Method of analysis of structure-ground interaction in earthquakes

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Publisher's version / Version de l'éditeur:
https://doi.org/10.4224/40001193
Technical Paper (National Research Council of Canada. Division of Building Research), 1971-04

METHOD OF ANALYSIS OF STRUCTURE-GROUND INTERACTION IN EARTHQUAKES

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DBR NRC

Ottawa April 1971

Price 75 cents NRCC 11920

METHOD OF ANALYSIS OF STRUCTURE-GROUND INTERACTION IN EARTHQUAKES

SUMMARY

An equivalent single-degree-of-freedom model is utilized to determine relative displacements and overturning moments under earthquakes for single-storey structures on an elastic ground. The important structural parameters are identified from a parameter study, and extensions of the method to similar structural configurations are suggested.

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MÉTHODE D'ANALYSE DE L'ACTION RÉCIPROQUE ENTRE LES STRUCTURES ET LE SOL DURANT LES SEISMES

SOMMAIRE

Un modèle équivalent à un seul degré de liberté est utilisé pour déterminer les déplacements relatifs et les moments de renversement sous les charges sismiques pour les structures à un seul étage sur un sol élastique. Les paramètres importants de structure sont identifiés par une étude des paramètres et une extension de cette méthode à des configurations structurales semblables est proposée.

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Technical Paper No. 340
of the
Division of Building Research

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ABSTRACT

A method of analysis is presented for determining elastic structure-ground interaction effects of single-storey structures under earthquake loads. An equivalent single-degree-of-freedom (S.D.F) model is derived, by means of which relative displacements and overturning moments can be found directly from response spectra of known earthquakes or from other derived or assumed response spectra. The method is illustrated by sample calculations of responses for the interaction system, the equivalent S.D.F. model, and an S.D.F. system merely with reduced natural frequency.

A study for a wide range of parameters under earthquake-type disturbances establishes that, for the interaction systems considered, inter-storey displacements are reduced but overturning moments may increase relative to an S.D.F. system with identical fundamental frequency and inter-storey damping.

Although the results are obtained specifically for circular bases and frequency-dependent foundation parameters, the method is shown to be applicable to bases with other geometries and stiffness properties.

Comparisons are made of the magnitudes of resonance peaks for relative displacement from the "exact" theory employed here and from approximate theories developed by Balan et al (21). An approximate expression for finding overturning moments in structures with ground interaction is derived and numerical comparisons with the exact theory are conducted. Finally, a numerical comparison is made between the equivalent damping as obtained from relative displacements and overturning moments. It is found that the approximate theories give satisfactory results for tall slender structures but are generally inaccurate for low structures. The equivalent damping computed from overturning moments compares favourably with that obtained from relative displacements.

METHOD OF ANALYSIS OF STRUCTURE-GROUND INTERACTION IN EARTHQUAKES

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A. INTRODUCTION

The performance of a structure founded on a deformable medium is of considerable interest in earthquake engineering and has received some attention. Merritt and Housner (1) have presented a study for single- and multi-storey buildings subjected to various earthquakes. Only rocking of the base was considered and no foundation damping was included. Parmelee (2) studied single-storey structures under steady-state ground motion; his extension to multi-storey buildings, however, seems questionable (3). Perelman, Parmelee and Lee (4) and Parmelee et al (5) have studied, for limited range of parameters, the response of single-storey buildings to artificially generated earthquakes. Kobori, Minai and Suzuki (6) also studied various simple interaction systems under sinusoidal ground movement. Many specific problems with foundation interaction have been investigated by others (see especially Refs. 7 and 8).

Most previous investigations may be grouped into two categories: those that compute the structural response to specific base disturbances from which the influences of interaction effects are deduced; and those for which steady-state conditions are assumed. To isolate the influence of various parameters on the response of structures to arbitrary disturbances, however, it seems necessary to attempt a more fundamental approach to interaction studies. As there are a large number of parameters that may significantly affect response, a complete investigation using response studies would be prohibitive, even with relatively short-duration, artificially-generated base motions.

The approach taken in this study is to derive an equivalent single-degree-of-freedom (S.D.F.) model representing relative displacement and overturning moment for single-storey interaction systems (a three degree-of-freedom system). It is shown that the phenomenon of structure-ground interaction under arbitrary ground disturbances can be reduced to a relatively simple analytical method of finding a revised resonance frequency and an equivalent damping coefficient. The dynamic response of this "interaction system" when subjected to a specific ground disturbance can then be found from well-established procedures for S.D.F. systems, such as numerical integration and response spectrum techniques.

The use of an equivalent S.D.F. system to represent the interaction structure overcomes a basic drawback inherent in some previous interaction studies that used earthquake-type ground disturbances, where comparison of the response for the interaction structure with that of the fixed-based structure showed that sometimes the response is larger, and sometimes smaller (1). Such results are explainable in that the frequency change that accompanies structure-ground interaction may in itself result in a substantial change in response - either a reduction or an increase, depending on the locations of the spectral peaks of the particular disturbance relative to the natural frequency of the interaction system. Certain aspects of the interaction phenomenon can therefore be masked by the characteristics of the specific ground disturbance chosen.

With the equivalent S.D.F. approach, the properties of the structure are separated from the influence of specific random-type disturbances. A study of the system itself can thus be conducted without the results being influenced by the properties of the particular earthquake chosen. Furthermore, the significant parameters in the interaction process can be readily identified and evaluated.

B. INTERACTION MODEL

The interaction system under consideration is shown in Figure 1. This model was also used by previous investigators (2, 6). The initial formulation of the problem follows closely that of Parmelee (2). Additional interpretations of expressions are given, however, and the present derivation is extended beyond that in Ref. 2.

For purposes of this derivation both the masses m_o and m_1 are circular in plan with radius r. The corresponding differential equations of motion under any arbitrary base disturbance are:

horizontal translation of base mass

$$m_1 \ddot{U}_H + m_o \ddot{U}_B + P = 0 \qquad (1)$$

horizontal translation of top mass

$$m_1\ddot{U}_H + c\dot{U}_m + kU_m = 0 \tag{2}$$

rotation about point b

$$I_1\ddot{\Phi} + I_o\ddot{\Phi} + m_1 h\ddot{U}_H + M = 0 \tag{3}$$

where $$I_o = m_o \frac{r^2}{4} + m_1 \frac{r^2}{4},$$

$$I_1 = m_1 h^2 ,$$

and dots above a variable represent differentiation with respect to time. The remaining symbols are defined in Figure 1.* Under the influence of a steady-state ground displacement $u_g = We^{ipt}$, the resulting complex amplifications X, Y, and Z of the displacement components U_B, Φ and U_m are given by

$$\left.\begin{aligned} U_B &= We^{ipt}\ X = u_g\ (X_1 + iX_2) \\ \Phi &= We^{ipt}\ Y = u_g\ (Y_1 + iY_2) \\ U_m &= We^{ipt}\ Z = u_g\ (Z_1 + iZ_2) \end{aligned}\right\} \tag{4}$$

The forces between the base and the half-space are given by

$$P = P_o e^{ipt} = u_g\ (X - 1)\ A \tag{5}$$

$$M = M_o e^{ipt} = u_g\ YB \tag{6}$$

A and B are dynamic stiffness coefficients which relate the generalized forces and the corresponding displacements under sinusoidal excitation. For a circular base

$$A = Gr\ (K_H + i a\ C_H) \tag{7}$$

$$B = Gr^3(K_R + i a\ C_R) \tag{8}$$

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*A list of symbols is also given under "Nomenclature," p. 30.

where G = shear modulus of ground

r = radius

a = non-dimensional frequency = pr/V_s

p = circular frequency, rad/sec

V_s = shear wave velocity of ground

i = $\sqrt{-1}$

K_H, K_R = horizontal and rotational stiffness factors

C_H, C_R = horizontal and rotational damping factors

Substitution of the above relations in Eqs. (1) to (3) and simplification gives:

$$\begin{pmatrix} \left(1-\frac{\omega_0{}^2}{p^2}\right) & -\frac{2\lambda\omega_0}{p} & 1 & 0 & 1 & 0 \\ -\frac{2\lambda\omega_0}{p} & -\left(1-\frac{\omega_0{}^2}{p^2}\right) & 0 & -1 & 0 & -1 \\ 1 & 0 & \left(1+\frac{1}{\alpha}-\frac{\omega_H^2}{p^2}\right) & -\frac{2\lambda_H\omega_H}{p} & 1 & 0 \\ 0 & -1 & -\frac{2\lambda_H\omega_H}{p} & -\left(1+\frac{1}{\alpha}-\frac{\omega_H^2}{p^2}\right) & 0 & -1 \\ 1 & 0 & 1 & 0 & \left(1+\frac{1+\alpha}{4\eta}-\frac{\omega_R^2}{p^2}\right) & -\frac{2\lambda_R\omega_R}{p} \\ 0 & -1 & 0 & -1 & -\frac{2\lambda_R\omega_R}{p} & -\left(1+\frac{1+\alpha}{4\eta}-\frac{\omega_R^2}{p^2}\right) \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \\ X_1 \\ X_2 \\ hY_1 \\ hY_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{\omega_H^2}{p^2} \\ \frac{2\lambda_H\omega_H}{p} \\ 0 \\ 0 \end{pmatrix} \quad (9)$$

where
$$\frac{\omega_H^2}{p^2} = \frac{K_H}{a^2\beta}, \quad \frac{\omega_R^2}{p^2} = \frac{K_R}{a^2\beta\eta} \tag{10}$$

$$\lambda_H = \frac{C_H}{2(\beta K_H)^{\frac{1}{2}}}, \quad \lambda_R = \frac{C_R}{2(\beta\eta K_R)^{\frac{1}{2}}} \tag{11}$$

and
$$\alpha = \frac{m_o}{m_1}, \quad \beta = \frac{m_1}{\rho r^3}, \quad \eta = \left(\frac{h}{r}\right)^2, \quad \omega_o^2 = \frac{k}{m_1}$$

λ = relative inter-storey damping ratio,

ρ = density of ground

It should be noted that, in general, ω_H^2, ω_R^2, λ_H and λ_R in Eqs. (10) and (11) are frequency-dependent quantities. ω_H can be interpreted as the horizontal resonant frequency of the base alone, ω_R as the rocking frequency of the mass m_1 with moment of inertia $I_1 = m_1h^2$. λ_H and λ_R are the corresponding relative damping ratios for horizontal and rocking motions, respectively.

Solution for the steady-state amplification vector yields

$$\begin{bmatrix} Z_1 \\ Z_2 \\ X_1 \\ X_2 \\ hY_1 \\ hY_2 \end{bmatrix} = \begin{bmatrix} \text{6 x 6 matrix} \\ \text{from Eq. 9} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \dfrac{\omega_H^2}{p^2} \\ \dfrac{2\lambda_H\omega_H}{p} \\ 0 \\ 0 \end{bmatrix} = T_{u_g}^d \tag{12}$$

where the matrix inversion indicated may be carried out numerically in a computer.

