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Publisher's version / Version de l'éditeur:

North American Contribution to the Second International Conference on Permafrost, 1973

National Research Council Canada
Conseil national de recherches Canada

THERMAL CONDITIONS IN PERMAFROST - A REVIEW OF NORTH AMERICAN LITERATURE

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REPRINTED FROM
PERMAFROST
NORTH AMERICAN CONTRIBUTION TO THE SECOND INTERNATIONAL
CONFERENCE ON PERMAFROST
HELD IN YAKUTSK, SIBERIA, JULY 1973
P. 3 - 25

TECHNICAL PAPER NO. 410
OF THE
DIVISION OF BUILDING RESEARCH

OTTAWA

PRICE 50 CENTS

NRCC 13846

THERMAL CONDITIONS IN PERMAFROST - A REVIEW OF NORTH AMERICAN LITERATURE

This review of published information on thermal conditions in permafrost covers, primarily, the ten year period between the 1st International Permafrost Conference held in 1963, and the 2nd, held in Yakutsk, U.S.S.R., in 1973. The subject is covered in two parts - "The Surface Boundary Condition" (prepared by LWG) and "Subsurface Thermal Conditions" (prepared by AHL). The use of computerized numerical methods for ground temperature calculations is reviewed in an appendix prepared by L.E. Goodrich. Subjects covered are: the climatological balance; components of the surface heat balance; dependence of ground temperature on surface conditions; effect of surface changes; steady ground heat flow and the depth of permafrost; seasonal ground temperature variations; effects of long term variations of surface temperature; effects of lateral variation in surface temperature; and ground thermal properties.

LES CONDITIONS THERMIQUES DANS LE PERGELISOL - UN RESUME DES ECRITS NORD-AMERICAINS

Le présent exposé de synthèse sur la littérature touchant les conditions thermiques des couvertures de pergélisol couvre surtout la décennie entre la première conférence internationale sur le pergélisol tenue en 1963 et la deuxième, tenue à Yakutsk (U.R.S.S.) en 1973. L'exposé comporte deux parties; la première, par L.W. Gold, est intitulée "The Surface Boundary Condition" et la deuxième, par A.H. Lachenbruch, est intitulée "Subsurface Thermal Conditions". Dans l'annexe, L. E. Goodrich passe en revue les méthodes numériques automatisées servant au calcul de la température du sol. Les sujets traités sont: le bilan climatologique, les composantes du bilan de chaleur en surface, la dépendance de la température du sol vis-à-vis des conditions en surface, l'effet des changements en surface, la stabilité du flux de chaleur interne et la profondeur du pergélisol, les variations saisonnières de la température du sol, l'effet des variations à long terme de la température en surface, les effets d'une variation latérale de la température en surface et les propriétés thermiques du sol.

Reprinted from
PERMAFROST: The North American Contribution to the Second International Conference
ISBN 0-309-02115-4
NATIONAL ACADEMY OF SCIENCES
Washington, D.C. 1973

THERMAL CONDITIONS IN PERMAFROST—A REVIEW OF NORTH AMERICAN LITERATURE

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INTRODUCTION

Increasing interest and activity in Alaska and northern Canada, particularly concerning oil and gas exploration and development of other natural resources, have resulted in a more general appreciation of permafrost as a thermal condition of the ground that must be given careful consideration and often requires special treatment. Appropriate methods, for example, had to be developed for excavating and handling ice-rich ore from the iron mines of Labrador and northern Quebec. The settlement that occurs as a result of the thawing of permafrost had to be taken into consideration in the design, construction, and operation of dikes for the Nelson River power development in northern Manitoba. Potential settlement and stability of ice-rich ground are causing challenging problems for the design of future oil and gas lines in Alaska, the Mackenzie Valley of the Canadian Northwest Territories, and Arctic Archipelago. Concern for the arctic environment is causing both government and industry to increase greatly the rate of accumulation of information on northern terrain and to give more attention to developing suitable methods for constructing and operating in permafrost areas.

Thermal conditions in permafrost will be discussed in this paper, in two sections–"The Surface Boundary Condition" (prepared by LWG) and "Subsurface Thermal Conditions" (prepared by AHL). The use of computerized numerical methods for ground temperature calculations is reviewed in an appendix prepared by L. E. Goodrich. For those topics that were reviewed in the First International Permafrost Symposium, the discussion emphasizes studies reported in the 10 years that have elapsed.

To most people, "permafrost" means earth material in which the moisture is predominantly in the solid state as ice, both winter and summer. A variety of more precise definitions is adopted as needed in special applications with rather little confusion. In general discussions of temperature it is useful to define permafrost as the region of perennially negative Celsius earth temperatures, and this will be adequate for most of our discussion. This is at least a necessary condition for permafrost by almost any definition, and it emphasizes the fact that permafrost is primarily a thermal state. It will, of course, include dry earth materials and those in which part or all of the moisture is liquid as a result of freezing-point depression by solutes, surface forces, or confining pressures in excess of 1 atm.

In principle, the ground thermal regime can be determined for any time at a given location if the boundary conditions are specified. A particularly complex set of conditions exists at the surface where the earth is in contact with the atmosphere. At this boundary the temperature is continually changing in order to maintain a balance between heat lost or gained by radiation, evaporation, convection, conduction, and precipitation. The most satisfactory method of establishing this upper boundary condition appears to be by direct measurement of the temperature at some depth with respect to the surface. Although attempts are being made to determine the time dependence of the surface temperature by measuring the components of the surface heat exchange, the greatest benefit of these measurements has been to provide a general qualitative to semiquantitative appreciation of the influence of relief and surface factors, such as vegetation, type of soil, snowcover, and presence of water, on the relationships between ground temperature and weather or climate. This appreciation, along with an understanding of the dependence of the ground thermal regime on the properties of the soil, is necessary to predict the consequences of surface disturbance such as that associated with human activity.

Superimposed on the effects of changes in surface temperature are those due to heat flow from the interior of the earth. Although this amount of heat is small relative to the cyclical contributions at the surface, it has a significant influence on the temperature gradient at depths greater than that to which the annual temperature changes propagate and, therefore, on the thickness of permafrost. Attention has been given to measuring geothermal heat flow in Alaska, northern Canada, and the Arctic Basin primarily because of its scientific interest. Such measurements in permafrost areas are usually easier to interpret than ones made in nonpermafrost areas, because they are not usually disturbed by groundwater movement.

A description of temperature changes and their effects in porous ground is complicated by movement of the fluid and gaseous phases of water. Additional complicating effects are caused by the phase change that occurs in bulk water at 0 °C and at lower temperatures for water under the influence of the surface of the solid. Good progress has been made in taking into account phase changes in ground temperature calculations and in understanding their effects on the ground thermal regime, but both the qualitative and quantitative understanding of the contribution and effects of water and vapor movement are still in a primitive state.

Knowledge of thermal properties is a basic requirement for a proper interpretation and description of thermal conditions in permafrost. The need to calculate the temperature changes that occur in permafrost due to given imposed boundary conditions has revealed the lack of knowledge concerning these properties and their dependence on soil type, density, water content, and ice content. Increasing attention is now being given to their measurement both in the laboratory and in the field.

THE SURFACE BOUNDARY CONDITION

Climatological Balance

The average temperature and heat content of the ground are determined by the cyclical exchange of heat and moisture at the earth's surface. These thermal characteristics at a given site are primarily representative of the local climate, which, in turn, is a function of the large-scale atmospheric circulation, and of the organic and inorganic structure of the surfaces. They are affected to a secondary degree by local effects, such as type of surface material, surface slope, and availability of water.

The exchange of heat and moisture at the surface is driven by the radiation received from the sun. In the absence of climatic change, the net annual change in the heat content of the ground must, on the average, be zero; the net annual gain in solar radiation just equals the sum of the net long-wave radiation, sensible heat loss, and evaporative loss. Similarly, from a short-term climatological point of view, it can be assumed that there is no change in the amount of water stored in the ground and that the average annual precipitation equals the sum of runoff, evaporation, and evapotranspiration.

It is useful for many scientific and engineering problems to have an appreciation of the climatological values of the components of heat exchange. Incoming global solar radiation is measured on a regular basis at eight stations in the permafrost region of Canada and at three in Alaska. Titus and Truhlar[103] have summarized the Canadian data in map form. Radiation values calculated from sunshine records for 15 additional Canadian stations and data from four Alaskan stations were also used in the preparation of the maps.

Vowinckel and Orvig[109] and Hay[44,45] calculated incoming solar radiation, as well as the other components of the radiation balance, and prepared maps presenting mean values. Hay's calculations showed that no land area in Canada and Alaska has a net negative annual balance.

McFadden and Ragotzkie[82] have carried out extensive measurements of the average surface albedo in northern areas from an aircraft flying at a height of about 300 m. Their measurements show the significant influence of surface characteristics on the annual variation in the radiation balance. The tundra region, because of its lack of trees, has the largest seasonal range in albedo from an average high of about 80 percent during the period of snowcover to an average low of about 20 percent during the snow-free period. Kung *et al.*,[64] who were associated with the same project as McFadden and Ragotzkie, present maps of the average albedo of North America, including the permafrost regions, for various times of the year. They discuss the dependence of albedo on surface texture.

McFadden and Ragotzkie,[82] pointed out that melting of snow occurs rapidly in spring in the tundra and that the amount of solar radiation absorbed by the surface can increase 600 percent within 3 weeks during the period of high daily incoming solar radiation. Melting is less rapid in the boreal forest region because of the sheltering effect of the tree cover, even though the average albedo during the period of snowcover is lower than for the tundra. Consideration was given by McFadden and Ragotzkie to the effect of these albedo conditions on climate. Observations in the U.S. Tundra Biome Project[105] are providing more detailed information on radiation effects during the snowmelt period on the tundra and the very rapid change in ground temperature associated with them.

At present, very few evaporation or runoff measurements are being made in the Canadian and Alaskan Arctic; none are being made in the Canadian Arctic Archipelago. Estimates of average values of the components of the heat and moisture balance at the surface are based primarily on standard meteorological observations. Hay[44] and Hare and Hay[42] have studied the large-scale water balance. Maps of mean annual precipitation and runoff have been prepared based primarily on information from the Canadian Atmospheric Environmental Service, U.S. Environmental Data Service, and the *Hydrologic Atlas of Canada.*[49] They have also considered the evaporation field both as the difference between precipitation and runoff and as calculated values with reference to a study on evaporation from small lakes carried out by Ferguson *et al.*[36] The mean annual convective heat field was determined as the difference between the net radiation and evaporation fields. Their analysis indicates a discrepancy in northern areas that they consider to be due to an undermeasurement of snowfall. Similar difficulties in measuring snowfall in northern exposed areas using standard measurement techniques have been reported in the Soviet literature.

The marked similarity in the regional dependence of the isolines for various climatic and surface elements is evidence of their interaction. Hare[41] and Hare and Ritchie[43] discuss these interactions with respect to the boundaries of the tundra and boreal forest regions. Factors of major importance are the control exerted by forest cover on the albedo, particularly during the period of snowcover and spring melt, and the average position of the arctic front, which determines the distribution of water vapor and cloud cover. The regional dependence of the boundaries of the discontinuous and continuous permafrost zones[19,37] demonstrates the importance of these interactions for the ground thermal regime.

Components of the Surface Heat Balance

Measurement of the components of the surface heat balance is required to confirm and improve climatological estimates based on standard weather observations and to increase knowledge of the relative importance of factors controlling the ground thermal regime. Few such measurements have been made in the permafrost areas of Canada and the United States in the past 10 years.

The relative values of the components of the heat balance are quite sensitive to the characteristics of the surface, particularly albedo, availability of water, and type and areal extent of the cover. Ahrnsbrak[1] found for the tundra west of Hudson Bay that the sensible heat loss was appreciably greater than the evaporative loss during summer periods. This observation is consistent with the low annual precipitation that occurs in that region. Further south at sites immediately adjacent to Hudson Bay, Rouse and Stewart[92] found the evaporative loss to be about equal to or a little greater than the sensible heat loss in lichen-heath terrain. Their observations indicated that the lichen surface had a high resistance for the movement of moisture through it. Brown[17] measured a potential evaporation of about 20 cm of water for a grass site from 30 June to 17 September 1960 at Norman Wells, N.W.T. The corresponding values for nearby moss, lichen, and wooded sites for the same period were between 10 and 13 cm.

