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ONE-DIMENSIONAL MATHEMATICAL MODEL OF THE LOWER FRASER RIVER

SUBMITTED BY
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SYNOPSIS

A study of the tidal motion in the Fraser River was made using a one-dimensional mathematical model. The equations of motion and continuity were solved by a direct explicit finite-difference scheme and the model was calibrated to reproduce water elevations throughout the river. A proposal for the improvement of navigation conditions and the alleviation of flood danger in the Lower Fraser River, the Boundary Bay Diversion Scheme, was studied using the mathematical model and the results were compared with those previously obtained from a hydraulic model constructed at the National Research Council. It was concluded that a one-dimensional mathematical model could supply much of the information required to assess the effectiveness and feasibility of large-scale engineering works in the tidal reach of the Fraser River.

1.0 INTRODUCTION

The Fraser River System is the largest and most important in British Columbia, draining approximately 90,000 square miles, or a quarter of the Province, and providing access to New Westminster Harbour, a freshwater port open to ocean navigation the year round (Fig. 1). In the past, two hydraulic models have been constructed encompassing the Lower Fraser River to investigate some of the schemes proposed for the improvement of navigation conditions up to New Westminster and the alleviation of flood danger in low lying areas.(1)(2)*

A mathematical model was developed to study the tidal propagation in the Fraser River under wide variations in the rate of discharge and to verify some of the results with those of the latest hydraulic model. The one-dimensional shallow-water equations for unsteady flow in open channels were solved by a direct explicit finite-difference scheme in which the differential equations are replaced by algebraic equations and solved directly. The mathematical model formulated was found to reproduce accurately the existing tidal elevations throughout the river.

The model was then used to study the Boundary Bay Diversion Scheme, a large-scale engineering project that would affect the water elevations and velocities throughout the Lower Fraser River.

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* Figures in raised parenthesis refer to the reference numbers in Appendix II.

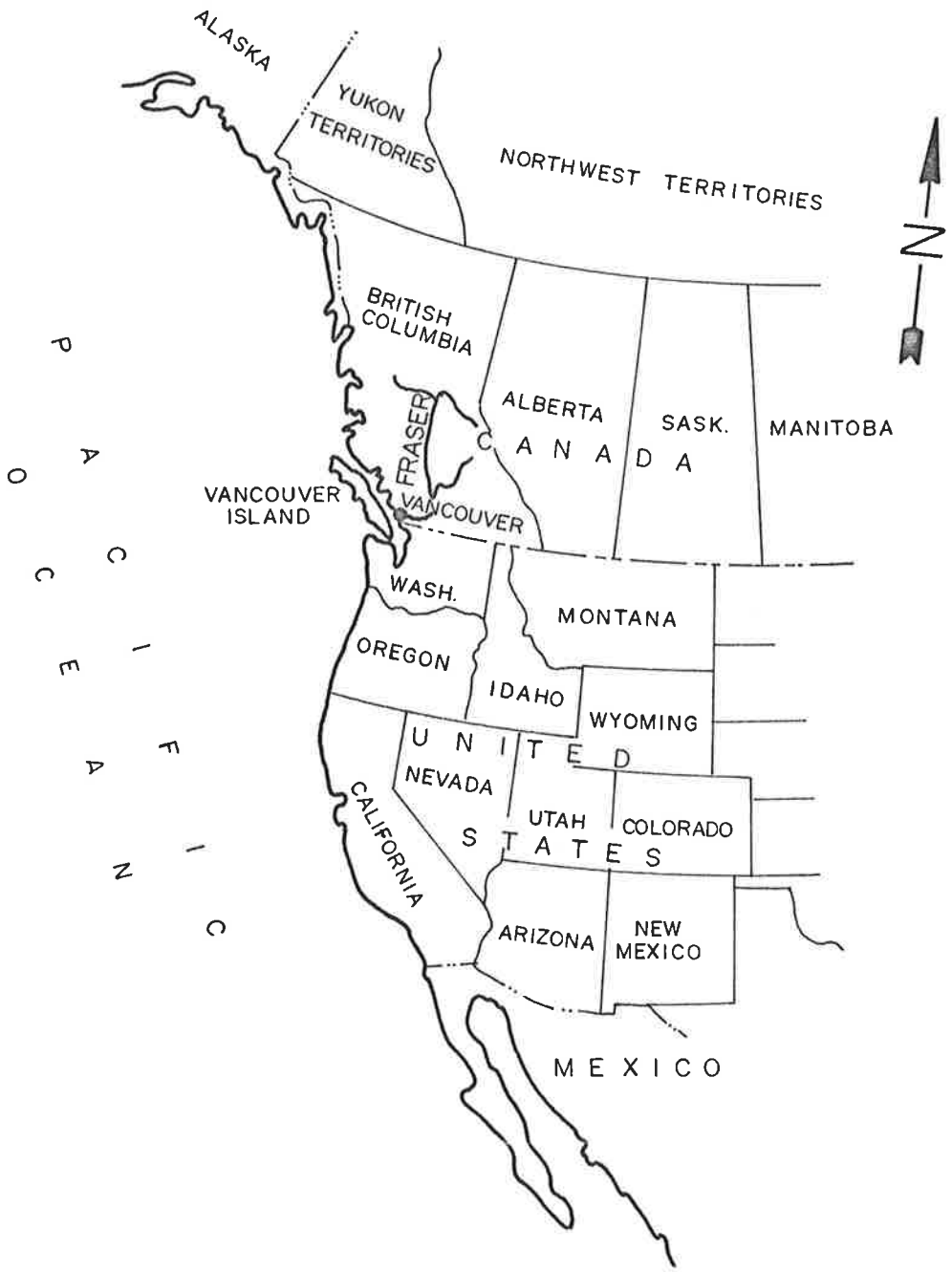


FIG. 1 (A) LOCATION SKETCH

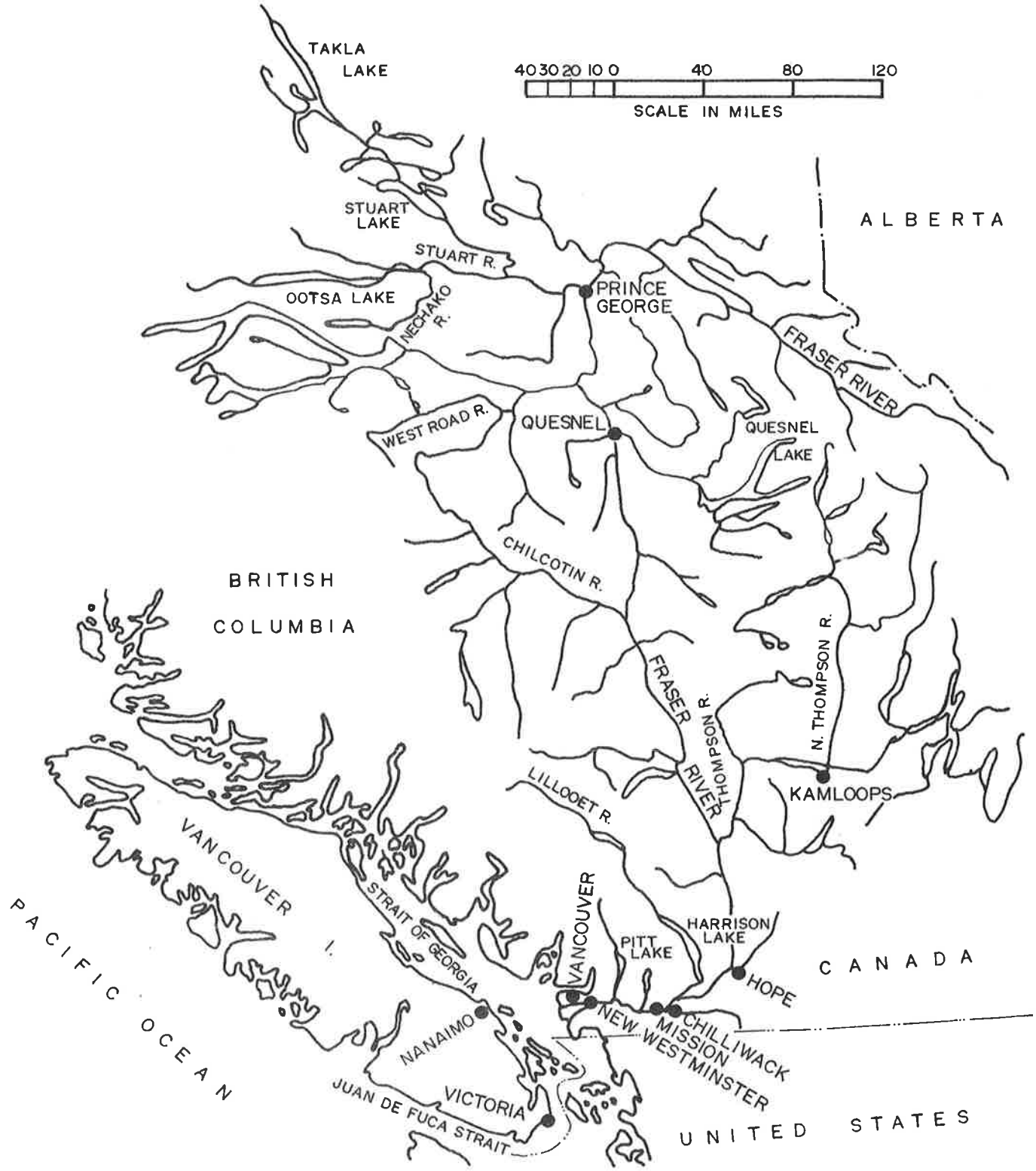


FIG. I(B) THE FRASER RIVER DRAINAGE BASIN

2.0 LOWER FRASER VALLEY

At the village of Hope the Fraser River turns westward and flows through an ever widening flood plain to the Strait of Georgia. This area is known as the Lower Fraser Valley (Fig. 2). Except for the occasional hills or mountain, the fan-shaped lowland has a fairly uniform surface, the result of flood deposits left by the river.

From Hope to the mouth, the natural river channel has an average width of 2,000 feet expanding to flood-channel proportions of more than three miles in certain areas and drops approximately 125 feet. From a physiographical viewpoint, the river can be divided into three distinct sections:

1. An upper section east of the Sumas River that is steep and braided where spring freshets deposit vast amounts of coarse material.
2. A middle section between New Westminster and the Sumas River that has the characteristics of a mature stream with a flat, deep, reasonably regular channel.
3. A lower section below New Westminster comprising the Fraser River Delta where the river divides into several branches (Fig. 3).

Five major tributaries join the Lower Fraser River; the Harrison, Sumas, Chilliwack, Stave, and the Pitt Rivers.

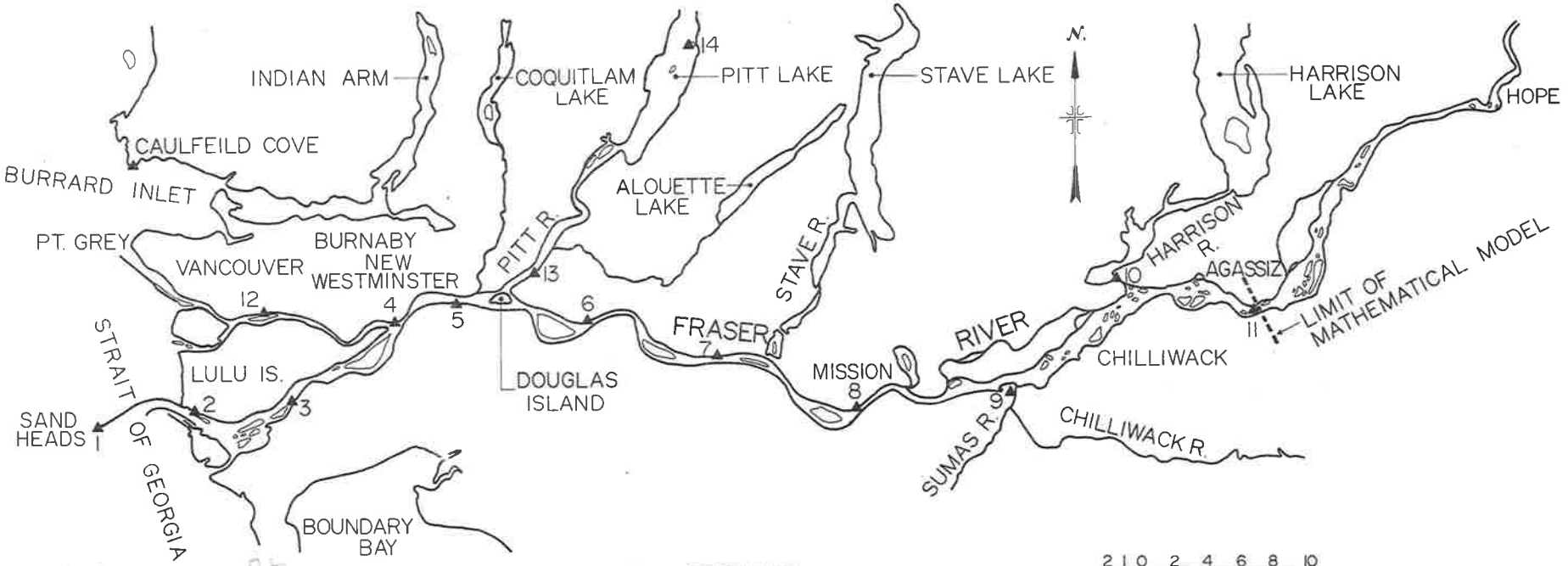


FIG.2 THE LOWER FRASER RIVER

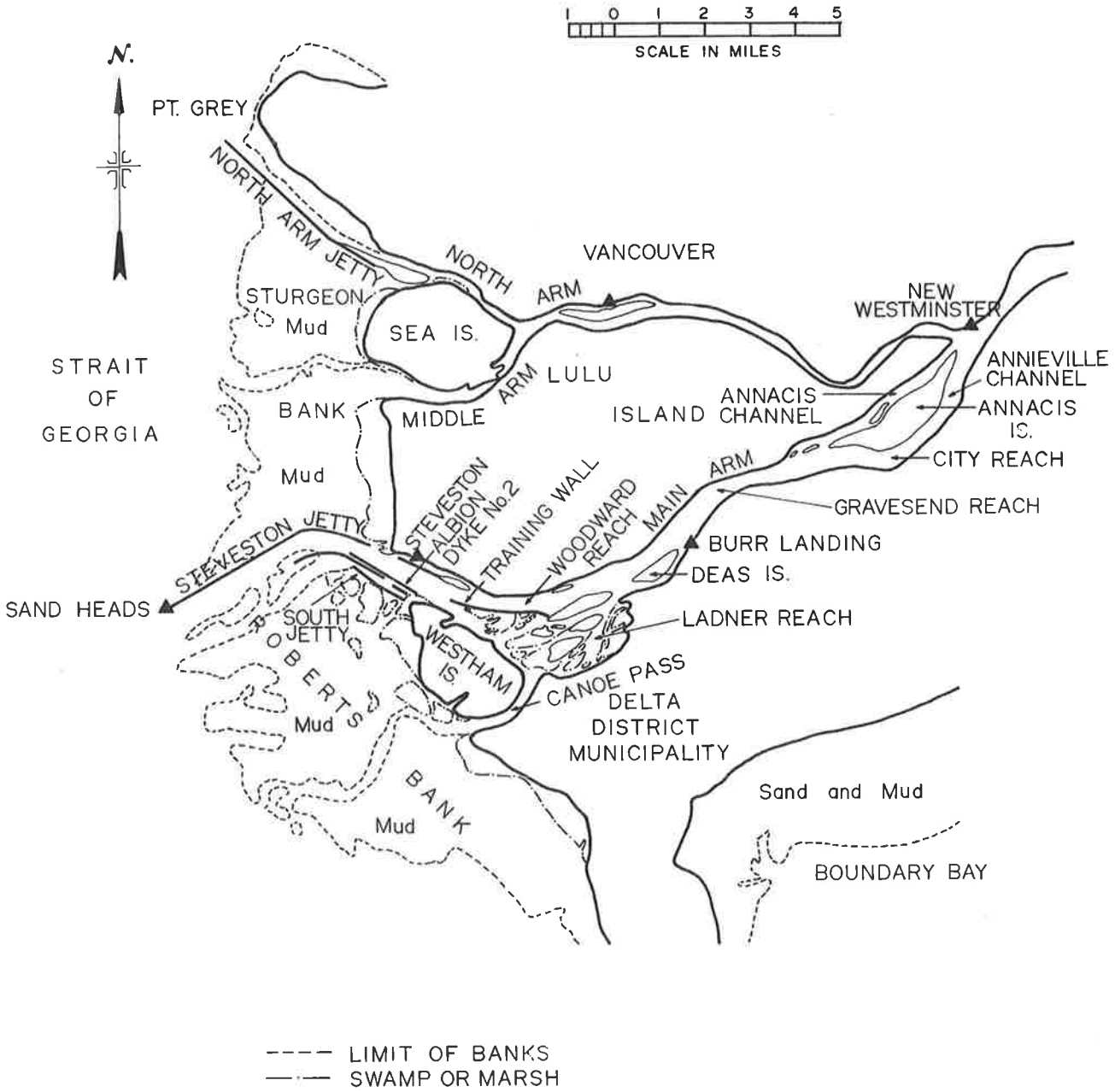


FIG.3 THE FRASER RIVER DELTA

2.1 LIMITS OF THE MODEL

The region studied by means of the mathematical model stretches from Sand Heads to Agassiz a distance of approximately 70 miles.