$T^d_{u_g}$ represents the displacement response vector of the interaction system to a steady-state ground displacement u_g at any frequency and is commonly called the transfer function for the system (9). From the definition of frequency response curve (10) it may be seen that for a continuous range of frequencies the frequency response curves representing displacement ratios may be evaluated from Eq. (12).

The transfer function as well as the frequency response curves for other time derivatives may be obtained if Eq. (12) is multiplied by the proper power of ip corresponding to the order of time derivatives represented by the variables. For example, the transfer function $T^d_{\ddot{u}_g}$ for the displacement vector subjected to base accelerations is given by

$$T^d_{\ddot{u}_g} = \frac{1}{(ip)^2} T^d_{u_g} = \left(\frac{-1}{p^2}\right) T^d_{u_g} \tag{13}$$

The description of the dynamic foundation behaviour is contained in the terms A and B in Eqs. (7) and (8). For a spring and dashpot assembly, the terms GrK_H and Gr^3K_R represent the spring stiffness, and GrC_H and Gr^3C_R the damping coefficient. For a circular disc on an elastic half-space, the steady-state dynamic flexibility is given in Ref. 11.

$$A = \frac{Gr}{f_{1H} + if_{2H}} = Gr\left[\frac{f_{1H} - if_{2H}}{\left(f_{1H}\right)^2 + \left(f_{2H}\right)^2}\right] \tag{14}$$

$$A = \frac{Gr^3}{f_{1R} + if_{2R}} = Gr^3\left[\frac{f_{1R} - if_{2R}}{\left(f_{1R}\right)^2 + \left(f_{2R}\right)^2}\right] \tag{15}$$

from which, in conjunction with Eqs. (7) and (8),

$$K_H = \frac{f_{1H}}{\left(f_{1H}\right)^2 + \left(f_{2H}\right)^2}, \quad C_H = \frac{-\dfrac{f_{2H}}{a}}{\left(f_{1H}\right)^2 + \left(f_{2H}\right)^2} \tag{16}$$

$$K_R = \frac{f_{1R}}{(f_{1R})^2 + (f_{2R})^2}, \quad C_R = \frac{-\frac{f_{2R}}{a}}{(f_{1R})^2 + (f_{2R})^2} \tag{17}$$

A similar derivation was originally presented by Hsieh (12). The values for f_1 and f_2 used in this study are those obtained by Bycroft for a circular disc on an elastic halfspace (11). The variations of K_H, K_R, C_H and C_R for the circular base as a function of the frequency parameter a are shown in Figure 2 by solid lines. The dash-dotted line indicates constant magnitude approximations, which will be discussed later.

Having defined the properties of the structure and the foundation under steady-state base motions, one may compute the dynamic response to any arbitrary base disturbance by means of a superposition of Fourier components.

C. RESPONSE CALCULATIONS BY FAST FOURIER TRANSFORM

Interaction Systems - The response calculations for the interaction systems were performed by means of the discrete Fourier transform method given by Eqs. (18) and (19).

$$A_r = \Delta t \sum_{k=0}^{N-1} q_k \exp\left(-\frac{2\pi ikr}{N}\right); \quad r = 0, \ldots N-1 \tag{18}$$

$$\mathrm{Re}(X_k) = 2\,\Delta f \sum_{r=0}^{\frac{N}{2}} A_r\, T_r \exp\left(\frac{2\pi irk}{N}\right); \quad k = 0, \ldots N-1 \tag{19}$$

where

q_k = discrete record, with duration $t_d = N\Delta t$

N = total number of discrete points in the record

T_r = transfer function at discrete frequency variable r

Δf = frequency increment in Hz, $\Delta f = \frac{1}{t_d}$

and $\mathrm{Re}(X_k)$ = real part of X_k = response at discrete time variable k.

A_r as given in Eq. (18) is the discrete Fourier transform of the disturbance, and X_k of Eq. (19) is the inverse discrete Fourier transform of $A_r T_r$.

The computations indicated in Eqs. (18) and (19) were carried out with the aid of the fast Fourier transform computational algorithm (13, 14, 15). This algorithm requires a digitized input at equal time intervals and a total number of points equal to 2^n, where n is an integer.

The accuracy of the method was checked by performing response calculations using the transfer function for an S.D.F. oscillator (9) and comparing it with the results of a numerical integration procedure (16). For both methods, 27.3 sec of the El Centro, 1940 Earthquake, N-S component (Figure 3) was digitized into 4096 points. The velocity and displacement response comparisons show deviations of less than 1 per cent over the whole 27 sec of the record. The procedure, however, requires that sufficient damping be present in the system to reduce the free oscillations to acceptable magnitude by the end of the real time computations. If the free oscillations are not damped out, they will reappear at the beginning of the computations.

D. EQUIVALENT S.D.F. MODEL FOR RELATIVE DISPLACEMENT

With the introduction of rocking and relative horizontal motion of the base, the original S.D.F. system has become a three-degree-of-freedom system. Three modal shapes can therefore be expected, but in this study only the effects due to the lowest mode are considered. For the range of structural parameters considered herein, the contribution of the second and third modes to the total response of single-storey structures can be assumed negligible in most earthquake disturbances.

1. Properties of Frequency Response Curves

The dynamic characteristics of a linear system are completely determined by the frequency response curves. For a particular response parameter, e.g. relative displacement, the frequency response curve is defined as the ratio of response to disturbance under steady-state conditions as a function of frequency. Although, in general, both a real and an imaginary component are present, for small amounts of damping it is satisfactory to consider merely the vectorial sum of the real and imaginary components, i.e. the amplitude frequency response curve. The latter will be used exclusively henceforth.

Because of superposition, two dynamical systems with the same frequency response curve will have the same response when subjected to identical random disturbances. For the problem at hand it is therefore necessary to transform the response curves of a component of the interaction system into a response curve of an equivalent S.D.F. model with identical resonant frequency and good agreement of amplitudes over the entire frequency range.

2. Determination of Fundamental Resonant Frequency

The fundamental resonance frequency for the interaction system may be computed by determining the eigenvalues of the system once a standard eigenvalue problem has been formulated. This is accomplished by using only first, third and fifth rows and columns of the matrix in Eq. (9), corresponding to the real terms in the displacement vector; the right hand vector is set to zero because free vibrations are implied.

If frequency-dependent stiffness parameters are present, as in the case considered, they can be introduced by successively approximating the stiffness parameters corresponding to the eigenvalue computed in the previous cycle. Having thus obtained the fundamental frequency ω_1, one may substitute it in the transfer function, Eq. (9), to obtain the real and imaginary part of the response parameter. Because the computation is very sensitive to small frequency variations, a few trial calculations with small positive and negative excursions from the resonance frequency may be necessary to obtain the true resonance peak values.

Alternately, a numerical search of the response curves may be employed to detect the peak amplitude located at the resonance frequency of the fundamental mode, ω_1. This latter method was used to obtain the numerical results presented here.

3. Equivalent S.D.F. Model

Agreement of the amplitudes of the frequency response curve is obtained as follows. Examination of the equations of motion for relative displacement of the interaction system shows that at zero frequency the ratio of relative displacement of the top mass m_1 to the imposed ground acceleration, $U_m/\ddot{u}_g$, is equal to $m_1/k = 1/\omega_o^2$. As, however, the displacement response curve of an S.D.F. system with a resonance frequency ω_1 will have a zero frequency displacement amplitude of $1/\omega_1^2$, the equivalent S.D.F. model with frequency ω_1 is obtained by multiplying the amplitudes for the interaction system by $(\omega_o/\omega_1)^2$ (See Figure 4). Agreement of the response curves at resonance is achieved by computing an equivalent damping coefficient λ_e from the relation

$$\lambda_e = \frac{1}{2M_e}, \tag{20}$$

where

$$M_e = M_I \left(\frac{\omega_o}{\omega_1}\right)^2$$

M_I = peak amplitude of frequency response curve for the interaction parameter.

It should be noted that M_e and M_I are dimensionless, i.e. they do not correspond to the dimensions shown in Figure 4. Because all amplitudes of the frequency response curve have been increased by $(\omega_1/\omega_o)^2$, the response computed with the above equivalent S.D.F. is thus too large by the factor $(\omega_1/\omega_o)^2$.

With these three parameters, i.e. fundamental resonance frequency, multiplication factor $(\omega_o/\omega_1)^2$, and equivalent damping, the displacement response curves of the interaction system multiplied by $(\omega_o/\omega_1)^2$ and the equivalent S.D.F. model agree closely over the complete frequency range, even when the frequency-dependent foundation parameters are considered.

4. Specific Response Calculations

Figures 5 and 6 present relative displacement responses for two specific structures, Nos. 1 and 2, with parameters as given in Table I. The base disturbance consists of the record of the El Centro, California, 1940 earthquake, N-S component shown in Figure 3. For Structure No. 1 the solid line in Figure 5 represents the response of the interaction system, as obtained by the Fourier transform method, whereas the dotted line represents the response of an S.D.F. system with natural frequency Ω = 7.59 rad/sec and damping ratio λ = 2 per cent. The interaction response using the equivalent S.D.F. with

$(\omega_1/\omega_o)^2 = (7.59/10.0)^2 = 0.576$, $\lambda_e = 1.37$ per cent and $\omega_o = 7.59$ rad/sec gives results that are indistinguishable from the true interaction response throughout the full 27 sec.

Similar calculations for Structure No. 2 are presented in Figure 6. The solid line (Figure 6(a)) represents the interaction response obtained by numerical integration (16) using the equivalent S.D.F. model shown in Table I. The dotted curve again represents the relative displacement for the S.D.F. with $\lambda = 2$ per cent and $\Omega = 7.60$ rad/sec. For this structure the Fourier transform calculation was not successful because the small amount of damping in the system did not reduce the free oscillations sufficiently before the calculations were terminated. For illustrative purposes, however, and to obtain at least a qualitative picture of the interaction response obtained by the Fourier transform method, the curve is presented in Figure 6(b). The undamped oscillations may be observed at the beginning of the calculated response.

E. PARAMETER STUDY

To examine the behaviour of the interaction system a parameter study has been carried out with the following ranges of parameters:

$$\begin{aligned}
\omega_o &= 5 \text{ to } 20 \text{ rad/sec} \\
m_1 &= 1{,}000 \text{ to } 4{,}000 \text{ lb sec}^2/\text{in.} \\
m_o &= 1{,}000 \text{ lb sec}^2/\text{in.} \\
h &= 20 \text{ to } 80 \text{ ft} \\
r &= 15 \text{ and } 20 \text{ ft} \\
\lambda &= 1,\ 2 \text{ and } 5 \text{ per cent} \\
V_s &= 300,\ 500, \text{ and } 800 \text{ fps.}
\end{aligned}$$

Poisson's ratio ν and density of the ground ρ have been kept constant at zero and 120 lb/ft^3, respectively. A variation in the two parameters would be reflected primarily in changes in the shear wave velocity V_s and the rocking stiffness k_ϕ, both of which are considered as variables in this study. The types of structure covered by these parameters would include elevated water towers and other tall slender structures that can be idealized as S.D.F. systems. Although the assumed values for shear wave velocity of the ground are low for the usual type of formulation materials, the method of analysis is applicable for all higher values of shear wave velocity.