Kelley and Weaver[61] carried out a heat balance for a study of ground temperatures in well-drained tundra near Barrow, Alaska. They found the total net radiation for the year to be 49 800 J cm^{-2}, of which 3 340 J cm^{-2} was utilized in melting the snowcover. This value is close to the climatological estimate given by Hare and Hay[42] for the Barrow area. Observations of the components of the surface heat balance are also being measured at Barrow as part of the Tundra Biome Project.[114]

Vowinckel and Orvig[110,111] are calculating annual values for the components of the heat balance for specific sites from standard radiosonde and synoptic weather observations combined with information concerning such characteristics of the ground as soil depth and vegetation type. These values should provide an appreciation of the relative and absolute values of the components of the exchange, and how they depend on factors such as precipitation, runoff, and type of surface cover.

Brown[22,23] is conducting a detailed study of ground temperature and weather at Thompson, Manitoba, located at the southern edge of the discontinuous-permafrost region. His observations show that the thermal characteristics of the surface are a major determining factor in the occurrence or nonoccurrence of permafrost at a given location.

The U.S. Tundra Biome Project at Barrow demonstrated that models of the heat balance must take into account not only the characteristics of the surface at the measurement site but also other factors such as the horizontal inhomogeneity caused by lakes, polygonal ground, marshes, and topographic highs.[105] These areal features also have a marked influence on the ground thermal regime and changes induced in it by daily and annual weather and climatic temperature cycles (e.g., ref. 74).

Dependence of Ground Temperature on Surface Conditions

It is now well established that a difference of about 1–6 °C exists in northern latitudes between the average annual air and ground surface temperatures. Observations have shown that this difference is due to the effects of such near-surface factors as snowcover, vegetation, time-varying ground thermal properties, relief, slope and orientation of the surface, and surface and subsurface drainage. In conformance with this, the southern boundary of the discontinuous-permafrost zone is found to coincide approximately with the −1 °C annual air temperature isotherm; continuous permafrost can be expected north of the −6 °C isotherm.[16,19,21] Most of the investigations carried out during the past 10 years on the factors that determine the occurrence of permafrost in the discontinuous zone and the thickness of the active layer in the continuous zone, i.e., the ground temperature conditions in permafrost areas, have been qualitative to semi-quantitative in nature.

If the nonlinear contribution to the heat flow of freezing and thawing of water is negligible, the depth at which the amplitude of a cyclical temperature disturbance of angular frequency (ω) is reduced by a given amount in a semi-infinite medium is proportional to $\sqrt{\alpha/\omega}$, where α, the effective diffusivity of the medium, equals $K/\rho c$, where K, ρ, and c are its thermal conductivity, density, and mass specific heat, respectively. For a given frequency, therefore, the total heat capacity of the depth affected is proportional to $\rho c\sqrt{\alpha} = \sqrt{\rho c K}$. This quantity, sometimes called the conductive capacity or contact coefficient, is of particular significance for calculations of periodic heat flow in layered systems. Typical value of it for near surface materials are presented in Table I.

Table I provides an appreciation of the ability of natural materials to respond or adjust to imposed temperature changes. New snow, for example, offers a significantly higher resistance to change than wet sand and a much higher resistance than turbulent water. As neutral and unstable air can accept heat more readily than most surface materials,

TABLE I Representative Values of the Conductive Capacity[a]

	$\sqrt{\rho c K}$ $\left(\frac{\text{cal}}{\text{cm}^2\ \text{s}^{1/2}}\right)^b$
New snow	0.002
Old snow	0.012
Dry sand	0.011
Wet sand	0.04
Sandy clay	0.037
Organic soil	0.04
Wet marshy soil	0.038
Ice	0.05
Still water	0.039
Stirred water	
great stability; moderate current	0.32
moderate stability; strong current	7.0
homogeneous; strong current	17.0
Still air	1.3×10^{-4}
Stirred air	
very stable	0.01
neutral	0.1
very unstable	1.0

[a]After Priestley.[85]

[b] $\frac{1\ \text{cal}}{\text{cm}^2\ \text{s}^{1/2}} \simeq \frac{42\ \text{kJ}}{\text{m}^2\ \text{s}^{1/2}}$.

the heat carried away by the air will be appreciably greater than that conducted into the ground. During the period of the year when the ground is being warmed, for example, the total heat flow into it is usually less than 10 percent of the net radiation received at the surface. It is for this reason that it is so difficult, if not impractical, to determine the ground thermal regime from observations of only the components of the surface heat exchange.

Not only does the conductive capacity of the surface cover have a great influence on the thermal response of the ground but it also affects the ability of a particular surface type to thermally modify the air above. Because the atmosphere undergoes considerable turbulent mixing and usually has a larger conductive capacity than ground, lateral variations in average annual air temperature are associated with greater distances than corresponding variations in the average annual ground temperature, i.e., the atmosphere has a greater averaging capability. Most weather observations, therefore, are representative of appreciably larger areas than ground temperature measurements.

Lachenbruch[68] shows that, for a layered system, the ratio of the amplitude of a temperature disturbance at the interface between the two layers to that imposed on the surface of the upper layer depends on the thickness and thermal properties of the layer, the frequence of the disturbance, and the ratio $\sqrt{K_1 \rho_1 c_1} / \sqrt{K_2 \rho_2 c_2}$, where subscripts 1 and 2 refer to the upper and lower layers, respectively. Qualitatively, if a semi-infinite medium is covered by a layer with a lower conductive capacity, the decrease with depth of the amplitude of a temperature disturbance imposed on the surface will be greater than if the layer were not present. Conversely, if the layer has a higher conductive capacity, the decrease of the amplitude with depth will not be as great.

At present, it is not possible to interpret unambiguously and correlate the semi-quantitative observations of ground temperatures and the occurrence of permafrost because of insufficient knowledge concerning the heat transfer properties of ground and surface materials and how these properties depend on such factors as water content, thermal state, and density. In the boreal forest and taiga areas of the southern part of the discontinuous zone, permafrost is usually encountered only in peatland or bog areas in association with peat plateaus of palsas. It does not generally occur in low areas with water at or near the surface or in high, well-drained regions.[16,18,20] This clearly indicates the important influence of the thermal properties of moss, lichen, and peat on the ground thermal regime. The thermal conductivity of dry moss and peat is about a factor of 10 smaller than their value when wet. The difference in their resistance to heat flow for a dry summer condition and a wet, frozen winter condition can be sufficient to cause permafrost to be established in marginal areas.

The availability of water to the surface is particularly important with respect to the ground thermal regime, because evaporation is such an effective method of dissipating heat. Brown[17] found at Norman Wells that evapotranspiration from sedge, moss, and lichen was appreciably less than from grass covers. The surface temperature of the moss and lichen was often several degrees higher than the air temperature because of reduced evaporative cooling. He also observed that the thickness of the active layer under sedge was greater than under the moss or lichen covers, indicating that sedge had a larger effective thermal diffusivity. The depth of thaw, of course, decreased with increasing thickness of peat, and there was evidence that this was also true of the average annual ground temperature.

Brown[22] found no permafrost in Precambrian rock outcrops in the Thompson, Manitoba, area where the average annual air temperature is –3.3 °C. At Yellowknife, N.W.T., with an average annual air temperature of –5.5 °C, permafrost does not occur in exposed Precambrian rock, but is found in rock covered with overburden. The range in average annual ground temperature at Yellowknife at the 15-m depth was 1–1.8 °C in Precambrian granite, 0.8–1 °C in beach and till deposits, and –0.5 to –1.0 °C in sedge and spruce peatlands. Permafrost was found in all terrain, including exposed Precambrian rock on Devon Island, but the rock had the thickest active layer.

The thermal conductivity of rock is about the same in summer as in winter; thus, a conductivity effect cannot contribute to any difference that exists between the average annual ground surface and air temperatures. Other factors must be responsible, of which snowcover is probably the most important. A second factor is higher surface temperatures in summer for exposed rock than for vegetated ground, due to reduced evaporative cooling.

The Subarctic Laboratory at Schefferville, Quebec, run by McGill University (average annual air temperature −4.3 °C), has given particular attention to the effect of snowcover, vegetation, and relief on the ground thermal regime.[3,52,101,102] This region is much more exposed than the discontinuous zones studied in western Canada. A strong interdependence was found between permafrost, exposure, relief, snowcover, and vegetation. Permafrost was always observed beneath exposed, wind-swept ridges. Areas with sufficient shelter to allow brush to grow were always found to be warmer in summer and appeared to have a higher average annual ground temperature than surrounding areas completely lacking vegetation. Snowcover was found to be the major factor determining the presence of permafrost. In general, no permafrost was found in areas where the thickness of the snow exceeded about 40 cm.

Under some conditions, the contribution of one surface factor can be masked by that of a second. Price[88] found from a study in the Yukon that the active layer was shallower on slopes facing southeast than those facing north, because there was better development of the vegetation on slopes with a southern exposure. In this case, plant cover was more important than exposure in determining the depth of thaw.

Observations on the ground thermal regime, although few in number and still relatively primitive in scope and approach, are beginning to define the effect of surface factors such as relief, vegetation, drainage, soil type, and snowcover.[35] They indicate clearly that changes in ground temperature can be caused not only by variations in weather and climate but also by modifications to the characteristics of the surface. The surface disturbance and changes associated with man's activity in the Arctic in recent years and the realization of the need to preserve the permafrost condition in many situations have caused considerable attention to be focused on this aspect of the ground thermal problem.

Effect of Surface Changes

Several studies have been undertaken to document the thermal effects of surface changes and to establish the practice that must be followed to prevent the terrain damage that will unacceptably interfere with natural processes, transportation, and performance of structures (e.g., ref. 2).

Terrain damage due to degradation of permafrost is significant only in those soils that have a high ice content and a surface cover whose characteristics are sensitive to mechanical disturbance. Sandy and gravelly upland areas with little vegetative cover, particularly in the high Arctic, are relatively insensitive thermally to transient surface activity, although traces of that activity may linger for a very long time due to the slow rate of recovery in the northern environment. Serious damage occurs primarily in terrain with a significant cover of peat overlain by mosses, lichens, sedge, dwarf shrub, and similar, sensitive, arctic organic surface vegetation.

Surface disturbance can affect the ground thermal regime in both a symmetric and an asymmetric manner. In the symmetric case, only the amplitude of the annual surface temperature variation is changed, and the average annual ground temperature remains the same. An increase in amplitude will cause the depth of the active layer to increase and possible subsidence due to melting of ice in permafrost. Most surface disturbances, however, cause an asymmetric effect, modifying the difference between the average annual air and ground surface temperature in addition to possibly changing the amplitude in the annual surface temperature variation. Because the geothermal gradient is appreciably smaller than the maximum average temperature gradient in soil near the surface in summer, a change in the average value of the annual ground surface temperature will (in sufficient time) have a greater effect on the thickness of permafrost than a comparable change in its amplitude. In many areas, this change can be large enough to cause complete degradation of the underlying permafrost.

Attention is being given to the consequences of past disturbances of both human and natural origin. Observations of seismic lines for which the organic layer was removed by bulldozer have indicated increases in the thickness of the active layer of 150 percent or more, the amount depending on the thickness removed and its effective thermal resistance. Increase in the thickness of the active layer was often accompanied by extensive settlement and erosion.[10,11,77,79,112] Bliss and Wein[11] observed in the Mackenzie Delta area that, for seismic lines through spruce-alder forest, the depth of the active layer increased 62 percent; through the more fragile willow-alder cover, it was about 117 percent. Because of the serious terrain damage that has occurred, the practice of preparing seismic lines with a bulldozer has been stopped.

Winter roads constructed on frozen ground have a much smaller effect on the thickness of the active layer than bulldozed roads, although some increase may occur due to compaction. Bliss and Wein[11] also found that damage due to such roads is less through wet sedge areas than over upland terrain covered with small shrubs that are more susceptible to compaction in winter.

