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COOLING LOAD CALCULATIONS BY THERMAL RESPONSE FACTOR METHOD

BY

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CALCUL DE LA CHARGE THERMIQUE IMPOSEE AU SYSTEME DE CLIMATISATION SELON LA METHODE DE CALCUL DES FACTEURS DE LA REPONSE THERMIQUE

SOMMAIRE

Les auteurs présentent une méthode d'utilisation de l'ordinateur pour le calcul du diagramme de charge thermique d'une pièce climatisée et des températures des parois de la pièce. On peut également utiliser cette technique pour calculer la température de l'atmosphère de la pièce au cas où la charge thermique dépasserait la capacité du système de climatisation.

Les données relatives au bâtiment et celles concernant les conditions ambiantes sont complètement séparées, mais il est nécessaire de les exprimer sous forme de séries chronologiques. Les caractéristiques thermiques dynamiques du bâtiment sont exprimées par l'ensemble des facteurs de réponse du système et les auteurs exposent une méthode de détermination des facteurs de réponse du système constitué par la pièce dans un article complémentaire. Les données relatives aux conditions ambiantes comprennent les effets de l'heure, de la date, de la latitude et de l'orientation du bâtiment, et les caractéristiques de transmission calorique et adsorption du verre des vitres. Le calcul des éléments de la réponse du système aux conditions ambiantes ne nécessite que la multiplication de l'ensemble des facteurs de réponse par la représentation en séries chronologiques appropriées de l'environnement. Cette opération constitue une méthode numérique de transformation de convolution entre les fonctions de transfert du système et les facteurs d'excitation.

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Cooling Load Calculations by Thermal Response Factor Method

Computers are becoming indispensable tools in most fields of science and engineering. It seems appropriate, therefore, to re-examine traditional air-conditioning design procedures to discover which phases of the work can be done better by a computer.

When a computer is first considered, a designer naturally thinks of programming the machine to do the same calculations that he has previously done by hand. This, however, can yield only the advantage of speed over traditional calculations, and it is arguable whether this alone is worth the effort involved in preparing a program and data for machine computation of cooling load. Programs of this type have been prepared, but they are not widely used. One should not infer, however, that the computer has little or nothing to offer the air-conditioning designer; it has a great potential, but it must be used to answer the right questions.

What does the system designer need to calculate? First, he must find the heating and cooling load profile for design days at different times of the year. Air-conditioning handbooks indicate that the cooling load profile is quite different from the instantaneous heat gain profile because of heat storage in the internal parts of the building, but none of the hand calculation techniques can take full advantage of this phenomenon. Secondly, the designer should be able to predict the temperature of the room surfaces during the various design days, since comfort depends as much on surrounding surface temperature as on air temperature.

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Current design methods make the assumption that internal walls, floor, and ceiling temperatures are the same as the room air temperature. This is obviously not true for situations with radiant heat gains because the radiant energy cannot affect the room air directly; it must first heat opaque surfaces until they are warmer than the room air and thus can transfer heat to the room air by convection. This assumption is necessary for hand calculation, but unnecessary with a computer. Consequently, a computer can give a more accurate estimate of room conditions during peak cooling load periods.

It has been established that most people are reasonably comfortable when the comfort index (combination of room air temperature and surface temperature) is within a few degrees of the optimum. The designer should, therefore, try to provide a system that will never allow conditions to fall outside a specified comfort range rather than one that provides a constant air temperature. This approach has not been common until now because the prediction of a room comfort index for design conditions is impractical without a computer. With a computer it is entirely practical.

CHARACTERISTICS OF DIGITAL COMPUTERS

Digital computers can add, subtract, multiply, divide, and test whether a number is greater than, less than, or equal to some other number. More complicated operations like raising numbers to powers, evaluating special functions such as exponential or sine, and evaluating derivatives or integrals of functions can only be performed if the process can be approximated by a sequence of arithmetic operations. The best method of solving any problem on a digital computer is the one that requires the lowest number of arithmetic operations to obtain the required degree of accuracy.

