Temperatures Under Ice Rinks
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TECHNICAL NOTE

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SUBJECT TEMPERATURES UNDER ICE RINKS

The temperature at any depth beneath the cooled floor of an ice rink can be calculated quite easily if some simplifying assumptions are made. These are:

(1) At the time that the ice is first made in the fall, the temperature under the rink is the same at all depths; this may be represented as T_o.

(2) The temperature of the rink floor is kept constant at T_∞ during the entire period that there is ice in the rink.

(3) The rink is large enough for the temperatures under the centre area to be uninfluenced by the temperatures near the perimeter.

(4) The material under the floor is dry, i.e. there is no latent heat released when the temperature drops below 32°F.

(5) The thermal conductivity and heat capacity are the same at all depths.

If these assumptions are valid the temperature, T_x, at depth X measured downward from the under surface of the floor slab is given by the expression

$$\frac{T_x - T_\infty}{T_o - T_\infty} = \operatorname{erf}\left(\frac{X}{2\sqrt{\alpha t}}\right)$$

where

α = thermal diffusivity of the material under the floor

t = time measured from the start of the ice season

erf = the error function, which is given in Table I.

The quantity $X/2\sqrt{\alpha t}$ should be dimensionless, hence X, α and t must be expressed in consistent units, e.g., X in feet, α in sq feet/hour and t in hours.

The following example illustrates the use of this formula:

Find the depth of the 32°F isotherm at the end of a 28-week curling season.

$$T_o = 56°F, \quad T_\infty = 20°F, \quad \alpha = 0.024\ ft^2/hr.$$

Solution: $$\frac{T_x - T_\infty}{T_o - T_\infty} = \frac{32 - 20}{56 - 20} = \frac{12}{36} = 1/3.$$

The corresponding value of $X/2\sqrt{\alpha t}$ is 0.305 (found by interpolating in the error function table), hence

$$\begin{aligned} X &= 0.305 \times 2\sqrt{\alpha t} \\ &= 0.305 \times 2\sqrt{0.024} \times 4700 = \underline{\underline{6.5\ ft}}. \end{aligned}$$

The frost penetration under a cooled slab can be reduced by placing insulation between the cooling pipes and the ground. The graphs in Figure 1 give the relationship between the dimensionless quantities that characterize the situation. In this case h is the conductance of the insulation and K is the conductivity of the soil. For example, 2 inches of styrofoam insulation would have a conductance of about 0.13 Btu/hr ft^2 °F and the K for crushed rock is about 0.67 Btu/hr ft °F.

Reworking the previous example with 2 inches of insulation gives:

$$\text{Fourier Modulus} \left(\frac{h}{K}\right)^2 \alpha t = \left(\frac{0.13}{0.67}\right)^2 \times 0.024 \times 4700 = 4.25.$$

Thus, for a temperature ratio

$$\frac{T_x - T_\infty}{T_o - T_\infty} = \frac{32 - 20}{56 - 20} = 1/3$$

the Biot Modulus $\frac{hx}{K} = 0.3$ (from Figure 1A).

Thus, $X = \frac{0.3 \times 0.67}{0.13} = \underline{\underline{1.5 \text{ ft}}}$.

The problem might have been to determine the thickness of insulation that would keep the soil entirely unfrozen. In this case, it is necessary to find the value of the Fourier Modulus that corresponds to the temperature ratio of 1/3 when the Biot Modulus = 0 (i. e., for X = 0).

The graph shows $\left(\frac{h}{K}\right)^2 \alpha t = 2$ in this case.

Thus, $h^2 = \frac{2k^2}{\alpha t} = \frac{2 \times (0.67)^2}{0.024 \times 4700} = 0.008$

so $h = 0.09$ Btu/hr ft^2°F

which corresponds to about 3 inches of styrofoam.

The graph is rather small and hard to read with much accuracy, so for critical calculations, it is preferable to use a formula. The surface temperature (X = 0) is given by

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{Y^2} \operatorname{erfc} Y$$

where Y^2 = Fourier Modulus

The function $e^{Y^2} \operatorname{erfc} Y$ is given in Table I. By interpolation, it is found that $Y = 1.43$ when the temperature ratio equals 1/3.

Thus, $h = \frac{1.43\ K}{\sqrt{\alpha t}} = \frac{1.43 \times 0.67}{\sqrt{0.024 \times 4700}} = 0.09 \frac{\text{Btu}}{\text{hr ft}^2\ °\text{F}}$ as before.

The data used in this example are taken from observations that have been made at a curling rink that is uninsulated. For an insulated rink, the value of T_o may be somewhat lower than the 56°F used in these calculations. If it is assumed to be only 50°F, it would require 4 inches rather than 3 inches of styrofoam to keep the ground unfrozen.