Investigations are being conducted by Radforth[90] on the effect of vehicles on the terrain in summer in order to establish information required for land use regulations. These observations show how the rate of deterioration of the surface depends on the nature of the cover, moisture content, depth of thaw, weight of vehicle, aggressiveness of track, etc. Similar observations are being made in Alaska.[15,91] Brown *et al.*[14] have studied the consequences of specified surface disturbances at sites near Livengood, Alaska, and along the proposed Alaskan pipeline route. Removing the entire peat layer increased the thickness of the active layer for the undisturbed condition from 32 to 75 cm. If the entire peat layer was removed and replaced by a mulch, or if the living cover was sheared, the thickness of the active layer increased to about 60 cm. The depth of thaw under trails bulldozed in the spring was found to be 100 percent greater than in adjacent undisturbed sites. Very significant depression of the surface occurred due to melting of ice and erosion. The increase in depth of thaw under recently burned-over areas was found to be about 50 percent.

Kallio and Rieger[59] investigated the consequences of surface modifications at Fairbanks, Alaska. The upper vegetation was removed from three plots; two were cultivated—one with grass and the second with potatoes. For the site that was cleared only, the period of thaw was the same as for a fourth, undisturbed control site, but the depth of thaw was greater. In the grass and potato sites, thawing began earlier, and freezing was completed later than the control site. A permanent, unfrozen zone developed under the potato plot. The average annual ground temperature was greater than 0 °C above the permafrost table for all plots; the highest value was at the site planted with potatoes.

Observations made in burned-out areas showed an increase in the thickness of the active layer of between 30 and 60 percent.[10,14,46,107] Heginbottom found the depth of thaw was significantly greater (80 percent) under bulldozed firelines because of greater damage to the organic cover. Fire in the northern environment was reviewed recently at a symposium held at the University of Alaska.[104]

Water is one of the most effective modifiers of the ground surface condition. If a body of freshwater is sufficiently deep, the temperature at the bottom will always be 0 °C or higher. This creates a thawed zone whose depth and extent depends on the size of the body of water.[25,67] If the water is eroding, as, for example, along the shore of a river or the coast of the Arctic Ocean, it continuously encroaches upon permafrost and induces a thawing condition.[80,98] In the case of the river, erosion may be associated with exposure of thawed ground on the opposite bank. As the erosion process continues, vegetation will grow on the exposed ground on the slipoff side and the frozen condition will be restored, resulting in a gradation in maturity in both the permafrost and surface cover.[86,98] In some cases the influence of water is periodic (e.g., floodplains), subjecting the soil not only to a transient change in surface temperature but also to the effects associated with flood damage.[106,107]

Other types of natural process that affect the ground thermal regime are sedimentation, solifluction (particularly overriding of the surface by material involved in downslope movement), and vegetation changes. A review of information on degradation of permafrost due to surface effects is given by Mackay.[77,79]

Methods of controlling changes to the ground temperature caused by structures is another matter under review. The effects on ice-rich ground of removal or compaction of the organic cover may be lessened or completely prevented by using rigid insulation or an appropriate thickness of nonfrost-susceptible material.[7,16,40,63,87,93] An extensive study has been undertaken at Fairbanks, Alaska, on the effect of gravel and paved surfaces on the ground temperature. One of the paved surfaces was painted white to evaluate the effect of changing the albedo. Degradation of the permafrost was greatest under the gravel section and least under the paved surface painted white.[7] Ferrians *et al.*[38] present a good review of problems that have occurred in Alaska due to surface disturbance associated with the construction of roads, railways, bridges, and buildings.

Most of the information collected in the past 10 years on the effect of surface changes on the ground thermal regime has been of a qualitative nature. It is quite possible such studies will never be more than semiquantitative because of the great natural variability in the weather and characteristics and properties of surface and near-surface materials. Attention should be given, however, to experiments and observations that will determine the range of the characteristics and properties that are important and the factors on which they depend. Such information is necessary from the science point of view to develop a full understanding of the permafrost condition and the processes that affect it and from the engineering point of view to give a rational basis for design and the conduct of human activity in permafrost areas.

SUBSURFACE THERMAL CONDITIONS

General

To understand the distribution of permafrost and the factors controlling it, it is necessary to understand the distribution of temperature in those portions of the earth in which it is likely to occur. The distribution of temperature and heat flux over the ground surface is controlled by complex and poorly understood processes, as discussed above. Nevertheless, the earth's surface is relatively accessible for measurements, and useful generalizations for these quantities can be made there. On the other hand, our direct knowledge of the thermal regime of the subsurface is limited to measurements in isolated boreholes and a few mines. In the absence of heat transfer by moving fluids, the temperature throughout the interior of a solid can be obtained by heat conduction theory from a thorough knowledge of its surface conditions and very limited information about its interior. In temperate and tropical climates, the application of heat conduction theory to the thermal regime of the outer few hundred metres of the earth is severely limited by the effects of heat transferred by groundwater circulating through pores and fractures. In regions of continuous permafrost, however, the groundwater is largely immobilized either as ice or by surface forces, and heat conduction theory can often be applied with confidence to within a metre or so of the surface. As a result of this fact and of the importance of earth temperature in permafrost terrane, there is probably no other area of earth science in which known or easily derived analytical results from classical heat conduction theory yield more useful information. The richest English-language source of these results is the well-known book by Carslaw and Jaeger.[29] Whereas these analytical results yield insight and an overall understanding of the gross aspects of temperatures in permafrost, the more refined calculations usually required for engineering design are generally intractable analytically, and they must be treated by numerical methods. In such cases, however, the related exact analytical results provide useful insight and an invaluable means of verifying the computer programs.

The main features of the temperature regime in permafrost can be understood in terms of four rather distinct heat-transfer problems:

1. The steady one-dimensional flow of heat from the earth's interior to its isothermal surface, and the depth of the bottom of permafrost;
2. The periodic seasonal variation of surface temperature and the depth to the top of permafrost;
3. Long-term variations of surface temperature, the evolution of permafrost, and the effects of climatic change; and,
4. Lateral variations in the surface temperature and the effects of bodies of water, variable surface cover, topography, and engineering structures placed on the surface.

For steady-state problems or transient ones that do not involve appreciable movement of an ice-rich permafrost boundary, the governing equations are linear, and the individual solution to each of the four problems may be superimposed to obtain a complete description of the thermal regime. Even where this condition is violated for one or more of the problems, useful approximate results can often be obtained by superposition. In the following paragraphs, each of the four problems will be outlined briefly, referring where possible to the more recent work on each in North America.

Steady Heat Flow and the Depth of Permafrost

This is probably the most important of the four problems, and, in spite of the fact that it involves only simple arithmetic and forms the starting point for almost every discussion of earth temperatures, it is probably the most widely misunderstood. Other discussions relating primarily to permafrost are included in the references.[32,54,57,70,72–75,100]

If the earth is losing heat steadily by conduction through its outer layers at the rate q^* per unit area, then in any horizontal layer with thermal conductivity K_1 the Fourier heat conduction law requires:

$$q^* = K_1 G_1, \tag{1}$$

where G_1 is the thermal gradient in the layer, i.e.,

$$G_1 = \frac{dT}{dz}, \; z \text{ in layer 1.} \tag{2}$$

Through any other layer with conductivity K_2 and gradient G_2, the same amount of heat q^* must be flowing; hence:

$$q^* = K_1 G_1 = K_2 G_2 = \text{etc.}, \tag{3}$$

and the gradient in each layer varies inversely with its conductivity and directly with the regional heat flow. The temperature drop across layer 1 of thickness Δz_1 is:

$$\Delta T_1 = G_1 \, \Delta z_1 = \frac{q^*}{K_1} \, \Delta z_1 . \tag{4}$$

If this homogeneous layer should extend from the ground surface, $z = 0$, where the long-term mean surface temperature is $-T_0$ °C, to the base of permafrost, $z = z_p$, where the temperature is 0 °C, then the gradient will be uniform, and the temperature $T(z)$ is given by:

$$T(z) = \frac{q^*}{K} z - T_0, \; 0 < z < z_p, \tag{5}$$

and the permafrost depth z_p is:

$$z_p = \frac{K}{q^*} T_0, \; -T_0 \text{ in °C.} \tag{6}$$

Refinements of this simple picture are easily established, e.g., if the permafrost is stratified with n layers, then K in Eq. (6) is replaced by the harmonic mean conductivity given by:

$$K = \left\{ \frac{1}{z_p} \sum_{i=1}^{n} \frac{\Delta z_i}{K_i} \right\}^{-1} . \tag{7}$$

It is frequently assumed, tacitly, that permafrost depth (z_p) can be estimated from the surface temperature $-T_0$, perhaps because that is the only quantity in Eq. (6) observable at the earth's surface. However, Eq. (6) expresses the important fact that permafrost depth is equally sensitive to thermal conductivity K and heat flow q^*. It is seen from Eq. (3) that the heat flow, q^*, can be determined from an equilibrium measurement of the gradient G in any deep layer penetrated by a borehole if the conductivity, K, of that layer is also determined from a piece of core. This may be any convenient horizon; whether or not it lies within the permafrost zone is irrelevant to the problem. It is fortunate that q^* can be determined with very little subsurface information, because very little is available.[57,75,99] It is also fortunate that data from all of the continents indicate that, for the most part, q^* varies only by a factor of about 2 (from 1 to 2×10^{-6} cal/cm^2 s). Over lateral distances on the order of 100 km, the variation is usually much less. By contrast, the thermal conductivity, K, in Eq. (6) can vary by a factor of 3 or more over relatively short distances.

The relation in Eq. (6) is illustrated in Figure 1, which presents generalized temperature observations at four Alaskan arctic coastal locations (data are from ref. 12, 72, 74 and Lachenbruch, Sass, Munroe, and Moses, unpublished). For this discussion, only the linear portions of the observed profiles and their dashed extrapolations should be considered. It is seen that the correlation between surface temperature, $-T_0$, and permafrost depth, z_p, is poor. At Cape Thompson, where $-T_0$ is about $-7\ ^\circ$C, the permafrost is 25 percent deeper than at Cape Simpson, where the long-term mean surface temperature is $-12\ ^\circ$C. Permafrost is 60 percent thicker at Prudhoe Bay than at Barrow in spite of the warmer surface conditions at Prudhoe. Existing data indicate that the heat flow, q^*, is about the same at all four sites (probably within 10 percent of 1.4×10^{-6} cal/cm^{2}s); hence, variations in permafrost thickness depend mainly on K and T_0 [Eq. (6)]. The profiles of Figure 1 are quite predictable from Eq. (6) when measured and estimated values of thermal conductivity (K) are considered for each location. The increased gradient near the bottom of the Prudhoe Bay profile corresponds to a decrease in conductivity in accordance with Eq. (3). At this depth, ice is replaced by water; more will be said of this later [Eq. (35), below]. In the other holes, the moisture content is too low to display this effect.

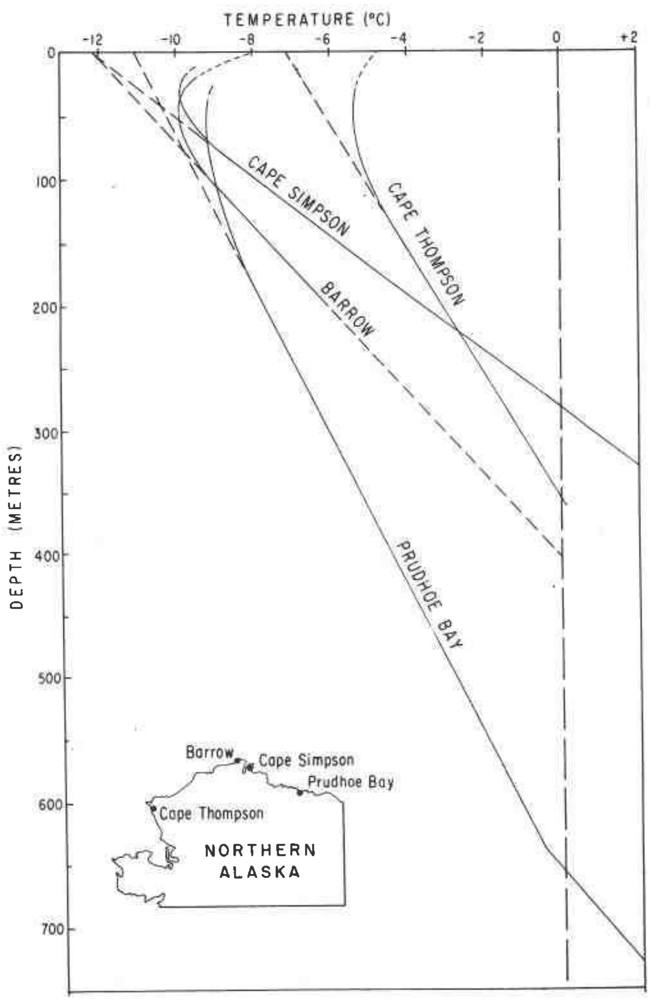

FIGURE 1 Generalized profiles of measured temperature on the Alaskan arctic coast (solid lines). Dashed lines represent extrapolations. See Eq. (5), (17), and (36).

