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Chu, W. T.

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by W.T. Chu

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SOMMAIRE

Comme suite à l'analyse modale détaillée de Chien et Soroka (1) quant à la corrélation croisée spatiale en salle de réverbération, une analyse similaire est appliquée à l'excitation sonore en bandes étroites dans le cas d'un régime permanent.

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LETTERS TO THE EDITOR

SPATIAL CROSS-CORRELATION OF REVERBERANT SOUND FIELDS

According to Chien and Soroka [1], the derivation of a narrow-band correlation function by Morrow [2], using a modal approach, included the assumption of zero cross-correlation of different modes at two different points. Chien and Soroka comment that this assumption is physically reasonable but not obvious. The purpose of this letter to the editor is to clarify this assumption and rederive the narrow-band correlation function in a detailed manner analogous to that of reference [1].

Consider a rectangular room of sides L_x, L_y, L_z and volume $V = L_x L_y L_z$. The sound field at the point $r = (x, y, z)$ due to a simple source $Q_0\,\mathrm{e}^{-\mathrm{i}\omega t}$ at $r_0 = (x_0, y_0, z_0)$ can be written as [1]

$$p = \frac{\mathrm{i}\rho\omega Q_0}{V}\,\mathrm{e}^{-\mathrm{i}\omega t}\sum_N \frac{\psi_N(r)\,\psi_N(r_0)}{\Lambda_N(k^2 - k_N^2 + 2\mathrm{i}k\delta_N)}, \tag{1}$$

where the origin of co-ordinates is at one corner of the chamber, N stands for the trio of numbers l, m, n and $\Lambda_N = 1/\varepsilon_l\varepsilon_m\varepsilon_n$, $\varepsilon_0 = 1$ and $\varepsilon_i = 2$ for $i \neq 0$, $\psi_N(x, y, z) = \cos k_x x \cos k_y y \cos k_z z$, $k_x = l\pi/L_x$, $k_y = m\pi/L_y$, $k_z = n\pi/L_z$, $k_N^2 = k_x^2 + k_y^2 + k_z^2$, $\delta_N = \beta(\varepsilon_l/L_x + \varepsilon_m/L_y + \varepsilon_n/L_z)$, $k = \omega/c$ is the wave number, ρ is the density of the medium, and β is the normal admittance of the chamber surface, assumed to be real and small. It is noted that only terms near $k_N = k$ contribute significantly because the damping term $2k\delta_N$ is usually small.

When the simple source emits a narrow-band signal rather than a single frequency, the resultant sound field can be considered as the combination of many fully excited normal modes each vibrating at its own different characteristic frequency. If one considers only the fully excited modes (i.e., puts $k = k_N$), the resultant sound field can be written as

$$p_{\Delta\omega} = \frac{\rho c}{2V}\sum_N \frac{\mathrm{e}^{-\mathrm{i}\omega_N t}\psi_N(r)\,\psi_N(r_0)}{\Lambda_N\delta_N}\,Q(\mathrm{i}\omega_N), \tag{2}$$

where $Q(\mathrm{i}\omega)$ represents the Fourier spectrum of the random source signal [3]. In terms of complex notation, the spatial cross-correlation function $\overline{p_1 p_2}$ can be obtained as $\frac{1}{2}\mathrm{Re}(p_1 p_2{}^*)$ with the understanding that the final result should no longer be a function of time. The bar denotes time averages and * denotes complex conjugate. Using equation (2) to form the product gives

$$\mathrm{Re}\,(p_1 p_2{}^*)_{\Delta\omega} = \left(\frac{\rho c}{2V}\right)^2 \left(\sum_N \frac{\psi_N(r_1)\,\psi_N(r_0)}{\Lambda_N\delta_N}\right)\left(\sum_M \frac{\psi_M(r_2)\,\psi_M(r_0)}{\Lambda_M\delta_M}\right)\cos(\omega_N - \omega_M)t$$
$$\times\, Q(\mathrm{i}\omega_N)\,Q^*(\mathrm{i}\omega_M). \tag{3}$$

The above condition requires that $\omega_N = \omega_M$ (i.e., $N = M$ in equation (3)) before it can truly represent the correlation function $\overline{p_1 p_2}$. Physically what this means is that harmonic waves at different frequencies are uncorrelated. Except for some degenerate modes, which are assumed to be few for a well designed room, this observation corresponds to Morrow's assumption of zero cross-correlation of different modes. Thus it follows that

$$(\overline{p_1 p_2})_{\Delta\omega} = \tfrac{1}{2}\left(\frac{\rho Q c}{2V}\right)^2 \sum_N \frac{\psi_N(r_1)\psi_N(r_2)\psi_N^2(r_0)}{\Lambda_N^2\delta_N^2}, \tag{4}$$