1. Reduction of Resonance Frequency

For the range of parameters shown above, the reduction of resonant frequency for the interaction systems is shown in Figure 7. The ordinates indicate the reduction in resonant frequency relative to the fixed base frequency, whereas the abscissa is the ratio of static rocking frequency to the fixed base natural frequency, ω_Φ/ω_o .

Here $\qquad \omega_\Phi^2 = \dfrac{k_\Phi}{I}$

$\qquad k_\Phi = \dfrac{8Gr^3}{3(1-\nu)}$ = rocking stiffness of circular plate under static conditions,

and $\qquad I$ = total moment of inertia = $m_1h^2 + m_1\dfrac{r^2}{4} + m_o\dfrac{r^2}{4}$ for the circular configuration of top and bottom mass.

The lower, heavier curve in Figure 7 corresponds to the theoretical relation of frequency reduction for a single-storey building, considering only rocking and relative displacement, as derived by Merritt and Housner (1). This curve also establishes the theoretical lower bound for frequency reduction of the three-degree-of-freedom interaction system. Values of frequency reduction for the above ranges of parameters fall within the region bounded by the two curves.

Because the ratio of frequency reduction ω_o/ω_1 is central to the derivation of the equivalent S.D.F. model and the determination of interaction response, the results presented in Figure 7 have the following implications. Through the ratio of rocking frequency to fixed-based natural frequency, ω_Φ/ω_o, the quantitative dependence of ground-structure interaction effects under random-type ground disturbances is established as a function of major parameters such as ground stiffness, geometry of the base, geometry and mass distribution of the structure, and lateral stiffness of the structure.

2. Comparison of Magnitudes of Resonance Peaks

The factor $(\omega_o/\omega_1)^2$ having been established, the other parameter required for a complete quantitative description of the equivalent S.D.F. model is the equivalent damping ratio λ_e. For the particular structural parameters and foundation properties shown, λ_e may be found from Figures 8 to 12 and Eq. (20). Detailed results are presented for the Bycroft foundation model, the Bycroft foundation with zero damping, and a foundation having constant stiffness and damping coefficients. The curves in Figures 8 to 12 represent envelopes of the resonance peaks M_I of the frequency response curves for the particular structural parameters given.

(a) Bycroft Foundation Model

For the foundation properties presented in Figure 2, Figures 8 and 9 show the ratio of peak amplitudes M_I of the frequency response curve for the interaction system to the peak amplitudes M_S of the S.D.F. system. The abscissa is $a_o = \omega_o r / V_s$,

where $\omega_o = \left(\frac{k}{m_1}\right)^{\frac{1}{2}}$ = natural frequency of fixed-based structure.

With small values of a_o it may be seen that for tall structures the peak amplitude of the frequency response curve exceeds that of an S.D.F. system with the same amount of structural damping.

In order to interpret some influences of the foundation properties, it is useful to refer to the S.D.F. stiffness and damping terms presented in Figure 2. It may be observed that in the rocking mode the foundation damping coefficient for small values of a_o is practically zero; hence the large values of peak resonance amplitudes are not unexpected.

The effects of varying inter-storey damping are demonstrated in Figure 10 for $\lambda = 1$ per cent and $\lambda = 5$ per cent, using the foundation parameters of Figure 2. The general variation of peak amplitudes is similar, although the closer coupling of the top mass with increased inter-storey damping is reflected in larger amplification ratios than for structures with less damping. Further results of M_I/M_S for $m_1 = 1000$ lb sec^2/in. and $\lambda = 5$ per cent are presented in Figures 11 and 12. The curves for $r = 15$ ft in Figure 11 are seen to exceed those of the S.D.F. oscillator, i.e. the fixed based structure, by substantial amounts, whereas the curves for $r = 20$ ft in Figure 12 remain approximately equal to the S.D.F. for a large variation of a_o.

(b) Bycroft Foundation with Zero Foundation Damping

In order to assess the influence of foundation damping on the amplitudes of the frequency-response curves, this particular foundation model is chosen as one extreme condition. Figure 13 shows the same ratio M_I/M_S of the peaks of the response curves for the case where the foundation stiffness terms K_H and K_R are those shown in Figure 2 and Eqs. (14) and (15), but the damping terms C_H and C_R in Figure 2 and Eqs. (16) and (17) are set equal to zero. All peak response amplitudes may be seen to exceed those of the corresponding S.D.F. oscillator.

(c) Constant Foundation Parameters

In investigating the effects of foundation flexibility it is frequently assumed that stiffness and damping properties are independent of frequency. Such an approximation is shown in Figure 2 by the dash-dotted lines. The parameter study for the ratio M_I/M_S, using the constant foundation parameters, is shown in Figure 14. It may be seen that the peak values in the range of small a_o's have been substantially reduced compared with those in Figure 9 with the frequency-dependent foundation parameters.

This would be expected because with smaller values of a_o more damping is present under the constant approximation than for the frequency-dependent case. The effect of the approximation on the reduction of natural frequency, however, is quite small. For the parameter range studied, the largest increase in the fundamental resonant frequency over the case with frequency-dependent parameters is 4 per cent and occurs for $\omega_1 r/V_s = 0.35$. The deviations decrease for smaller values of $\omega_1 r/V_s$. For this approximation of constant foundation properties the bounds for frequency reduction in Figure 7 are therefore still valid.

(d) Bycroft Foundation with Additional Rocking Damping

In addition to the three types of foundations dealt with above, namely the Bycroft foundation, Bycroft foundation with zero foundation damping, and a foundation with constant spring stiffness and constant viscous damping magnitude, the foundation-type that results from adding a constant amount of damping to the geometric damping of the Bycroft foundation will be investigated here. Such additional amounts of damping may arise from hysteretic energy loss in the foundation material, or from an artificially introduced damping mechanism in the foundation. The theoretical basis for such damping models may be found in Ref. 17. Only additional damping in the rocking mode will be considered since rocking was seen to exert the dominant influence on the response of the interaction system for the parameters investigated. For the horizontal base motion, the standard Bycroft curves shown in Figure 2(a) have been used. The variation of foundation damping in the rocking mode is shown in Figure 2(c). The solid line represents the variation of C_R in Eq. (17) for the circular Bycroft foundation, whereas the dotted lines signify the values for C_R with additional rocking damping of 0.1 and 0.3.

Using the various damping values, a parameter study was conducted for the peak values M_I of the frequency response curves for relative displacements U_m and for overturning moments M/m_1h. The results are presented as the ratio of M_I/M_S, the ratio of the peak values of the interaction system to the single-degree-of-freedom, fixed-based system. Figure 15 and 16 show the variation of M_I/M_S for relative displacement as a function of the frequency ratio a_o for the Bycroft foundation and the structural parameters given on the diagrams. On the same diagram the variation of relative displacement for damping of $C_R + 0.1$ and $C_R + 0.3$ are given by the dashed and dashed-dotted lines, respectively.

It may be seen that for the relative displacements, substantial reductions occur in the amplitudes of resonance peaks. This implies a reduction of the equivalent damping factor λ_e which in turn means that substantial reductions in displacements can be computed by the method of the equivalent S.D.F. described in a previous section. The resonance frequencies are affected only slightly by the relatively small amounts of damping considered here, and for practical purposes they may be assumed to remain constant.

It should be emphasized that the peak amplitudes of the frequency response curves for relative displacement do not, by themselves, give any measure of the response that may be expected from a random-type base input.

F. DETERMINATION OF MAXIMUM RESPONSE FROM SPECTRA

With the aid of the equivalent S.D.F. model, the maximum response for an interaction system may be determined from established response spectra of known earthquakes or other disturbances. The procedure is illustrated below with Structure No. 2, Table I, for the response spectrum (18) of the El Centro, 1940 earthquake, N-S component shown in Figure 17.

(1) Find reduced fundamental frequency ω_1 (either from computations outlined above or from the graph in Figure 7): $\omega_1 = 7.60$ rad/sec.

(2) Determine peak amplitude M_I/M_S of frequency response curve from Figure 9: $M_I/M_S = 1.51$.

(3) Multiply amplitude M_I/M_S by $(\omega_o/\omega_1)^2 = 6.95$ and determine effective damping ratio from Eq. (20). By direct proportionality with $\lambda = 2$ per cent,

$$\lambda_e = (0.02)\ \frac{1}{(1.51)\ (6.95)} = 0.00192 = 0.192 \text{ per cent.}$$

(4) Enter response spectrum with damping ratio λ_e and natural frequency ω_1 (or the corresponding period $T = 1.21$ sec) and read maximum spectral response: $S_D \simeq 7$ in.

(5) Divide spectral value by $(\omega_o/\omega_1)^2$ to obtain true maximum interaction response: maximum relative displacement $\simeq 1$ in. This value agrees with the response calculation shown in Figure 6 and for which results are tabulated in Table II.

The procedure is only slightly more complex than that using the response spectrum for S.D.F. systems. With the aid of the equivalent S.D.F. model, it is thus possible to construct a modified response spectrum for a particular structure to account for influence of structure-ground interaction.

1. Response for Different Types of Foundation Damping

It is of interest to compare the responses of a particular structure for two assumptions of foundation damping: (1) Bycroft foundation, as represented by the solid lines in Figure 2; and (2) Bycroft foundation with damping coefficients C_H and C_R in Eqs. (16) and (17) set to zero. Such a comparison for Structures No. 1 and 2 is indicated in Table II. The magnitudes of the peak frequency response amplitudes have been expressed in terms of equivalent damping λ_e for the equivalent S.D.F. model. This provides a uniform basis of comparison for the different foundation models and permits the evaluation of the maximum response directly from a response spectrum. The values of S_D are the relative displacements for the interaction system as obtained from Figure 17 by means of the equivalent S.D.F. model and the multiplication factor $(\omega_1/\omega_o)^2$. U_{max} represents the maximum displacement obtained from response calculations. The interpolation between adjacent spectral curves to find S_D is subject to considerable uncertainty, but for practical purposes the error involved can be ignored.

A useful approximation in the application of the response spectrum follows. For structures with small amounts of inter-storey damping (say up to 5 per cent) the equivalent damping λ_e will be small for reasonably large reductions of frequency, as is evident from Eq. (20). Consequently, one can use the spectrum curve for zero damping and divide the response by $(\omega_o/\omega_1)^2$. This will give a conservative estimate of spectral response for all cases. The approximation will improve with smaller inter-storey damping and larger frequency reduction ratio $(\omega_o/\omega_1)^2$.