Seasonal Temperature Variations

Next in importance to the steady heat flow, which determines the base of permafrost, is the periodic variation in surface temperature from summer to winter. It determines the position of the top of permafrost and generates the active layer upon which man's activities take place. This variation is also responsible for periodic thermal stress in the surficial permafrost layers that causes them to crack with profound geomorphic consequences.

It is common knowledge that the temperature at the earth's surface is strongly influenced by systematic diurnal and annual periodicities—predictable results of the earth's rotation and revolution. Superimposed on these are the incompletely understood effects of energy transmission between the outer limits of the earth's atmosphere and its solid surface. Variations in the latter cause perturbations in these periodicities (warm spells and cold snaps, mild and severe summers and winters, and so on) that, over intervals of a few years, generally appear to occur in random sequence. Thus, the depth of thawing in permafrost terrains, too, will tend to vary randomly from year to year as will the mean temperature at the ground surface for any successive 12-month period. Over longer periods, systematic variations (climatic changes) may be observable; their effects on permafrost is the subject of the next section. In discussing the seasonal effects on permafrost temperature, it is useful to assume at first that the surface temperature fluctuations are truly periodic with a period (P_y) of one year and that they vary about an annual mean value denoted by $-T_0'$. The periodic variation at the surface, denoted by $\theta\left(0, \frac{2\pi t}{P_y}\right)$

will give rise to temperature variation with the same period, $\theta\left(z, \frac{2\pi t}{P_y}\right)$, at any depth z beneath the surface. In this discussion the effects of steady flux, q^*, at depth are unimportant, and the temperature $T(z,t)$ at any depth z at time t is given by:

$$T(z,t) = -T_0' + \theta\left(z, \frac{2\pi t}{P_y}\right). \tag{8}$$

Denote by $\theta_m(z)$ the maximum value attained each year by θ at any depth z. In virtually all permafrost areas, some thawing occurs at the surface in summer and, hence, $\theta_m(0) > T_0'$, where it is understood that temperatures are measured in degrees Celsius. As the seasonal fluctuations attenuate with depth, $\theta_m(z)$ will generally decrease with z. The depth to the top of permafrost z_a (equivalent in this case to the depth to the bottom of the active layer) is therefore given implicitly by:

$$\theta_m(z_a) = T_0'. \tag{9}$$

The gross features of the temperature field in the zone of annual variation are given by the simple heat conduction model of a semi-infinite homogeneous medium ($z > 0$) with sinusoidal surface temperature, $\theta\left(0, \frac{2\pi t}{P_y}\right) = A_0 \sin \frac{2\pi t}{P_y}$; in which case Eq. (8) becomes:

$$T(z,t) = -T_0' + A_0\, e^{-z\sqrt{\frac{\pi}{\alpha P_y}}} \sin\left(\frac{2\pi t}{P_y} - z\sqrt{\frac{\pi}{\alpha P_y}}\right), \tag{10}$$

and Eq. (9) becomes:

$$A_0\, e^{-z_a\sqrt{\frac{\pi}{\alpha P_y}}} = T_0'; \tag{11a}$$

$$z_a = \sqrt{\frac{\alpha P_y}{\pi}} \ln \frac{A_0}{T_0'}, \tag{11b}$$

where the thermal diffusivity (α), the density (ρ), and the mass specific heat (c) are related by:

$$\alpha = \frac{K}{\rho c}.$$

These much-discussed results (see, e.g., ref. 29 and 58) describe the exponential attenuation of the seasonal wave with depth and the linear lag in phase. A refinement in which the surface temperature $\theta\left(0, \frac{2\pi t}{P_y}\right)$ is represented by a Fourier series describes the rapid attenuation of high frequency components in surficial layers and consequent smoothing of the waves to simple sinusoidal form with increasing depth. For a typical value of α (0.01 cm^2/s), Eq. (10) indicates an attenuation of the annual wave by 10^{-1} for every 7-m depth. It is seen that comparable attenuation of a diurnal wave (of period $P_y/365$) would occur in 35 cm. As the surface amplitude, A_0, of the annual wave is typically on the order of 15–30 °C and that of the diurnal wave is commonly considerably less, the annual wave can generally be expected to dominate, even near the base of the active layer. From the foregoing the seasonal variation will typically be $\sim 10^{-2}$ °C at a depth of 20 m, and the phase lag there will be about 1 year. For thermal computations at greater depths, the surface temperature can be considered to be simply $-T_0'$. From Eq. (5), the neglected effect of uniform heat flow, q^*, at 20 m is typically less than 1 °C. The geothermal gradient, of course, may be added to the seasonal effects, but in the seasonally varying layer, it usually is not warranted by other uncertainties.

Although extremely useful for insight and order-of-magnitude estimates, the simple solution [Eq. (10)] generally fails in more refined applications (e.g., to estimates of z_a or α from temperature observations) because of effects of inhomogeneity, latent heat, and longer term aperiodicities.

For surface conditions, similar to those on the Alaskan arctic coast ($A_0 \sim 16$ °C, $-T_0' \sim -8$ °) and $\alpha \sim 8 \times 10^{-3}$ cm^2/s, Eq. (11b) gives the value of $z_a \simeq 2$ m. This is a reasonable approximation of observed depths of a dry active layer there or of the depth of a gravel fill required to maintain permafrost at its lower surface. A simple refinement to account for the change in properties at the base of the active layer yields in place of Eq. (11b) the transcendental equation [ref. 68, Eq. (21)]:

$$z_a = \sqrt{\frac{\alpha_1 P_y}{\pi}} \ln\left[\frac{A_0}{T_0'}\left(\frac{1+M}{\sqrt{S}}\right)\right], \tag{12a}$$

where

$$M = \frac{\sqrt{K_1 \rho_1 c_1} - \sqrt{K_2 \rho_2 c_2}}{\sqrt{K_1 \rho_1 c_1} + \sqrt{K_2 \rho_2 c_2}} \tag{12b}$$

and

$$S = 1 + 2M\, e^{-2z_a\sqrt{\frac{\pi}{\alpha_1 P_y}}} \cos 2z_a\sqrt{\frac{\pi}{\alpha_1 P_y}} + M^2\, e^{-4z_a\sqrt{\frac{\pi}{\alpha_1 P_y}}}. \tag{12c}$$

The subscript 1 refers to the surficial layer and 2 refers to the underlying permafrost. Equation (12) illustrates the point made in a previous section that the "insulating" property of an active layer (or gravel fill) is sensitive to the properties of the underlying permafrost that is being insulated. It has been shown[68] that, if the permafrost were typical icy silt, Eq. (12) would indicate that the 2 m dry active layer (or gravel fill) thickness obtained above from Eq. (11b) would be too large by about 50 percent. Equation (12) has provided a rationale for design of many gravel roads on the Alaskan arctic slope.

A further analytical refinement accounting for three layers shows that a gravel fill thickness can be reduced substantially by a thin layer of rigid insulation at its base.[68] However, a comparison of Eq. (11b) and (12) shows that even simple refinements lead quickly to cumbersome results. Furthermore, none of these solutions takes account of the often important effects of latent heat of freezing and thawing moisture in the active layer; as a result, they indicate falsely that the active layer should thicken indefinitely as the mean temperature, $-T_0'$, approaches 0 °C in the subarctic. Although no analytical solutions are known for heat conduction in a two-phase medium with moving interface and periodic surface temperature, simple approximations based on Neumann's solution are often successful in estimating frost and thaw penetration in a wet active layer.[26,27,58] Quite naturally in recent years considerable attention has been given to the application of numerical methods to these difficult and important problems.[48,58,60,84]

An important characteristic of temperatures in permafrost beneath a wet active layer is the asymmetry generated in the thermal wave by the effects of latent heat.[73] The active layer warms readily past the freezing point in the spring, but its unfrozen lower portion is maintained at the freezing point throughout the autumn freezeup. As a result, the top of permafrost is maintained at 0 °C until it comes in contact with the downward growing seasonal frost. By that time, the surface temperature is generally very low, and large gradients in the newly frozen active layer cause anomalously rapid cooling in the surficial permafrost immediately after freezeup. This rapid cooling may play an important role in the generation of thermal tension and ice-wedge cracks in permafrost. An analysis of this thermal stress is one of the more important applications of a study of the seasonal temperature variation (θ).[69] It is likely that this stress depends on the rate of cooling$\left(\frac{d\theta}{dt}\right)$, as well as the amount of cooling ($T_0' - \theta$). For if it did not, the stress would be thermoelastic, and too large by an order of magnitude to explain the common observation that active ice wedges generally do not crack every year[69] (see Mackay and Black, this volume). For the reasonable assumption of power law type flow deformation (with power m) the thermal tension (σ) in permafrost is given approximately by:

$$\sigma \simeq \eta(\theta)\left[\gamma \frac{d\theta}{dt}\right]^{\frac{1}{m}}, \tag{13}$$

where γ is the coefficient of expansion and η is a quasi-viscous parameter very sensitive to θ. Hence an understanding of the stress and cracking in permafrost will probably require a detailed knowledge of θ and its time derivative.

Long-Term Variations of Surface Temperatures

It has been shown that if heat is flowing steadily from the earth's interior through homogeneous permafrost materials, the temperature profile beneath the zone of annual fluctuations is a straight line [Eq. (5)] that intersects the surface at the mean annual surface temperature $-T_0$. If the delicate thermal balance that determines $-T_0$ changes systematically, the net flux across the earth's surface will change, and the mean surface temperature will change to accommodate it. Suppose that such a change results in a shift in the mean surface temperature from an initial long-standing stable value, $-T_0$, to a new stable value $-T_0 + \Delta T_0$. After thermal equilibrium is achieved in the permafrost, the new temperature profile, of course, will be given by:

$$T(z) = \frac{q^*}{K} z - T_0 + \Delta T_0, \tag{14}$$

and the permafrost will have thinned from the bottom by $\Delta T_0/G$. (For G = 20 °C/km, ΔT_0 = 3 °C; this would amount to 150 m.) During the transition between these two steady states, the temperature profile will trace a family of curves bracketed by the lines defined by Eq. (5) and (14). We assume the change at the surface takes place between time $t = 0$ and $t = t_0$, according to some function $\Delta T(0, t)$, and denote the total change $\Delta T(0, t_0)$ by ΔT_0. The corresponding effect at depth is denoted by $\Delta T(z, t)$ and the transient profile is then given by:

$$T(z, t) = \frac{q^*}{K} z - T_0 + \Delta T(z, t). \tag{15}$$

A useful general discussion of this problem is given by Carslaw and Jaeger.[29] For a surface temperature change of the form:

$$\Delta T(0, t) = D\, t^{\frac{1}{2}n}, \quad n = 0, 1, 2 \ldots., \tag{16}$$

where D is a constant. The complete temperature disturbance is given by:

$$\Delta T(z, t) = \Delta T(0, t)\, \frac{i^n \mathrm{erfc}\, \dfrac{z}{2\sqrt{\alpha t}}}{i^{n-1} \mathrm{erfc}\, 0}, \quad 0 < t < t_0, \tag{17}$$

where $(i^n\, \mathrm{erfc})y$ is the repeated integral of the error function for argument y. The net unbalanced downward heat flux, $F(t)$, through the earth's surface associated with this change is given by:

$$F(t) = \frac{\Gamma\left(\frac{1}{2}n+1\right)}{\Gamma\left(\frac{1}{2}n+\frac{1}{2}\right)\sqrt{\alpha}} K\, Dt^{\frac{1}{2}(n-1)} \tag{18a}$$

$$= \frac{\Gamma\left(\frac{1}{2}n+1\right)}{\Gamma\left(\frac{1}{2}n+\frac{1}{2}\right)\sqrt{\alpha t}} K\, \Delta T(0, t). \tag{18b}$$