Unlike rivers such as the St. Lawrence which widens and deepens gradually before reaching the sea, the Fraser River changes abruptly from a relatively narrow, shallow channel to flow directly into the Strait of Georgia over a series of mud flats. The seaward limit of the model was actually taken in an area that can only be truly represented by a two-dimensional model. For the Main Arm, a one-dimensional model should have been started at the Steveston gauging station where the flow is contained between two banks. However, at Steveston and the corresponding entrances to the other three channels, the tidal records would be influenced by large-scale engineering works within the tidal reach of the river. The only location where the tidal elevations were available and would be independent of changes within the river was located on an arc passing through Sand Heads and the end of the North Arm Jetty or approximately along the outer boundary of Roberts and Sturgeon Banks. Here the tide is nearly identical to the tide at Caulfield Cove, a reliable tidal gauge with a long period of records available.

As an approximation, the flow over the mud flats was considered one-dimensional by dividing the area into

four channels as shown in Figure 4. This schematization could have been adjusted for the Main Arm, which carries approximately 70% of the total discharge, if normal calibration procedures failed to reproduce the water elevations at Steveston.

The upper limit of the model was taken at Agassiz where hydrograph records indicate the water level to be independent of tidal variations.

2.2 TIDE IN THE RIVER SYSTEM

The tide entering the Strait of Georgia through the Strait of Juan de Fuca is of the mixed type with considerable diurnal inequality having two daily high waters and two low waters. The tidal range at Sand Heads averages between 8.5 feet to 12.5 feet.

Under average conditions the tide progresses as far upstream as the mouth of the Sumas River with the greatest reduction in range occurring from Sand Heads to New Westminster. During high discharge periods the tide can proceed no further than Port Hammond.

A large amount of tidal energy branches off the Fraser River and propagates through the Pitt River into Pitt Lake which experiences a tidal range of 1½ to 2 feet.

2.3 FRESHWATER DISCHARGE

The discharge of the Fraser River is large due to the comparatively high precipitation received by the drainage

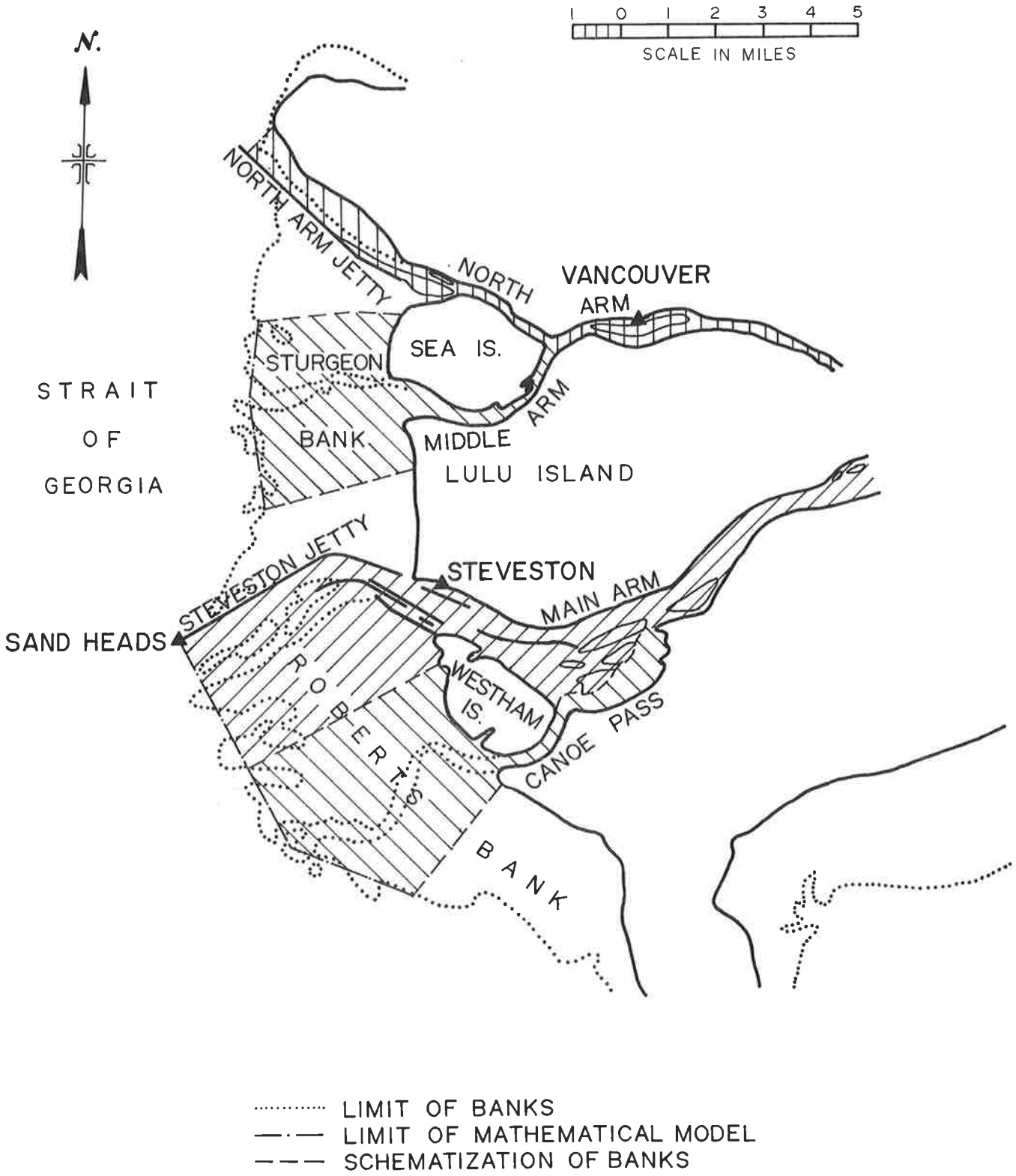


FIG. 4 SCHEMATIZATION OF ROBERTS AND STURGEON BANKS

basin. Wide variations in discharge result from the vast winter reservoirs of snow held in the mountains. The minimum and maximum average monthly discharges are 25,000 c.f.s. occurring during the winter and 240,000 c.f.s. occurring during the spring freshet. Extreme values of 12,000 c.f.s. and 536,000 c.f.s. have been recorded.

Floods are a danger in low-lying areas of the Lower Fraser Valley when the discharge at Hope exceeds approximately 430,000 c.f.s.

3.0 BASIC EQUATIONS

The basic equations considered are the one-dimensional shallow-water equations for unsteady flow in open channels.

3.1 Equation of Continuity

The equation of continuity:

$$\frac{\partial Q}{\partial x} + B \frac{\partial h}{\partial t} - \frac{Q_T}{dx} = 0 \qquad (1)$$

where

- Q is the main channel discharge
- B is the water surface width
- h is the water surface elevation above a specific datum
- Q_T is the tributary discharge
- x is the distance along the channel
- t is the time.*

* The symbols are defined as they appear and in Appendix I.

The datum used was the Canadian Geodetic Datum (G.S.C.).

3.2 Equation of Motion

The equation of motion in the x-direction:

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + g \frac{\partial h}{\partial x} + g S_f = 0 \qquad (2)$$

where

$\bar{u}$ is the average velocity over the vertical in the x-direction

g is the acceleration due to gravity

S_f is the friction slope

The convective term $\bar{u} \frac{\partial \bar{u}}{\partial x}$ in tidal rivers is usually small when compared with the other terms of the equation [3]. For the Fraser River the order of magnitude of the convective term is approximately one tenth of the other terms and has therefore been neglected.

The friction slope, S_f, has been represented by most investigators as either Manning's or Chezy's empirical formula in problems of this type:

$$S_f = \left(\frac{n}{1.49}\right)^2 \frac{u|u|}{R^{4/3}} \qquad \text{Manning}$$

$$S_f = \frac{u|u|}{C^2 R} \qquad \text{Chezy}$$

where

n is the Manning coefficient

R is the hydraulic radius

C is the Chezy coefficient.

For this model Chezy's equation was used.

The hydraulic radius, R, which is difficult to determine for a river was replaced by the hydraulic depth, d.

Expressing equation (2) in terms of Q:

$$\frac{1}{gA}\frac{\partial Q}{\partial t} - \frac{Q}{gA^2}\frac{\partial A}{\partial t} + \frac{\partial h}{\partial x} + \frac{Q\,|Q|}{A^2C^2d} = 0$$

where

$Q = \bar{\bar{u}}\,A$

$\bar{\bar{u}}$ is the average velocity over the cross-section

A is the cross-sectional area

d is the hydraulic depth.

Expanding the term $\frac{\partial A}{\partial t}$:

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial B}\frac{\partial B}{\partial t} + \frac{\partial A}{\partial h}\frac{\partial h}{\partial t} .$$

Neglecting the term $\frac{\partial A}{\partial B}\frac{\partial B}{\partial t}$ with respect to the value of the term $\frac{\partial A}{\partial h}\frac{\partial h}{\partial t}$ and considering the term $\frac{\partial A}{\partial h}$ equal to B when the bank storage is omitted[4]:

$$\frac{1}{gA}\frac{\partial Q}{\partial t} - \frac{QB}{gA^2}\frac{\partial h}{\partial t} + \frac{\partial h}{\partial x} + \frac{Q\,|Q|}{A^2C^2d} = 0 . \qquad (3)$$

3.3 Assumptions

The following assumptions have been made in the development of equations (1) and (3):

1. The effects of wind, Coriolis acceleration, and barometric pressure variations have been neglected.

2. The centrifugal forces in bends and at obstructions have been neglected.

3. The coefficients of velocity distribution have been neglected by equating them to unity.

4. The tributary discharges in the equation of continuity have been assumed constant with time.

5. The local channel slope has been neglected from the equation of motion.

6. The density has been assumed constant by neglecting any salinity intrusion and any suspended sediment.

4.0 MATHEMATICAL MODEL

A direct explicit finite-difference scheme in a rectangular staggered grid was chosen as the method of solution because it is the simplest to program for a river containing several channels and islands. In order to solve the differential equations (1) and (3), the various terms of these equations are replaced by finite-difference quotients thus reducing the differential equations to a system of algebraic equations.

Central differences are used in both the x-direction and the t-direction:

$$f'(x) = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} + e_t \qquad (4)$$

where

e_t is the truncation error.

The centering scheme is shown in Figure 5. It will be noticed that Q is calculated at even grid points in the x-direction and the t-direction, while h is calculated at odd grid points.

Using equation (4), the momentum equation (3) centered at 2m and 2n+1 becomes:

$$\left(\frac{1}{gA}\right)^{2n+1}_{2m} \frac{Q^{2n+2}_{2m} - Q^{2n}_{2m}}{2\Delta t} - \left(\frac{B}{gA^2}\right)^{2n+1}_{2m} Q^{2n+2}_{2m} \left\{\frac{h^{2n+1}_{2m-1} - h^{2n-1}_{2m-1} + h^{2n+1}_{2m+1} - h^{2n-1}_{2m+1}}{4\Delta t}\right.$$

$$\left. + \frac{h^{2n+1}_{2m+1} - h^{2n+1}_{2m-1}}{2\Delta x} + \left(\frac{1}{A^2C^2d}\right)^{2n+1}_{2m} Q^{2n+2}_{2m} \left|Q^{2n}_{2m}\right| \right\} = 0 \qquad (5)$$

where the subscripts refer to the distance steps and the superscripts refer to the time steps.

The non-linear term Q^2 has been linearized by setting:

$$Q^2 = Q^{2n+2}_{2m} \left|Q^{2n}_{2m}\right|$$

Similarly the continuity equation (1) centered at 2m+1 and 2n becomes:

$$\frac{Q^{2n}_{2m+2} - Q^{2n}_{2m}}{2\Delta x} + B^{2n}_{2m+1} \frac{h^{2n+1}_{2m+1} - h^{2n-1}_{2m+1}}{2\Delta t} - \frac{Q_T}{2\Delta x}{}_{2m+1} = 0 \qquad (6)$$

Rearranging equation (5) to give an expression for Q^{2n+2}_{2m} :

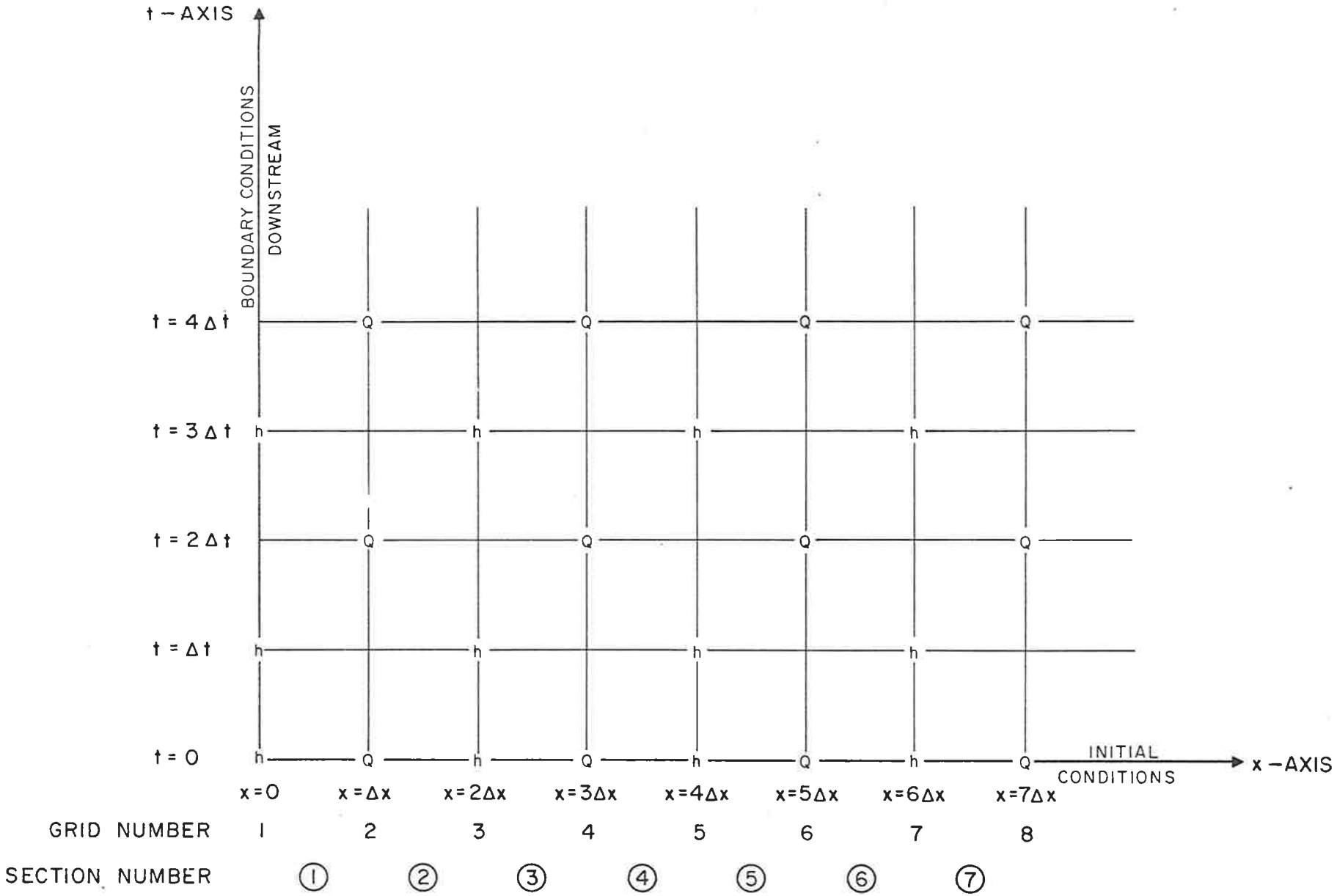


FIG. 5 CENTERING SCHEME

Since the channel dimensions of the Lower Fraser River are relatively constant a value of 6,000 feet for Δx was used throughout the model except for the two sections of Douglas Pass where 4,200 feet was used due to the short length of the channel.

Next, the channel cross-sections had to be schematized so that the data required for a solution to equations (7) and (8) could be represented in a simple mathematical form in the computer program (Fig. 12). The schematization adopted is illustrated in Figure 13.

At each grid the following characteristic dimensions were specified:

A_{CD} is the cross-sectional area below chart datum

B_{CD} is the width at chart datum

B_T is the total width

d_T is the maximum elevation of the bank areas

CD is the depth to chart datum from G.S.C.