2. Generalization of Maximum Response Comparisons

A general conclusion regarding the response magnitude of interaction systems can be obtained from an examination of Figures 8 to 14, which indicate that over a considerable range of values of a_o the peaks of the frequency response curves are smaller than those of the S.D.F. oscillator with the same natural frequency. For these cases the maximum response of the interaction system is a priori less than that of the S.D.F. case. Where the resonance peaks exceed those of the S.D.F. case, the response of the interaction system may exceed that of the S.D.F. oscillator, particularly under steady-state excitations with frequency close to the resonance frequency of the structure. Considering, however, the random nature of the earthquake excitation, the contributions over the whole frequency response curve have to be included. If the earthquake and the resulting response are idealized as weakly stationary processes, the mean square response is given in Ref. 19.

$$\overline{x}^2 = \int_0^\infty \left| H(p) \right|^2 D(p) \, dp \tag{21}$$

where

$\left| H(p) \right|$ = amplitude of transfer function, and

$D(p)$ = power spectral density of the excitation.

As $\left| H(p) \right|^2$ is highly peaked at the resonant frequency ω_1, D(p) may be approximated by $D(\omega_1)$ = constant. Consequently, the comparison of the response of two structures can be made on the basis of the ratio of the areas under their respective frequency response curves,

$$\int \left| H(p)_I \right| dp \Big/ \int \left| H(p)_S \right| dp \tag{22}$$

For the parameter ranges considered, such a comparison of areas has been carried out for the interaction system with zero foundation damping relative to an S.D.F. oscillator with the same inter-storey damping. This represents the most unfavourable case as far as magnitude of relative displacement response is concerned. For all cases, the ratio has been equal to or smaller than 1.00; for structures with a high aspect ratio h/r, the ratio of the areas is of the order of 0.1.

On the basis of random vibration theory it is demonstrated that the mean relative displacement response for an interaction system may be expected to be less than or equal to the response of an S.D.F. oscillator with the same natural frequency, irrespective of the particular viscous damping magnitudes in the foundation. For any particular interaction structure and a given base motion, this statement can be verified with the aid of the equivalent S.D.F. model and the response spectrum.

G. OVERTURNING MOMENTS

The overturning moment M acting on the elastic half-space resulting from the dynamic response of the structure may be obtained from Eq. (3):

$$M = -\left(I_o \ddot{\Phi} + I_1 \ddot{\Phi} + m_1 h \ddot{U}_H\right) \tag{23}$$

Upon substitution of steady-state amplification factors (Eq. 4) and appropriate structural constants, the frequency response (or transfer function) for overturning moment relative to ground acceleration is given by

$$\frac{M}{m_1 h \ddot{u}_g} = -\left[\left(1 + \frac{1+\alpha}{4\eta}\right) hY + X + Z\right] \tag{24}$$

For a rigidly based structure, $Y = 0$, $X = 1$;

thus
$$\frac{M}{m_1 h \ddot{u}_g} = -[1 + Z] \tag{25}$$

X, Y and Z may be computed from Eq. (12), where the addition has to be carried out with due regard to signs and real and imaginary components. A typical amplitude frequency response curve for the overturning moment of Structure No. 1 is presented in Figure 18.

1. Equivalent S.D.F. for Overturning Moment.

At zero frequency the frequency response curve may be seen to have an amplitude of 1.0, since $Y = Z = 0$, and $X = 1$ in Eq. (24). An S.D.F. system with the same natural frequency as the interaction system would have amplitude $1/\omega_1^2$. Consequently, the equivalent S.D.F. system is obtained by multiplying the overturning moment curve by $1/\omega_1^2$; the equivalent damping is determined from a comparison of the magnitude of resonance peaks from Eq. (24) relative to Eq. (25) at the resonance frequency ω_1. The value for the overturning moment ratio M/m_1h obtained from response calculations is then too small by the factor ω_1^2.

For the particular set of parameters shown, the variation of the peak amplitudes of the overturning moment response curves are presented in Figure 19 for the Bycroft foundation.

2. Maximum Overturning Moment from Response Spectra

Determination of the maximum overturning moment under a particular base motion may be obtained directly from the response spectrum in a manner similar to that described for relative displacement:

(1) Following a determination of the fundamental resonant frequency, ω_1, the amplitudes of the overturning moment response curve are multiplied by $1/\omega_1^2$ and the equivalent damping λ_e is determined.

(2) The corresponding value of maximum relative displacement is read from the response spectrum and multiplied by ω_1^2 to obtain the value of the ratio for overturning moment, M/m_1h.

Figure 19 shows that the peak amplitudes of the frequency response curves for a wide range of parameters are greater than those for a rigidly based structure. Unlike the case for relative top-storey displacement, the magnitudes of the resonance peaks for overturning moment give a qualitative indication of the magnitude of the overturning moment response under an arbitrary base disturbance. This may be concluded because the frequency response curves for the interaction system and the rigidly based structure have very nearly the same amplitudes except near the resonance frequency. Thus under a steady-state or random-type disturbance, the response of the structure with the smaller resonance peak represents a lower bound to the response of the structure with the larger resonance peak. It may therefore be deduced from Figure 19 that for some tall structures with foundation interaction the overturning moment will be larger than that for a rigidly based S.D.F. structure with the same natural frequency and inter-storey damping.

Again, an approximation of the maximum overturning moments of tall structures is obtained by taking the undamped spectral displacement S_D and multiplying it by $\omega_1{}^2$ to get M/m_1h; this gives a conservative estimate.

3. Response Calculations for Overturning Moments

Figure 20 shows response calculations for overturning moments for Structures No. 1 and 2 of Table I. For both interaction systems the overturning moment, shown by solid lines, is larger than for a rigidly based structure of the same natural frequency, as represented by the dotted curve. The responses obtained for Structure No. 1 from the equivalent S.D.F. model and the "exact" one by Fourier transform with transfer function of Eq. (24) differ by no more than 6 per cent. As already described for relative displacement, the Fourier transform method was unsuccessful in calculating the response for Structure No. 2; thus only the results for the equivalent S.D.F. model are presented.

Agreement of the equivalent S.D.F. model with the actual overturning response curve is not as good as for relative displacement. An examination of the mode shapes of the interaction system indicates that above the fundamental resonance frequency the relative base displacement has phase opposite that of the rocking and relative displacement, whereas below that frequency all are in phase. Because the magnitude of the relative base displacement is small compared with the relative top mass and rocking displacements, the deviation between the equivalent S.D.F. model and the true overturning frequency response curves can be neglected for most random-type motions.

H. EXTENSION OF METHOD

Although the results presented herein apply specifically to circular bases on an elastic half space and to the range of structural parameters investigated, the method of obtaining equivalent S.D.F. models for relative displacement and overturning moment is quite generally applicable for linear interaction systems. The curves for frequency reduction apply to other geometrical shapes as well as to circular ones provided (1) that the static stiffness of the foundation on the elastic ground is known, and (2) that frequency dependent stiffness and damping properties do not differ drastically from those of the circular ones. For example, the results apply closely to square and rectangular bases, whose S.D.F. stiffness and damping properties are presented in Ref. 20.

Flexural Type Structures - The detailed derivation and specific results presented pertain to structures that undergo a shear-type deformation. The results, however, are applicable also in a general way to structures that deflect primarily in the flexural mode, such as cantilever tower structures with substantial top masses provided the moment of inertia I_o of the top mass about its own horizontal centroidal axis is small compared to the term $m_1 h^2$. It is also understood that in a flexural-type structure lateral excursions remain small so that linear vibration theory remains valid.

J. COMPARISONS WITH APPROXIMATE ANALYSES

Approximate methods of analyses are important for the following reasons: first, approximations frequently provide a simple and quick answer to the solution of complex problems and secondly, since they are simpler than the more "exact" solutions, approximations often enable one to get a better conceptual grasp of the essential features of a problem. Often, however, approximations are limited by the degree of accuracy in which they are able to describe the real situation. Furthermore, the range of applicability of approximations may be limited.

The ground-structure interaction model used by Parmelee (2) and described above under "Interaction Model" is chosen as the standard for purposes of evaluating the approximate theoretical formulations.

This ground-interaction model contains all the essential features which appear to play important roles under earthquake-type disturbances. These features are: (1) the necessary number of degrees of freedom for an interaction structure - relative interstorey displacement, relative base displacement, and rocking; (2) interstorey damping, and (3) frequency dependent foundation parameters as given by the Bycroft coefficients. The above model is also referred to herein as the "exact" theory.

Two approximate theories for structure-ground interaction are investigated here: the first is the case of the undamped system, treated by Balan et al. (21) and, for a simpler case, by Merritt and Housner (1). The second is an approximate treatment of overturning moments that results from simplifications in the derivation of the "exact" theory.

1. Validity of Undamped Interaction Relations

Balan et al (21) have investigated the relationship among the relative displacement U_m, rocking displacement $h\Phi$ and translational displacement u for the model shown in Figure 1. The result is valid for the undamped case or for the case with proportional damping:

$$u\,\omega_T^{\,2} = (h\Phi)\,\omega_\Phi^{\,2} = U_m \omega_o^{\,2} \, . \tag{26}$$

where

$$\omega_T^{\,2} = \frac{k_T}{m_o}$$

$$\omega_\Phi^{\,2} = \frac{K_\Phi}{m_1}$$

$$\omega_o^{\,2} = \frac{k}{m_1}$$

and

$$K_\Phi = \frac{k_\Phi}{h^2} \, .$$

k_T is the horizontal translational stiffness of the base on the half-space, and K_Φ is the rocking stiffness of the base on the half-space. For the interaction model shown in Figure 1, $\omega_\Phi^{\,2}$ becomes

$$\omega_\Phi^{\,2} = \frac{k_\Phi}{I}$$

where $I = I_o + I_1 + m_1 h^2$ = total amount of inertia. Since, however, foundation damping constitutes an essential part of the dynamic behaviour of the interaction system, and since the damping matrix is in general nonproportional, the validity of Eq. (26) needs to be verified for its applicability to the realistic, damped, structural model. Such a check was carried out from the parameter study of the magnitudes of resonance peaks.

For steady state conditions, Eq. (26) can be re-written as

$$u\,\omega_T^{\;2} = hY\omega_\Phi^{\;2} = Z\,\omega_o^{\;2} \tag{27}$$

so that

$$hY = \left[\frac{\omega_o}{\omega_\Phi}\right]^2 Z \tag{28}$$

Using Merritt and Housners' relationship (1)

$$\left(\frac{\omega_o}{\omega_\Phi}\right)^2 = \left(\frac{\omega_o}{\omega_1}\right)^2 - 1 \tag{29}$$

Eq. (28) becomes

$$hY = \left[\left(\frac{\omega_o}{\omega_1}\right)^2 - 1\right] Z \tag{30}$$

The numerical comparisons between approximate and exact theoretical relationships are performed on structures with the parameters given in Table III. All structures are circular in plan and the dynamic behaviour of the base on the elastic half-space is described by Bycroft's curves (11), which are shown by the solid lines in Figure 2.

A numerical search for the peak quantities of relative displacement Z, rocking displacement hY, and overturning moment M was conducted. The amplification vectors for the above quantities were evaluated from Eqs. (9) and (24) as a function of frequency for the above three quantities. The comparisons between the accurate values hY and the approximations given by Eq. (26) are presented in Tables V and VII for the Structures No. 3 and No. 4, whose parameters are given in Table III. The agreement between these quantities is expressed as a per cent difference in columns (11) of Tables V and VII.