By superimposing these results for different values of n, an arbitrary history of surface temperature or unbalanced flux can be represented in terms of a power series, and its effects on permafrost temperature can be calculated directly.[29] General formulations can also be made in terms of Fourier series using results like Eq. (10) or in terms of step functions as described by Birch.[8,55]

Results of this kind can be used directly or inversely to study the evolution or temperature history of permafrost. In both cases, we make the reasonable assumption that the geothermal flux, q^*, is uniform; hence, the initial temperature is given by Eq. (5). The direct method poses no difficulty in concept, and the growth, deterioration, and temperature structure of permafrost can be calculated directly for any assumed hypothetical temperature or flux history at the surface. If thickening or thinning of ice-rich permafrost is involved, useful analytical results will not be available, but there are no serious obstacles to direct calculation by numerical methods. In the more interesting inverse problem, we are given an observed temperature profile [Eq. (15)] at some point in time ($t = t_0$), and from it we attempt to reconstruct the temperature history, $\Delta T(0, t)$, or unbalanced flux history, $F(t)$, at the surface. Quite apart from any analytical difficulties, the inverse problem contains two basic conceptual ambiguities: the uncertainty in identifying the transient contribution $\Delta T(z, t_0)$ [Eq. (15)] in the observed temperature profile and the uncertainty in selecting from a broad range of temperature histories, $\Delta T(0, t)$, consistent with the empirically determined disturbance, $\Delta T(z, t_0)$.

The inverse problem is simplest for surface changes in which $F(t)$ is at least of the order of the geothermal flux, q^*, and which have occurred so recently that their effects have not penetrated to the base of permafrost. In such cases, if the permafrost is homogeneous, the first type of ambiguity is not serious, and the analytical difficulties associated with latent heat at the base of permafrost do not occur. In Figure 1 the effects of such a change in its early stages can be seen in all four profiles. There, $\Delta T(z, t_0)$ is clearly represented by the horizontal distance between the upward extrapolated dashed lines and the curved solid lines. It is seen that ΔT_0 evidently ranges from 2 to 4 °C and that $-T_0$ is about -12°C at Barrow and Cape Simpson, -11 °C at Prudhoe Bay, and -7 °C at Cape Thompson. Recent changes such as these have been reported from many areas,[6,30,50,51] but for reasons already given, they can probably be studied best by conduction theory in regions of continuous permafrost.[34]

Ambiguity of the second type is illustrated by Figure 2, where the climatic disturbance $\Delta T(z, t_0)$ for Cape Thompson (Figure 1) is shown in the inset.[74,75] The climatic histories, inferred from Eq. (17) for n = 0, 1, 2, 3, and 4, are

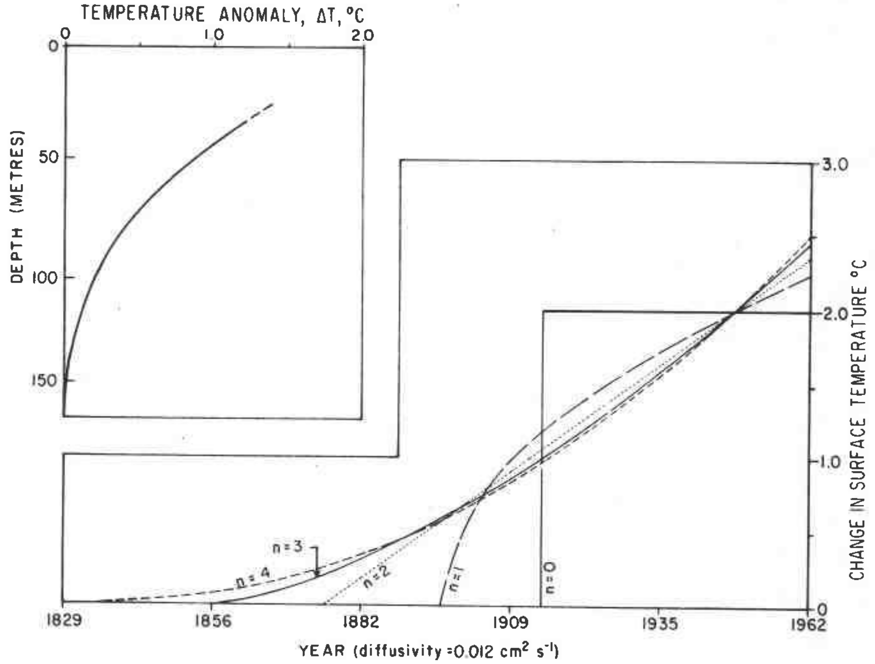

FIGURE 2 The measured climatic temperature anomaly at Cape Thompson (inset) and surface temperature histories consistent with it [Eq. (17)].

also illustrated in Figure 2. The cases $n = 0, 1, 2, 3$, each reproduce the observations (inset) with a standard error (0.01 °C) close to the limit of uncertainty of the measurements, and the case $n = 4$ is only slightly worse. Although it is not possible to select from among them, all of the models clearly indicate that the climatic warming is essentially a product of the last 100 years.

In our search for a cause of this secular change, it would be useful to know the history of the unbalanced flux, $F(t)$, at the surface, and it is seen from Eq. (18a) that each of the models has quite different consequences in this respect. The case $n = 0$ implies a very large initial flux imabalance that subsequently decays as $t^{-½}$; $n = 1$ implies a constant flux imbalance throughout the period of surface warming, and larger values of n imply that $F(t)$ increased progressively during the event.

For the case $n = 1$ (Figure 2), it is seen that ΔT_0 is slightly greater than 2 °C and that $t_0 \sim 65$ years (equivalent to $\alpha t_0 \sim 2.5 \times 10^7$ cm²). Using the measured conductivity value ($K \simeq 7 \times 10^{-3}$ cal/cm s °C), it is seen from Eq. (18b) that the observed effect could be accounted for by a uniform heat imbalance F of about 80 cal/cm² per year for the past six decades. Thus, it is remarkable that this temperature anomaly, conspicuous to a depth of 150 m, results from a cumulative input of heat at the earth's surface sufficient only to melt 0.6 m of ice since the turn of this century.

The foregoing numerical example serves to illustrate the potential value of analysis of permafrost temperatures to the study of secular changes in heat balance at the earth's surface. A straightforward method of reducing ambiguity (of the second type) in determining the form of $F(t)$ is illustrated in Figure 3. Here data are presented from the well designated as "Barrow" (Figure 1), where temperatures were monitored for a period of 8 years.[72,73,75] This permitted a determination of both the temperature anomaly $\Delta T(z, t_0)$ and its time derivative:

$$\left.\frac{d\Delta T(z,t)}{dt}\right|_{t=t_0}.$$

It can be shown from differentiation of Eq. (17) that the ambiguity in the determination of n is sharply reduced if both the temperature anomaly and its time derivative are constrained by depth observations. The dots in Figure 3a represent the empirically determined values of the temperature anomaly and its time derivative; the solid curves represent the corresponding calculated values for the temperature history shown in Figure 3b. Although, as in the previous example, several values of n [Eq. (17)] described the temperature anomaly, a specific history rather like n=3 or the broken line illustrated was required to account for the combined data. It is seen that the broken line (Figure 3b) provides an acceptable fit, implying a substantial increase in F

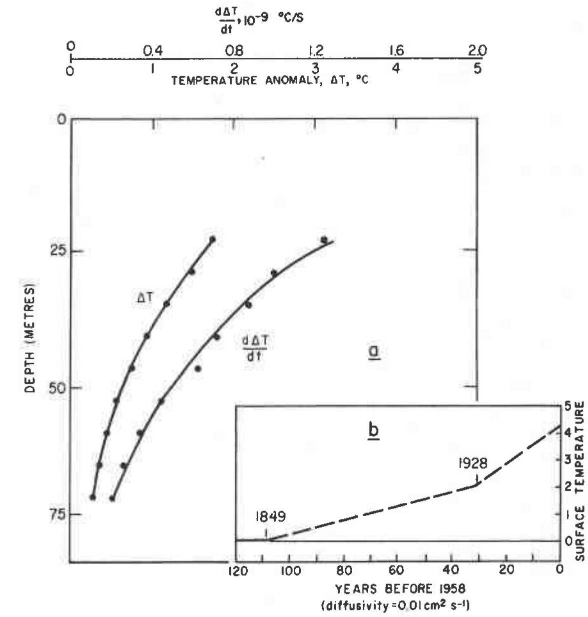

FIGURE 3 Climatic temperature anomaly at Barrow, Alaska. Dots in *a* represent observed values of anomaly (ΔT) and its observed rate of change. Solid lines represent corresponding computed values for the temperature history at the surface shown in *b*.

during the climatic event. In analyses of these second-order effects, it is important that the medium be homogeneous or that the variation in properties with depth be known.

For each of the stations represented in Figure 1, a more recent secular cooling is indicated by near-surface observations, but the data are partially obscured by seasonal variations, and no new principles would be illustrated by discussing them. It was pointed out in an earlier discussion that the difference in gradient at the four stations is a result of the difference in thermal conductivity, K, at each; those with the greater K have the smaller G. This observation, obtained from the problem of steady flow, is qualitatively consistent with the present discussion if the climatic changes at each are roughly synchronous. It is seen from Figure 1 that the maximum depth of the transient disturbance $\Delta T(z, t_0)$ increases from station to station as the gradeint decreases, i.e., as K increases. It is seen from Eq. (17) that the depth of the disturbance is determined by the value of the quantity $\frac{z}{\sqrt{\alpha t_0}} = \frac{z\sqrt{\rho c}}{\sqrt{K}\sqrt{t_0}}$. The quantity ρc varies little among permafrost materials and, hence, the depth z at which the climatic disturbance vanishes should increase with $(K)^{½}$. As q^* is relatively uniform, the depth should vary as $G^{-½}$ [Eq. (1)] approximately as observed (Figure 1).

As an illustration of the first type of ambiguity in the inverse problem, we consider the important problem of estimating the age of permafrost from a temperature profile.

Here the difficulty is distinguishing between the steady and transient contributions [first and third terms on the right in Eq. (15)] to observed gradients in profiles that are nearly linear deep in the permafrost. To take an extreme example, suppose permafrost is generated in a time interval t_0 from an initial state in which the mean surface temperature $(-T_0)$ [Eq. (5)] was 0 °C. If we assume that the surface temperature decrease was the step function $n = 0$, then Eq. (17) yields:

$$\Delta T(z, t_0) = \Delta T_0 \, erfc \frac{z}{2\sqrt{\alpha t_0}}, \; t = t_0 \qquad (19a)$$

$$\simeq \Delta T_0 \left[1 - \frac{z}{\sqrt{\pi \alpha t_0}}\right], \; \frac{z_p}{2\sqrt{\alpha t_0}} < 0.3. \qquad (19b)$$

In this example, ΔT_0 is, of course, a negative quantity. Substituting Eq. (19b) in Eq. (15) yields for an observation at $t = t_0$:

$$T(z, t_0) \simeq \left[\frac{q^*}{K} - \frac{\Delta T_0}{\sqrt{\pi \alpha t_0}}\right] z + \Delta T_0, \; \frac{z_p}{2\sqrt{\alpha t_0}} < 0.3. \qquad (20)$$

Similar results apply for other values of n. Hence, long before thermal equilibrium is approached in permafrost, the transient disturbance [represented by the second term in brackets, Eq. (19b) and (20)] becomes linear, and we no longer have the curvature to distinguish its effect from the steady flux. Denoting by M the fractional departure from equilibrium at the ultimate base of permafrost (z_p), we obtain from Eq. (19b):

$$M \equiv 1 - \frac{\Delta T(z_p, t_0)}{\Delta T_0} \simeq \frac{z_p}{\sqrt{\pi \alpha t_0}}$$

$$t_0 \simeq \frac{z_p^2}{\pi \alpha M^2}, \text{ provided } M \lesssim 0.3, n = 0$$

$$\simeq \frac{2.5 \times 10^3}{M^2} \text{ years at Prudhoe Bay.}$$

Thus at Prudhoe Bay where the profile is linear, we cannot distinguish between a permafrost age of 25 000 years (M = 30 percent) and one of 250 000 years (M = 10 percent). The best way to resolve this ambiguity is to compare the computed heat flow in permafrost with values determined at great depths, or at nearby locations where the properties are different.[55,95] On the basis of fragmentary evidence of this sort, and when the effects of latent heat are considered, it seems likely that permafrost in the Prudhoe Bay area has been continually frozen on the order of 10^5 years or longer.