To make these dimensions representative of the reach of river considered in each calculation, they were weighted as follows:

$$A_{CDx} = 0.25\ A_{CDx-1} + 0.50\ A_{CDx} + 0.25\ A_{CDx+1}$$

$$B_{CDx} = 0.25\ B_{CDx-1} + 0.50\ B_{CDx} + 0.25\ B_{CDx+1}$$

$$B_{Tx} = 0.25\ B_{Tx-1} + 0.50\ B_{Tx} + 0.25\ B_{Tx+1}$$

$$d_{Tx} = 0.25\ d_{Tx-1} + 0.50\ d_{Tx} + 0.25\ d_{Tx+1}$$

$$CD_x = 0.25\ CD_{x-1} + 0.50\ CD_x + 0.25\ CD_{x+1}$$

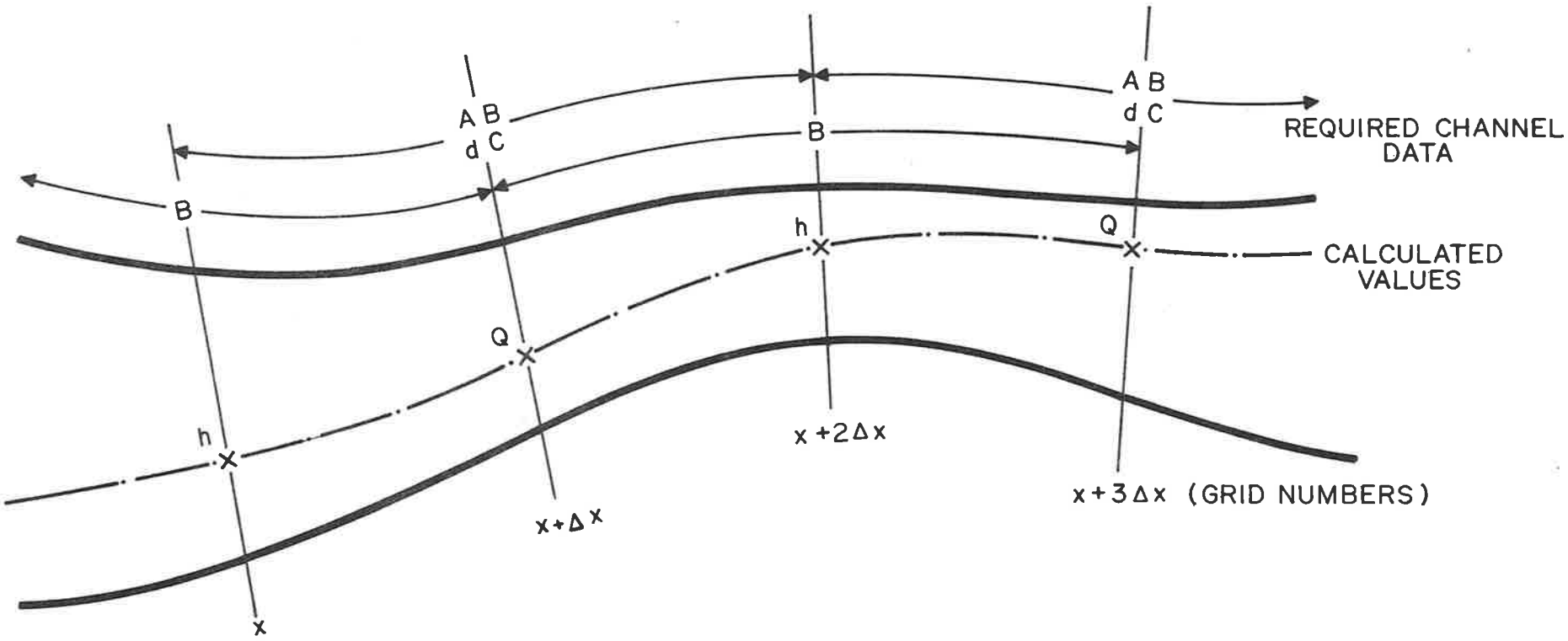


FIG.12 CHANNEL DATA

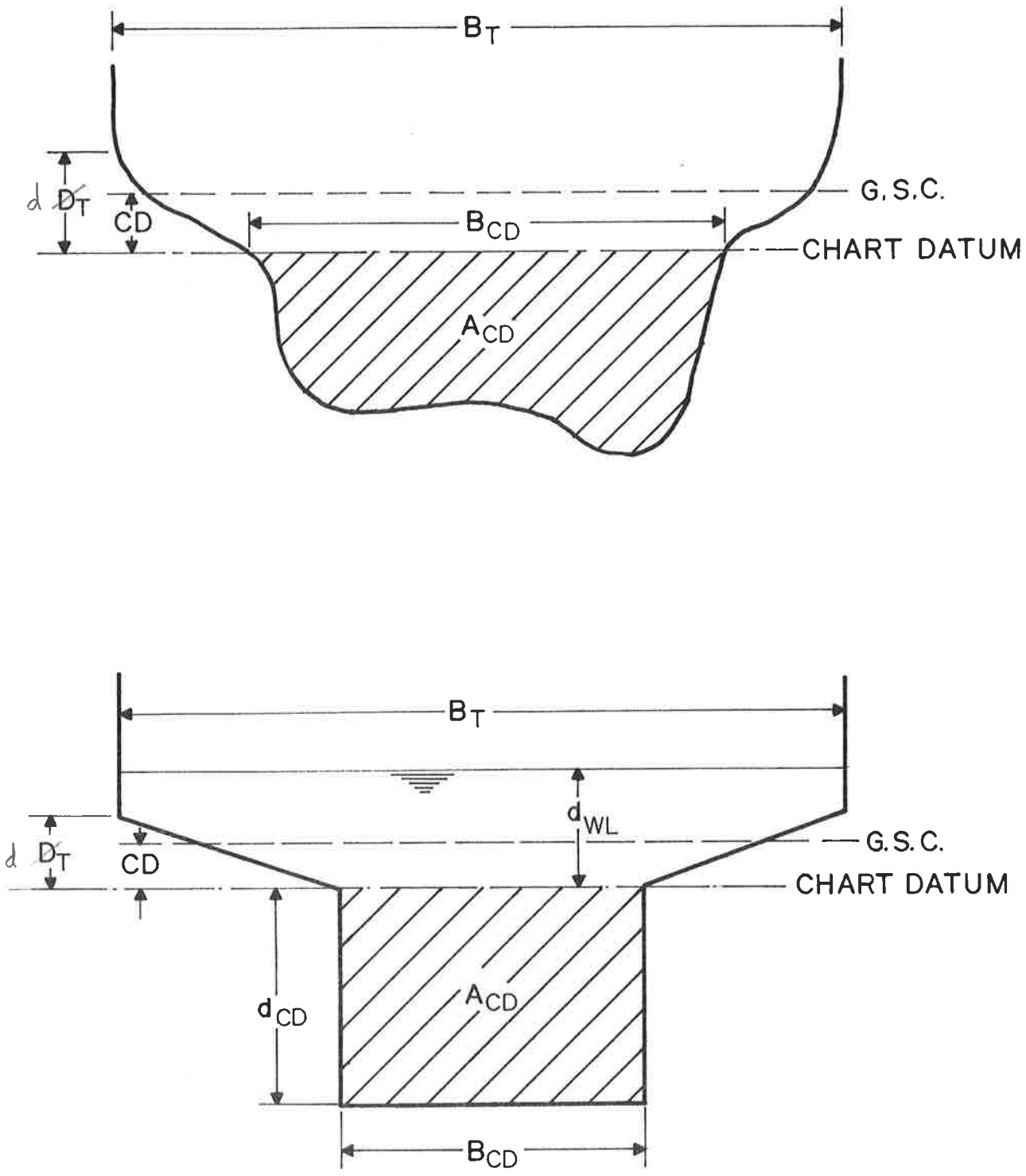


FIG.13 SCHEMATIZED CROSS-SECTION

Two other dimensions were then defined:

d_{CD} is the average depth below chart datum

$$d_{CD} = \frac{A_{CD}}{B_{CD}}$$

d_{WL} is the depth of water above chart datum.

Chart datum was taken between one and two feet below the low water level of August 11-12, 1961. This level corresponds closely to the chart datum elevation given between Sand Heads and Douglas Island on the Canadian Hydrographic Charts.

The schematization can be further qualified by dividing the channel into conveying and storage sections. Three possible approaches exist:

1. The whole channel conveys.
2. Partial storage over the berms.
3. Complete storage over the berms.

All three schemes were tested on the model. The one chosen as the most accurate representation considered the whole channel to convey.

The required data was obtained by using the following formulae:

For $d_{WL} \leqslant$ chart datum

$$B = B_{CD}$$

$$A = A_{CD} + B_{CD} d_{WL}$$

$$d = d_{CD} + d_{WL}$$

For $d_{WL} < d_T$

$$B = B_{CD} + (B_T - B_{CD}) \frac{d_{WL}}{d_T}$$

$$A = A_{CD} + B_{CD} d_{WL} + (B_T - B_{CD}) \frac{d_{WL}^2}{2 d_T}$$

$$d = d_{CD} + d_{WL}$$

For $d_{WL} \geq d_T$

$$B = B_{CD} + (B_T - B_{CD})$$

$$A = A_{CD} + B_{CD} d_{WL} + (B_T - B_{CD}) \left(d_{WL} - \frac{d_T}{2}\right)$$

$$d = d_{CD} + d_{WL}$$

The quantities obtained from these formulae represent the average values over the appropriate section of the river under consideration for either an h or Q calculation.

5.2 Tidal Data

The water elevations throughout the river are recorded at numerous automatic gauging stations. The gauges used in this study are shown in Figure 2. The information from these gauges was available in the form of continuous water level tracings. Hourly readings were taken from these curves and converted to the G.S.C. datum.

6.0 DEVELOPMENT OF THE MODEL

6.1 Time Increment

A time increment of one half a degree of tide or 124.2 seconds was used and the model was stable.

6.2 Initial Conditions

At t = 0 it was necessary to define the initial elevation and discharge at alternate grids as shown in Figure 5. For the first run, a water surface profile along the length of the river was constructed by consulting the gauge records at t = 0 and from this interpolating the elevation at the required sections. A discharge profile was similarly constructed from the results of the hydraulic model. To ensure cyclic convergence from these initial conditions the diurnal tide at Sand Heads was repeated four times. In subsequent runs for the same tide where only small modifications were made to the data, the initial conditions were obtained from the end conditions of the previous run. Convergence could then be obtained after two cycles.

If the model were to be used for forecasting purposes the same procedure would be followed. Some approximation would have to be made for the water levels and discharges. The discharges could initially be set to zero. These assumptions would be satisfactory provided an adequate number of cycles were used to achieve convergence.

6.3 Friction Factor

The Chezy coefficients were originally obtained from a logarithmic formula in which C is a function of the channel depth and roughness:

$$C = 32.7 \log_{10} \frac{12d}{a}$$

where

d is the appropriate depth

a is the actual bottom roughness

The values of bottom roughness were estimated from surveys of the bed sediment sizes conducted on the river[8]. Each value represents the bottom roughness throughout a particular section.

This method proved unsatisfactory for two reasons:

1. The values of bottom roughness had to be varied by a factor of up to 500 to obtain the initial calibration.
2. The variations in C due to changes in the depth were small.

Therefore, the mean value of C obtained from the initial runs was used for subsequent runs.

6.4 Output

The main output from the model consisted of plots of the water level, discharge, and velocity as a function of time for the fourteen stations along the river. The time period plotted corresponded to the last tidal cycle at Sand Heads. Samples of these plots are shown in Figures 14, 15 and 16.

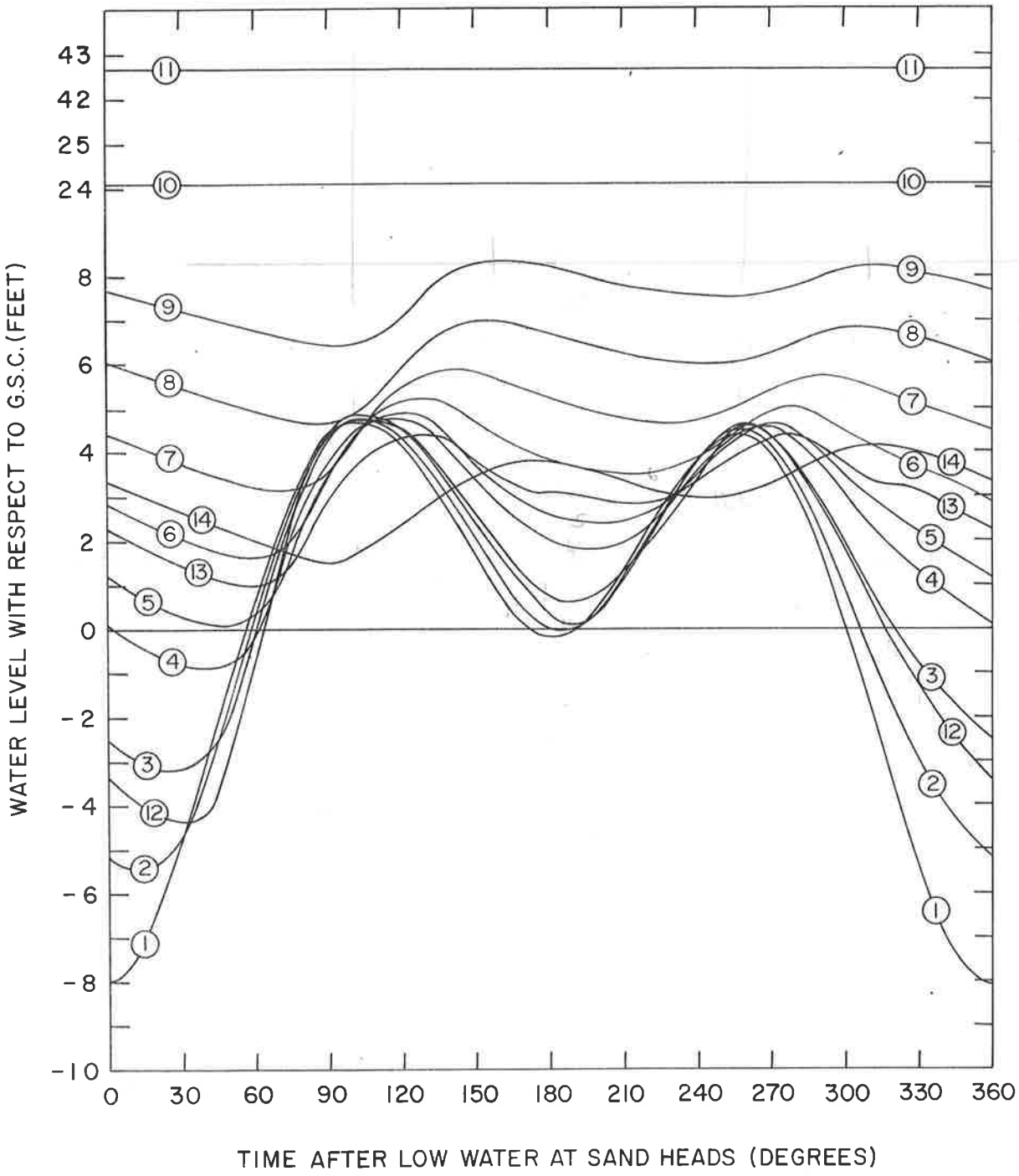
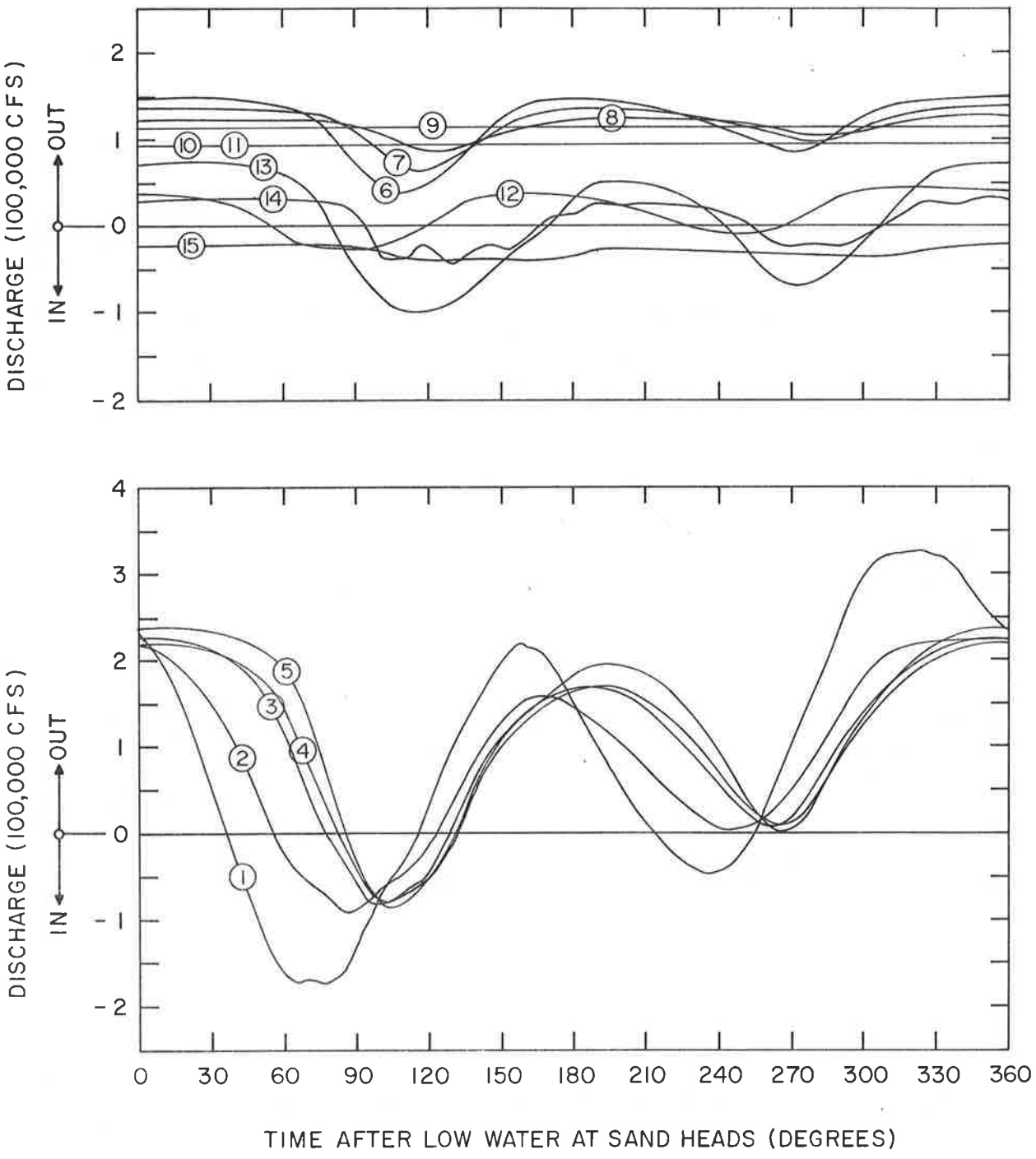
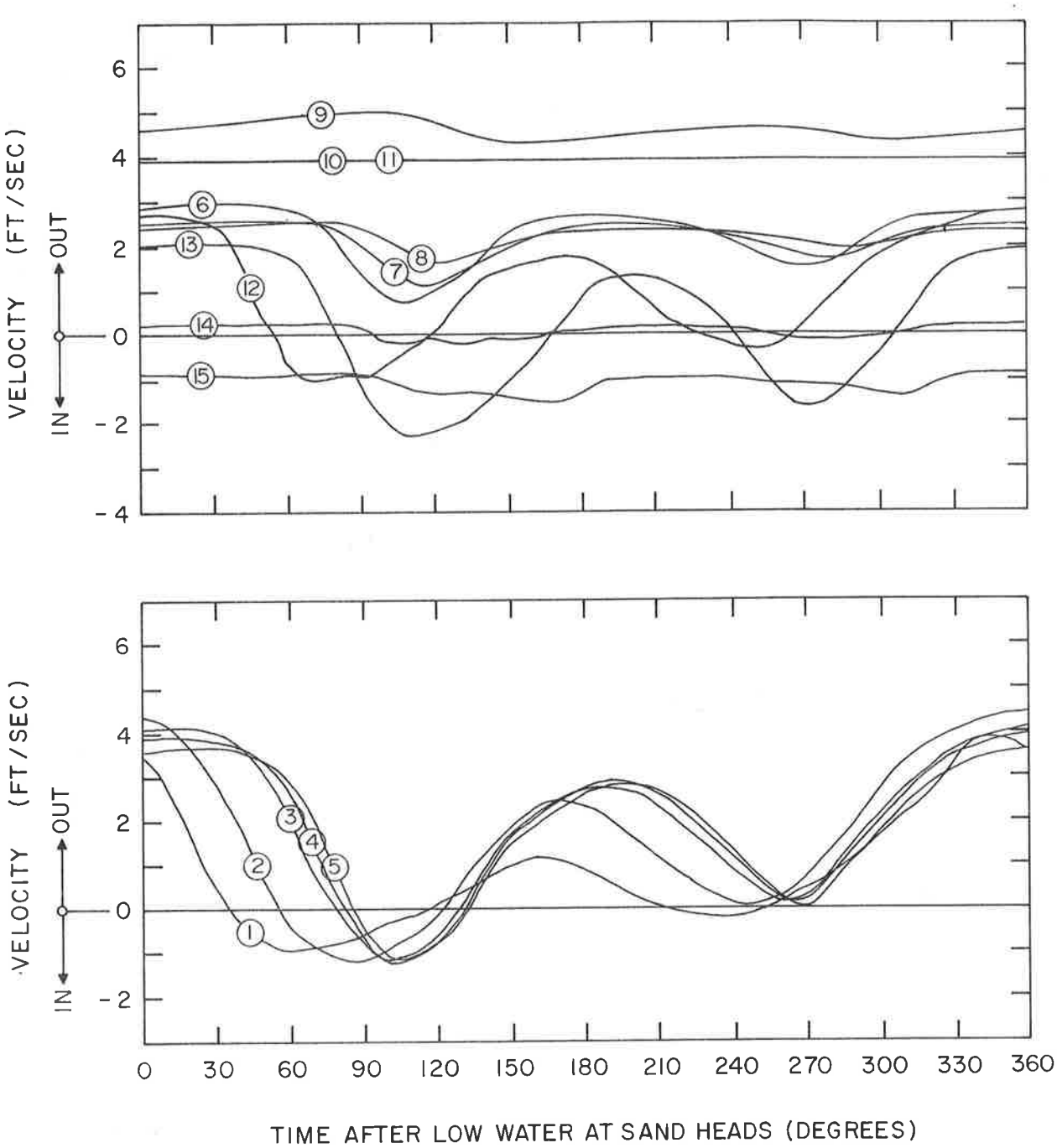