Two numerical techniques are available for calculating transient heat conduction in building components: one using finite difference approximations of derivatives to convert differential equations into sets of algebraic equations; the other using the superposition principle and response factors. As finite difference calculations involve far more arithmetic than the response factor method, it is clearly advantageous to use the response factor technique. Response factors can only be used, however, if the system can be represented by linear and invariable equations. Mitalas[1] has shown that linear equations are adequate to describe the heat transfer processes that occur in an air-conditioned room.

PRINCIPLE OF SUPERPOSITION

The characteristics of any physical system can be described by giving the relation between an excitation function and the system's response to it. Such a relation is particularly useful if the system is linear and invariable. Linearity implies that the magnitude of the response is linearly related to the magnitude of the excitation; invariability means that equal excitations applied at different times always produce equal responses.

Linearity and invariability are prerequisites of any superposition method of calculating a system's performance, e.g. the Fourier series or Laplace transform methods. The superposition principle can be stated as, "When a number of changes are taking place simultaneously in any system, each one proceeds as if it were independent of the others, and the total change is the sum of the effects due to the independent changes." There are, therefore, three steps in calculating the response of a linear invariable system:

1. Resolve the excitation into a series of simple components.
2. Calculate the response for each component.
3. Add the responses for the separate components.

TIME - SERIES

A time-series is just a series of numbers or quantities representing the values of a function at successive equal intervals of time. For example, a table of numbers giving the outside air temperature every hour on the hour is a time-series representation of the outside air temperature with a time interval of one hour.

The main points about time-series are:

1. Any function of time can be represented by a time-series.
2. The resolution of a function into a time-series is very simple.

Each term in a time-series can be thought of as the magnitude of a triangular pulse centered at the time in question with a base equal to twice the interval between successive terms. The sum of such a set of overlapping triangles is a continuous function made up of straight line segments (Fig. 1). The accuracy of the representation depends on the size of the time interval between successive terms.

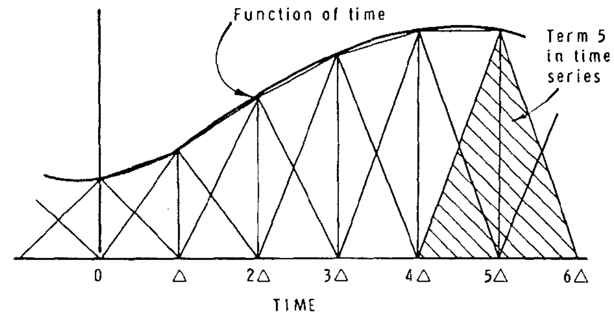


Fig. 1 Time-series representation of a continuous function

The concept of a time-series was first presented by Tustin,[2] in 1947. He showed that time-series are very similar to polynomials in that they can be added, subtracted, multiplied and divided. The commutative and distributive laws of ordinary arithmetic also apply for time-series.

RESPONSE FACTORS

The response of a linear, invariable system to a unit time-series excitation function (1, 0, 0, . . .) is called the unit response function, and the time-series representation of this unit response function is the set of response factors. For example: if the cooling load for a room subjected only to a unit time-series of solar radiation on the floor is as shown in Fig. 2, the cooling load response factors, r_j, for variations in solar radiation on the floor are the values of this unit response function at times $j\Delta$, $j = 0, 1, 2, 3, \ldots$ ($j = 0$ corresponds with the time of the peak in the unit time-series).

Thus, the cooling load resulting from this radiation input, I, is, in time-series form

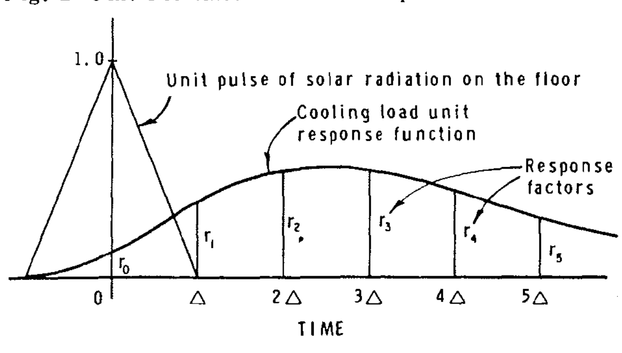
$$CL = R \cdot I \qquad (1)$$

The general term in the cooling load time-series is

$$cl_n = \sum_{j=0}^{\infty} r_j \cdot i_{n-j} \qquad (2)$$

Eq (2) is really the definition of the product of the two time-series on the right side of Eq (1).