Another possible approximation as given by Eq. (28) is compared with the theoretically exact values in Tables V and VII. Again the discrepancy is represented by a per cent difference in column (14).

2. Overturning Moments

The quantity which is of particular interest to the foundation engineer is the overturning moment, which arises from the moment of the inertia forces of the structure about the base. The complete transfer function for overturning moment is given by Eq. (24).

In an attempt to simplify the treatment of overturning moments, the approximate relationship between the relative displacement Z and overturning moment M is investigated. Substitution of Eq. (30) into Eq. (24), and letting X = 1, gives

$$\frac{M}{m_1 h \ddot{u}_g} = \left[\left(1 + \frac{1+\alpha}{4\eta}\right)\left(\left(\frac{\omega_o}{\omega_1}\right)^2 - 1\right) Z + Z + 1\right]$$

$$= 1 + Z\left[\left(\frac{\omega_o}{\omega_1}\right)^2\left(1 + \frac{1+\alpha}{4\eta}\right) - \frac{1+\alpha}{4\eta}\right]. \qquad (31)$$

$\frac{1+\alpha}{4\eta}$ is small for large $\eta = \left(\frac{h}{r}\right)^2$; thus the term $\left[\left(\frac{\omega_o}{\omega_1}\right)^2 - 1\right]\left(\frac{1+\alpha}{4\eta}\right)$

becomes negligible. Therefore

$$\frac{M}{m_1 h \ddot{u}_g} \cong 1 + Z\left(\frac{\omega_o}{\omega_1}\right)^2 \qquad (32)$$

and the equivalent S.D.F. model for this approximate overturning moment is

$$\left[1 + Z\left(\frac{\omega_o}{\omega_1}\right)^2\right] \cdot \frac{1}{\omega_1^2} \qquad (33)$$

This follows the procedure described previously under "Overturning Moments."

The displacement magnification factor for relative displacement is given by

$$\frac{U_m}{\ddot{u}_g} = \frac{U_m}{\omega^2 u_g} = \frac{1}{\omega^2} \cdot Z$$

since $Z = \frac{U_m}{u_g}$ by definition. Thus the equivalent S.D.F. for relative displacement is

$$\frac{1}{\omega^2} \cdot Z \cdot \left(\frac{\omega_o}{\omega_1}\right)^2 \tag{34}$$

as was described under "Equivalent S.D.F. Model for Relative Displacement." If amplitudes of resonance peaks (which occur at the frequency ω_1) are compared, Eq. (34) becomes

$$\frac{1}{\omega_1^2} \cdot Z \cdot \left(\frac{\omega_o}{\omega_1}\right) \tag{35}$$

The expressions for the equivalent S.D.F. for overturning moment and relative displacement, Eqs. (33) and (35) respectively, are seen to differ by the constant 1.0 within the parentheses. This constant can in most cases be assumed negligible in comparison with Z (which equals 25 for $\lambda = 2$ per cent for a S.D.F. system, for example). It may therefore be concluded that the equivalent damping λ_e for relative displacement and overturning moment are approximately equal. Thus it is only necessary to perform a parameter study of resonance peaks for relative displacement, as the results for overturning moments can be computed therefrom. For a quantitative comparison of agreement between equivalent damping obtained from relative displacement and overturning moments see Tables IV and VI.

3. Discussion of Results of Approximate Analyses

The comparison of results for Balan's approximate relationships between the displacement components and the numerical results of the theoretical interaction model shows that in all cases investigated Eq. (30) gives poorer agreement with the exact rocking amplitude hY than Eq. (28). Equation (30) is arrived at from Eq. (28) by substituting Eq. (29) and is consequently subject to two sets of approximations. Thus a larger discrepancy is not surprising. The per cent errors for Eq. (30) are seen to be unacceptably large for low structures, whereas results for tall structures are quite accurate. A similar trend is evident for the results of Eq. (28), although the agreement with the theoretical results is much better. The per cent error is still large, however, for "soft", low structures. The large discrepancies are due mainly to the neglect of the relative base displacement in the approximate expressions, i.e. X was set equal to 1.0. This base displacement becomes relatively more important for low structures and hence the errors may be expected to be large.

The approximation of Eq. (31) for the amplitudes of resonance peaks of overturning moments is seen to give reasonable agreement for the structures investigated, as is shown in Column (8) of Tables IV and VI. The agreement is excellent for the structures with the 2 per cent structural damping. For the 5 per cent structural damping, greater deviations are evident for the low soft structures, although even these may give acceptable magnitudes for some structural design or analysis purposes.

The comparison of equivalent damping factors in Tables V and VII shows that λ_e as obtained from relative displacements is smaller than or equal to the λ_e from overturning moments for all cases investigated. The largest deviation is obtained for stiff structures. Use of the equivalent damping for relative displacements as an approximation for the equivalent damping of overturning moments seems to give acceptable answers since variations in equivalent damping of 10 to 15 per cent produce only small differences in maximum response.

K. CONCLUSION

A method of analysis is presented that permits the response determination of single-storey structure-ground interaction systems under earthquake-type disturbances. The procedure is based on the derivation of an equivalent S.D.F. model which permits the use of well-known numerical integration and response spectrum techniques in determining relative displacement and overturning moments. This approach has the advantage over previously used methods that the properties of the interaction structure can be studied separately from the response characteristics of particular random-type base disturbances.

For the range of parameters investigated it is shown that the most important interaction parameter is the ratio of rocking frequency of the structure to its fixed-base natural frequency. Thus the quantitative dependence of dynamic interaction effects on ground stiffness, base and structural geometry, distribution of mass, and lateral stiffness of the structure is established.

It is also demonstrated that under random type motions the relative displacement for a single-storey interaction system can be expected to be equal to or less than that for an S.D.F. oscillator with the same natural frequency and inter-storey damping. Maximum overturning moments, however, are generally larger than those for a rigidly based structure of the same natural frequency.

Using the concepts of the equivalent S.D.F. model and the transfer function for the interaction structure a study is presented of quantitative comparisons with approximate methods of determining rocking amplitudes, overturning moments and equivalent damping factors for single-storey dynamic structure-ground interaction effects. The approximate relationships for overturning moments as well as equivalent damping are derived here.

The results show that the approximate calculation of rocking amplitude using the relationships of Balan et al (20) gives good agreement with the theoretical model for relatively tall structures, but gives poor agreement for low, "soft" structures.

The approximate relationship for overturning moments gives satisfactory agreement throughout the range of structures investigated. The use of the equivalent damping factor of relative displacement for the computation of overturning moments also gives satisfactory results.

Graphical results of peak amplitudes of the frequency response curves for relative displacements and overturning moments show that additional foundation damping decreases the resonance peaks substantially, which means that the resulting response in relative displacement and overturning moments also decreases markedly. The curves may be used to evaluate these reduced responses from response spectra by the method of the equivalent S.D.F. model. These results suggest the possible use of artificial damping mechanisms to reduce both relative displacements and overturning moments under earthquake loadings.

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NOMENCLATURE

a	=	non-dimensional frequency = pr/V_s
a_o	=	non-dimensional S.D.F. resonance frequency = $\omega_o r/V_s$
A	=	dynamic stiffness for horizontal displacement on a half-space
B	=	dynamic rocking stiffness on a half-space
c	=	inter-storey damping coefficient
C_H, C_R	=	S.D.F. damping coefficients for horizontal and rocking displacements, respectively
D	=	power spectral density of a random excitation
f_{1H}, f_{2H} f_{1R}, f_{2R}	=	variables for steady-state dynamic behaviour of weightless disc on an elastic half-space for horizontal and rocking motion
f	=	frequency, Hz
G	=	shear modulus of ground
g	=	subscript denoting "ground"
h	=	storey height
I	=	total moment of inertia = $I_o + I_1$; subscript designating interaction system
I_o	=	moment of inertia of top and bottom mass
I_1	=	second moment of mass m_1 about the base = $m_1 h^2$
k	=	storey stiffness; discrete time variable in discrete Fourier transform
K_H, K_R	=	S.D.F. stiffness coefficients for horizontal and rocking displacements, respectively
k_Φ	=	static rocking stiffness
m_o	=	base mass
m_1	=	top mass
M, M_o	=	moment on base under arbitrary and steady-state motion, respectively
M_e	=	peak magnitude of frequency response for equivalent S.D.F. model

M_I, M_S	=	peak magnitude of frequency response for interaction system and S.D.F. oscillator, respectively
p	=	frequency, rad/sec
P, P_o	=	horizontal force on base under arbitrary and steady-state motion, respectively
r	=	radius of base; discrete frequency variable in discrete Fourier transform
S	=	subscript designating S.D.F. system
S_D	=	spectral displacement
t	=	time variable
t_d	=	duration of discrete base disturbance
T	=	period, sec.
$T_{u_g}^{d}$	=	transfer function for displacement vector d and ground displacement u_g
T_r	=	transfer function at discrete frequency variable r
u_g, $\ddot{u}_g$	=	steady-state ground displacement and acceleration, respectively
u	=	relative horizontal displacement of base mass with respect to free-field ground motion
U_{max}	=	maximum displacement from response calculations
U_B	=	total base displacement of interaction system
U_H	=	total displacement of top mass of interaction system
U_m	=	relative inter-storey displacement of interaction system or S.D.F. system
V_s	=	shear wave velocity of ground
W	=	amplitude of steady-state ground disturbance
X, Y, Z	=	complex amplification factors for base displacement, rocking and inter-storey displacement, respectively
α	=	mass ratio = m_o/m_1
β	=	non-dimensional top mass = $m_1/\rho r^3$
Δ	=	small increment of a variable

η	=	aspect ratio squared = $(h/r)^2$
λ	=	relative inter-storey damping ratio
λ_e	=	equivalent S.D.F. damping ratio
λ_{eM}	=	equivalent S.D.F. damping ratio computed from over-turning moments
λ_H, λ_R	=	relative damping ratios for horizontal and rocking motion, respectively, of base mass
ν	=	Poisson's ratio of ground
Φ	=	angular variable, radians
ρ	=	mass density of ground
Ω	=	natural frequency of S.D.F., rad/sec
ω	=	natural frequency of fixed-based structure, rad/sec = $(k/m)^{\frac{1}{2}}$
ω_1	=	fundamental frequency of interaction system, rad/sec
ω_H	=	resonant frequencies of disc on elastic half-space for horizontal motion, rad/sec
ω_R	=	resonant rocking frequency for circular structure with moment of inertia $I_1 = m_1 h$, rad/sec
ω_Φ	=	rocking frequency = $(k_\Phi/I)^{\frac{1}{2}}$

TABLE I

PARAMETERS FOR SAMPLE CALCULATIONS

Parameter	Unit	Structure No. 1	Structure No. 2
(a) Structural Parameters			
m_1	lb sec^2/in.	1000	4000
m_o	lb sec^2/in.	1000	1000
h	ft	40	80
r	ft	15	15
V_s	fps	300	800
ω_o	rad/sec	10	20
ω_1	rad/sec	7.59	7.60
λ	%	2.0	2.0
(b) Equivalent S.D.F. Model for Relative Displacement			
ω_1	rad/sec	7.59	7.60
$\left(\frac{\omega_1}{\omega_o}\right)^2$		.576	.144
λ_e	%	1.37	.192
(c) Equivalent S.D.F. Model for Overturning Moment			
ω_1	rad/sec	7.59	7.60
ω_1^2		57.6	57.7
λ_e	%	1.39	0.20

