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ON WAVEFORMS PRODUCED BY FORWARD SCATTER

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ABSTRACT

A simple theoretical model is used to account for most of the characteristics of the flutter signals generated in a bi-static doppler detection system such as a McGill Fence. The development is a generalization of optical slit theory to include the coherent detector case. Very good agreement with experimental results has been obtained. In particular it is shown that it is possible to make an estimate of the length of the target from a knowledge of waveforms. The development is sufficiently general as to apply to other systems such as are used in meteor observations.

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INTRODUCTION

A great deal of useful information about the characteristic flutter signals associated with bi-static doppler systems can be obtained from simple geometry. A more or less vertical plane containing both transmitter and receiver may be considered to constitute a sort of fence erected for the detection of flying objects. It has been found advantageous for purposes of discussion to establish rectangular coordinates, Fig.1, such that the transmitter is located at $-X_o$ and the receiver at $+X_o$, then the X axis measures distances along the fence, the Y axis across the fence, and the Z axis measures altitude. In addition to the direct wave travelling along the X axis, a scattered wave is to be considered to travel from $-X_o$ via $P(x,y,z)$ to $+X_o$. The relative phase difference between the component waves is determined by the difference in transmission path length. If the point P varies either due to motion of the scattering body or in the process of a summation, then in general, the phase ϕ will also vary. Only in the special case of P moving on an ellipsoidal surface (with focii at $-X_o$ and $+X_o$) will ϕ remain fixed, otherwise the variation of ϕ with time is responsible for the flutter signals.

An over simplified model, based on single point scattering fails to put in evidence some interesting properties of flutter waveforms that have been observed repeatedly, namely a series of minima evenly spaced with respect to a center of symmetry located at $Y_o = 0$ or the instant of crossing the fence. A pair of scattering points moving coherently does predict minima but in the wrong places. It is interesting to note that if we think of these points as defining the ends of a cigar-shaped body moving in the direction of its own length L then the location of the minima can be used to determine L. A better model analogous to the slit in optics has successfully predicted flutter waveforms in considerable detail. This theory can be used to estimate the length of the scattering body, under favourable circumstances.

It will be shown in the development of the slit theory that the relative phase of the signal from a scattering point in the vicinity of the fence is βY^2 and from this it is easy to see that a point target would give rise to a waveform proportional to the real part of $\exp j(\beta Y^2 + \mu)$ where μ is an arbitrary angle. The frequency of this waveform, given by $d(\beta Y^2)/dt$ would be $2\beta Y\, dY/dt = 2\beta YV$. i.e. linear with Y for constant velocity, but having no minima.

A target consisting of two scattering points located at $Y_o - d/2$ and $Y_o + d/2$ would generate a waveform given by

$$\mathrm{Re}\left[\exp j\beta\,(Y_o + \frac{d}{2})^2 + \exp j\beta\,(Y_o - \frac{d}{2})^2\right]$$

and a minimum should occur whenever

$$(Y_o + \frac{d}{2})^2 - (Y_o - \frac{d}{2})^2 = \frac{\pi(1 + 2n)}{\beta} \qquad n = 1,2,3\ldots$$

or

$$2Y_o = \frac{\pi}{\beta d}\,(1 + 2n).$$

The location of these minima can be described as an odd series, whereas a typical waveform, Fig.2, has minima located by an even series which will later be shown to be

$$2Y_o = \frac{\pi}{\beta d}\,(2n).$$

SLIT THEORY

The general equation for the voltage contribution from an elementary wavelet may be taken as proportional to the real part of $\exp j(\omega t + \phi)$. The total scattered signal from a body large compared to the wavelength λ will be given by a summation wherein the variations of ϕ play a prominent part. It has been found worthwhile to analyse a somewhat idealized situation where the scattering body is a cigar-shaped target and is moving in the direction of its own length L and more or less across the fence. The body may be considered equivalent to an optical slit of opening $L \sin \theta$ moving with velocity equal to the Y component of target velocity.