Fig. 2 Unit excitation and unit response functions



In this paper capital letters indicate time-series; lower case with a subscript indicate particular terms in such a time-series. The subscript indicates the time in units of Δ from some arbitrarily assigned origin for time.

The response factor method of calculating transient heat flow was used first by Nessi and Nisolle [3]. They employed the superposition of unit step functions rather than triangular pulses. The first application of triangular pulses was by Hill, [4] in 1957. Just prior to that Brisken and Reque [5] presented a paper using the response factors due to rectangular pulses.

The differences between the present application of time-series and response factors and the Brisken and Reque approach are discussed in detail in Ref 6, which deals with the determination of room response factors. The response factor method is convenient for room thermal performance calculations because it requires only that the excitation functions be expressed as time-series, and it shows very clearly the influence of each excitation on the final result.

COOLING LOAD AND SURFACE TEMPERATURE CALCULATIONS

When a system response is calculated by an expression such as Eq (1), the data can be separated into two independent categories:

(1) the unit response functions for the room;
(2) the excitation functions.

The room response functions contain the pertinent information about the room: its size, type of construction, color, emissivity of the surfaces, etc. The excitation functions, on the other hand, take account of the geographical location and orientation of the room, the time of day and date, and the nature of the surroundings. The only exceptions to this subdivision are the excitation functions for absorbed and transmitted solar radiation. These take account of two aspects of the building design: the transmission and absorption factors for the window glass, and any outside shading that may be provided.

There are, obviously, as many sets of response factors as there are combinations of excitation functions and responses of interest. The response factors need to be determined only once for a particular room. They can then be combined with the appropriate excitation functions to obtain the response in a specific case. The ease of separating the room characteristics from the characteristics of the external environment is one of the major advantages of the method, for it permits the separate calculation of building and environmental data.

The response factor method becomes practical only when a computer is used to carry out the large number of multiplications and additions. But when a computer is available, this approach provides the information that the system designer really needs. The following example illustrates the results that can be obtained.

Example Problem

Determine the cooling load profile for August 1 for a west-facing office in a multi-story building at 40 deg North latitude. The room is 20 by 20 by 10 ft, with the whole outside wall of plate glass. The floor is a 6-in. concrete slab, and the ceiling a very light sheet suspended from the under side of the floor above. The partitions are 3-in. slabs of light-weight concrete. The room is bounded on both sides and above and below by identical rooms, and there is a corridor, which is kept at 75F, on the other side of the back wall. It may be assumed that all the room surfaces are "gray", with an emissivity of 0.9, and that furniture covers half the floor surface.

Base the excitation functions on the data in the cooling load chapter of the ASHRAE Guide And Data Book. The total energy input to the room through the lighting system is 7000 Btuh, half of which is transferred directly to the room air and the other half as radiation absorbed by the various room surfaces. The lights are on from 07:30 to 17:30, local solar time.

Solution

The time-series for the excitation functions, based on a 1-hr interval, are given in Table I. The conditions have been assumed to be periodic, with a fundamental period of one day. (Non-periodic conditions can be handled equally well.)

The cooling load response factors for the various excitations are given in Table II. These have been calculated by the procedure outlined in Ref 6. The room-surface temperature response factors were calculated at the same time as the cooling load factors, but they are so extensive that they have not been tabulated. Each component of the cooling load has been calculated for each hour of the day, and all the components have been added for the total cooling load profile. For example, the component of load at 16:00 hr due to radiation absorbed by the floor surface is

$$
\begin{aligned}
(\text{cooling load due to floor excitation})_{16} &= 17.7 \times 80.75 \\
&+ 18.0 \times 69.30 \\
&+ 15.7 \times 51.48 \\
&+ 14.1 \times 25.88 \\
&\quad\vdots \qquad \vdots
\end{aligned}
$$

An hour later it would be

$$
\begin{aligned}
(\text{cooling load due to floor excitation})_{17} &= 17.7 \times 64.67 \\
&+ 18.0 \times 80.75 \\
&+ 15.7 \times 69.30 \\
&+ 14.1 \times 51.48 \\
&\quad\vdots \qquad \vdots
\end{aligned}
$$

and so on.