If a climatic warming results in a change from $-T_0 <$ 0 °C to $-T_0 + \Delta T_0 > 0$ °C, permafrost will be unstable, but the history of its disappearance will be extremely sensitive to the distribution of moisture content with depth and local variability of ΔT_0. Melting surfaces will encroach on the transient permafrost from above and below, and it will soon become isothermal at its melting point. The length of time the permafrost will persist can vary greatly from place to place, depending on the amount of latent heat that must be supplied from below by q^* and from above by $F(t)$. The result is discontinuous permafrost that is at, or close to, 0 °C at all depths (Figure 3),[70] a condition produced throughout much of the subarctic by recent climatic warming. Under these conditions, moving groundwater can become an important heat transfer agent, and the applicability of conduction theory is limited.

Lateral Variations in the Surface Temperature

To this point, we have considered only one-dimensional problems, i.e., permafrost temperatures have been treated as if they depend only on distance beneath the ground surface. Clearly, this can be true only if the temperature over the earth's surface at any time can be considered uniform for lateral distances equal to several times the permafrost depth. It has been emphasized above that the temperature at the earth's solid surface is sensitive to moisture conditions, thermal properties, and geometric parameters of the surface and near-surface materials. Thus, the mean annual temperature and the thermal response of the surface to the passage of the seasons can vary significantly in a restricted locality as one passes laterally from dry to wet habitats, gravel beaches, ponds, lakes, rivers, or bodies of salt water.[12,13] Even at a fixed point on the surface, these conditions might change significantly with time, e.g., as plant communities change or as shorelines shift, river channels migrate, or lakes are drained by thermal erosion of ice wedges. In general, engineering modifications of the surface, such as clearing timber, compressing the tundra mat, or constructing a heated building, roadway, or airstrip, also cause perturbations to the temperature over restricted areas of the earth's surface.[39]

If we are interested in the temperature 50 m beneath a lake basin at a point 500 m from the shoreline, the effects of lateral heat flow to the cold shoreline will be negligible, and the one-dimensional considerations previously discussed will suffice; e.g., a rapid draining of the lake can be considered as a climatic change. However, if our concern is the effect of the lake on the depth of permafrost a few hundred metres beneath the same point, the cold shoreline and lateral heat flow will play an important role, and a two- or three-dimensional model will be indicated. Many important natural phenomena—such as the orientation of ice-wedge polygons,[69] the distribution of permafrost beneath the ocean's edge, or the formation of lake basins and pingos—cannot be understood without a consideration of the controlling two- or three-dimensional heat conduction problems. The same can be said of many engineering problems, such as the distribution of water supplies in continuous permafrost areas or the thaw settlement of a gravel pad or heated building.

As in the previous problems, a simple heat conduction model describes the gross aspects of this problem and provides physical insight into complex thermal conditions. It will be useful to consider the surface temperature to have a normal or ambient regime described by two parameters—the mean annual temperature, $-T_0'$, and the amplitude, A_0, of seasonal fluctuations, which is assumed to be sinusoidal. Then, the normal temperature at any time and depth will be given by Eq. (10). Assume that at $t = 0$ a disturbance originates in a region (R) of the surface and that it shifts the mean annual temperature by δT_0 and the amplitude by δA_0. Then, the temperature at the surface ($z = 0$) is described by:

$$T(x,y,0,t) = -T_0' + \delta T_0 + (A_0 + \delta A_0)\sin\frac{2\pi t}{P_y}, \qquad (x,y,0) \text{ in } R,\ t \geqslant 0 \tag{21a}$$

$$= -T_0' + A_0 \sin\frac{2\pi t}{P_y}, \qquad (x,y,0) \text{ not in } R,\ -\infty < t < +\infty; \quad (x,y,0) \text{ in } R,\ t < 0. \tag{21b}$$

These perturbations at the surface in R will result in a disturbance of the normal temperature given by Eq. (10) at any point (x,y,z) beneath the surface. (The effects of temperature in R varying linearly with time are discussed by Birch.[8]) The mean shift at the surface δT_0 will cause a similar shift at any subsurface point denoted by $\delta T(x,y,z,t)$ and referred to as the "principal disturbance." The amplitude change at the surface δA_0 will cause a periodic disturbance at depth denoted by $\delta\theta(x,y,z,t)$ and referred to as the "seasonal disturbance." (The special case of no seasonal change in R, as in a permanently ice-covered lake or thermostatically controlled building is obtained by setting $\delta A_0 = -A_0$.)

Discussion of the seasonal disturbance can be found elsewhere.[67] It is much less important in general considerations than the principal disturbance or mean shift, which for homogeneous materials is given quite generally by [ref. 67, Eq. (26)]:

$$T(x,y,z,t) = \frac{\delta T_0}{2\pi}\iint_R \left[\frac{r}{\sqrt{\pi\alpha t}}e^{\frac{-r^2}{4\alpha t}} + erfc\,\frac{r}{2\sqrt{\alpha t}}\right] d\Omega. \tag{22}$$

Here, $d\Omega$ is the element of solid angle subtended at the point of interest (x,y,z) by an element of surface area $dxdy$ in R. The distance between this surface element and the field point (x,y,z) is denoted by r. Equation (22) leads to simple integrated results for regions, R, of the polar coordinate system[67] (a circle, annulus, sector, etc.) and for a two-dimensional half plane or strip.[66] They give the distribution of temperature in time and space beneath lakes, buildings, ocean shorelines, or linear features, such as rivers, roadways, and gravel spits.

If the lateral dimensions of the disturbed surface region, R, are small relative to the distance r of the point of interest (x,y,z), then r can be represented by its mean value over R and:

$$\delta T(x,y,z,t) \simeq \left[\frac{r}{\sqrt{\pi\alpha t}}e^{\frac{-r^2}{4\alpha t}} + erfc\,\frac{r}{2\sqrt{\alpha t}}\right]\delta T_0\,\frac{\Omega}{2\pi}, \tag{23}$$

where Ω is the solid angle subtended by R at (x,y,z). If R is too large relative to r to permit Eq. (23), it can always be broken into subregions R_i, $i = 1, 2, 3 \ldots$ that will permit it. There are two other valid reasons for breaking the disturbed surface region into subregions: if parts, R_i, have different mean temperatures $\delta_i T_0$ (e.g., a furnace room of a heated building, shallow and deep parts of lakes, etc.) or if the boundaries of R have been changing so that in each subregion, R_i, the disturbance $\delta_i T_0$ has been in existence a different time t_i (e.g., a migrating river channel, transgressing sea, or a wing added to a heated building). Therefore, Eq. (22) leads to the extremely useful and general result:

$$\delta T(x,y,z,t) = \sum_i \left[\frac{2}{\sqrt{\pi}}\beta_i e^{-\beta_i^2} + erfc\,\beta_i\right]\frac{\Omega_i}{2\pi}\,\delta_i T_0, \tag{24a}$$

where $$\beta_i = \frac{r_i}{2\sqrt{\alpha t_i}}. \tag{24b}$$

The expression in brackets is a transient factor representing the fractional approach to equilibrium of the disturbance in R_i at the point (x,y,z) at time t_i. Effects are negligible (i.e., the factor is $\lesssim 0.05$) for $\beta \gtrsim 2$ and close to equilibrium (factor $\gtrsim 0.95$) for $\beta < 0.5$ (ref. 67, Figure 22). The equilibrium result ($\beta \to 0$) is well known from potential theory:

$$\delta T(x,y,z) = \sum_i \frac{\Omega_i}{2\pi}\,\delta_i T_0. \tag{25}$$

Analytical results for Ω are available for the rectangle [see, e.g., ref. 67, Eq. (32)], as well as the strip and half plane and for figures in polar coordinates. The effects of arbitrary regions can be approximated by combining them.[24,28,66,67]

Figure 4 illustrates the effect, δT, of three important types of anomalous surface regions. The mean annual temperature on the sea bottom beyond the shore-fast ice (2-3 m isobath) is generally close to 0 °C at high latitudes. During the ice-covered period, it is close to the freezing point of seawater (approximately −1.8 °C); during the short ice-free period, however, it can be much larger (up to 10 or 12 °C;

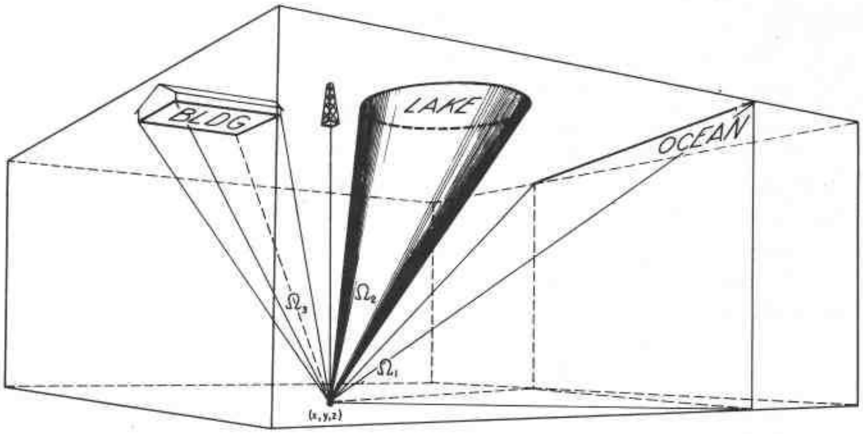

FIGURE 4 Representation of the solid angle, Ω, that determines the disturbance at any point (x,y,z) caused by thermally anomalous surface features. See Eq. (24) and (25).

Brewer, unpublished). The mean annual value at Barrow has been estimated at about $-\frac{1}{2}$ °C[66] and at Cape Thompson, farther south, it is probably about $+\frac{3}{4}$ °C[74] (see inset Figure 1). Thus equilibrium permafrost would extend out under the ocean at Barrow, and, at Cape Thompson, it would not. As the mean annual temperature of the emergent surface at high latitudes is on the order of -5 to -15 °C, $\delta_1 T_0$ (Figure 4) is of this magnitude, and the effect of the ocean can be quite large at points (x,y,z) where it subtends a large solid angle Ω_1. Near Barrow, the measured permafrost temperatures were found to be consistent with Eq. (25), indicating that the shoreline has been relatively stable there in recent times.[73] Near Cape Thompson, however, temperatures are too low for equilibrium conditions, and application of the transient theory[74] yields an estimate for the rate of shoreline transgression of the past few 1 000 years. Disequilibrium was indicated also near aggrading shoals at the mouth of the Mackenzie River.[78]

Lakes whose depth exceeds the maximum fresh-ice thickness (∿ 2 m at high latitudes) must have positive mean bottom temperatures (in °C). They are underlain by perennially thawed sediments and, like the oceans, are associated with a large surface anomaly, $\delta_2 T_0$. Shallower lakes that freeze to the bottom typically have much lower mean temperatures and produce smaller anomalies.[13] As with the ocean, the history of lakes can be studied by analyzing their transient anomalies in nearby boreholes. Two lakes studied at Barrow and one in the Mackenzie Delta were consistent with equilibrium theory.[56,73]

The mean temperature at the base of a heated structure is likely to be 5 or 10 °C greater than that of a deep lake, and the associated surface anomaly $\delta_3 T_0$ will be correspondingly larger. For the proportions shown in Figure 4, however, the effect of a building constructed in historic time would be negligible at (x,y,z) because β_3 [Eq. (24)] would be very large.

