference 2), he determined that there was no difference between the results of the two methods. Since the OEC test apparatus had been previously configured for constant speed, this arrangement was retained. Another difference between the experiment methods was in the measurement of phase angle. Fridsma calculated phase angles using a wave probe located level with the centre of gravity, and one beam off the centreline of the model. At the OEC, a sonic probe was used to measure wave height and at this position it was found that the measurement was unreliable, due to spray. The method was modified by locating the wave probe ahead of the model, and correcting the phase angle for the lag between the wave measured at the probe and the centre of gravity.

Figure 2 shows a comparison of the pitch and heave responses for the OEC model plotted against the Fridsma model. The largest responses occurred at the low wave frequencies, and the lowest occurred at the high ones. It can be seen that there is good agreement between the two sets of results. Figure 3 shows a similar comparison of phase angles. The accuracy of the OEC phase angle method was tested by measuring the wave profile simultaneously at the location of the wave probe and the position of the centre of gravity, but without the model present. It was estimated that the method agreed to within 15 degrees when the carriage was stationary and 20 degrees when the carriage was

Figure 2, Comparison of Pitch and Heave Amplitudes, Regular Head Waves, $V/\sqrt{L}=4.0$

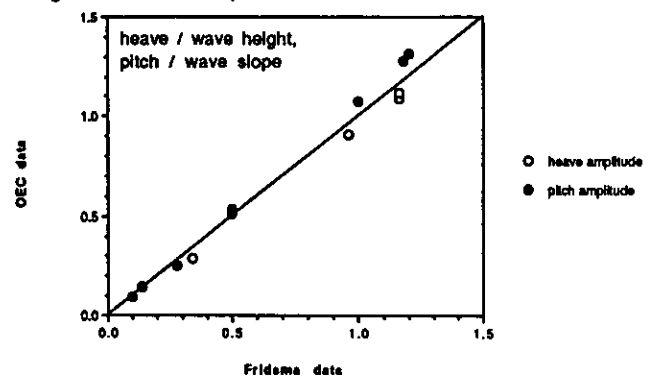


Figure 3, Comparison of Pitch and Heave Phase Angles, Regular Head Waves, $V/\sqrt{L}=4.0$

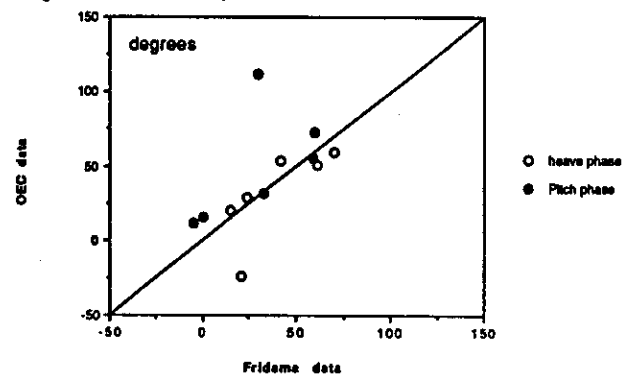
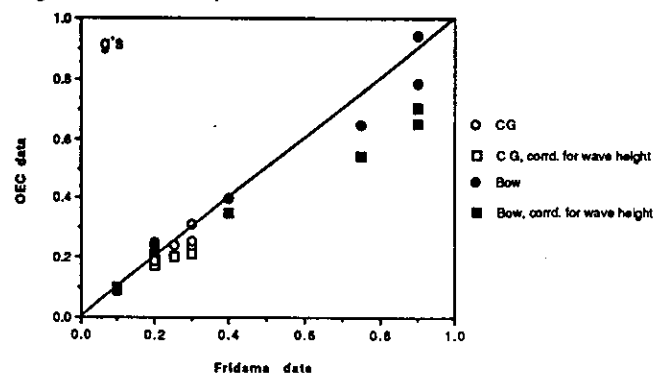


Figure 4, Comparison of Acceleration Amplitudes, Regular Head Waves, $V/\sqrt{L}=4.0$



moving. It can be seen that the agreement with the Fridsma data is all within this range, except for one data point for pitch phase angle and one for heave phase angle. Both points were obtained in a single experiment, at the highest wave frequency. It is possible that the difference between the results was due to the fact that the model was influencing the wave profile, which was not allowed for in the OEC analysis.