The relative phase ϕ is given exactly by

$$\frac{2\pi}{\lambda}\left[\sqrt{(X_o + X)^2 + Y^2 + Z^2} + \sqrt{(X_o - X)^2 + Y^2 + Z^2} - 2X_o\right]$$

and if we restrict ourselves to $|Y|$ less than a tenth of $(X_0 \pm X)$ i.e., a 10^o beam, each of the square root quantities may be expanded by the binomial theorem and good approximations obtained by taking only the first two terms. With a little reduction we get

$$\frac{2\pi\ (Y^2 + Z^2)}{\lambda X_0 \left[1 - (X/X_0)^2\right]} .$$

For simplicity we now assume Z to be fixed (constant altitude) and X to be varying so slowly that variations in $(X/X_0)^2$ are small compared to unity; then the variations in phase will be given by

βY^2 where $$\beta = \frac{2\pi}{\lambda X_0 \left[1 - (X/X_0)^2\right]} .$$

If we now sum across the width of the slit (projected length of the target) from $Y_2 = Y_0 - \frac{L}{2} \sin \theta$ to $Y_1 = Y_0 + \frac{L}{2} \sin \theta$ the total scattered signal will be proportional to the magnitude or absolute value of

$$\exp\ (j\omega t) \int_{Y_2}^{Y_1} \exp\ (j\beta Y^2)\ dy.$$

This is the quasi stationary case. Because of the many orders of magnitude difference between the speed of light and any expected target velocity the quantity $\exp\ (j\omega t)$ may be taken outside the integral.

The resulting scattered signal eventually arrives at a diode envelope detector in the receiver where either one of two things may occur. In the absence of any direct wave from the transmitter, the diode detector will give an output substantially equal to the absolute value of the integral. However if there is a suitable reference signal, either as a direct wave or supplied from a stable local oscillator, the diode becomes a coherent detector, and the output is then more nearly equal to the real part of the integral. In general it can be said that the "real part" will be a more rapidly varying quantity, with many oscillations. The "magnitude" will appear as the envelope of the oscillations that make up the "real part".

The integral itself gives rise to Fresnel functions $C(u)$ and $S(u)$, where $u = \sqrt{2\beta/\pi}\, Y$, which when plotted on the complex plane yield a well-known curve -- Cornu's spiral. In relation to the spiral, the complex signal voltage will be the directed line joining the points representing Y_2 and Y_1. Thus, we can readily see that for a fixed $L \sin \theta = Y_1 - Y_2$ the two points will chase each other around the spiral maintaining constant separation along the curve. The straight line distance between points however may vary considerably depending on the curvature of the spiral at the region of interest.

In the neighbourhood of $Y_o = 0$ and for $L \sin \theta$ small compared to $\sqrt{\pi/2\beta}$ the resultant voltage vector will have a maximum amplitude and minimum angular velocity (frequency).

For Y_1 and Y_2 both less than $-\sqrt{\pi/2\beta}$ or both greater than $+\sqrt{\pi/2\beta}$ the Fresnel integrals may be approximated by

$$C(u) = \pm \frac{1}{2} + \frac{\sin \frac{\pi}{2} u^2}{\pi u}$$

$$S(u) = \pm \frac{1}{2} - \frac{\cos \frac{\pi}{2} u^2}{\pi u}$$

and the integral from Y_2 to Y_1 simplifies to

$$\left. \frac{\exp j(\beta Y^2 - \frac{\pi}{2})}{2\beta Y} \right]_{Y_2}^{Y_1} .$$

It is now evident that the magnitude of this quantity will have a minimum whenever

$$\beta Y_1^{\,2} - \beta Y_2^{\,2} = 2n\pi$$

or

$$2Y_o = \frac{2n\pi}{\beta L \sin \theta} .$$

Thus we should expect a series of minima with uniform spacing. The first minimum either side of the fence may not be accurately given by the approximate formula; however in practice this formula appears to be at least as good as the original assumptions regarding the equivalent slit.