The total cooling load for each hour is shown by the top curve on Fig. 3. This indicates that

Table I. Room Excitation Time-Series for 1-Hr Interval

Time	Window SAT F	Ceiling Btu/ft^2hr	Floor and Furniture Btu/ft^2hr	North End Wall Btu/ft^2hr	South End Wall Btu/ft^2hr	Back Wall Btu/ft^2hr	Air Temp Corridor and Room F
1	72.8						
2	72.8						
3	71.9						
4	71.0						
5	71.0						
6	71.9	0.76	0.76	0.82	0.82	0.62	↑
7	73.9	1.66	1.66	1.80	1.80	1.36	
8	76.6	3.36	9.86	3.56	3.56	2.94	
9	80.1	3.92	10.42	4.16	4.16	3.40	
10	83.4	4.32	10.82	4.60	4.60	3.72	
11	87.4	4.58	11.08	4.88	4.88	3.92	
12	90.4	4.66	11.16	4.97	4.97	4.02	75.0
13	99.6	5.13	25.88	7.99	5.48	4.41	
14	105.3	5.23	51.48	12.50	5.59	4.49	
15	110.0	5.14	69.30	15.50	5.50	4.42	
16	109.1	4.79	80.75	10.74	4.96	4.12	
17	105.2	4.02	64.67	4.27	11.02	23.73	
18	96.6	1.59	15.30	1.72	17.87	38.43	↓
19	83.1						
20	81.2						
21	79.3						
22	77.4						
23	75.6						
24	73.7						

Table II. Cooling Load Response Factors for Time Interval of 1 Hr

Excitation / j	Ceiling	Floor	End Wall	Back Wall	Furniture	Window	Room Air	Corridor Air
	ft^2					Btu/hr F		
0	152.5	17.7	90.7	87.4	134.5	142.6	-850.1	6.5
1	32.3	18.0	37.7	27.9	12.1	13.2	208.5	18.5
2	21.2	15.7	14.2	9.8	5.6	6.5	66.5	9.2
3	17.4	14.1	6.8	5.1	3.6	4.4	44.4	3.8
4	15.0	12.5	4.1	3.3	2.7	3.4	34.6	1.9
5	13.2	11.2	3.0	2.5	2.3	2.9	29.1	1.1
6	11.7	9.9	2.5	2.1	2.0	2.5	25.3	0.8
7	10.4	8.8	2.1	1.8	1.7	2.2	22.3	0.7
8	9.2	7.8	1.9	1.6	1.5	1.9	19.7	0.6
9	8.188	6.960	1.659	1.425	1.360	1.720	17.513	0.490
.								
.	Ratio of successive terms = 0.889							
.								