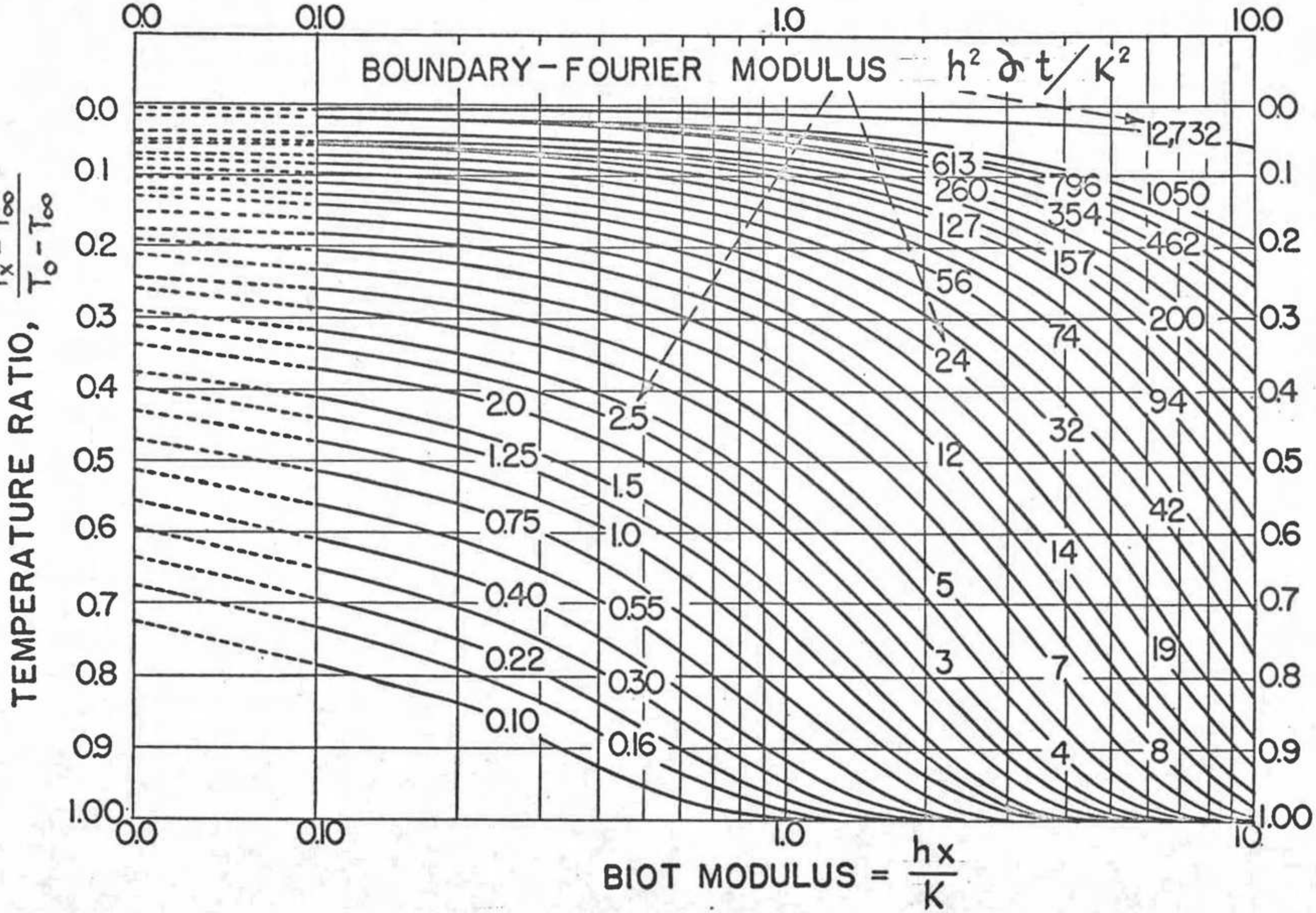
TABLE I

TABULATED VALUES OF SPECIAL FUNCTIONS

x	e^{x^2} erfc x	erf x
0	1.0	0
0.05	0.9460	0.056372
0.1	0.8965	0.112463
0.15	0.8509	0.167996
0.2	0.8090	0.222703
0.25	0.7703	0.276326
0.3	0.7346	0.328627
0.35	0.7015	0.379382
0.4	0.6708	0.428392
0.45	0.6423	0.475482
0.5	0.6157	0.520500
0.55	0.5909	0.563323
0.6	0.5678	0.603856
0.65	0.5462	0.642029
0.7	0.5259	0.677801
0.75	0.5069	0.711156
0.8	0.4891	0.742101
0.85	0.4723	0.770668
0.9	0.4565	0.796908
0.95	0.4416	0.820891
1.0	0.4276	0.842701
1.1	0.4017	0.880205
1.2	0.3785	0.910314
1.3	0.3576	0.934008
1.4	0.3387	0.952285
1.5	0.3216	0.966105
1.6	0.3060	0.976348
1.7	0.2917	0.983790
1.8	0.2786	0.989091
1.9	0.2665	0.992790
2.0	0.2554	0.995322
2.1	0.2451	0.997021
2.2	0.2356	0.998137
2.3	0.2267	0.998857
2.4	0.2185	0.999311
2.5	0.2108	0.999593
2.6	0.2036	0.999764
2.7	0.1969	0.999866
2.8	0.1905	0.999925
2.9	0.1846	0.999959
3.0	0.1790	0.999978