For constant target velocity V we may write $Y_o = Vt \sin \theta$ and so locate the minima on an amplitude-time or waveform graph. These should occur at

$$t = \frac{Y_o}{V \sin \theta}$$

$$= \frac{n\pi}{\beta VL \sin^2\theta} .$$

The waveforms in Fig.2 show this property in a very pronounced way.

AN APPLICATION

For the very special case of $X = 0$ and $\theta = \pi/2$ this reduces to

$$t = n \cdot \frac{\lambda}{2} \cdot \frac{X_o}{VL}$$

which of course may be rearranged to yield explicitly any particular parameter if all but it are known. Specifically, we conducted an experiment from which it was possible to verify the length of the aircraft.

The waveforms in Fig.2 were obtained on the N.R.C. dual-beam McGill Fence in which the two receivers are some 500 yards apart. The aircraft, a Dakota 64.5 feet long, flew at right angles to the fence midway between transmitter and receivers. From the shift of the two waveforms in time, 3.5 sec., and the spacing between fences, 230 m., the ground speed was $V = 66$ m/s or 128 knots. The system parameters are $\lambda = .6$ m. and $X_o = 28.7$ km. The average time between minima is about 6.6 sec. and from these data we get a slit length L of

$$\frac{\lambda}{2}\frac{X_o}{Vt} = \frac{.6 \times 28700}{2 \times 66 \times 6.6} = 19.8 \text{ m. or } 65 \text{ feet!}$$

DISCUSSION

A detail has been deliberately left out of the picture because it does not affect the main result. This is the question of the precise phase at $Y_o = 0$ of the scattered wave with respect to the direct wave. The most noticeable effect will occur in the region of $Y_o = 0$, i.e., the centre of symmetry or the instant of crossing the fence. Instead of merely taking the real part of the integral we should have taken a more general projection on a reference line whose orientation with respect to the spiral is determined by the total phase difference. This quantity is one that changes rapidly with path position and so for practical purposes may be considered random. However there will always be an envelope maximum at the point $Y_o = 0$ so long as we are careful to preserve the very low frequencies (in principle right down to d-c). The maximum at the center of each of the waveforms in Fig. 2 has been deliberately suppressed by means of an input filter having some 6 db. attenuation below 1 cps. This was done to keep the amplitude within the linear range of the recording devices.

Although many records of flutter waveforms show some form of envelope modulation few are as well behaved as those in Fig. 2 - nor should this be expected from the general case of curved flight. The directivity patterns of both the transmitting and receiving antennas also enter into the picture and although we have assumed here uniform illumination and reception this need not always be so. Actually the antennas used have nominal beam widths of 10^o and the received signals fall below noise level and are lost when the target is out of the beam. However we can put an additional interpretation on the recordings of flutter waveforms for certain special cases.

Here we restrict ourselves for simplicity to right angle crossings not too near either end of the fence - making $\sin\theta$ unity. The bearing of the target from either transmitter or receiver is given approximately in radian measure by $Y_o/(X_o \pm X)$. The scattering angle α is the sum of these angles and is

$$\alpha_{rad.} = \frac{2Y_o}{X_o\left[1 - (X/X_o)^2\right]} = \frac{\beta\lambda Y_o}{\pi}$$

or in terms of time

$$\alpha^o = \frac{360}{\pi} \cdot \frac{VX_o}{X_o\left[1 - (X/X_o)^2\right]} \cdot t$$

Thus we may consider the time axis of our recorded waveforms to be also an angular axis. If we use the scattering angle as our independent variable then our formula for locating minima becomes

$$\alpha_{min.} = \frac{n\lambda}{L \sin \theta} \quad \text{(radians)} \qquad n = 1,2,3..$$

which is a well known formula in optics. The integral giving the scattered signal could also be expressed in terms of the scattering angle α instead of the coordinate distance Y_0 in which case we would get a polar re-radiation pattern analogous to those used in antenna work.