FIG. 14 OUTPUT PLOT OF WATER LEVELS



①	SAND HEADS	⑧	MISSION
②	STEVESTON	⑨	SARDIS
③	BURR LANDING	⑩	HARRISON MILLS
④	NEW WESTMINSTER	⑪	AGASSIZ
⑤	PORT MANN	⑫	VANCOUVER
⑥	PORT HAMMOND	⑬	PORT COQUITLAM
⑦	WHONOCK	⑭	LITTLE GOOSE ISLAND
		⑮	DOUGLAS PASS

FIG. 15 OUTPUT PLOT OF DISCHARGES



1. SAND HEADS
2. STEVESTON
3. BURR LANDING
4. NEW WESTMINSTER
5. PORT MANN
6. PORT HAMMOND
7. WHONOCK
8. MISSION
9. SARDIS
10. HARRISON MILLS
11. AGASSIZ
12. VANCOUVER
13. PORT COQUITLAM
14. LITTLE GOOSE ISLAND
15. DOUGLAS PASS

FIG.16 OUTPUT PLOT OF VELOCITIES

7.0 CALIBRATION OF THE MODEL

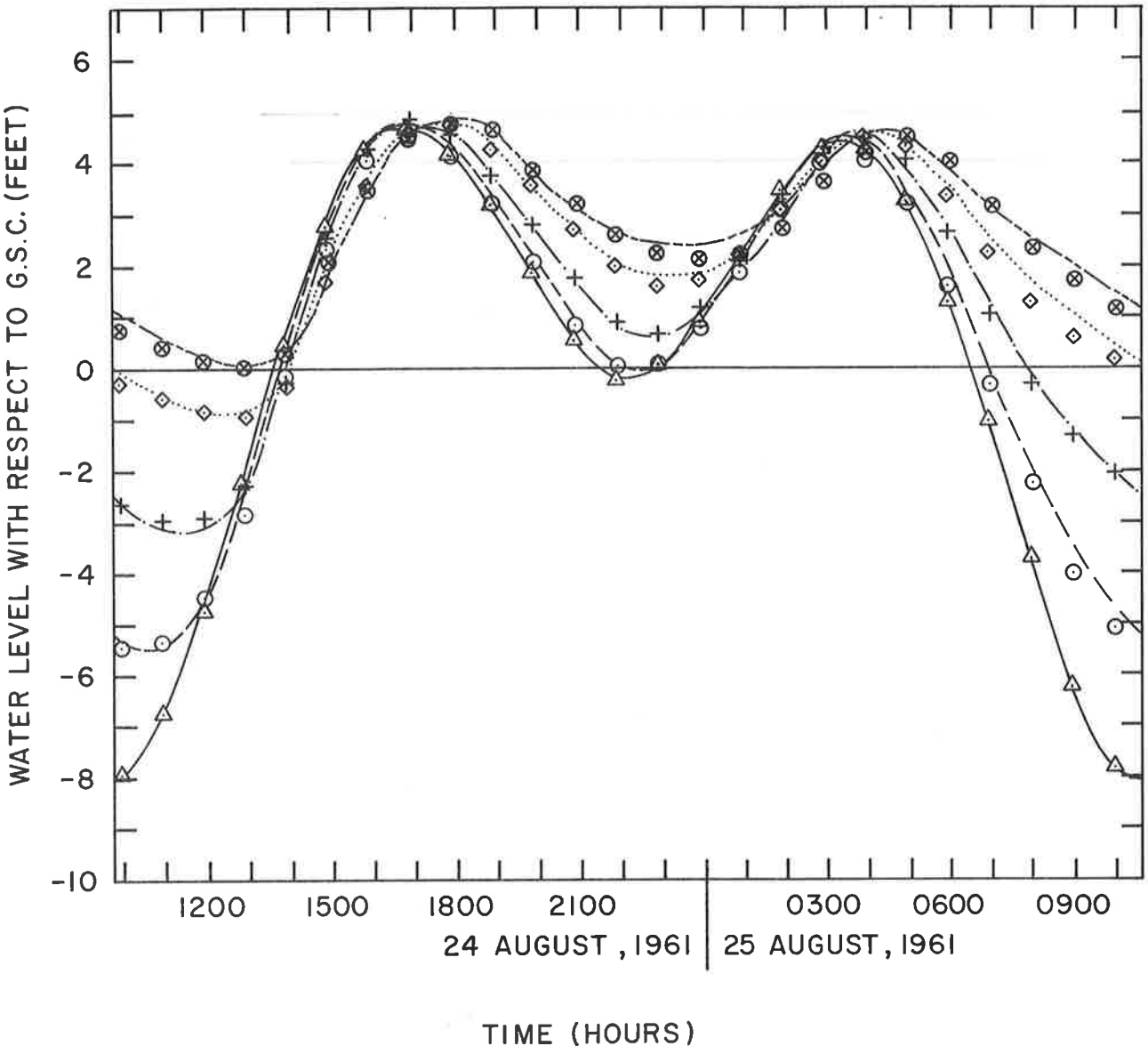
During the initial computer runs of the model, the program was debugged and checked for convergence and stability. The result was a working model of the Lower Fraser River. However, the model would not necessarily reproduce the water levels and discharges in the prototype.

The model had to be calibrated to the prototype. The boundary conditions used were those of August 24-25, 1961 for an average discharge at Hope of 93,370 c.f.s. The calibration involved the evaluation of several schematization schemes and the adjustment of the friction coefficients to reproduce the water elevations at the gauging stations of the prototype. The calibration results are shown in Figure 17.

8.0 ACCEPTANCE OF THE MODEL

Although the model parameters were optimized to reproduce a set of prototype tidal elevations, the model may still not have any physical correspondence with the natural river. Further comparisons between mathematical model and the prototype should include:

1. A comparison of the discharges or velocities along the river.
2. A comparison of the friction coefficients.
3. An examination of an independent tide using the same calibration.



STATION	COMPUTED	RECORDED
SAND HEADS	———	△
STEVESTON	– – – – –	⊙
BURR LANDING	—·—·—	+
NEW WESTMINSTER	·····	◇
PORT MANN	—— – ——	⊗

FIG. 17(A) CALIBRATION FOR AUGUST 24-25, 1961

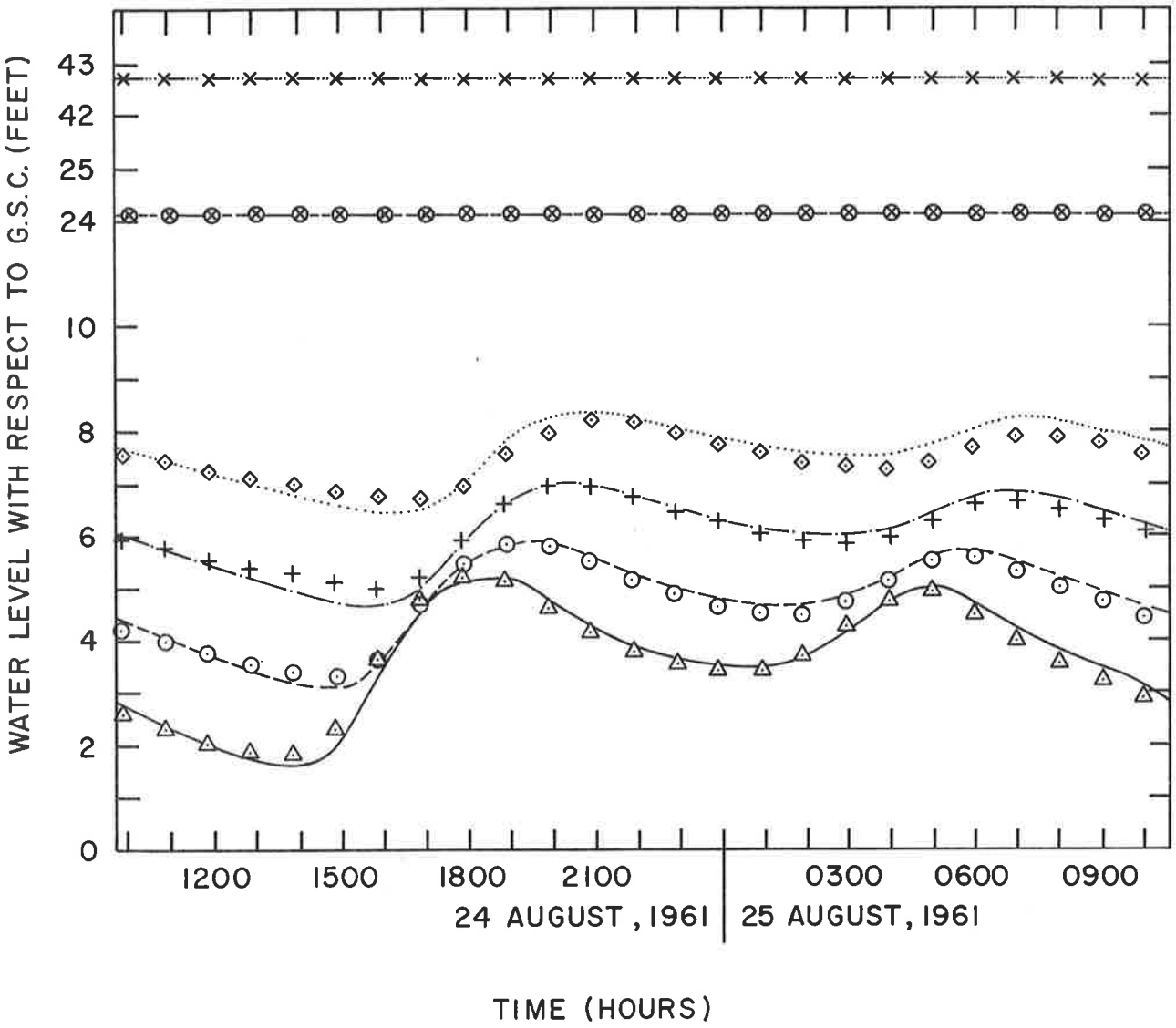
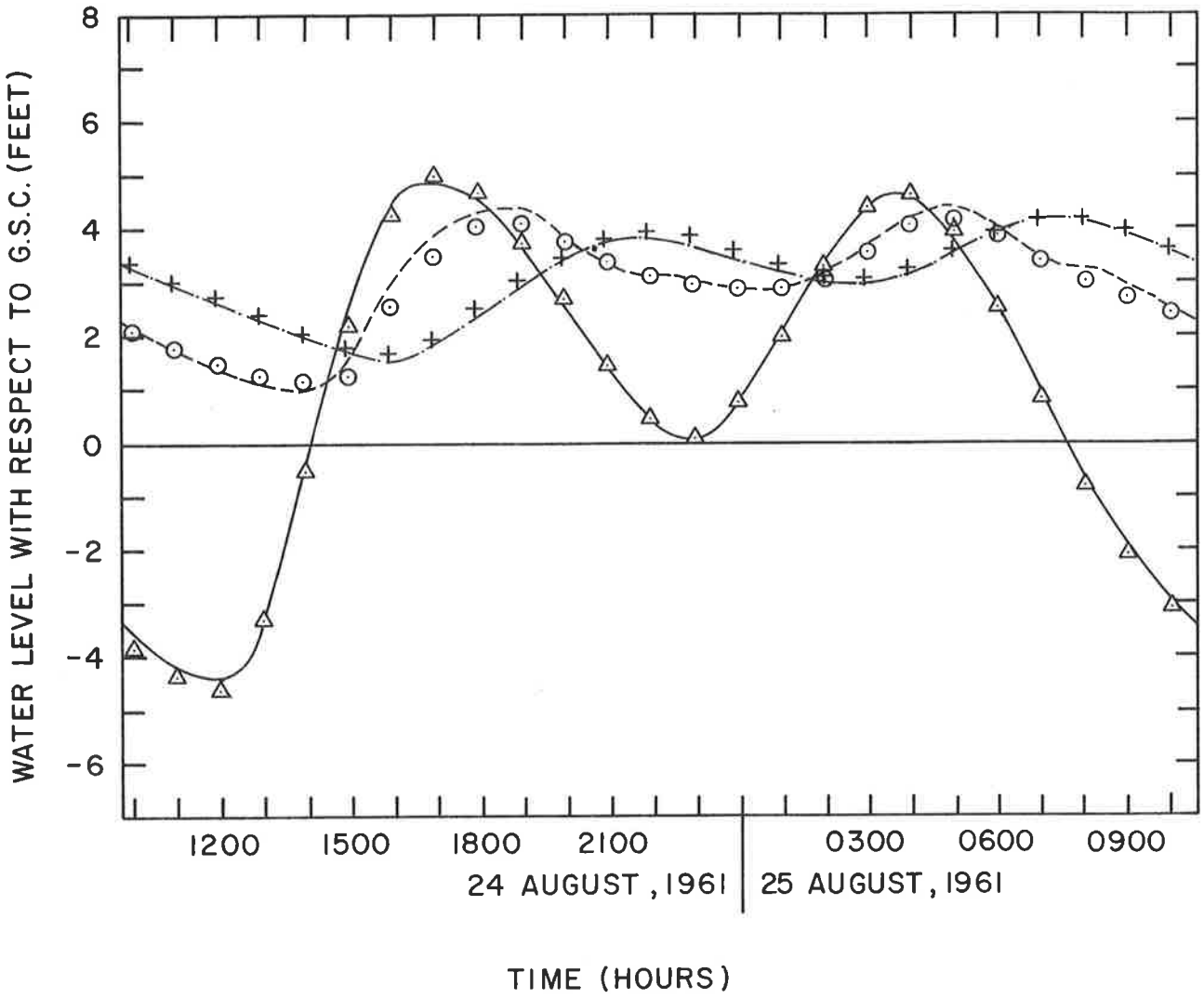