TABLE II

COMPARISONS OF RELATIVE DISPLACEMENTS FOR DIFFERENT FOUNDATION MODELS

Foundation	Equivalent Damping Ratio λ_e, %	Spectral Displacement S_D, in.	Computed Maximum Relative Displacement U_{max}, in.
Structure No. 1			
(1) Bycroft Model	1.37	3.2	3.05
(2) Bycroft Model with zero foundation damping	0.68	3.7	3.32
Structure No. 2			
(1) Bycroft Model	0.192	1.0	0.92
(2) Bycroft Model with zero foundation damping	0.104	1.0	0.96

TABLE III

PARAMETERS OF STRUCTURES USED FOR

COMPARISON STUDIES

Parameter	Units	Structure No. 3	Structure No. 4
Base mass m_o	lb sec^2/in.	1000	1000
Top mass m_1	lb sec^2/in.	4000	1000
Base radius r	ft	20	15
Interstorey damping λ	%	2	5

TABLE IV

COMPARISONS OF APPROXIMATIONS FOR OVERTURNING MOMENTS. STRUCTURE NO. 3

Column (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Fixed based resonance frequency ω_o, rad/sec	Fundamental frequency of interaction system ω_1, rad/sec	height h, ft	Shear wave velocity of ground V_s, ft/sec	Amplitude of rel. displ. at resonance frequency ω_1 Z	$1 + Z\left(\frac{\omega_o}{\omega_1}\right)^2$ at resonance frequency ω_1 Eq. (31)	$\frac{M}{m_1 h}$ at resonance frequency ω_1 from Eq. (24)	% difference in amplitudes of overturning moments $\frac{(7)-(6)}{(7)} \times 100$
5	4.748	20	300	21.80	25.0	24.7	+1.2
10	8.260	20		10.28	16.1	16.1	0
15	10.316	20		4.293	10.05	10.16	-1.1
20	11.430	20		2.166	7.65	7.66	-0.1
5	3.414	80	300	30.18	65.6	65.4	+ .3
20	4.512	80		6.174	121.6	123.5	-1.5
10	9.713	20	800	23.72	26.1	25.47	+2.4
20	17.873	20		15.98	21.0	20.88	+0.6
5	4.650	80	800	26.62	31.8	30.9	+2.9
10	7.818	80		29.38	49.1	48.5	+2.9
20	10.563	80		26.61	96.5	96.7	-0.2

TABLE V

COMPARISONS OF APPROXIMATIONS FOR ROCKING AMPLITUDES AND EQUIVALENT DAMPING COEFFICIENTS. STRUCTURE NO. 3

Column (1)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Fixed based resonance frequency ω_o, rad/sec	$Z\left[\left(\frac{\omega_o}{\omega_1}\right)^2 - 1\right]$ Eq. (30)	Rocking amplitude hY from Eq. (12)	% difference in rocking amplitude; $\frac{(10) - (9)}{(10)} \times 100$	ω_ϕ rad/sec	$Z\left(\frac{\omega_o}{\omega_\phi}\right)^2$ Eq. (28)	% difference in rocking amplitudes; $\frac{(10) - (13)}{(10)} \times 100$,	Equivalent damping factor λ_e for relative displacement using Column (5), Table IV, %	Equivalent damping factor λ_{eM} for overturning moments using Column (7), Table IV, %
5	2.40	1.54	+62	16.82	1.92	+25	2.06	2.02
10	4.81	3.22	+50	16.82	3.63	+9.6	3.02	3.10
15	4.77	3.33	+43	16.82	3.42	+2.7	5.51	4.94
20	4.49	3.15	+42	16.82	3.07	-2.5	7.55	6.55
5	34.5	33.4	+3.3	4.770	33.1	-1	0.774	0.763
20	114.5	111.3	+2.9	4.770	108.8	-2.2	0.414	0.405
10	1.51	0.92	+64	44.94	1.17	+27	1.99	1.97
20	4.01	2.66	+51	44.94	3.16	+19	2.50	2.40
5	4.18	4.05	+3.2	12.75	4.1	+1	1.62	1.62
10	18.72	18.1	+3.4	12.75	18.0	-0.5	1.04	1.03
20	68.6	66.5	+3.2	12.75	65.5	0	0.523	0.518

TABLE VI

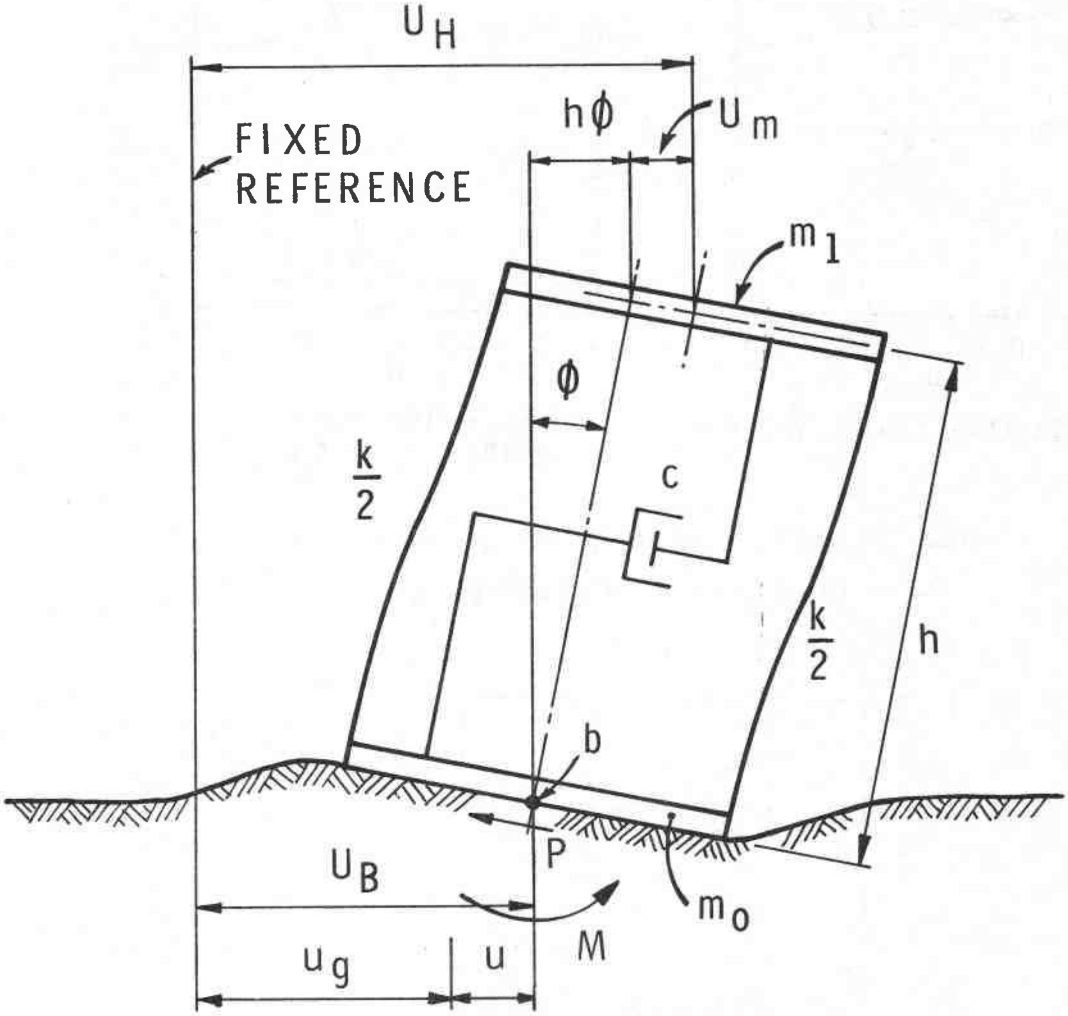
COMPARISONS OF APPROXIMATIONS FOR OVERTURNING MOMENTS. STRUCTURE NO. 4

Column (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Fixed based resonance frequency ω_0, rad/sec	Fundamental frequency of interaction system ω_1, rad/sec	Height h, ft	Shear wave velocity of ground V_s, ft/sec	Amplitude of rel. displ. at resonance frequency ω_1 Z	$1 + Z\left(\frac{\omega_o}{\omega_1}\right)^2$ at resonance frequency ω_1 Eq. (31)	$\frac{M}{m_1 h}$ at resonance frequency ω_1 from Eq. (24)	% difference in amplitudes of overturning moments $\frac{(7) - (6)}{(7)} \times 100$
5	4.88	20	300	10.18	11.7	10.90	+7.3
10	9.03	20		9.54	12.7	12.25	+3.6
15	12.06	20		7.056	11.9	11.85	+ .4
20	14.03	20		4.493	10.13	10.23	-1.0
5	3.88	80	300	12.73	22.1	21.36	+1.2
20	5.81	80		12.25	146.1	147.6	- .3
10	9.877	20	800	10.14	11.4	10.56	+8.0
20	18.90	20		10.09	12.3	11.67	+5.4
5	4.797	80	800	10.47	11.6	11.49	+1.0
10	8.560	80		11.67	16.9	16.09	+5.0
20	12.66	80		14.91	38.2	37.68	+1.4

TABLE VII

COMPARISONS OF APPROXIMATIONS FOR ROCKING AMPLITUDES AND EQUIVALENT DAMPING COEFFICIENTS. STRUCTURE NO. 4

Column (1)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Fixed based resonance frequency ω_o, rad/sec	$Z\left[\left(\frac{\omega_o}{\omega_1}\right)^2 - 1\right]$ Eq. (30)	Rocking amplitude hY from Eq. (12)	% difference in rocking amplitude; $\frac{(10) - (9)}{(10)} \times 100$	ω_Φ rad/sec	$Z\left(\frac{\omega_o}{\omega_\Phi}\right)^2$ Eq. (28)	% difference in rocking amplitudes; $\frac{(10) - (13)}{(10)} \times 100$	Equivalent damping factor λ_e for relative displacement using Column (5), Table VI, %	Equivalent damping factor λ_{eM} for overturning moments using Column (7), Table VI, %
5	.51	.42	+21	22.11	.52	+24	4.68	4.58
10	2.14	1.69	+26	22.11	1.96	+16	4.28	4.07
15	3.86	3.05	+26	22.11	3.25	+6.5	4.57	4.21
20	4.63	3.69	+25	22.11	3.68	+0.3	5.50	4.88
5	8.42	8.31	+1.3	6.203	8.26	-0.6	2.37	2.34
20	133.5	130.5	+2.3	6.203	127.5	-2.3	.34	.34
10	.25	.23	+10	59.09	.29	+26	4.80	4.73
20	1.21	.963	+26	59.09	1.16	+21	4.43	4.28
5	.915	.944	-3.1	16.58	.953	+1	4.39	4.35
10	4.25	4.24	+0.2	16.58	4.26	+0.5	3.14	3.10
20	22.3	21.95	+1.6	16.58	21.75	-0.9	1.35	1.31