The acceleration data are compared in figure 4. The measured data agrees reasonably well with Fridsma's values, but it should be noted that some of the waves were higher than the nominal values given by Fridsma. This was not important in the other comparisons, since response was expressed as a function of wave height. A correction for the effect of wave height, based on experiments carried out at different heights for a constant frequency, would indicate that the OEC model results are generally lower than Fridsma's, especially for bow acceleration at high wave frequencies.

A series of additional experiments was carried out to investigate this. It was found that at the very highest wave frequency, digital sampling rate could have an effect on the results, but could only explain, at most, half of the observed difference between the OEC and the Fridsma results. Other possible causes could be scale effects between the two sets of data, or different stiffnesses of the model and accelerometer mounting arrangements. The OEC model was constructed from laminated polystyrene foam, whereas the model used by Fridsma was built from plywood and frames.

For this particular model, no resistance in waves was reported by Fridsma, reference 1.

3.2 Irregular Head Waves

Fridsma tested his model in irregular waves at two loading coefficients, 0.384 and 0.480. Three speed-length ratios were tested, at single running trim angle of 4 degrees, in each of the three sea states described above. It was not possible to repeat the full test program at the OEC, since the maximum speed was not feasible for a model with a beam of 14.4 inches (366 mm). Thus only speed-length ratios of 2.0 and 4.0 could be compared. Time constraints meant that only one loading

condition could be tested, and the heavy displacement was selected. The model responses measured were pitch, heave, acceleration at the bow and centre of gravity, resistance, wave height and carriage speed. Data was recorded at 100 Hz, with 50 Hz low pass filters on all channels, in a similar manner to the regular waves.

Fridsma stated that his data was based on a minimum of 75 wave encounters, but typical records would contain 100 encounters. It was not possible to obtain all these encounters in a single tank length at the OEC and it was estimated that between six and eight lengths were required for each speed and sea state combination. Care was taken when performing the experiments to ensure that each run was made in a different portion of a single nominal sea state. Sea states were generated, to the correct ratio of significant wave height to beam, and tested by measuring them at a stationary point in the centre of the tank.

Fridsma's analysis had been done by digitizing the experiment records manually, relative to a predetermined base line. For resistance data, the mean of the total record was used, but for heave and pitch, the peak value, measured as the deviation from the signal mean, was the statistic of interest. For accelerations, the peak, measured relative to the zero value, was used. Probability distributions were then fitted to the observed response peaks, and parameters such as the significant and average of the 1/10th highest values were obtained from the fitted probability distributions.

The manual digitization was thought to be unnecessarily time consuming, and so the method was modified for the OEC analysis. A program to identify peak values in a digital time series was written, based on the methods outlined by Zselezky and McKee (reference 7). For this analysis calm water experiments were carried out to identify ambient noise levels in the signals, and the thresholds (or buffers) used were based on four times the root mean square of the noise value. These thresholds were 0.1 inches, 0.4 degrees and 0.1g for heave, pitch and accelerations respectively. Mean values only were used for analysis of the resistance and speed data.

It was decided to compare the OEC results with the Fridsma values of mean peak response obtained by direct measurement, rather than those obtained from the fitted probability distributions. This avoided any uncertainty in the fitted distributions, which was not allowed for by Fridsma. It was also desirable to give some estimate of the confidence of the measurements, based on statistical sampling theory. For the OEC data 95% confidence intervals for the mean peaks are also plotted, derived from number of peaks and their variance. No similar data was given by Fridsma.