Effects of topographic relief (not shown in Figure 4) can also be represented simply with Eq. (24) by replacing the relief by an equivalent temperature disturbance on a plane surface.[8,71] When applying Eq. (24) to calculate the temperatures beneath the ocean, e.g., to calculate distribution of offshore permafrost,[66,73,81] it is usually more convenient to consider the emergent land to be the anomalous surface region.

Summary of Temperature Effects

In summary, the gross aspects of the thermal regime in permafrost at high latitudes (or very high altitudes[53]) can be described in terms of simple heat conduction models. They provide insight needed to understand the relation between the temperature and distribution of permafrost and the dynamic climatologic and geomorphic processes that modify the surface of the terrain in which it occurs. The more important of the foregoing results can be combined into a summary equation for the temperature, T, in continuous permafrost:

$$T = \frac{q^*}{K} z - T_0 + \Delta T_0 \left[\frac{i^n \, erfc \, \dfrac{z}{2\sqrt{\alpha t_0}}}{i^{n-1} \, erfc \, 0} \right]$$

$$+ A_0 \, e^{-z\sqrt{\frac{\pi}{\alpha P_y}}} \sin\left(\frac{2\pi t}{P_y} - z\sqrt{\frac{\pi}{\alpha P_y}} \right)$$

$$+ \sum_{i=1}^{N} \left[\frac{r_i}{\sqrt{\pi \alpha t_i}} \, e^{\frac{-r_i^2}{4\alpha t_i}} + erfc \frac{r_i}{2\sqrt{\alpha t_i}} \right] \frac{\Omega_i}{2\pi} \, \delta_i T_0 . \qquad (26)$$

The first two terms represent effects of steady geothermal flux, q^*, to the surface whose long-term mean temperature is $-T_0$. The third term represents the important climatic change in the amount ΔT_0 of the last century (t_0). (For some purposes, it might be desirable to add a second term of this kind to account for the more recent secular cooling.)

The fourth term corresponds to seasonal fluctuations with surface amplitude A_0, and the fifth is the effect of N surface regions with anomalous temperatures $\delta_i T_0$ that have existed for a time t_i. The present mean surface temperature $(-T_0 + \Delta T_0)$ and A_0 may be estimated from surface observations. T_0 and q^*/K are obtained from temperature measurements in boreholes; K must be measured on laboratory samples or estimated from lithologic information. $\delta_i T_0$ is obtained from measurements or estimates of surface temperatures in anomalous regions. For superficial anomalies, it should be referred to the present mean surface temperature $(-T_0 + \Delta T)$; for effects at depths little affected by the recent climatic change, it should be referred to the previously existing long-term value $-T_0$. A trivial refinement of the fourth term in Eq. (26) can account for the two- and three-dimensional effects of secular change in the rare cases where it might be necessary [see, e.g., ref. 66, Eq. (8)].

Setting $T = 0\ ^\circ\mathrm{C}$ in Eq. (26) and replacing the sine factor in the second term by unity yield an implicit equation for the boundary surface of permafrost. A schematic illustration of such a surface is presented in Figure 5 for a case in which the mean sea bottom temperature is greater than $0\ ^\circ\mathrm{C}$.

Thermal Properties

In the preceding section, the general relations for temperature in permafrost contain two basic thermal parameters—ρc, the volumetric specific heat, and K, the thermal conductivity. These quantities are the subject of a very extensive literature, and we shall consider here only a few aspects, more or less unique to permafrost. In general, they relate to the presence of water and ice close to their transition temperature. When water changes to ice, its conductivity increases by a factor of 4, its volumetric specific heat decreases by a factor of ½, and it releases enough heat to raise the temperature of an equal volume of rock by 150 °C. Because of this behavior and because many thermal and mechanical problems in permafrost involve freezing and thawing, water content plays an important role in any thermal considerations of frozen and thawed earth materials.

The volumetric specific heat, ρc, is a measure of the amount of heat that must accumulate in a unit volume to produce a unit increase in temperature (ρ is the bulk density and c is the mass specific heat). For an aggregate of N constituents with constant phase composition, it is simply the sum of the values for each constituent $(\rho c)_i$ weighted by their respective volume fractions Φ_i; thus:

$$\rho c = \sum_{i=1}^{N} (\rho c)_i \, \Phi_i. \tag{27}$$

It is convenient that practically all common rock-forming minerals, including ice, have a value of ρc within 10 or 15 percent of 0.5 cal/cm^3 °C.[4] Water, of course, has a volume specific heat of 1.0 cal/cm^3 °C. If we denote the volume fraction of water in thawed saturated earth materials by Φ, Eq. (27) yields:

$$(\rho c)_t \simeq \frac{1}{2}(1 + \Phi), \tag{28}$$

and for the same material completely frozen

$$(\rho c)_f \simeq \frac{1}{2}. \tag{29}$$

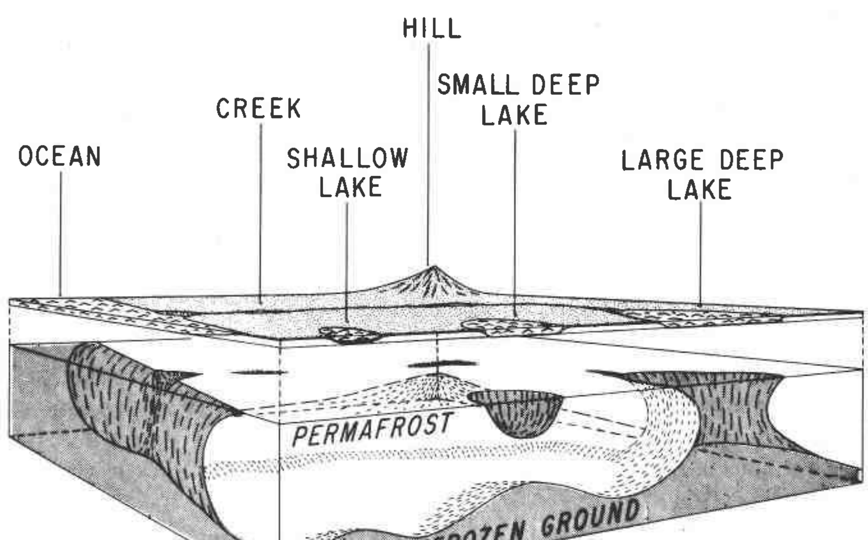

FIGURE 5 Schematic representation of permafrost distribution in a continuous-permafrost region where the mean ocean bottom temperature is greater than 0 °C. See Eq. (26).

It will be understood that the units of ρc in these and subsequent relations are cal/cm^3 °C. The subscript "t" will refer throughout to materials that are thawed and "f" to those that are completely frozen. Thus, for completely frozen or thawed materials, the volumetric specific heat is determined adequately for most thermal calculations by a knowledge of the moisture content alone. However, if the interstitial water is saline, or if the permafrost is so fine grained as to subject appreciable volumes of water to the effects of surface forces, it may contain temperature-dependent quantities of unfrozen water. In this case, Eq. (29) might be unsatisfactory.[76] This is primarily because ρc is defined in terms of a temperature change, which for such continuous two-phase systems involves a temperature-dependent contribution of latent heat.

The thermal conductivity, K, is a measure of the rate at which heat is driven through a medium under a unit thermal gradient. It is commonly measured either by the steady state ("flux-plate" or "divided-bar") method or by the transient ("probe" or "line-source") method. In the steady method, the temperature drop across a layer (or disk) of material of known conductivity is compared with that across an overlying layer of the unknown, under conditions of steady, linear flow perpendicular to the layers.[5,9,96,97,115] The conductivity is then determined from Eq. (3). (A variant using radial geometry was employed by Kersten.[62]) In the transient method, a long thin probe, containing an axial heater filament and a temperature sensor, is inserted in the medium of unknown conductivity. Heat is supplied to the filament at a constant rate, and the conductivity is determined by comparing the observed temperature rise with the appropriate radial heat conduction model.[85,108,113,116] The theory of the steady method is simple, but maintaining linear flow (i.e., eliminating radial losses) presents operational difficulties. By contrast, the theory of the transient method may be complicated, but it is relatively simple in operation. The transient method is generally preferred for *in situ* measurements in permafrost because of its convenience.[65]

It is important that, unlike the volume specific heat, the thermal conductivity does not require a changing temperature for its operational definition. (There is no steady-state method for measuring specific heat.) Thus, where temperature-dependent quantities of unfrozen water occur in permafrost, the transient method can give erroneous results, but the thermal conductivity of the system can still be defined uniquely by the steady technique.

Many formulas have been proposed to account for the thermal conductivity of an aggregate in terms of the constituent conductivities K_i and their respective volume fractions Φ_i.[83,116] Some are very cumbersome, and none has a completely satisfactory theoretical rationale. The major problem arises when the constituent conductivities K_i contrast by several orders of magnitude. Such is the case, for example, in an undersaturated soil containing appreciable volumes of air and highly conducting mineral grains. In a saturated soil, however, the maximum contrast is less than a factor of 20. It has been verified experimentally[96,117] that under these conditions no formula is more successful than the simplest one of all, the weighted geometric mean:

$$K = K_1{}^{\Phi_1} K_2{}^{\Phi_2} K_3{}^{\Phi_3} \; \ldots . \qquad (30)$$

If we denote the geometric mean conductivity of all the mineral constituents by K_r and the thermal conductivity of water by K_w (about 1.3 mcal/cm s °C near 0 °C), then Eq. (30) yields for saturated thawed material:

$$K_t \simeq K_w^{\Phi} K_r^{1-\Phi}$$
$$= (1.3)^{\Phi} K_r{}^{(1-\Phi)} \text{ mcal/cm s °C.} \qquad (31)$$

(The units of conductivity will not be repeated in the dimensional relations that follow.) Taking the conductivity of ice as 5.2 cal/cm s °C and neglecting the change in volume on freezing, the same material when completely frozen is described by:

$$K_f \simeq (5.2)^{\Phi} K_r{}^{(1-\Phi)}. \qquad (32)$$

The change in the value of Φ on freezing, of course, can be accommodated if the required precision warrants. Because the conductivity is defined by a steady-state process, it is also possible, in concept, to account for a volume fraction of unfrozen water $\Delta\Phi$ co-existing with a volume of ice $\Phi - \Delta\Phi$:

$$K_{f,t} = (5.2)^{\Phi - \Delta\Phi} (1.3)^{\Delta\Phi} K_r{}^{1-\Phi}, \qquad (33)$$

where the subscript "f,t" denotes that the water occurs in both states. It is assumed in Eq. (33) that the conductivity of water is not significantly influenced by the forces that depress its freezing temperature. It can be shown that Eq. (33) applies reasonably well to the interesting recent results of Penner.[85]

From Eq. (31) and (32), K can be approximated for saturated frozen and thawed earth materials from a knowledge of Φ and K_r, quantities that are normally easier to measure or estimate than natural state conductivity. K_r can be determined from Eq. (30) by analysis of the mineral constituents of a dry sample or drill cuttings and reference to tabulated conductivity values.[31,33,47,94] Common mineral conductivities vary from about 3 mcal/cm s °C for some micas and glass to about 18 for randomly oriented quartz. Values of K_r typically range from about 5 to 15.