FIG. 17 (B) CALIBRATION FOR AUGUST 24-25, 1961



STATION	COMPUTED	RECORDED
VANCOUVER	———	△
PORT COQUITLAM	– – – – –	⊙
LITTLE GOOSE ISLAND *	—·—·—	+

* ELEVATION OF ZERO GAUGE UNKNOWN

FIG. 17 (C) CALIBRATION FOR AUGUST 24-25, 1961

Since no discharge records were available the horizontal tide could not be verified.

A comparison between the friction coefficients used in the calibration of the model and the values estimated for the natural river is useful because it may reflect the accuracy of the finite-difference method and the schematization used in the model. The values of the Chezy coefficients used in the model are shown in Figure 18. All these values would normally be expected in a river such as the Fraser. The actual magnitude of the Chezy coefficients in the natural river is difficult to determine. However, their relative magnitude is indicated by the bottom roughness of the river bed. A plot of the bed sediment sizes along the river is shown in Figure 18. The close agreement between the two curves is a good indication that the model and the prototype will behave in a similar fashion.

If the model were now run using a different tide but approximately the same discharge, the results would probably be quite accurate. A more demanding test is to run the model for a spring discharge. On the physical model problems were encountered at the higher discharges. It was found necessary to use three different roughness patterns:

1. 0 to 150,000 c.f.s.
2. 150,000 c.f.s. to 300,000 c.f.s.
3. 300,000 c.f.s. and up.

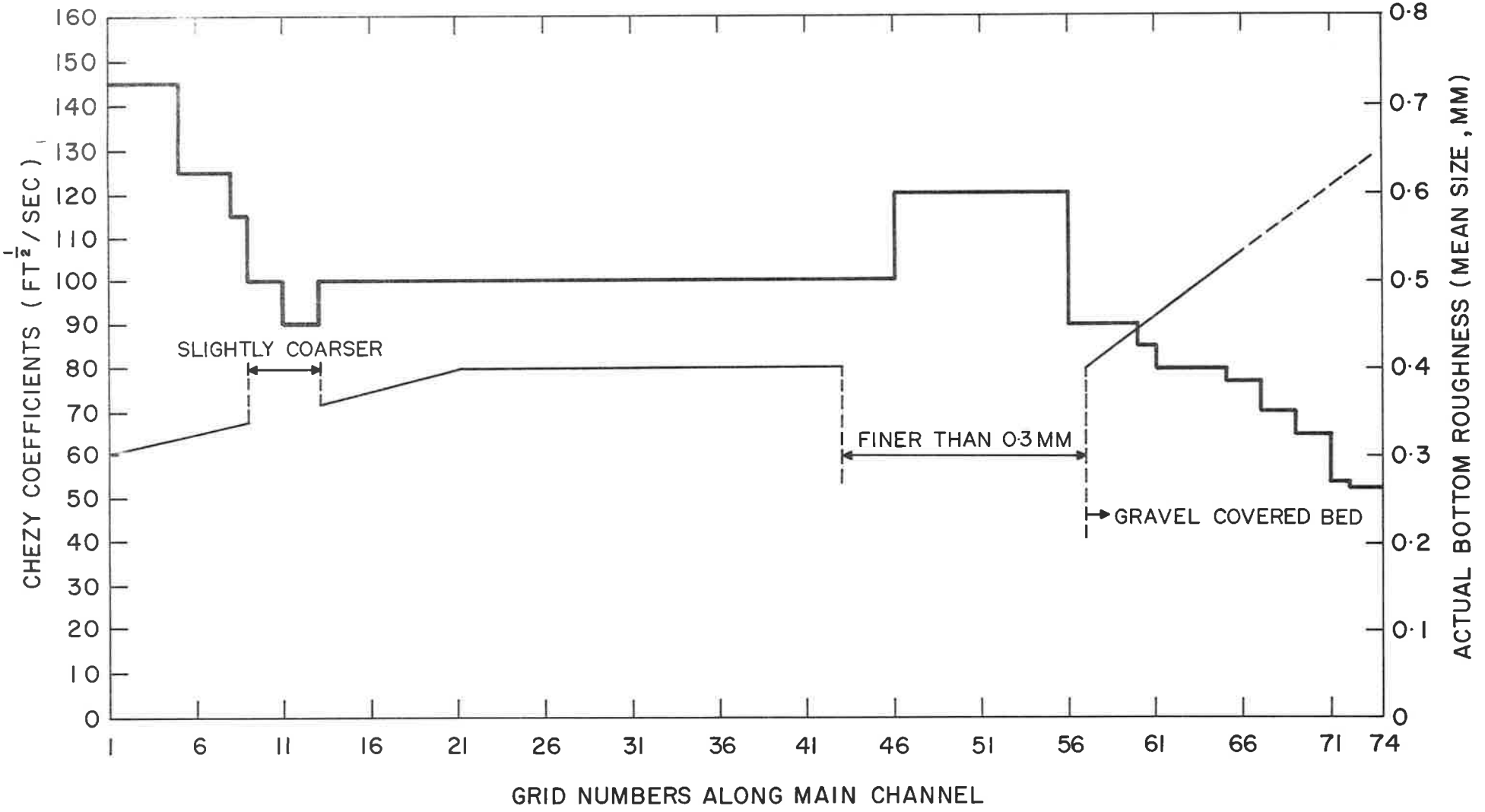
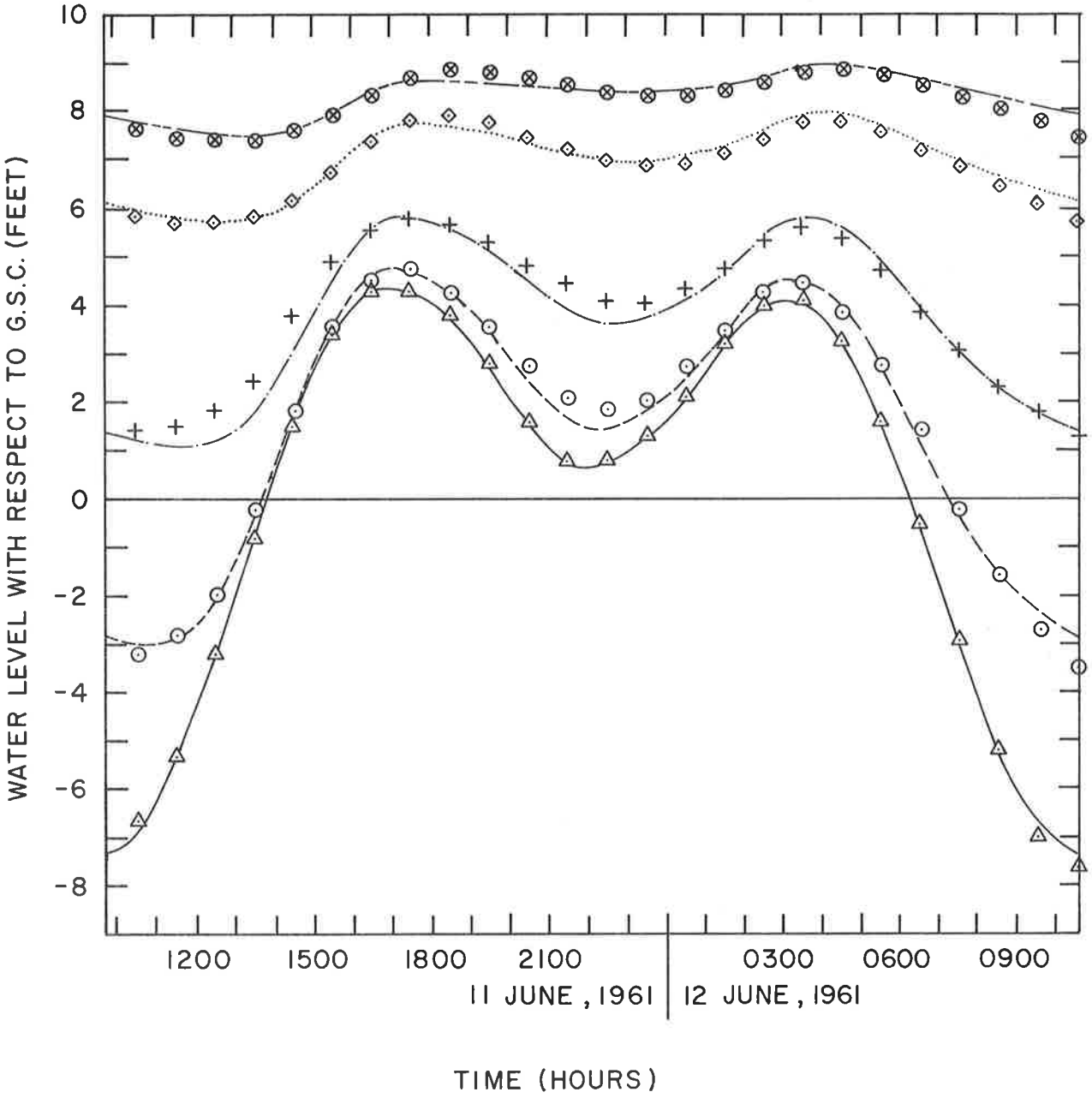


FIG.18 CHEZY COEFFICIENTS AND ACTUAL BOTTOM ROUGHNESS

The difficulty results from the dependence of the resistance coefficient, C, upon the discharge. In the tidal reach of the river, variations in C with discharge are relatively small compared with those occurring in the upper reaches less affected by the tidal motion. Here, increasing discharges bring about changes in the bottom roughness and configuration resulting in changes in the resistance coefficients which could not be represented in the physical model where roughness strips inserted in the bed were used to simulate the resistance of the channel.

For the mathematical model, the independent evaluation was therefore made using a moderate freshet discharge of 315,000 c.f.s. which occurred on June 11-12, 1961. The results of this run are shown in Figure 19. To interpret these results it must be remembered that other inaccuracies are present besides those inherent errors in the method of solution:

1. Inaccuracies in extracting the channel data from the charts.
2. Inaccuracies in the charts themselves due to surveying errors.
3. Inaccuracies in the charts due to the continuous changes in the channel from deposition, scour, and dredging.
4. Inaccuracies in the tidal records.



STATION	COMPUTED	RECORDED
SAND HEADS	———	△
STEVESTON	– – – – –	⊙
BURR LANDING	—·—·—	+
NEW WESTMINSTER	·····	◇
PORT MANN	——– –	⊗

FIG. 19 (A) RESULTS FOR JUNE 11-12, 1961

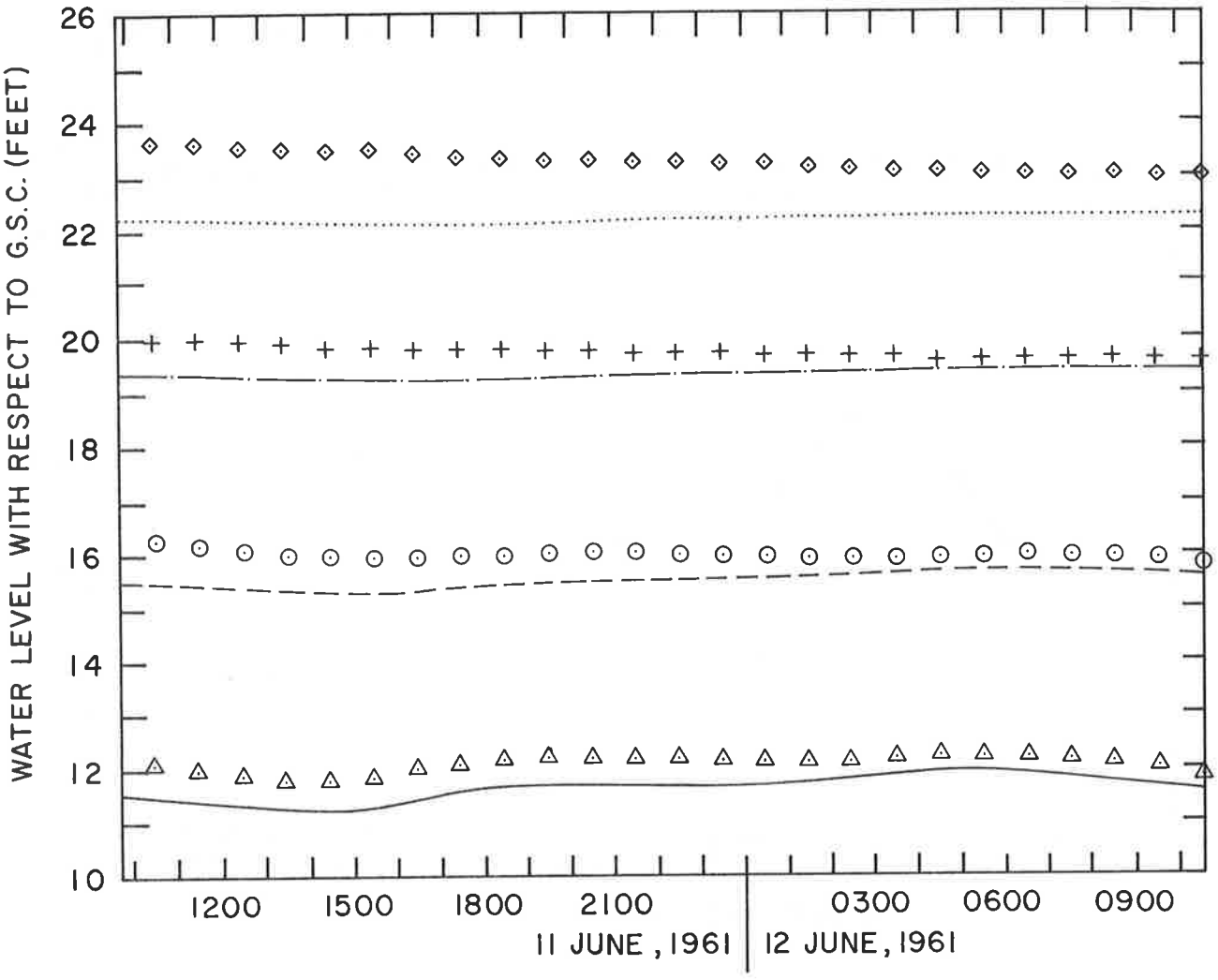
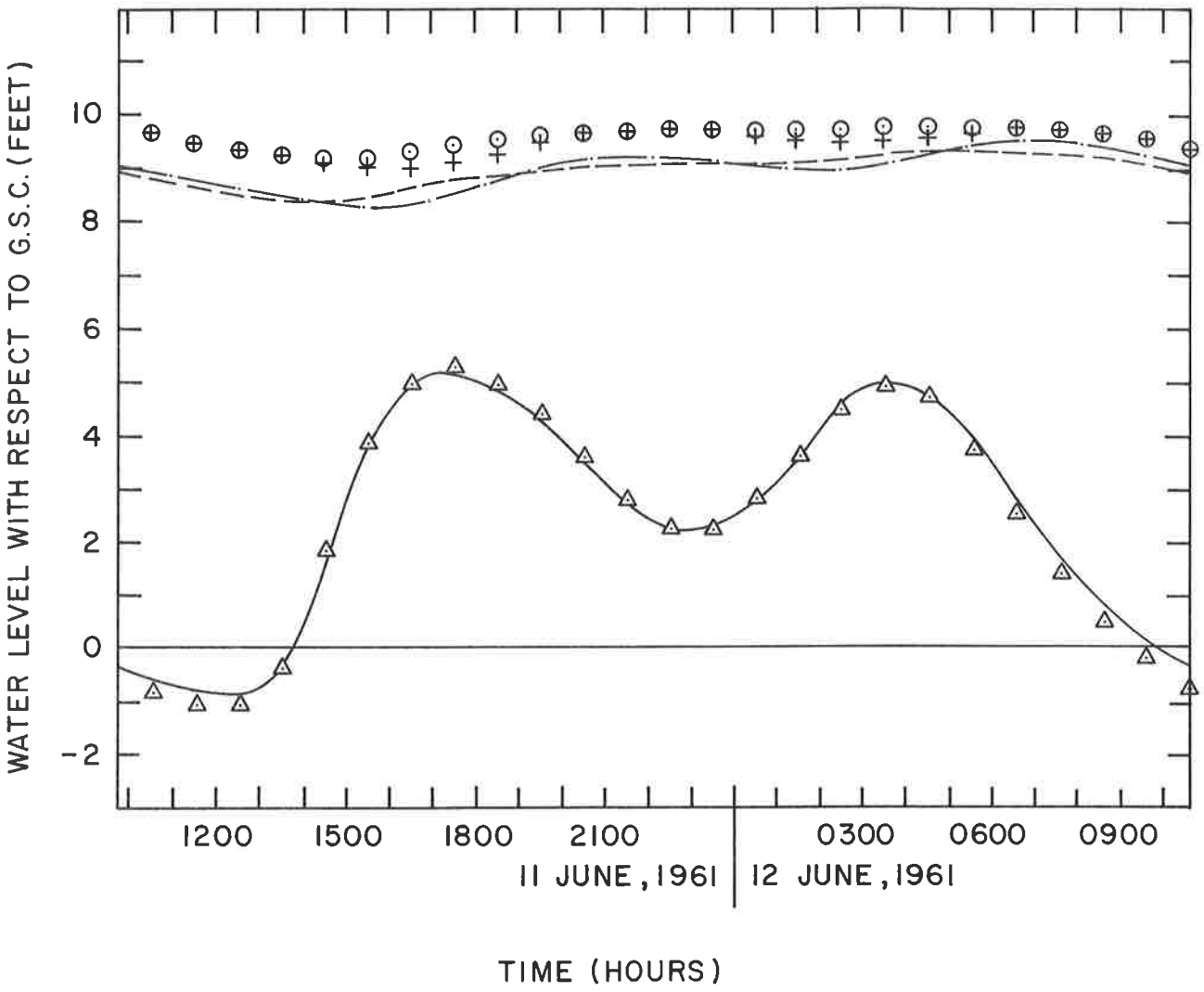