A comparison of the resistance in waves, non-dimensionalized by model displacement, is given in figure 5 and it can be seen that there is good agreement between the two sets of data. Mean peak heave and pitch angle are similarly compared in figures 6 and 7. Again it can be seen that there is good agreement between the the two sets of results.

Mean peak acceleration at the bow and centre of gravity are compared in figures 8 and 9. In this case the agreement is good for the accelerations measured at the centre of gravity, but is not as good for the bow acceleration at a speed-length ratio of 4. Experiments in regular waves had shown that sampling rate could affect the results. Since not all the responses could be measured at higher sampling rates, some experiments were carried out measuring only bow acceleration, and comparing results in the same waves at 100 Hz and 400 Hz sampling rates. The maximum amplitude observed at 400 Hz was slightly higher than at 100 Hz, but there was little effect on the average value, when all the peaks were combined. Another explanation for the results may be the difference in the analysis techniques used. Fridsma's data was digitized by hand, whereas the OEC data was digitized by computer. Zselezky and McKee (reference 7) discuss some of the complications of digitizing time histories, in that during the manual digitizing process, the eye tends to filter out some of the smaller peaks, whereas the numerical method does not. Thus the average value for the numerically digitized values tend to be lower, since they include a larger number of smaller peaks.

Figure 5, Comparison of Total Resistance (per unit displacement), Irregular Head Waves