Throughout this treatment attention has been concentrated on the Y or crossing component of motion. Presumably an analogous set of results hold for the Z direction but as yet vertical flight is uncommon.

The integral giving the waveforms to be expected from forward scattering also applies to certain back scattering problems - with obvious modifications. In the case of reflections from ionized meteor trails the lower limit in the definite integral will tend to remain at a very large negative value and the waveform will be mainly a function of the upper limit.

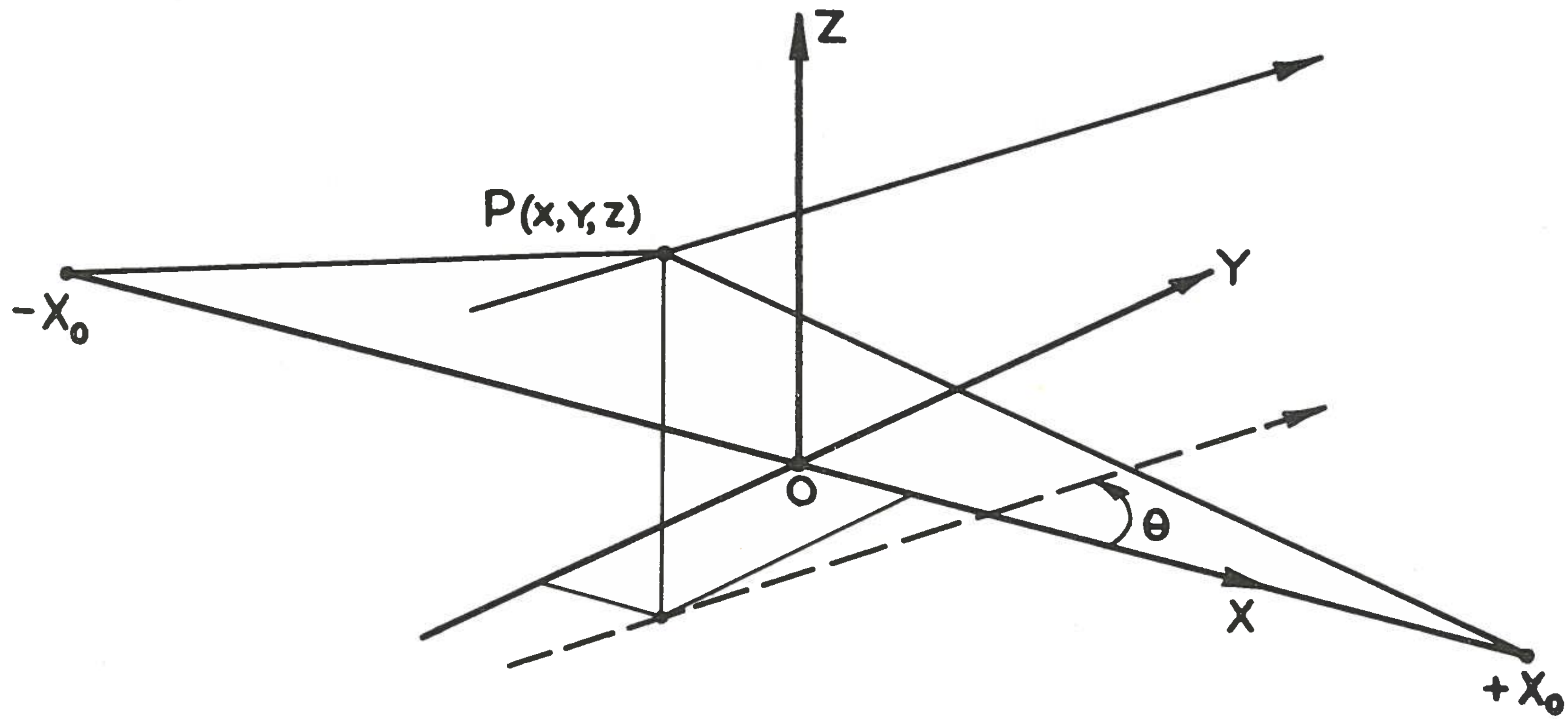
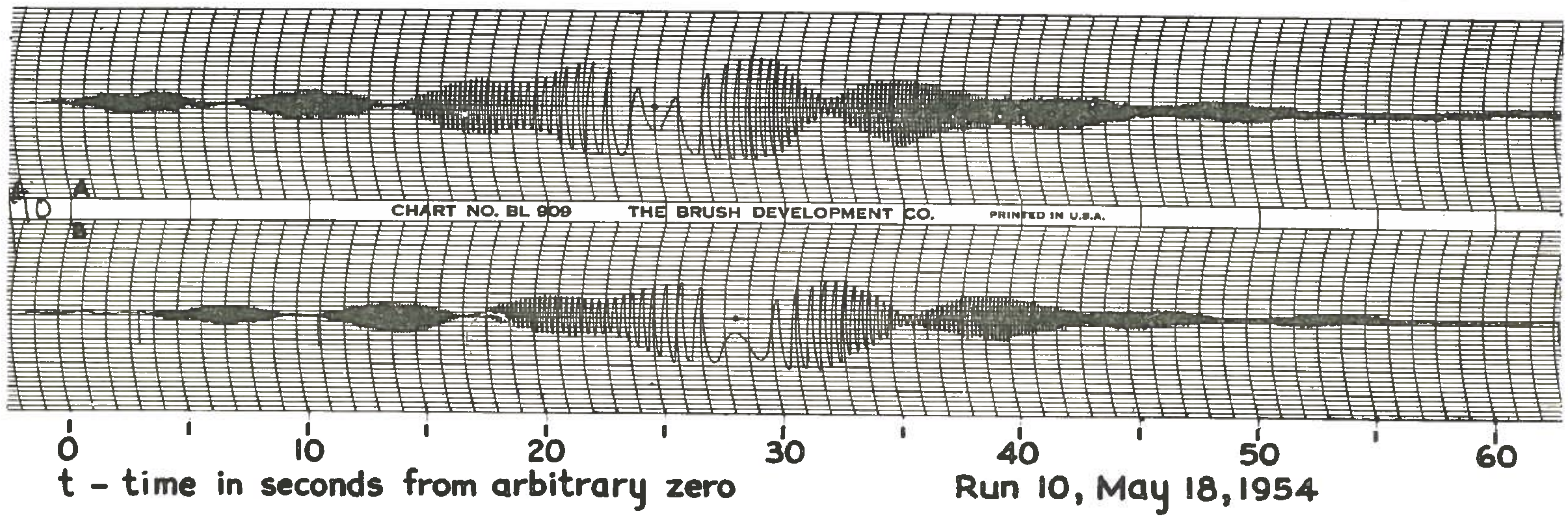