From: Table I in "Conduction of Heat in Solids", second edition, by H.S. Carslaw and J.C. Jaeger, Oxford University Press, 1959, p. 485.

SEMI-INFINITE SOLID



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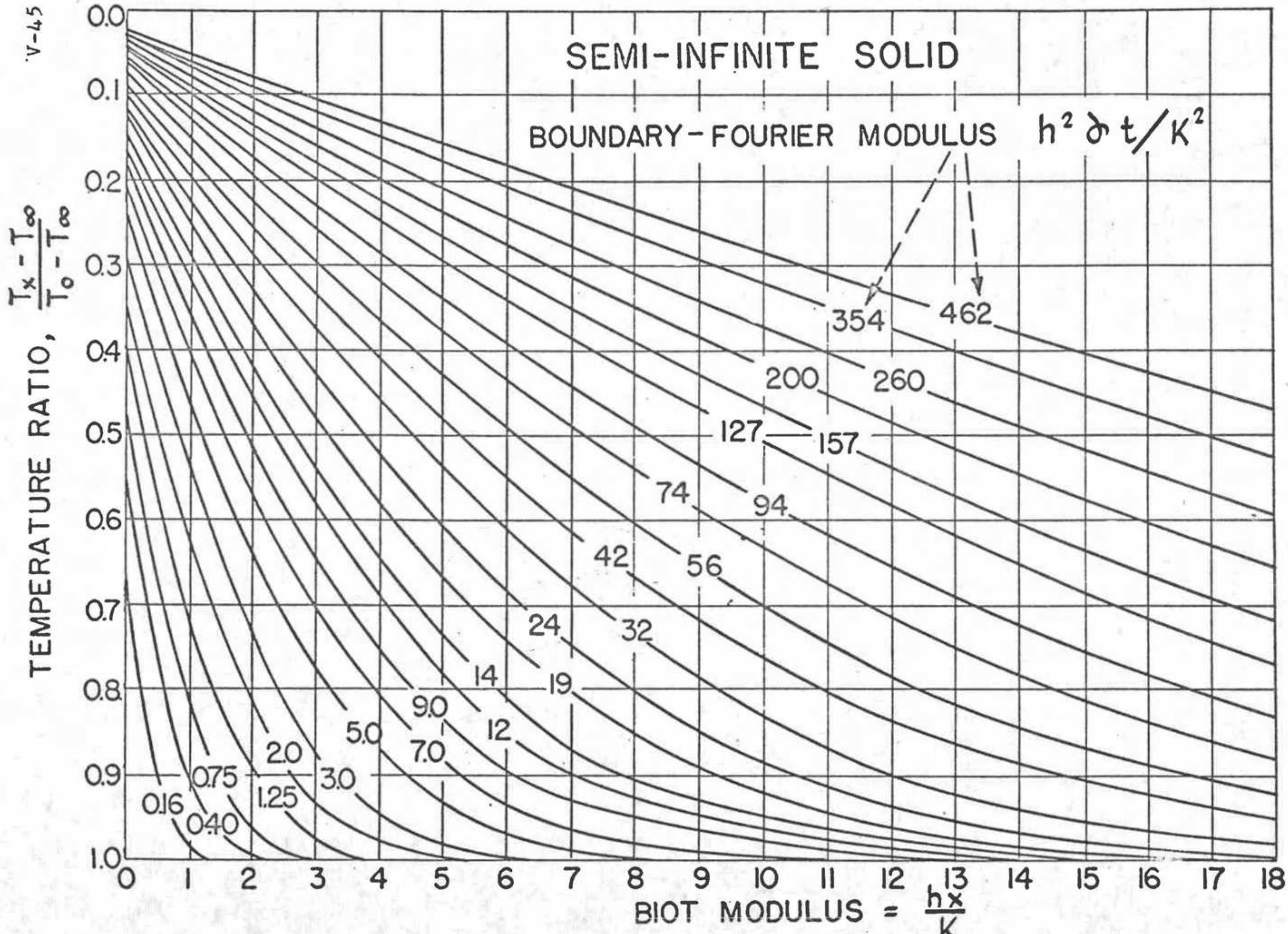

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