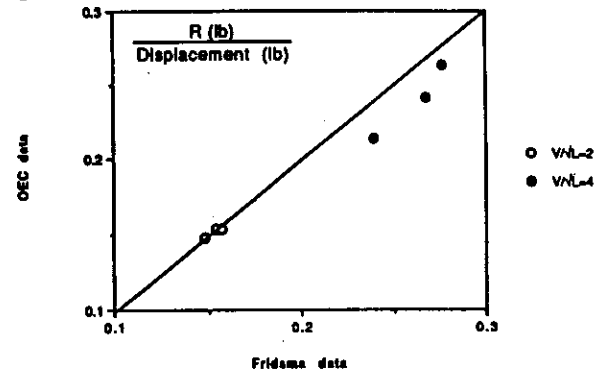


Figure 6, Comparison of Heave Amplitude in Irregular Head Waves

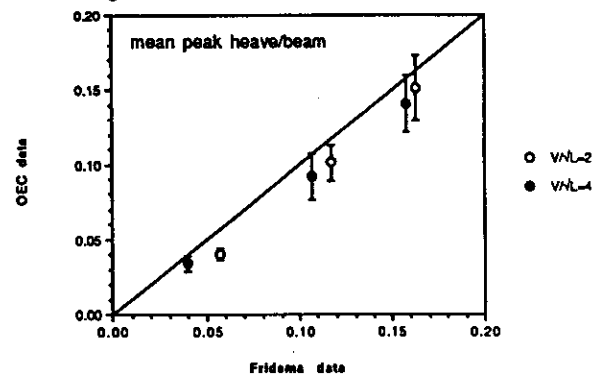


Figure 7, Comparison of Pitch Amplitude in Irregular Head Waves

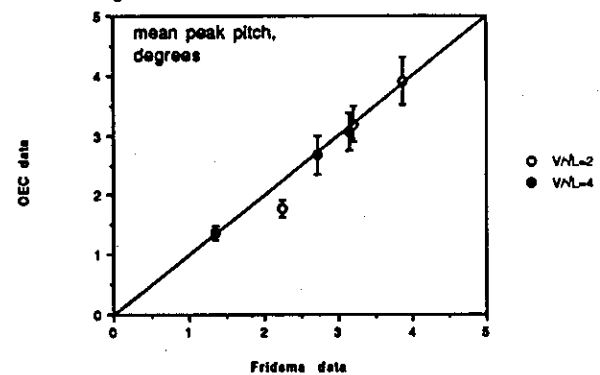


Figure 8, Comparison of Acceleration at C. G. in Irregular Waves

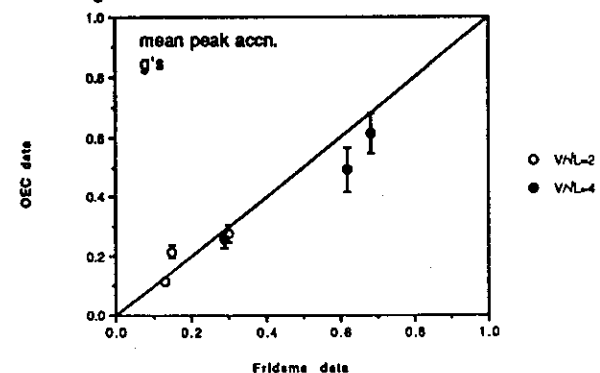
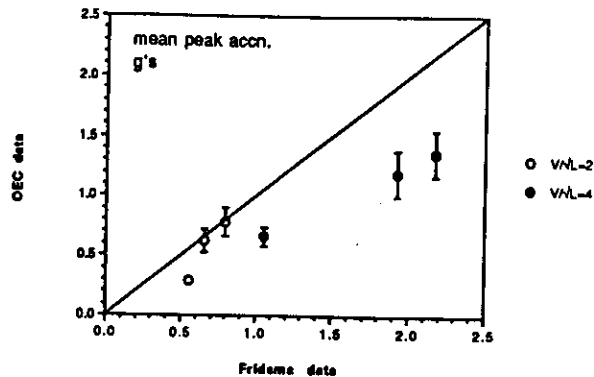


Figure 9, Comparison of Acceleration at Bow in Irregular Head Waves



Another factor to support this possibility is that Fridsma fitted exponential probability distributions to the acceleration peak data. This distribution has zero probability of a peak value less than zero. In fact, there is no physical mechanism preventing an acceleration peak value being below zero, defining zero as the calm water running value, and negative peaks were always observed in the OEC experiments. Thus the average value for the OEC data, including the small peaks and those below zero, should be lower than the value for the manually digitized data. Other possible causes of the difference are discussed in relation to the experiments in regular waves.

4. Extension to Fridsma Series, Responses in Irregular Head Waves at $V/\sqrt{L} = 4$

Once a satisfactory agreement between the results of experiments at the OEC and those published in references 1 and 2 had been obtained, it was decided to extend the Fridsma series as discussed above. The first models to be built and tested were the three models with a deadrise angle of 20 degrees. In our case, three separate models were constructed, rather than using transom extensions to the bow module. Calm water experiments indicated that 4 degrees was not a realistic running trim angle for the hull with a length to beam ratio of 2, and so a single value of 6 degrees was used for all the models. This portion of the paper presents results for the light displacement at a V/\sqrt{L} value of 4 only. Model experiment and analysis methods were exactly the same as those used for the comparison of the irregular wave experiment results with Fridsma.

The results of the experiments for all three models are compared in figures 10 to 14 for resistance, heave, pitch, acceleration at the centre of gravity and acceleration at the bow. Figure 10 shows the effect of length to beam ratio on resistance per unit displacement in waves. Calm water resistance values have been included in this figure, as a significant wave height of zero. Although the absolute values of resistance in waves are different between the three models, most of this is caused by differences in the calm water values. If the calm water values are subtracted, the added resistance due to the waves is approximately constant, and so it would appear that length to beam ratio does not significantly effect this parameter.

It is interesting to note that the minimum calm water resistance occurs for a length to beam ratio of three. Fridsma's data showed a progressive reduction in resistance coefficient with length to beam ratio, and this was continued with our models for length to beam ratio of 4 and 3. However a length to beam ratio of 2 shows an increase in calm water resistance. This probably because there is insufficient planing area for the short hull to generate the lift necessary to overcome the weight of the boat and reduce the resistance. It implies that the extrapolated loading coefficient may be too high for optimum calm water performance, but the value is realistic when compared actual boats.