INTERACTION SYSTEM

FIGURE 1

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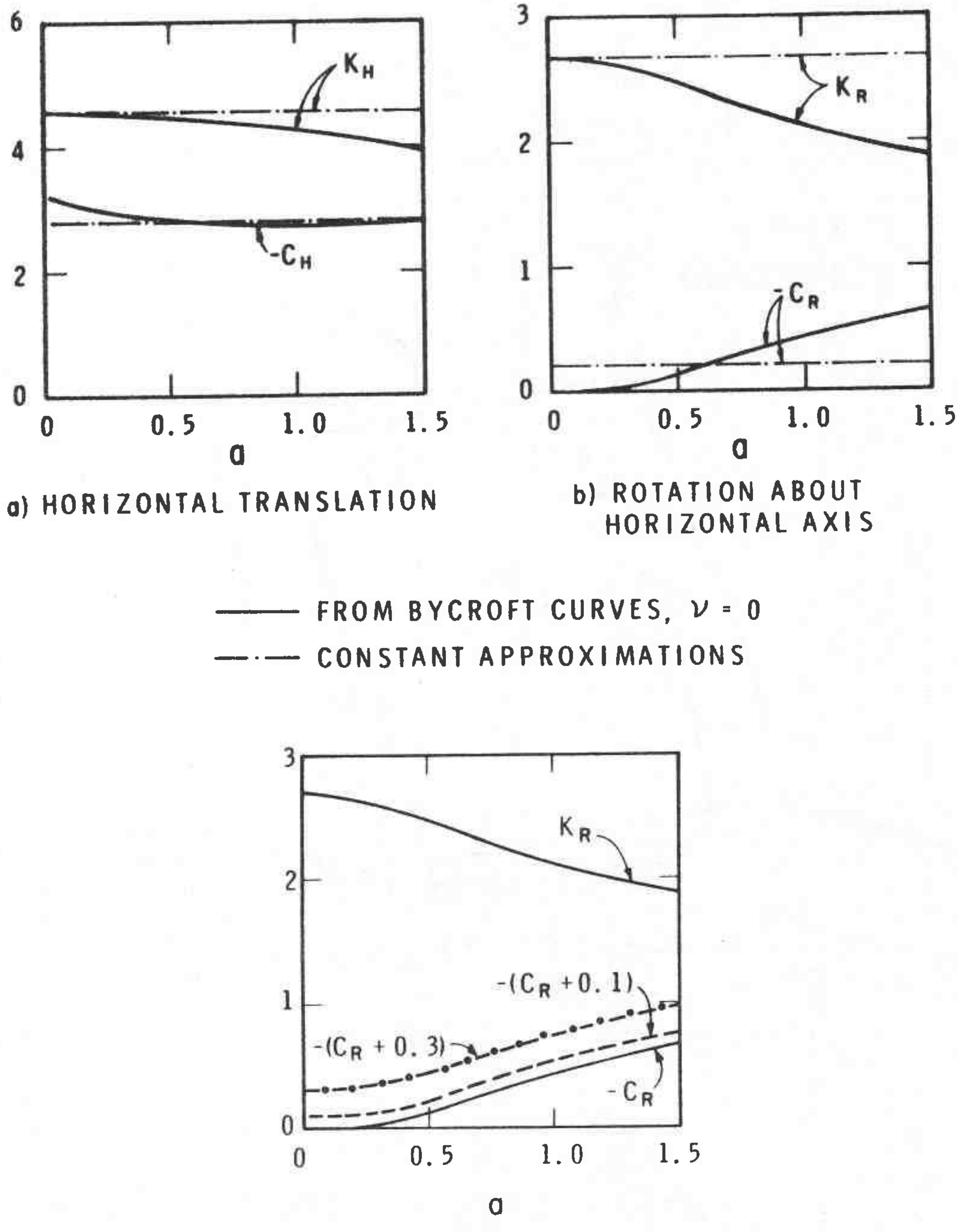