FIG. 19 (B) RESULTS FOR JUNE 11-12, 1961



STATION	COMPUTED	RECORDED
VANCOUVER	———	△
PORT COQUITLAM	- - - - -	⊙
LITTLE GOOSE ISLAND *	—·—·—	+

* ELEVATION OF ZERO GAUGE UNKNOWN

FIG. 19 (C) RESULTS FOR JUNE 11-12, 1961

5. Inaccuracies due to the position of the gauges in the schematization pattern. The gauge may have to be represented as much as 6,000 feet from its actual location.

6. Inaccuracies due to the position of the gauges in the river. Tidal gauges located on the banks of a river record variations in phase and velocity from the main stream and are subject to superelevation effects if located on bends of the river.

The model's response is considered satisfactory in reproducing the tidal motion in the river.

9.0 BOUNDARY BAY DIVERSION SCHEME

9.1 Outline of Proposal

A proposal for the improvement of navigation conditions and the alleviation of flood danger in the Lower Fraser River, the Boundary Bay Diversion Scheme, was previously investigated on a fixed-bed hydraulic model constructed at the National Research Council[2]. The same scheme was incorporated into the mathematical model so that the results of the two models could be compared.

The scheme consists of a diversion channel connecting the Main Arm to Boundary Bay as illustrated in Figure 20. The channel studied was rectangular; 1000 feet wide by 35 feet deep.

The scheme was easily included into the mathematical

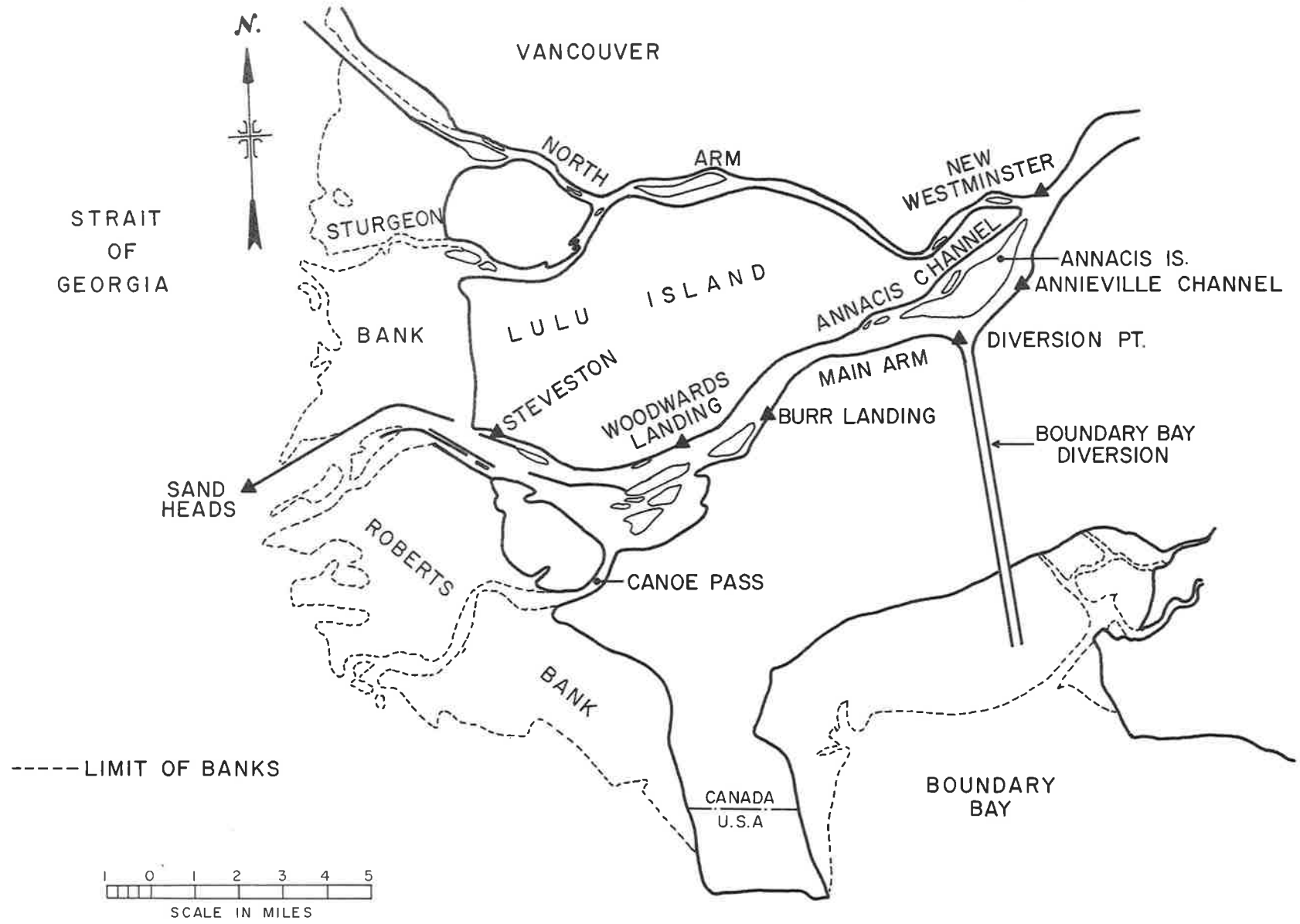


FIG. 20 BOUNDARY BAY DIVERSION SCHEME

model as another branch of the river. The amount of extra programming necessary was minimal (Fig. 10). The boundary condition imposed at the channel entrance was taken as the water surface elevation at Sand Heads. The initial water surface elevations and discharges along the channel were set equal to zero. The program was run for four cycles to achieve cyclic convergence from these initial conditions.

Tests were run, for the existing prototype conditions and for the conditions that would exist after the implementation of the Boundary Bay Diversion Scheme, to correspond to the tests run on the hydraulic model. Discharges at Hope of 100,000 c.f.s., 250,000 c.f.s., 420,000 c.f.s., and 536,000 c.f.s. were used.

9.2 Comparison of Results

The effect of the Boundary Bay Diversion Scheme on the tidal propagation in the Lower Fraser River is shown in Figures 21 and 22.

Since the channel dimensions and slope were fixed, the frictional parameter was critical in determining the flow through the channel. The mathematical results illustrated were obtained using a Chezy coefficient of 160. This value corresponds to the minimum Manning coefficient that would be expected in a straight uniform earth channel[9]. To reproduce the results of the hydraulic model, a smoother channel would have to be used.

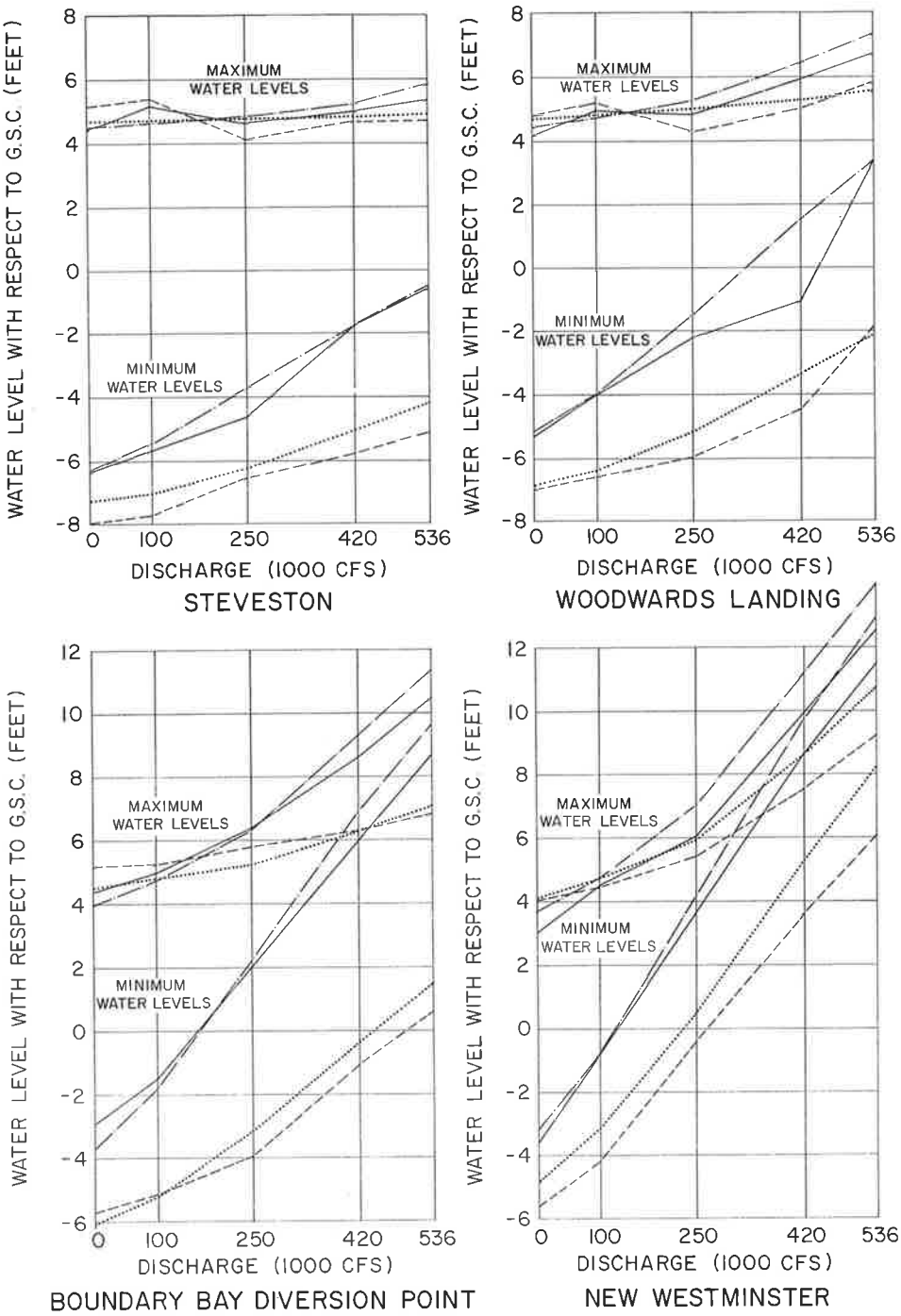
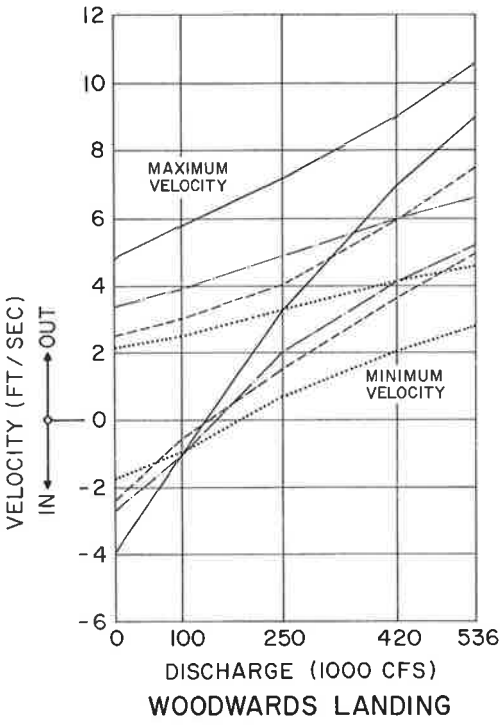
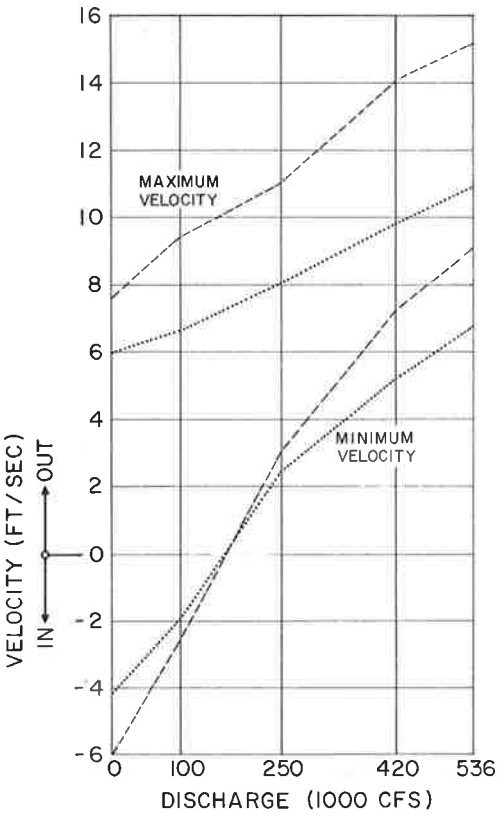


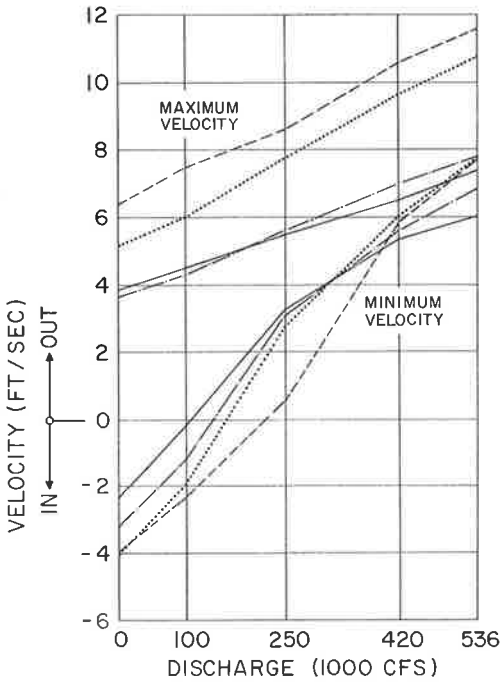
FIG. 21 CHANGES IN WATER LEVELS DUE TO BOUNDARY BAY DIVERSION



WOODWARDS LANDING



BOUNDARY BAY DIVERSION CHANNEL



ANNIEVILLE CHANNEL

HYDRAULIC MODEL
- NATURAL
- BOUNDARY BAY DIVERSION IN OPERATION

MATHEMATICAL MODEL
- NATURAL
- BOUNDARY BAY DIVERSION IN OPERATION

FIG. 22 CHANGES IN VELOCITIES DUE TO BOUNDARY BAY DIVERSION

This points up one of the main advantages of a mathematical model over a hydraulic model; the ability to investigate in detail the contribution made by one parameter over the expected natural range with a minimum of effort. The Boundary Bay Diversion Scheme could be tested for Chezy values from 80 to 160 and depending on the type of construction used for the channel and the material present the prototype response would be selected from the family of curves produced. A similar investigation on a hydraulic model would be quite difficult.