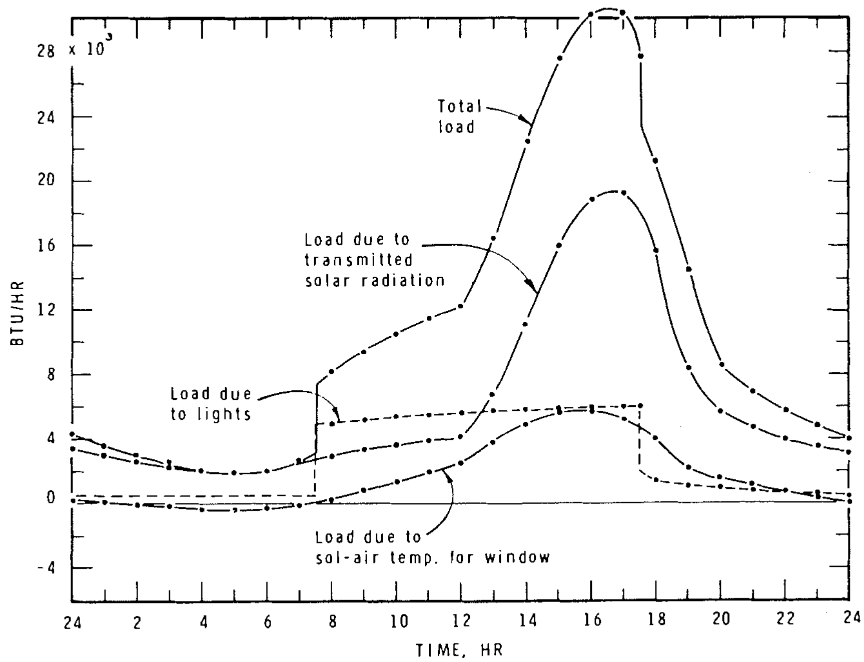


Fig. 3 Cooling load profile

the room needs a cooling unit with a capacity of 31,000 Btuh to keep the air temperature always at 75F. The other three curves on Fig. 3 show how much of the total load at any time is due to transmitted solar, lights, and the combined effect of solar energy absorbed by the glass and outside air temperature. The calculation did not include any component resulting from ventilation or occupants, but these can be computed in a straightforward way and added to the other components.

The curves in Fig. 3 show that much the largest portion of the peak cooling load is caused by solar radiation transmitted through the window. In this case it is particularly large because the whole wall is ordinary plate glass without any shading. The use of shading or heat-absorbing glass would alter the excitation functions, whereas changing the size of the window would change the room response factors.

Fig. 4 shows the temperature at some of the surfaces in the room. These were calculated in the same way as the cooling load, except that surface temperature response factors were used instead of cooling load response factors. The curves show the extent to which the temperature of room surfaces can deviate from the controlled room air temperature. It is obvious that occupants of this room would be uncomfortable during the afternoon in spite of the constant 75 F air temperature.

CALCULATION OF ROOM AIR TEMPERATURE

The response factor method can also be used to compute room air temperature if the capacity of