Fig. 1



Run 10, May 18, 1954

Dakota a/c Northbound at 2000 ft., midway.
Fence separation = 229 m.
B lags A – 3·5 sec.
∴ Ground Speed = 66 m/s.
Ave time between minima = 6·6 sec.
∴ Length of a/c = 19·8 m.

$Y_0 = 66t$ $\alpha^{0}_{min} = \cdot 26t$

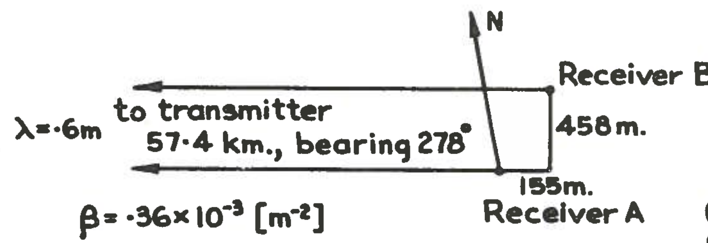


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Fig. 2

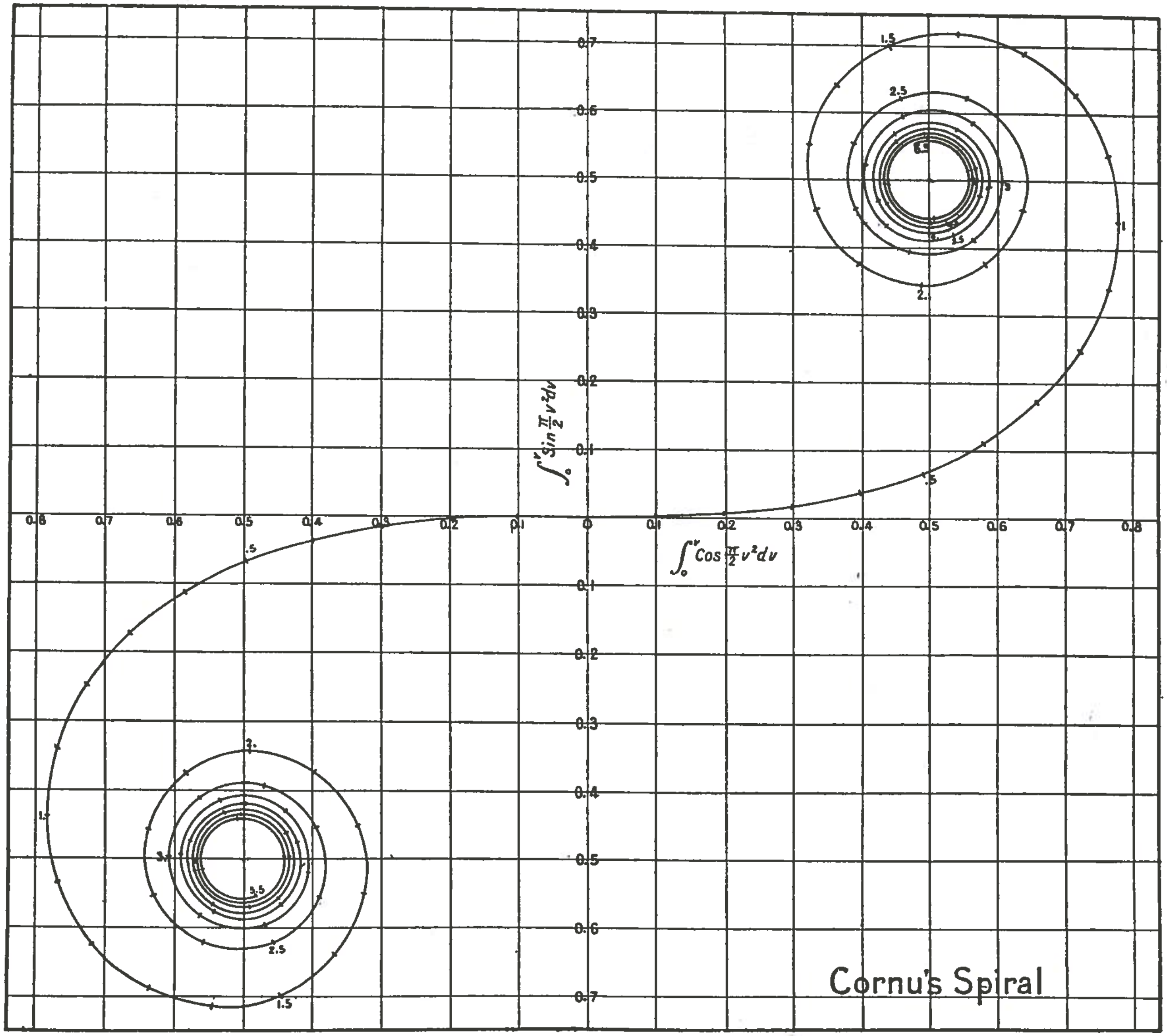

FIG. 3