However, it must be remembered that both the fixed-bed hydraulic model and the mathematical model only reproduce the results that would occur when the diversion was first put into operation. Scour and deposition would alter the existing channels until a new regime was established.

10.0 CONCLUSIONS

The mathematical model formulated was found to reproduce accurately the tidal elevations throughout the Lower Fraser River. The accuracy is considered sufficient for design purposes which makes the mathematical model a valuable aid in the analysis of the flow regime in the river under wide variations in the rate of discharge and to assess the effectiveness and feasibility of large-scale engineering works in the tidal reach of the river.

The mathematical model therefore represents an alternative to a large hydraulic model. The mathematical model can:

1. Establish the overall flow regime and identify key areas.
2. Supply boundary conditions to a large-scale mathematical or hydraulic model of a key area.
3. Give direction in planning the necessary field studies.
4. Separate and evaluate the contributions made by the individual natural forces in the river.

The result is a considerable saving in time and money.

APPENDIX I

Notation

A	Cross-sectional area.
A_{CD}	Cross-sectional area below chart datum.
a	Actual bottom roughness.
B	Water surface width.
B_{CD}	Channel width at chart datum.
B_T	Total channel width.
C	Chezy coefficient.
CD	Depth to chart datum from G.S.C.
C_o	Phase velocity of the tide.
c	Wave celerity.
d	Hydraulic depth.
d_{CD}	Average depth below chart datum.
d_{max}	Largest hydraulic depth occurring in the river.
d_T	Maximum elevation of the bank areas.
d_{WL}	Depth of water above chart datum.
e_T	Truncation error.
G.S.C.	Canadian Geodetic Datum.
g	Acceleration due to gravity.
h	Water surface elevation above a specific datum (G.S.C.)
n	Manning coefficient.
Q	Discharge

Q_{head}	Discharge at the end of Pitt Lake.
Q_T	Tributary discharge.
R	Hydraulic radius.
S_f	Friction slope.
t	Time.
u	Velocity in the x-direction.
$\bar{u}$	Average velocity over the vertical in the x-direction.
$\bar{\bar{u}}$	Average velocity over the cross-section in the x-direction.
x	Distance along the channel.
Δx	Distance increment.
Δt	Time increment.

APPENDIX II

References

1. Turner, E.S., "Fraser River Model Hydraulics Laboratory," Feature Article, NRC, ME 1951 (2).

2. "Investigation of Flood and Navigation Conditions in the Lower Fraser River," NRC, DME MH-106, March 1969.

3. Dronkers, J.J., "Tidal Computations for Rivers, Coastal Areas, and Seas," Journal of the Hydraulics Division, PASCE, Vol. 95, No. HY1, January 1969.

4. Dronkers, J.J., "Tidal Calculations," North-Holland Publishing Company, Amsterdam, 1964.

5. Strelkoff, T., "Numerical Solution of Saint-Venant Equations," Journal of the Hydraulics Division, PASCE, Vol. 96, No. HY1, January 1970.

6. Kamphuis, J.W., "Mathematical Model Study of the Propagation of Tides in the St. Lawrence River and Estuary," NRC, DME MH-105, August 1968.

7. Prandle, D., "Computational Methods for Tidal Flow Problems," Ph.D. Thesis, 1969, Victoria University of Manchester.

8. Blench, T., "Hydraulic Solutions for Dredging Problems of the Lower Fraser River," Report to: J.H. Parkin, Director, DME, NRC.

9. Daily, J.W. and Harleman, D.R.F., "Fluid Dynamics," Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1966.

$$Q_{2m}^{2n+2} = \frac{\frac{\Delta x}{\Delta t}\left(\frac{1}{gA}\right)_{2m}^{2n+1} Q_{2m}^{2n} - \left(h_{2m+1}^{2n+1} - h_{2m-1}^{2n+1}\right)}{\frac{\Delta x}{\Delta t}\left(\frac{1}{gA}\right)_{2m}^{2n+1} + 2\Delta x\left(\frac{1}{A^2C^2d}\right)_{2m}^{2n+1}|Q_{2m}^{2n}| - \frac{\Delta x}{2\Delta t}\left(\frac{B}{gA^2}\right)_{2m}^{2n+1}} \cdot$$

$$\cdot \left(h_{2m-1}^{2n+1} - h_{2m-1}^{2n-1} + h_{2m+1}^{2n+1} - h_{2m+1}^{2n-1}\right) \qquad (7)$$

Rearranging equation (6) to give an expression for h_{2m+1}^{2n+1} :

$$h_{2m+1}^{2n+1} = h_{2m+1}^{2n-1} - \left(\frac{1}{B}\right)_{2m+1}^{2n} \frac{\Delta t}{\Delta x}\left(Q_{2m+2}^{2n} - Q_{2m}^{2n} - Q_{T_{2m+1}}\right) \qquad (8)$$

The values used to calculate these two unknowns are shown in Figure 6.

4.1 Convergence and Stability

An explicit finite difference scheme based on the equations of continuity and motion will remain stable provided the Courant stability criterion:

$$\frac{\Delta x}{\Delta t} \geq u + c \qquad (9)$$

where

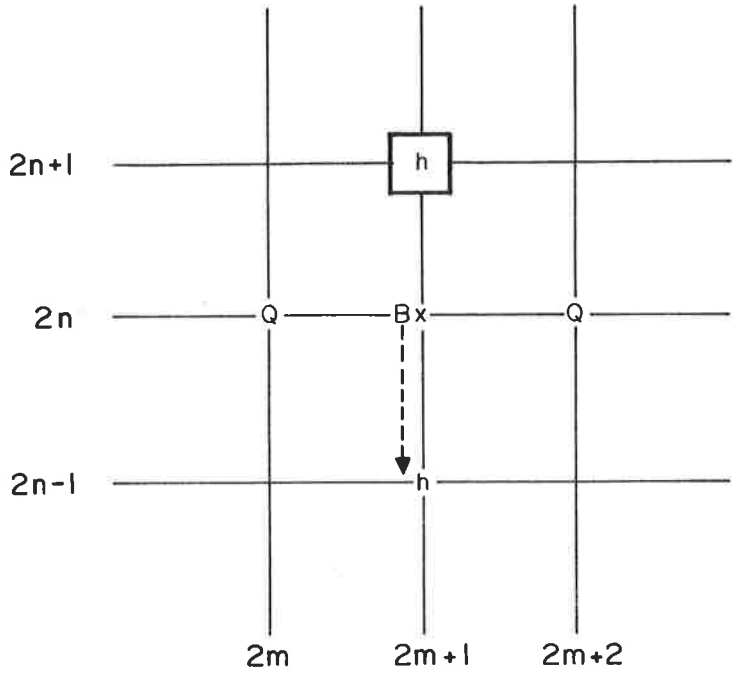
u is velocity in the x-direction

c is wave celerity

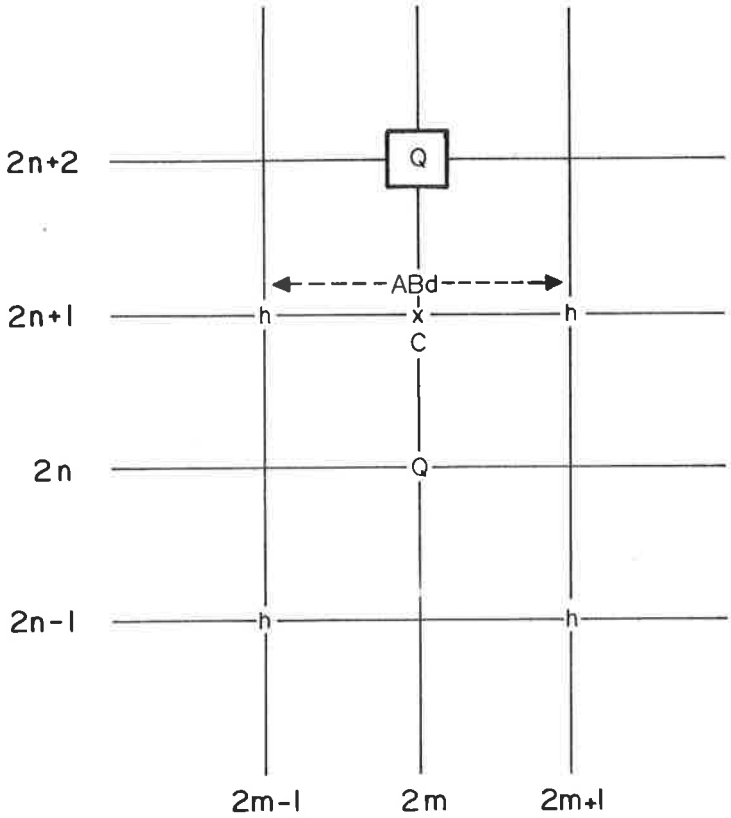
is adhered to and a staggered grid is used[5].

For the model, equation (9) was taken as:

$$\frac{\Delta x}{\Delta t} \geq C_o$$



KNOWN VALUES USED IN CALCULATING h



KNOWN VALUES USED IN CALCULATING Q

x CENTERING POINT

FIG.6 KNOWN VALUES USED IN CALCULATING h AND Q

where

$C_o = \sqrt{gd_{max}}$ is the phase velocity of the tide

d_{max} is the largest hydraulic depth occurring in the river.

4.2 Computation at a Junction

The computation at a junction is based on the two assumptions:

1. $Q_A = Q_B + Q_C$

2. $h_A = h_B = h_C$

where A, B, and C refer to different channels with A being the main branch.

Since h is calculated at odd grid points, by locating an odd grid point at each junction the only modification required will be to equation 8.

Diverging junction (with regard to the direction of computation):

$$h^{2n+1}_{2m+1} = h^{2n-1}_{2m+1} - \left(\frac{1}{B}\right)^{2n}_{2m+1} \frac{\Delta t}{\Delta x} \left(Q^{2n}_{2m+2,B} + Q^{2n}_{2m+2,C} - Q^{2n}_{2m,A}\right) \qquad (10)$$

Converging junction:

$$h^{2n+1}_{2m+1} = h^{2n-1}_{2m+1} - \left(\frac{1}{B}\right)^{2n}_{2m+1} \frac{\Delta t}{\Delta x} \left(Q^{2n}_{2m+2,A} - Q^{2n}_{2m,B} - Q^{2n}_{2m,C}\right) \qquad (11)$$

The term $Q_{T_{2m+1}}$ has been omitted from equations (10) and (11) since there is no tributary discharge at any of the

junctions in the Fraser River. The values used to calculate h_{2m+1}^{2n+1} are shown in Figure 7.

4.3 Computation at Douglas Island

All islands within the Fraser River (excluding the main islands in the delta) have been ignored except for Douglas Island which is located strategically at the junction of the Fraser and Pitt Rivers (Fig. 2). The river has been treated as a single channel at the location of other islands by combining the channel data.

The computation proceeds around Douglas Island as shown in Figure 8 using equations (10) and (11).

4.4 Computation at t = 0

From Figures 5 and 6 it can be seen that the computation centered at t = 0 cannot be based on central differences in time unless staggered initial conditions are used[6]. Forward differences in time were used:

$$f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x} + e_T \qquad (12)$$

Using equation (12), the momentum equation (3) centered at 2m and 2n+1 and solved for Q_{2m}^{2n+2} becomes:

$$Q_{2m}^{2n+2} = \frac{\frac{\Delta x}{\Delta t}\left(\frac{1}{gA}\right)_{2m}^{2n+1} Q_{2m}^{2n} - \left(h_{2m+1}^{2n+1} - h_{2m-1}^{2n+1}\right)}{\frac{\Delta x}{\Delta t}\left(\frac{1}{gA}\right)_{2m}^{2n+1} + 2\Delta x\left(\frac{1}{A^2C^2d}\right)_{2m}^{2n+1} |Q_{2m}^{2n}| - \frac{\Delta x}{\Delta t}\left(\frac{B}{gA^2}\right)_{2m}^{2n+1}} \cdot$$

$$\cdot \left(h_{2m-1}^{2n+1} - h_{2m-1}^{2n-1} + h_{2m+1}^{2n+1} - h_{2m+1}^{2n-1}\right) \qquad (13)$$