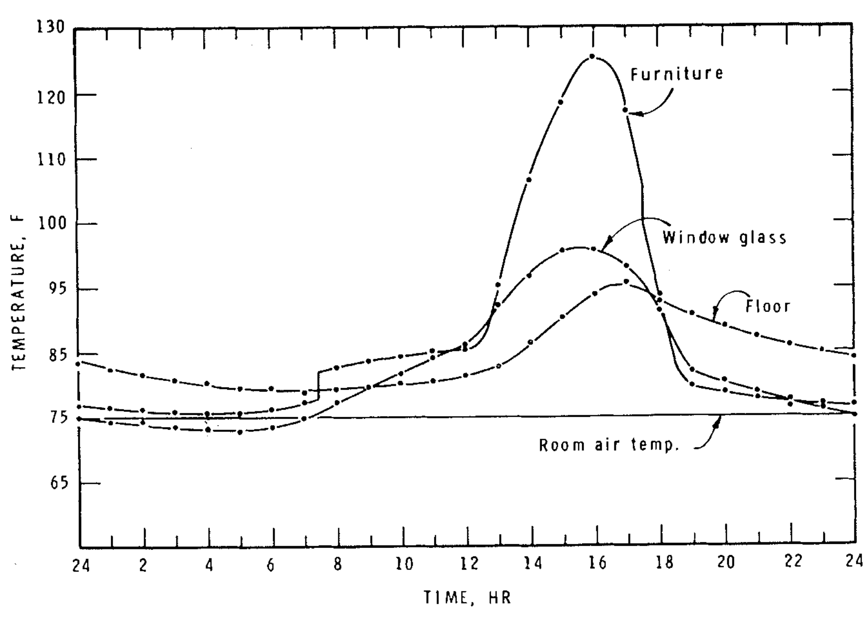


Fig. 4 Room surface temperatures

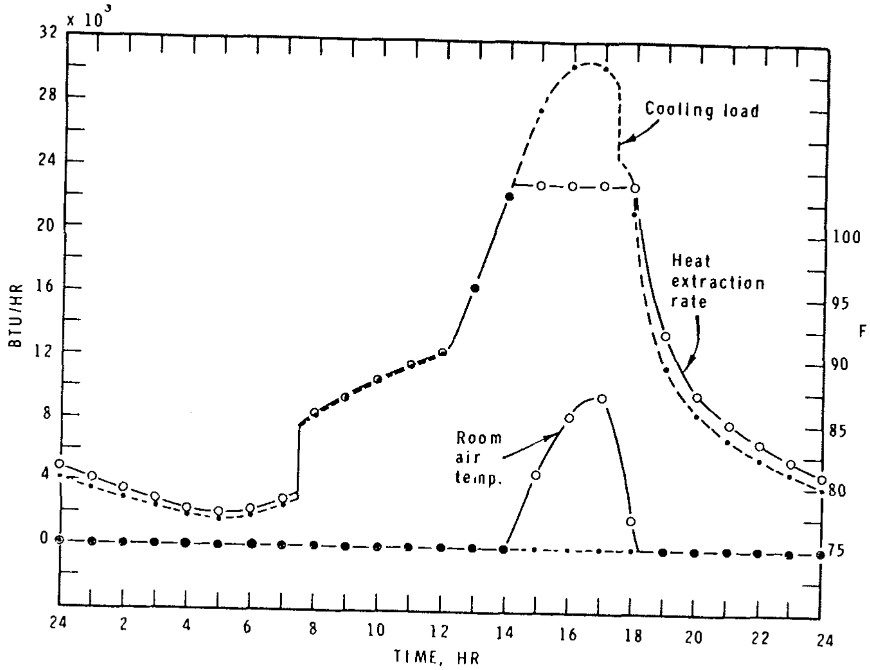


Fig. 5 Room air temperature when maximum heat extraction is less than peak cooling load

the cooling system is less than the peak cooling load or if the cooling system is shut off during some hours of the day. The data for this calculation are: the total cooling load, CL; the heat extraction, Q; and the cooling load response factors for a variation of room air temperature, designated P. The difference between the room air temperature and the constant value assumed for the cooling load calculation is θ. Each of these quantities is a time-series and the corresponding lower case letter with subscript indicates a term in the series.

Let

$$\theta_n^* = \frac{1}{p_o}\left\{q_n - cl_n - \sum_{j=1}^{\infty} p_j \cdot \theta_{n-j}\right\}$$

The deviation θ_n equals θ_n^* when the latter is positive, and $\theta_n = 0$ for all times when θ_n^* is negative if the cooling system is under thermostatic control. In the latter case it is assumed that the control system will adjust q_n to make the quantity inside the bracket zero if it can.

Fig. 5 shows how the air temperature would vary in the room considered in the previous example if the maximum extraction rate of the cooling system were only 75% of the peak cooling load. This type of calculation can be modified quite easily to allow for q_n as a function of θ_n if this relation is known for the room cooling unit.

ACCURACY.

No hard and fast statement can be made about the accuracy of the cooling load and surface temperatures calculated by the methods outlined above until they can be compared with values measured in an actual building. In the absence of test results, however, some indication of accuracy can be obtained by comparing results obtained by the response factor method with those obtained by other methods.

The cooling load for the room considered in the example was evaluated by two other methods: an analog computer simulation and finite difference calculations using a time increment of 1/2 hr. The response factor and analog results agreed to within about 1%, which is comparable with the accuracy of the analog components. The finite difference method gave a maximum cooling load about 10% less than that indicated by the other methods.

The following assumptions are implicit in all of these methods so that comparison in no way validates them:

1. Air temperature is uniform throughout the room.
2. The floor, ceiling, walls and windows are isothermal surfaces; no account is taken of deviations from one-dimensional heat flow near the intersections between various enclosing surfaces.
3. Convective heat transfer coefficients are constant over each isothermal surface.
4. Furniture has no heat storage capacity.

The only significant variation among the three methods is in the way they allow for transient heat conduction. The close agreement between the results obtained by the response factor method and the analog computer lends confidence that both methods are reasonably good in allowing for transient heat conduction. It is reasonable also to assume that the finite difference approximations for the partial derivatives lead to some error, and these results suggest that the finite difference results err on the non-conservative side. In any case, computation required by the finite difference method is so much greater than that required by the response factor method that there is no reason to use finite differences except when the system is non-linear.

CONCLUSION

The time-series and response factor techniques make it practical to use digital computers in air-conditioning calculations. The results are as accurate as those obtained by analog computer; but experimental work is needed to confirm the aptness of the assumptions involved in all methods of computing cooling loads and surface temperatures.

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DISCUSSION

CHAIRMAN STOECKER: We will hold discussion on this paper until after we have had a chance to hear the companion paper, and then we will permit the authors to answer the questions and comments together. (The companion paper follows. Ed.)