Combining Eq. (31) and (32) yields the useful simple result, depending only on moisture content:

$$\frac{K_f}{K_t} \simeq 4^{\Phi}. \qquad (34)$$

As an example of its application, let G_f denote the geothermal gradient in permafrost and G_t the gradient in the material immediately beneath its base. We assume that both gradients are uniform and that the difference between the materials results from the state of the water in each. Thus, from Eq. (3):

$$q^* = K_f G_f = K_t G_t. \tag{35}$$

Applying Eq. (34) yields an expression for Φ in terms of the observable gradient ratio:

$$\Phi = (\ln 4)^{-1} \ln \frac{G_t}{G_f}$$

$$= 0.72 \ln \frac{G_t}{G_f}. \tag{36}$$

In the profile illustrated for Prudhoe Bay (Figure 1), the gradient changes at a temperature corresponding approximately to the pressure-melting point of water in a hydrostatic system. Presumably, this gradient change is of the type described in Eq. (36). Substituting the observed value, $G_t/G_f \simeq 1.8$ yields $\Phi \simeq 42$ percent, which is consistent with fragmentary information obtained from frozen cores at this location. With this information, an analysis of K_r from drill cuttings leads to estimates of K_t and K_f [Eq. (31) and (32)] and q^* [Eq. (35)].

It has been shown that K and ρc frequently appear in the combinations $\alpha = K/\rho c$ and $\sqrt{K\rho c} \equiv K\sqrt{\alpha}$. The first is the thermal diffusivity, important in transient calculations, as it governs the rate of propagation of a temperature disturbance through a medium. The second is variously referred to as the "thermal inertia," "contact coefficient," or "conductive capacity." It governs the heat flux across a surface on which the temperature conditions are prescribed [Eq. (18)] or the interaction between dissimilar layers in transient problems [Eq. (12)]. Combining Eq. (28), (29), (31), and (32) yields useful approximations for these quantities for materials in the common saturated condition:

$$\alpha_f \simeq 2K_f \text{ cm}^2/\text{s}, \tag{37a}$$

$$\alpha_t \simeq \frac{2K_t}{1+\Phi} \text{ cm}^2/\text{s}, \tag{37b}$$

and

$$\frac{\alpha_f}{\alpha_t} \simeq (4)^{\Phi}(1+\Phi). \tag{37c}$$

It is noted that the contrast in diffusivity [Eq. (37c)] is even greater than the contrast in conductivity [Eq. (34)] when saturated materials freeze. Similarly:

$$(K\rho c)_f^{1/2} \simeq \left(\frac{1}{2}K_f\right)^{1/2}, \text{ cal/cm}^2\ {}^\circ\text{C s}^{1/2} \tag{38a}$$

$$(K\rho c)_t^{1/2} \simeq \left[\frac{1}{2}(1+\Phi)K_t\right]^{1/2}, \text{ cal/cm}^2\ {}^\circ\text{C s}^{1/2} \tag{38b}$$

and

$$\frac{(K\rho c)_f^{1/2}}{(K\rho c)_t^{1/2}} = \frac{2\Phi}{\sqrt{1+\Phi}}. \tag{38c}$$

It is seen that the thermal inertia is less sensitive to freezing than the conductivity. The ratio in Eq. (38c) governs the "heat-valve" effect wherein seasonally changing properties at the surface favor heat conduction out of the earth in winter to heat conduction into it in summer.

AREAS OF FUTURE RESEARCH

Methods of calculating ground temperatures for given boundary conditions are now relatively well developed (see Appendix). A major lack is detailed knowledge concerning the actual temperature distribution in permafrost regions and concerning the relative and absolute values of the climatological elements that control it. Obtaining this information is not so much a matter of research, which tends to provide detailed information at only a very small number of sites, but of the establishment of an adequate synoptic weather and ground temperature observation network. The importance of such a network in providing the appreciation of the permafrost condition required for engineering, land use, and scientific programs is often overlooked.

The ground thermal regime is dependent on the nature and characteristics of the surface cover. Little progress has been made in defining this dependence in a quantitative manner. Investigations should be undertaken to establish the relationship between the characteristics of the surface and near-surface material and the components of the surface heat exchange. Because the interest is primarily with respect to temperatures in the ground below the influence of diurnal variations, the relationships obtained should be for time periods of 1 week or more.

Man cannot operate in permafrost areas without affecting the surface in some way. It is imperative that a proper understanding be obtained of the consequences to the ground thermal regime due to such disturbances. Research is required to develop practical models for expressing these consequences in a quantitative way suitable for engineering and land use purposes. Such models would also be of great benefit to the scientist in his investigations of the mechanisms of heat and moisture transfer at the ground surface.

Ground temperature calculations depend on reliable values of the thermal properties of the ground. Measurements of these properties and of their temperature, ice, and water content dependence are required for many types of earth material.

A very challenging problem is to obtain evidence of past climate from ground temperature measurements. Good and numerous records are required for this method of investigating the past. The application of it to permafrost areas, where it has probably the best possibility of being successful, has only just begun to be exploited.

A knowledge of the steady flow of heat from the earth's interior is fundamental to estimates of the temperature and distribution of permafrost. Consequently, heat-flow measurements should be made whenever the opportunity arises until the regional trends in permafrost regions can be established.

Computerized numerical methods are proving to be powerful tools for modeling the ground thermal regime. Their development should be continued in parallel with complete and well-designed field observation programs. These methods should also be used for parametric investigations to help define the significance to the ground temperature of changes in the thermal properties of the ground and heat transfer characteristics of the surface. Such investigations would help clarify the factors determining the difference between the average annual air and ground surface temperatures.

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LIST OF CONVERSIONS

1 cal = 4.2 J
1 cal/cm^3 °C = 4.2 MJ/m^3·K
1 cal/cm·sec °C = 420 W/m·K
1 cal/cm^2·sec = 42 kW/m^2
1 cal/cm^2·sec$^{1/2}$ = 42 kJ/m^2·s$^{1/2}$

APPENDIX

COMPUTER SIMULATION

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For many problems of engineering or scientific interest, calculation of the ground thermal regime by analytic methods is either impossible or excessively cumbersome. In North America, the usual approach in such cases is to use computerized numerical methods.

Numerical models have been, and continue to be, developed by universities, government institutions, and industry. Recent interest in hot oil pipelines in the north has spurred efforts in the field of numerical modeling of the ground thermal regime, but much of this work remains proprietary. The following is an outline of the major considerations and problems relevant to modeling the ground thermal regime and the various numerical techniques used to treat them, illustrated by examples from recent literature.

Choice of the numerical method depends on the problem under consideration. For one-dimensional analyses, finite-difference methods are in common use. These fall into three categories: ordinary forward difference (explicit); backward difference (implicit); and alternating direction (explicit).

Although simple to program, the ordinary forward-difference method has a major drawback in that the time-steps chosen must be small enough to guarantee stable solutions. This requirement is often so stringent as to render the method inefficient for practical computations. This is particularly true when modeling surface boundary conditions of the heat-balance type encountered in geothermal problems. The method has been used, however, in a recent work by Dempsey[2] concerned with freezing and thawing in road pavements.

Backward-difference methods, although more complicated to program, are unconditionally stable. Longer time-steps may be used but since a greater number of calculations per time-step is required than for explicit methods, the computational advantage of implicit methods is not always ensured.

Alternating-direction methods are unconditionally stable and require few calculations per time-step. In the most widely known method of this type (that due to Saul'yev[12] and employed, for example, by Doherty[3] and Lachenbruch[9] in a model for the thermal regime around a buried pipeline in permafrost), accuracy considerations force the use of rather small time-steps. A new alternating-direction explicit method now being developed by the author retains the advantages of the Saul'yev method while giving an accuracy nearly identical with that of the implicit methods.

Although finite-difference methods, such as the alternating-direction implicit method of Peaceman and Rachford[11] used, for example, by Fleming[4] or the alternating-direction explicit method as used by Doherty,[3] are used for two-dimensional problems, the recent trend is toward the use of finite-element methods. In the latter, the differential equation and associated initial and boundary conditions are replaced by an equivalent variational problem, the solution of which may then be found approximately by numerical methods. The principal advantage of the technique, which is a consequence of the use of triangular-shaped node elements, is the ease, accuracy, and efficiency with which complicated geometrical boundaries can be treated. A recent program of this type, intended specifically for geothermal problems, is that of Hwang *et al.*[6] based on the fundamental work of Wilson and Nickell.[13]

Common to all models of interest to permafrost problems is some means of taking account of the effects of latent heat. The simplest approach is to assume that the phase change takes place over a small temperature range and introduce an effective heat capacity for the nodal volume undergoing phase change. Although easy to program, the method may require the use of a very fine space-grid in order to determine the position of a phase front with adequate accuracy. The time-step may then also have to be reduced to ensure that the phase boundary does not "miss" a node, thus creating an error in the results. With the time-step thus limited, there may be little or no advantage of an implicit over an explicit finite-difference formulation. The method produces distortion of the temperature field in the neighborhood of the phase boundary and keeping this within reasonable limits again requires suitably small space- and time-steps.

An example of an implicit one-dimensional formulation involving this type of latent heat treatment, but with the refinement of introducing a subgrid for evaluation of the effective heat capacity of the phase node, is the model of Nakano and Brown,[10] which was designed to study seasonal thaw in Fairbanks peat and silt. Another model using this approach is the one-dimensional simultaneous heat and mass transfer model of Harlan,[5] which was devised primarily for theoretical studies of moisture migration during soil freezing.

Although considerably more complicated to program much greater accuracy can be obtained for a given grid-spacing and time-step if, instead of introducing the artifice of an effective heat capacity for the node undergoing phase change, one models the actual boundary conditions at the moving phase boundary. This is achieved, for example, in the author's one-dimensional explicit finite-difference model, in the one-dimensional implicit model of Kazemi and Perkins,[7] and in the two-dimensional finite-element model of Hwang *et al.*[6]

All models mentioned either take account, or are potentially capable of taking account, of the effects of latent heat release over a temperature range, as occurs in fine-grained materials. Whether such refinement is either necessary or justified is debatable. Freezing in fine-grained soils in nature is usually accompanied by considerable moisture redistribution and volume changes whether or not one is dealing with an open or closed system; unless provision is made to properly account for the resulting heat transfer and modification to the thermal properties, the model is not physically consistent. Attempts to include some of the major effects of moisture redistribution during freezing of idealized theoretical materials have been made. The simultaneous heat and moisture transfer model of Harlan may be cited as an example.[5]

During thawing of permafrost materials, considerable consolidation may occur. Whether it is important to attempt to model such effects again depends on the problem to be considered. For example, if one intends to use the model for the optimum design of insulated roadways in which little or no thawing below the insulation is to be permitted, then thaw consolidation is not a consideration. On the other hand, models that attempt to describe the thermal regime around hot buried pipelines in permafrost most probably should take account of thaw settlement as well as movement of the pipe. Models that attempt to do this are under development, for example, the finite element analysis of Charlwood and Svec.[1]

All the numerical models mentioned are capable of handling space-, time-, and temperature-dependent thermal properties. Thermal conductivities for calculations are usually estimated from empirical formulas, such as those of Kersten[8] for inorganic clays and sands, assuming constant values of dry density and water content.

These formulas are incapable of accounting for the effects of ice inclusions and temperature-dependent phase composition that may be of major importance to temperature calculations in frozen permafrost materials. For thaw calculations in permafrost, however, this may be of little consequence, inasmuch as such problems are dominated by the heat flow in the thawed zone and are therefore relatively insensitive to changes in thermal properties of the frozen materials.

Although capable of handling a range of external boundary conditions, most thermal numerical models assume that surface temperature is a known function of time. Very often the assumption is made that surface temperature equals air temperature. Except for problems involving strong internal heat sources or sinks (hot or cold buried pipelines may be in this category), such an assumption may lead to errors that considerably outweigh those due to the approximate treatment of internal phenomena. A surface heat balance boundary treatment can be incorporated in most models, but adequate information on the various parameters involved and their variations with time is generally unavailable. This approach is attempted, for example, in the one-dimensional model of Dempsey for dry road pavements.[2]

For dry surfaces the difference between air and surface temperatures may be considerable, whereas wet surfaces more closely follow air temperatures, at least for time scales of the order of days or weeks, and, in this case, equating the two is not unreasonable. In view of the uncertainty of surface temperature values and since this is the single most important parameter in determining the near-surface ground thermal regime in many cases of interest, further refinement of the internal heat exchange aspects of numerical models is generally not warranted. For design calculations, it is seldom that one can do more than assume that surface temperature equals air temperature or some empirical function of it. This may nevertheless be adequate since such calculations are generally concerned with finding the response to possible extreme conditions.

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