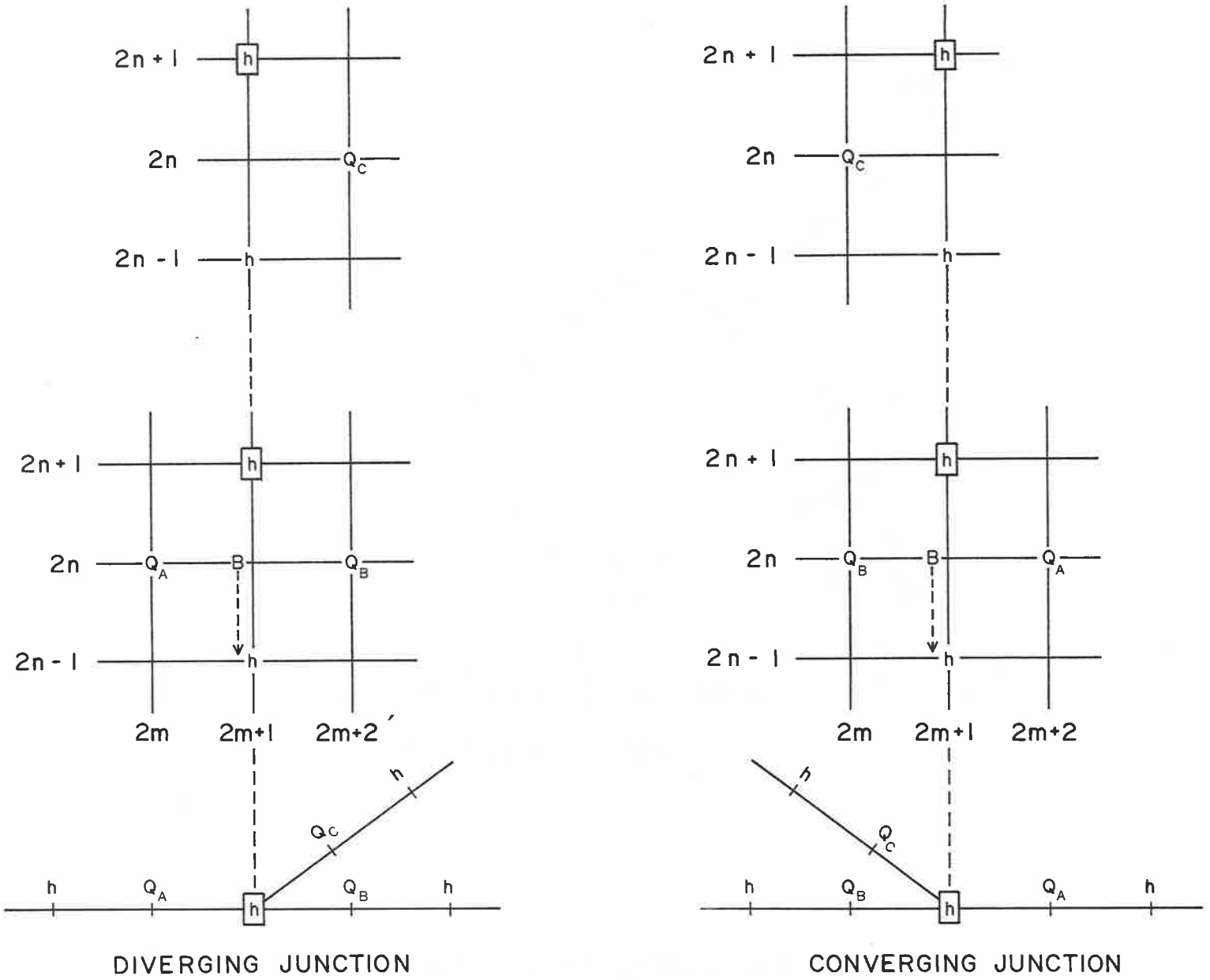


FIG. 7 COMPUTATION AT A JUNCTION

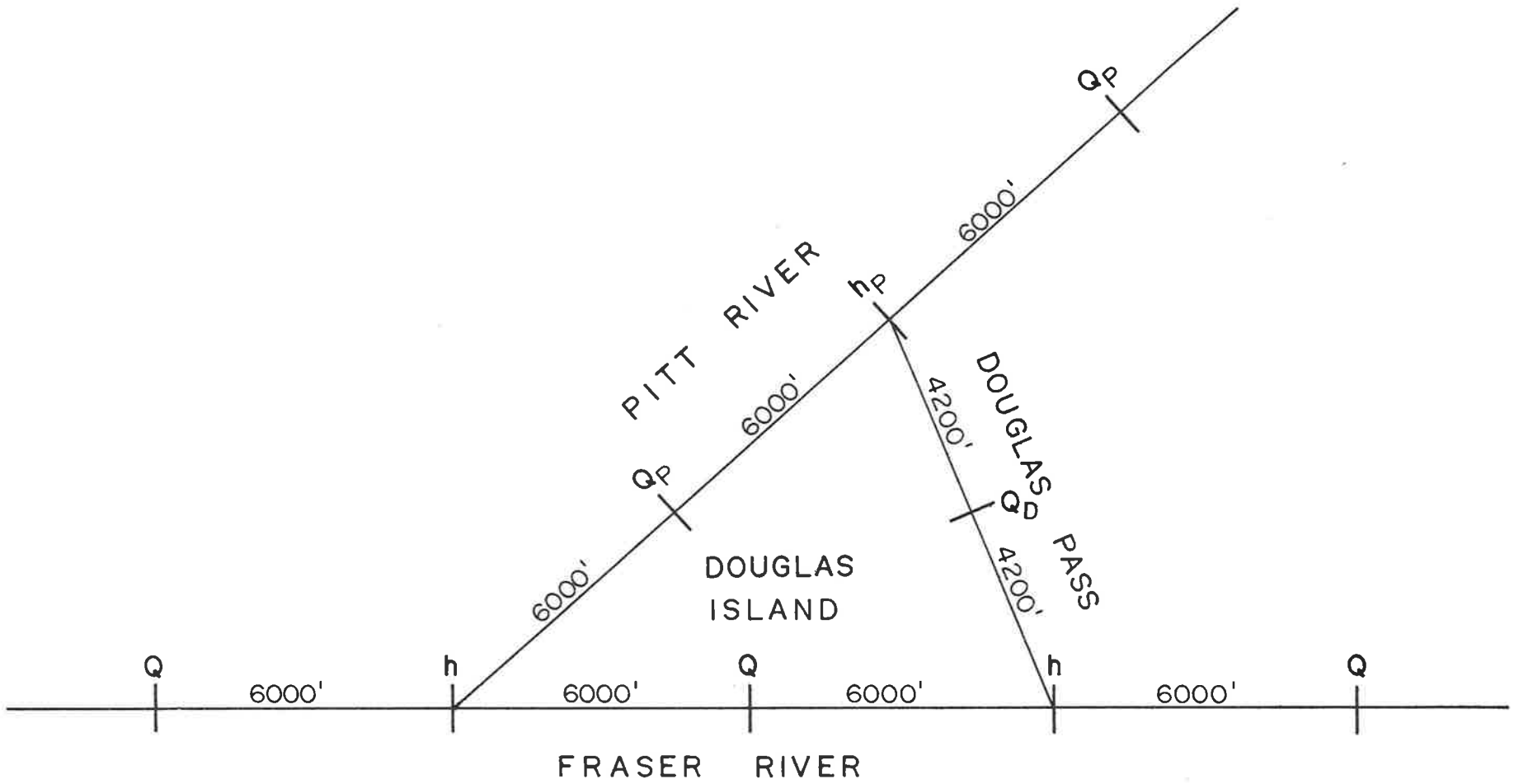


FIG. 8 COMPUTATION AT DOUGLAS ISLAND

Similarly, the continuity equation (1) centered at 2m+1 and 2n and solved for h_{2m+1}^{2n+1} becomes:

$$h_{2m+1}^{2n+1} = h_{2m+1}^{2n} - \left(\frac{1}{B}\right)_{2n+1}^{2n} \frac{\Delta t}{2\Delta x} \left(Q_{2m+2}^{2n} - Q_{2m}^{2n} - Q_{T_{2m+1}}\right) \qquad (14)$$

4.5 Boundary Conditions

4.5.1 Downstream

At the seaward boundary of the Main Arm, Canoe Pass, North Arm, and Middle Arm, the condition imposed was the water surface elevation at Sand Heads. In practice this was taken to be the recorded tidal level at Caulfield Cove. The Caulfield Cove gauge was chosen for the long period of records available and their similarity with the existing records at Sand Heads.

Since the mathematical model formulated required cyclic boundary conditions a Fourier Series Analysis was performed on the Caulfield Cove tidal curve and the resultant Fourier coefficients were then used as the input to the model thereby producing a continuous cyclic function.

4.5.2 Upstream

The upstream boundary condition was taken as the constant freshwater discharge at Agassiz. This value was obtained from the discharge records at Hope located 20 miles above Agassiz. Tributary discharge is negligible between these two stations.

4.5.3 At the Head of Pitt Lake

At the head of Pitt Lake a reflective boundary condition was used[7]:

$$Q_{head} = 0$$

where

Q_{head} is the discharge at the end of Pitt Lake.

4.6 Programming Techniques

A computer program to solve the algebraic equations derived was written in FORTRAN initially for an IBM-360 Model 50 ~~65~~ computer and later converted for an IBM-360 Model 67 installation. The FORTRAN notation used is shown in Figure 9. The order of computation forming the foundation of the program is illustrated in the flow diagram, Figure 10.

5.0 DATA

5.1 Channel Data

To formulate a mathematical model using finite-difference methods the river is schematized into sections with constant dimensions. The river was first subdivided lengthwise into a number of sections (Fig. 11). The length of each section was chosen by considering:

1. The Courant stability criterion (equation 9).
2. Variations in the channel width and depth.
3. Economic considerations involving both the amount of work and the amount of computation time required.

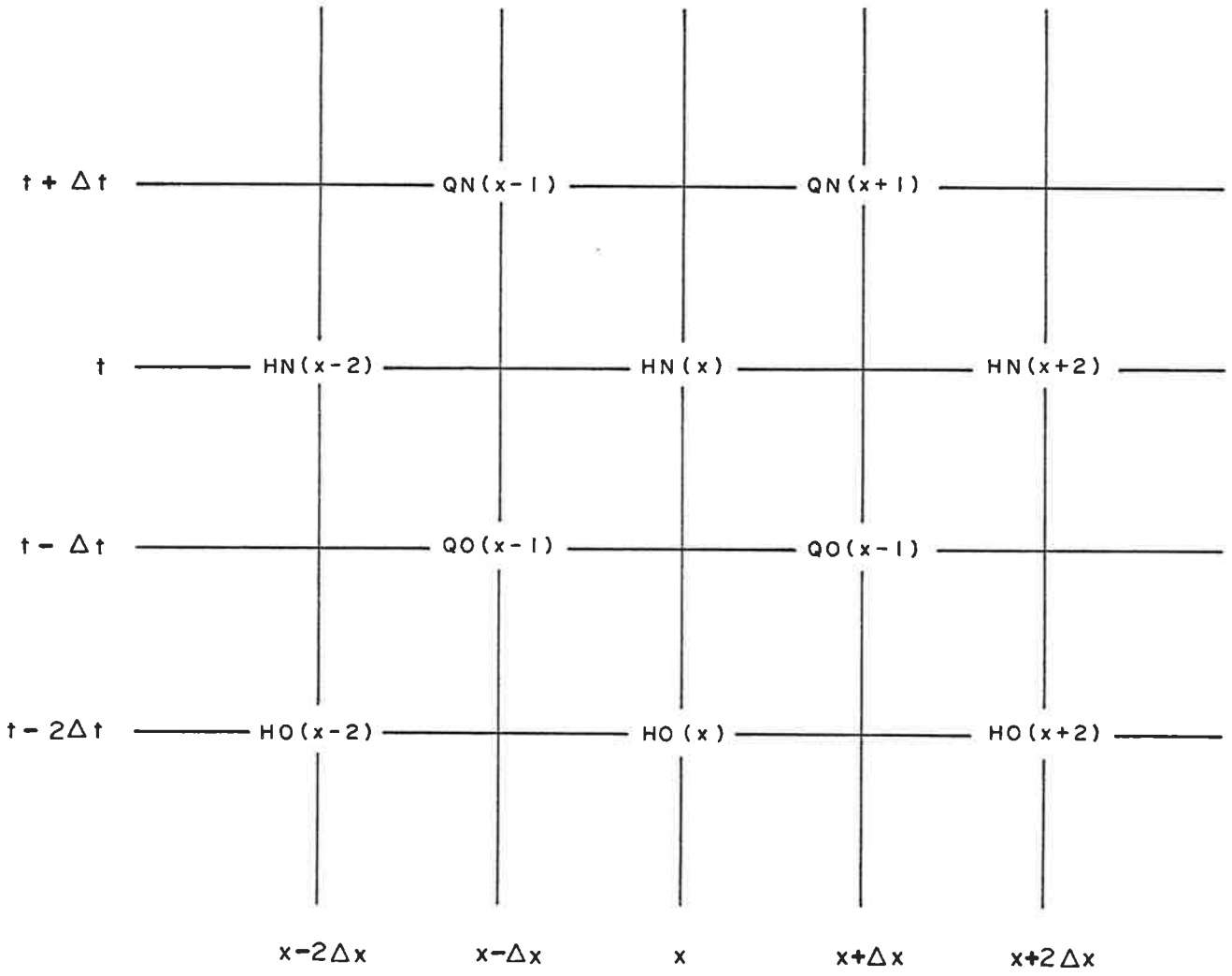


FIG. 9 FORTRAN NOTATION

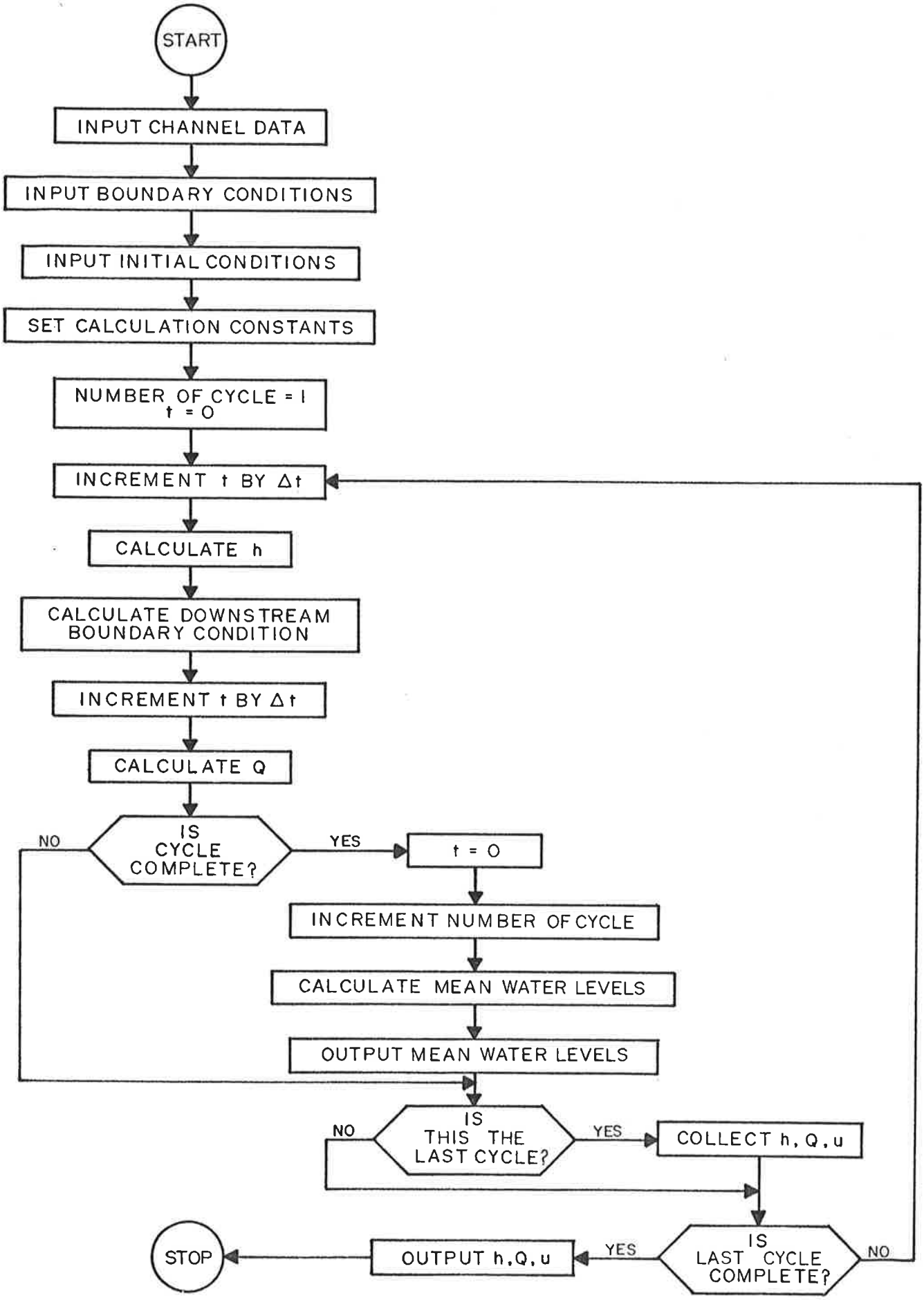


FIG.10 FLOW DIAGRAM

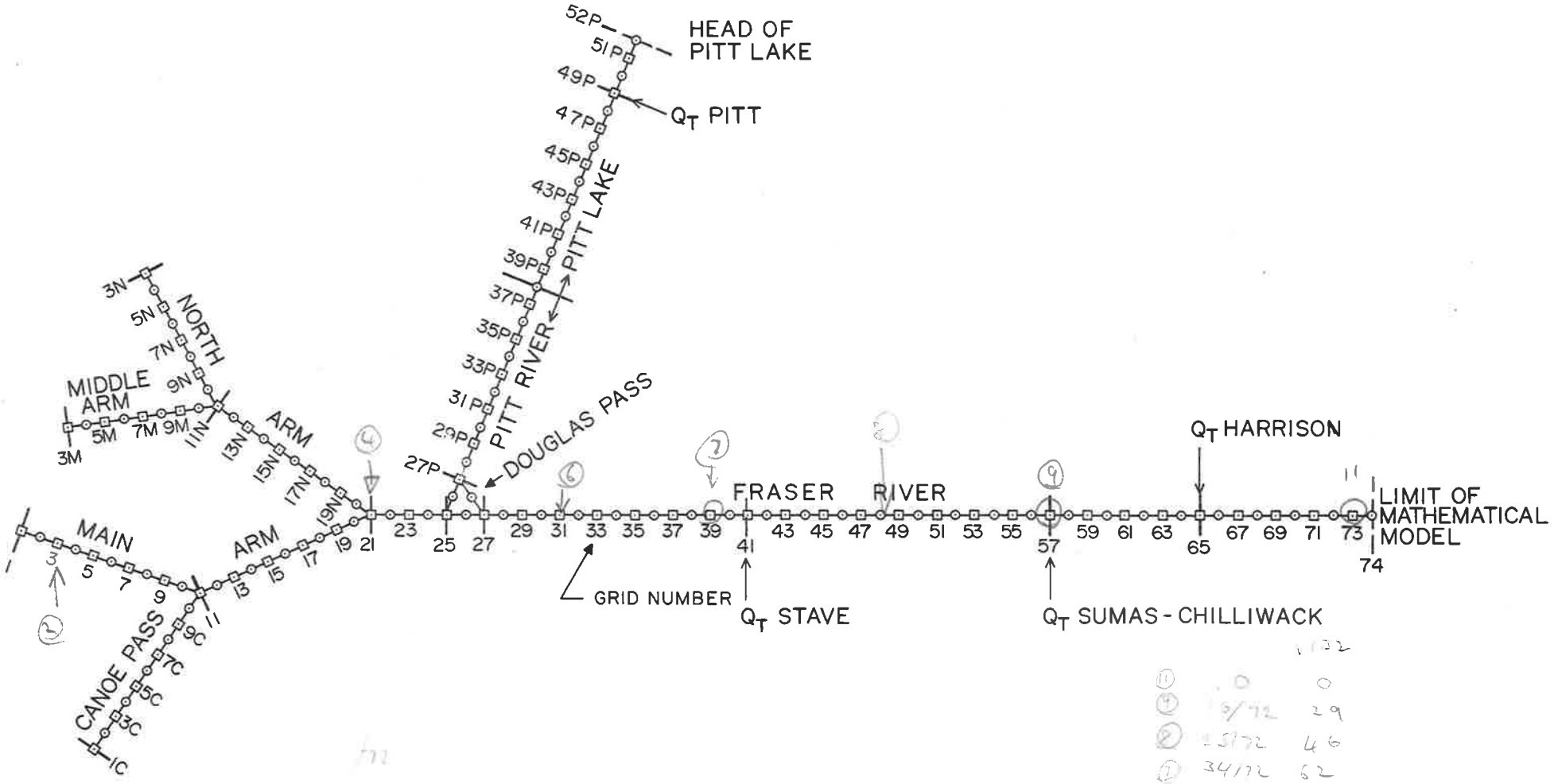


FIG. II SCHEMATIZED RIVER