Figure 11 shows a comparison of mean peak heave response (non-dimensionalized by model beam), and although the hull with L/b of 2 has the lowest response at the two largest wave heights, all the data are within 10 percent of the mean value for each wave height. We may conclude that length to beam ratio does not have a significant affect on heave response.

Length to beam ratio does have a major affect on mean peak pitch angle, as shown in figure 12. Here we can see that there is a steady increase in mean peak pitch as the length to beam ratio is reduced from 4 to 2. The acceleration data are compared in figures 13 and 14 for centre of gravity and bow locations respectively. Figure 13 shows that L/b of 3 has the lowest value of mean peak acceleration at the centre of gravity, throughout the range of wave heights,

Figure 10, Effect of L/b on Total Resistance in Irregular Head Waves, $V/\sqrt{L}=4$.

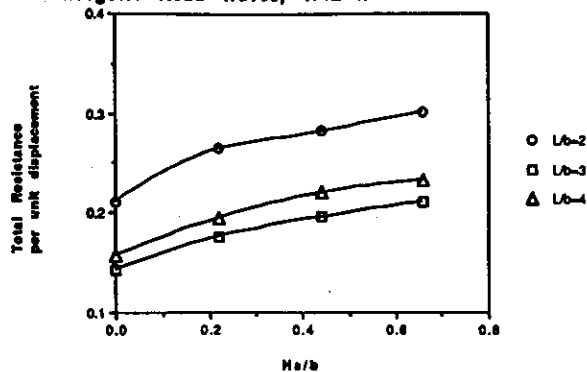


Figure 11, Effect of L/b on Heave Response in Irregular Head Waves, $V/\sqrt{L}=4$.

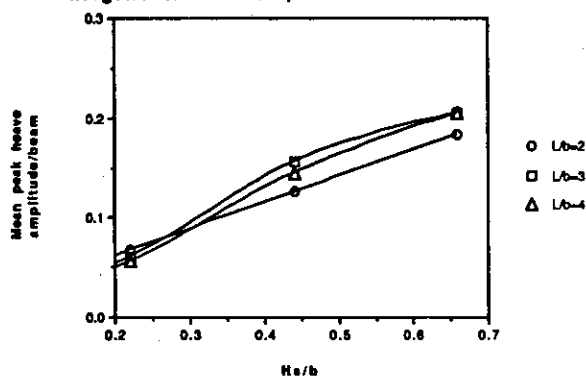


Figure 12, Effect of L/b on pitch response in Irregular Head Waves, $V/\sqrt{L}=4$.

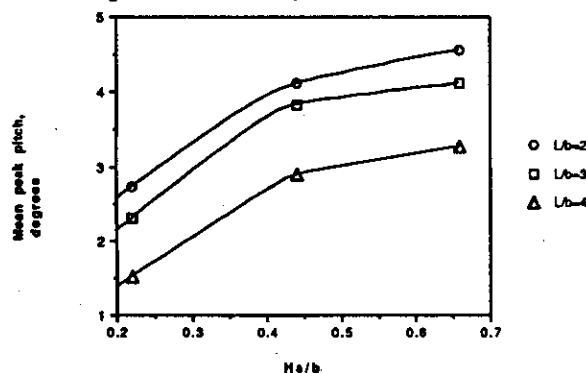


Figure 13, Effect of L/b on Acceleration at C. G. in Irregular Head Waves, $V/\sqrt{L}=4$.