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where $Q^2 = |Q(\mathrm{i}\omega_N)|^2$ for all N's allowable and β is a constant within the bandwidth of interest. At high frequencies N will still be very large. By following the same argument as given by reference [1] in expanding the characteristic function and in converting summation to integration, equation (4) can be changed to

$$\overline{(p_1 p_2)}_{\Delta\omega} = \frac{1}{2}\left(\frac{\rho Qc}{2V}\right)^2 \left(\frac{V}{\beta S}\right)^2 \int_{\Delta k} \langle \cos k_x(x_1 - x_2) \cos k_y(y_1 - y_2) \cos k_z(z_1 - z_2)\rangle \times \frac{k_c^2 V}{2\pi^2}\, \mathrm{d}k_N, \tag{5}$$

where $S = 2(L_x L_y + L_y L_z + L_z L_x)$ and $\langle\;\rangle$ denotes an average over the surface of an octant of the sphere of radius k_N and thickness $\mathrm{d}k_N$ in the wave number space. In the present case, since only the fully excited modes are considered, integration over the wave number space is no longer infinite but over Δk as indicated by the narrow-band nature of the source, and an averaged modal density as given by $k_c^2 V/2\pi^2$ was used. k_c is the wave number corresponding to the centre frequency of the narrow-band noise of the source. According to reference [1]

$$\langle \cos k_x(x_1 - x_2) \cos k_y(y_1 - y_2) \cos k_z(z_1 - z_2)\rangle = \sin(k_N r)/k_N r, \tag{6}$$

where $r^2 = (x_1 - x_2)^2 + (y_1 + y_2)^2 + (z_1 - z_2)^2$, and

$$\overline{(p_1 p_2)}_{\Delta\omega} = \left(\frac{\rho Qck_c}{\pi\beta S}\right)^2 \frac{V}{16} \int_{\Delta k} \frac{\sin(k_N r)}{k_N r}\, \mathrm{d}k_N. \tag{7}$$

The last integral has been considered by Cook [4]. He concludes that

$$\frac{1}{k_2 - k_1}\int_{k_1}^{k_2} \frac{\sin(kr)}{kr}\,\mathrm{d}k = \frac{\sin(kr)}{kr} + \text{terms of the order of } [(k_2 - k_1)/k]^2,$$

where $k = (k_2 + k_1)/2$. Therefore, for a narrow band,

$$\overline{(p_1 p_2)}_{\Delta\omega} = \left(\frac{\rho Qck_c}{\pi\beta S}\right)^2 \frac{\overline{V}}{16}\, \Delta k\, \frac{\overline{\sin(k_c r)}}{k_c r}$$

and the correlation coefficient is $\sin(k_c r)/k_c r$ as given by references [2] and [4].

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Division of Building Research,
National Research Council of Canada,
Ottawa,
Ontario, K1A OR6, Canada

W. T. Chu

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LASER–DOPPLER MEASUREMENT OF VIBRATING SURFACES: A PORTABLE INSTRUMENT

1. INTRODUCTION

Review articles on laser–Doppler Velocimetry (L.D.V.) serve to emphasize the variety of possible applications of the technique and the resulting variety of instruments (see, e.g., reference [1]). The use of L.D.V. for remote measurement of vibrating surfaces [2, 3] has led to a series of commercially available instruments [4, 5]. In the laboratory, however, there is still a need for a laser–Doppler vibration instrument which can be quickly and easily assembled from readily available parts and which self-aligns without resort to any fine adjustment. This note reports the continuing development of an instrument first described in reference [6] which, for most practical applications, compares well with commercially available models and is compact, robust and easy to assemble.

2. THEORY AND DESCRIPTION OF THE INSTRUMENT

The optical geometry is essentially that of a Michelson interferometer as shown in Figure 1. A beamsplitter, which consists of a polished microscope slide, splits the laser beam into

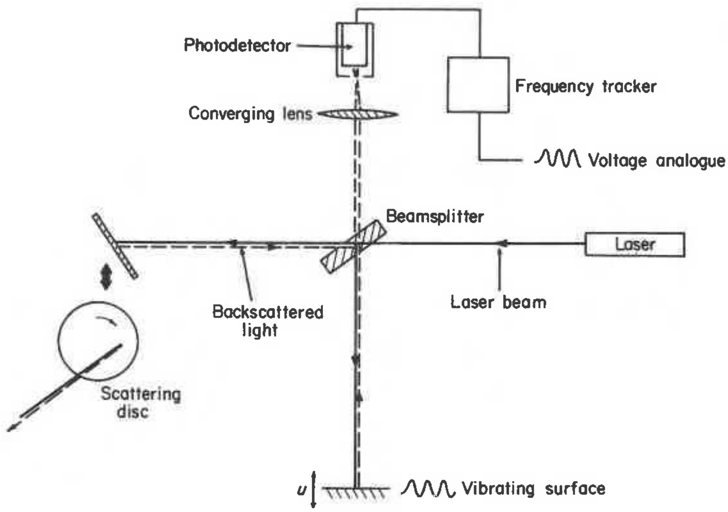

Figure 1. Schematic diagram of the vibration velocimeter.

two beams and directs them at the surfaces of a rotating scattering disc and the surface for measurement. The disc consists of a thin polished aluminium washer (3·5 cm diameter) driven by a small, constant speed, brushless d.c. motor (3000 r.p.m.). Backscattered light from the vibrating surface and the disc returns on-axis with the incident beams and is automatically mixed by the beamsplitter before collection by a converging lens. The lens then converges the light through a pin-hole onto the surface of a photodetector. With the disc stationary any movement of the vibrating surface produces a Doppler frequency shift (f_d) in the backscattered light which heterodynes with that from the disc into the photodetector.

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