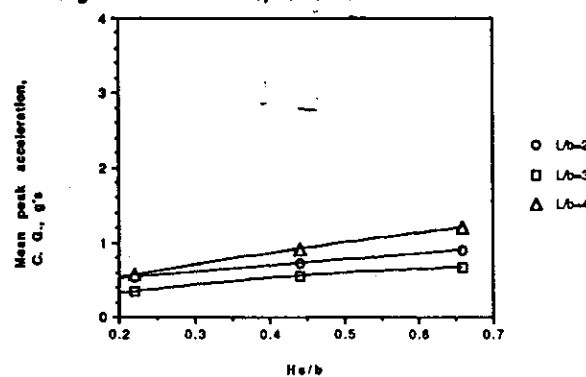
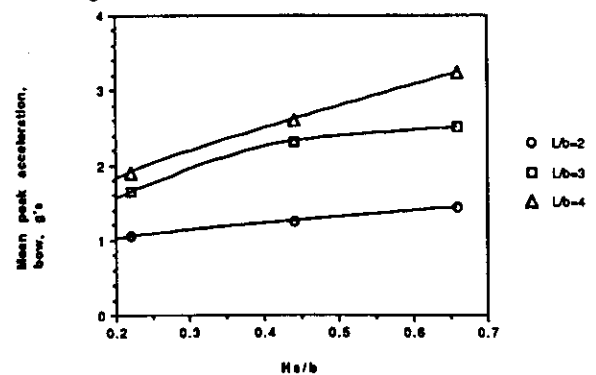


Figure 14, Effect of L/b on Bow Acceleration in Irregular Head Waves, $V/\sqrt{L}=4$.



whereas figure 14 indicates that a length to beam ratio of 2 consistently has the lowest mean peak bow acceleration.

The wave conditions are nominally the same between models, so the length of the hull, in relation to the expected (or average) wave length changes with length to beam ratio. The hull with the length to beam ratio of 2 is smaller in proportion to the expected wave length than the hull of length to beam ratio of 4. Also, the encounter frequency is reduced since the shorter model is running at a lower absolute speed for the same speed coefficient. These factors work in favour of the lower values of L/b , when comparing accelerations. It appears that the smaller models have a better tendency to contour the waves, and reduce the amplitude of the resulting slamming type accelerations. However, pitch motion amplitude increases as length to beam ratio is reduced. The increase of pitch amplitude is due to the shorter length of the hull, which must be deflected further to generate the same absolute value of buoyancy restoring force (and moment). The smaller hulls also have less area of planing surface, and so cannot make up this difference from increased hydrodynamic forces.

These results present some interesting tradeoffs for the designer. It appears that based on the simplified hull form, a length to beam ratio of less than three is undesirable. Whilst a hull with a lower length to beam ratio will result in a lower bow accelerations, it pays a penalty in terms of higher resistance, higher acceleration at the centre of gravity, and higher pitch amplitude. Since the centre of gravity is close to where the crew are likely to be sitting, this will result in a more uncomfortable ride. Only in cases when minimizing the bow acceleration is critical will

the lowest length to beam ratio be advantageous. A review of the OEC project data indicated that approximately 40 per cent of the designs had length to beam ratios less than 3. This may have been forced on the designer by considerations other than performance in waves.

5. Conclusions and Recommendations

This paper describes the results of model experiments on a simplified planing hull, first tested by Fridsma and described in references 1 and 2. When the results were compared, it was found that the agreement between the two data sets was good, after allowances were made for known differences in the experiment and analysis methods. The range of the original series was extended to cover length to beam ratios between 2 and 4, at a running trim of 6 degrees. This extension indicated that it is not desirable to reduce the length to beam ratio below three, when performance in waves at maximum speed is being considered.

This paper presents only a brief summary of the effect of length to beam ratio on the performance of simplified planing hulls in waves. Speed-length ratios of 2.0 and 3.0, at running trims of 6 degrees have been tested for the light displacement, but have not been included in this paper, due to space limitations. It is also planned to test some models at the heavy displacement for all three speed length ratios and carry out the experiments to determine the effect of deadrise at the length to beam ratio of 3. When all this data is collected, it is planned to review the effects of loading coefficient, deadrise angle and speed for low length to beam ratio planing hulls, and extend the design methods originally presented by Fridsma.

6. Acknowledgments

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