a) HORIZONTAL TRANSLATION

b) ROTATION ABOUT HORIZONTAL AXIS

c) ROTATION ABOUT HORIZONTAL AXIS

FIGURE 2

S.D.F. FOUNDATION PARAMETERS FOR CIRCULAR BASE

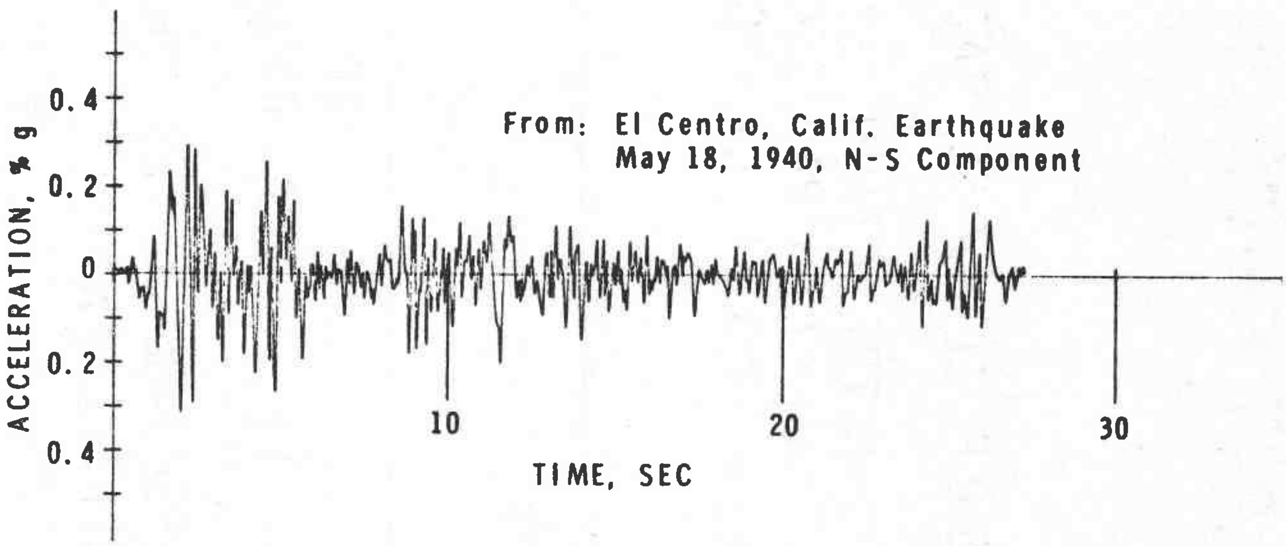

FIGURE 3 BASE ACCELERATION FOR SAMPLE CALCULATIONS

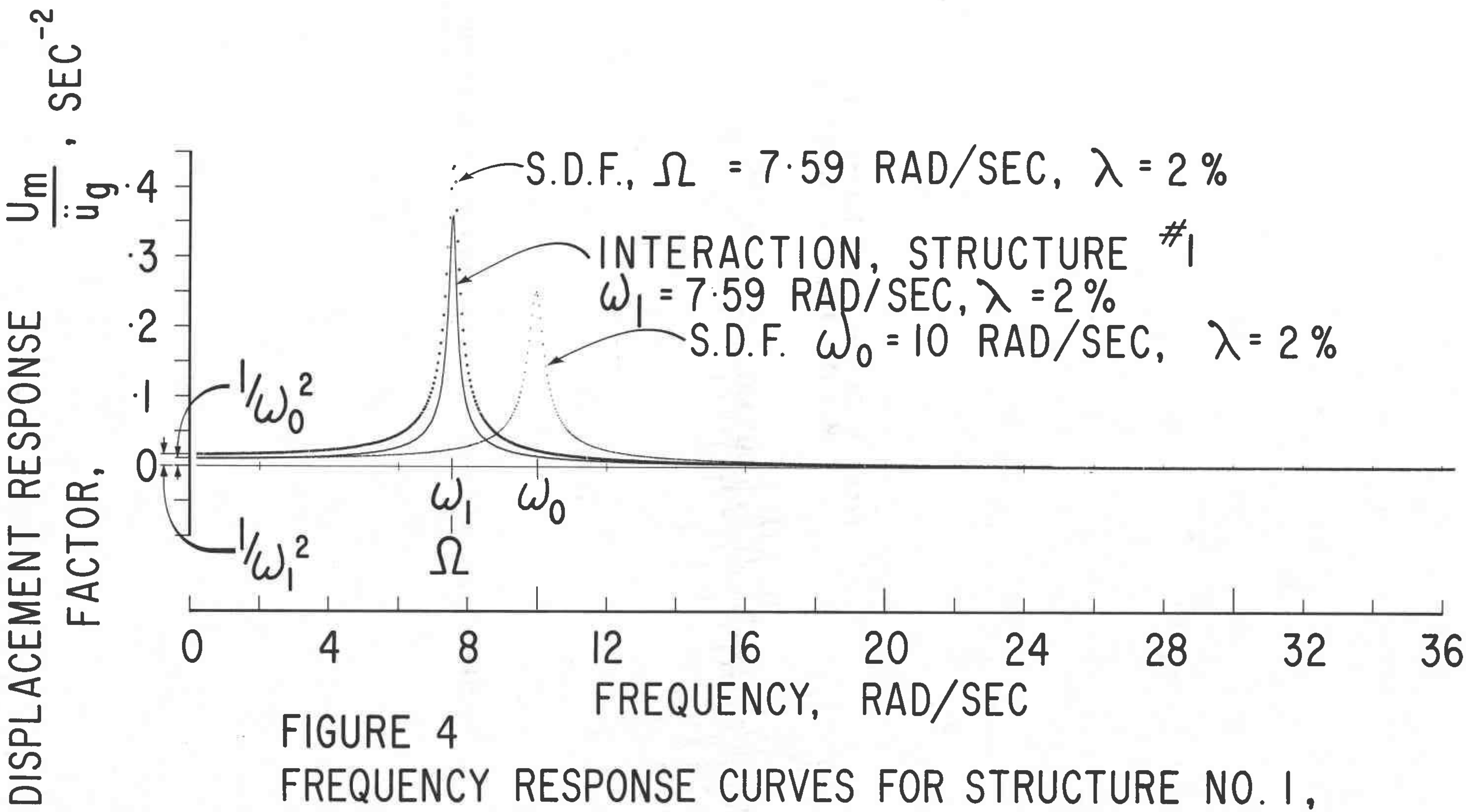

FIGURE 4
FREQUENCY RESPONSE CURVES FOR STRUCTURE NO. 1, RELATIVE DISPLACEMENT

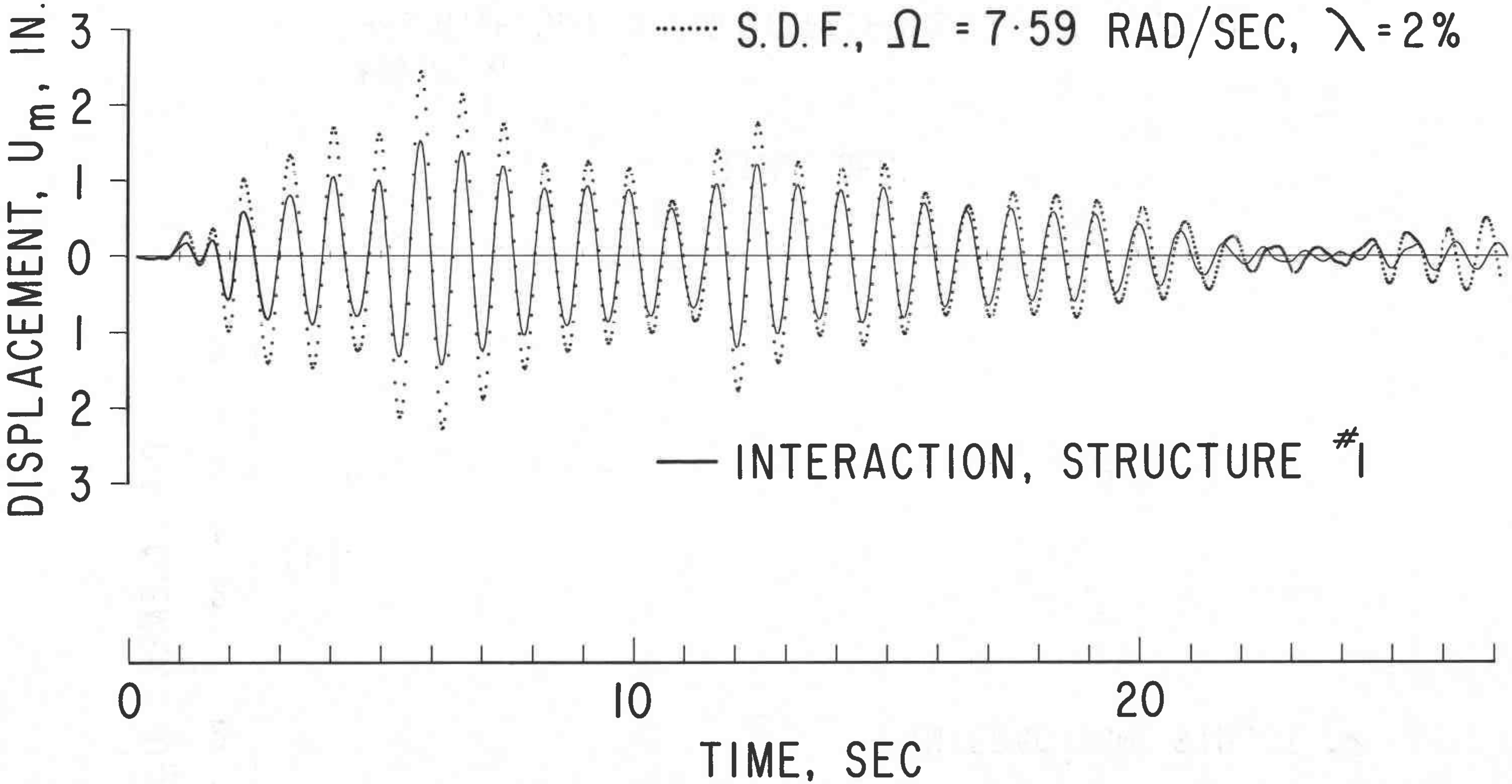

FIGURE 5
RELATIVE DISPLACEMENT RESPONSE FOR STRUCTURE NO. 1

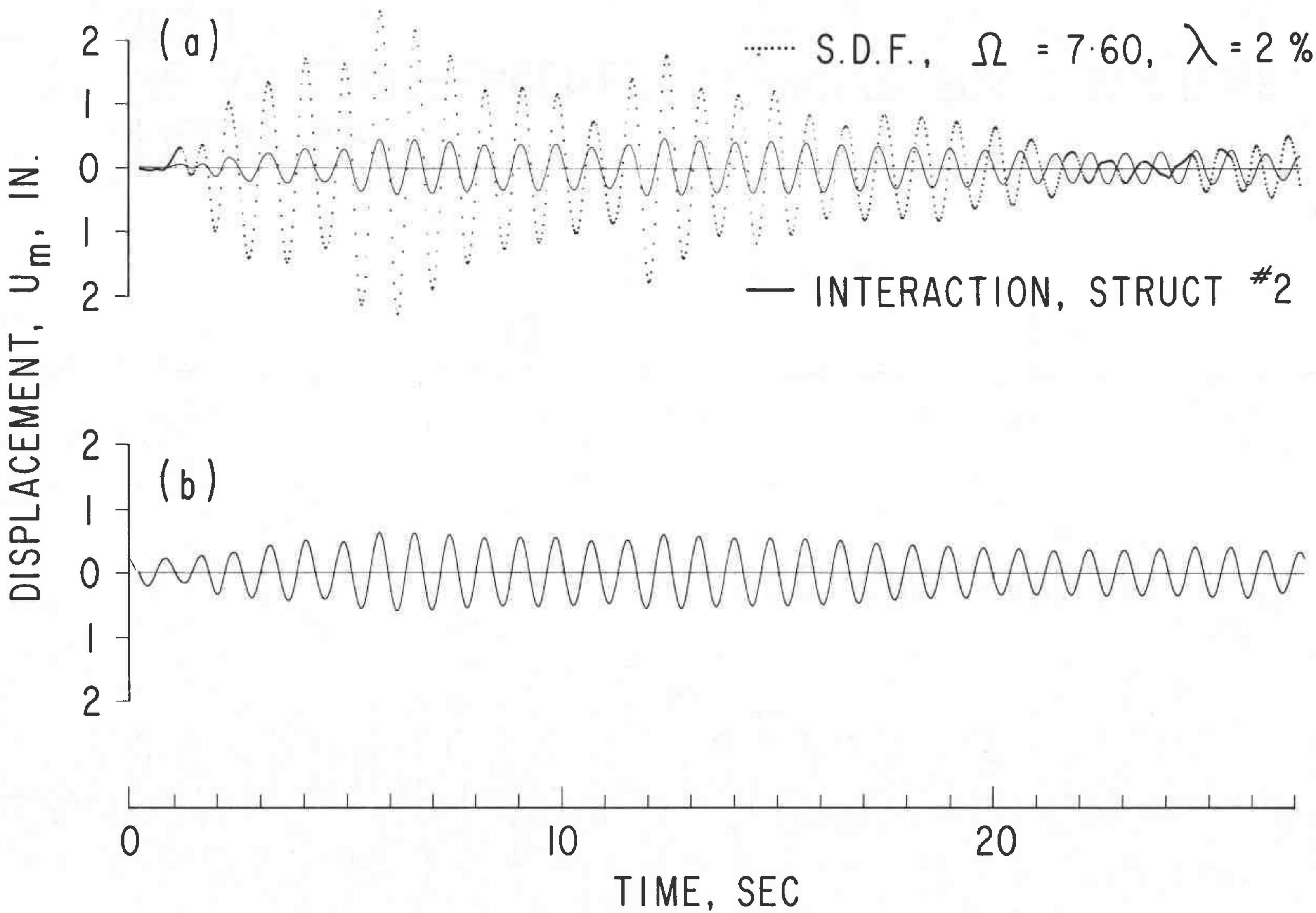

FIGURE 6
RELATIVE DISPLACEMENT RESPONSE FOR STRUCTURE NO. 2

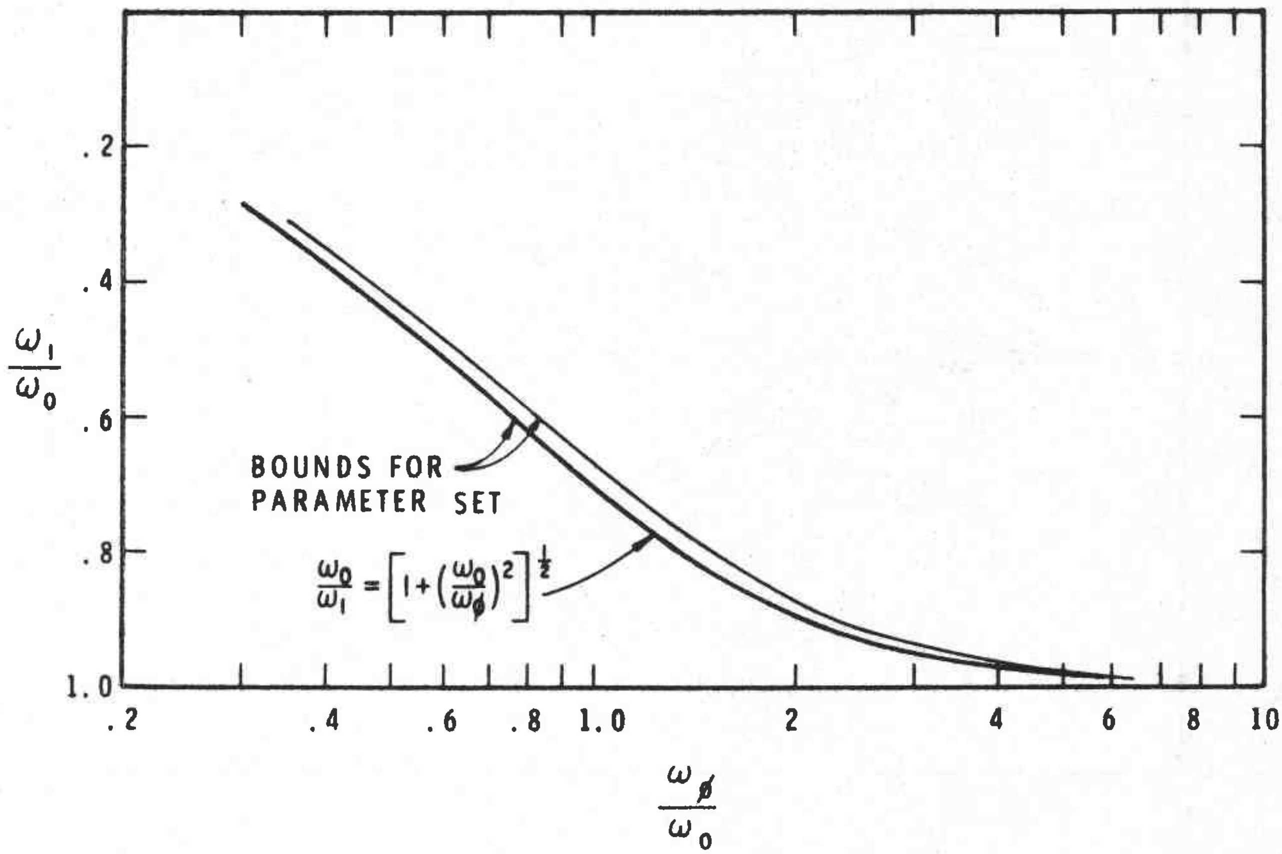

FIGURE 7 REDUCTION IN RESONANCE FREQUENCY FOR INTERACTION SYSTEMS

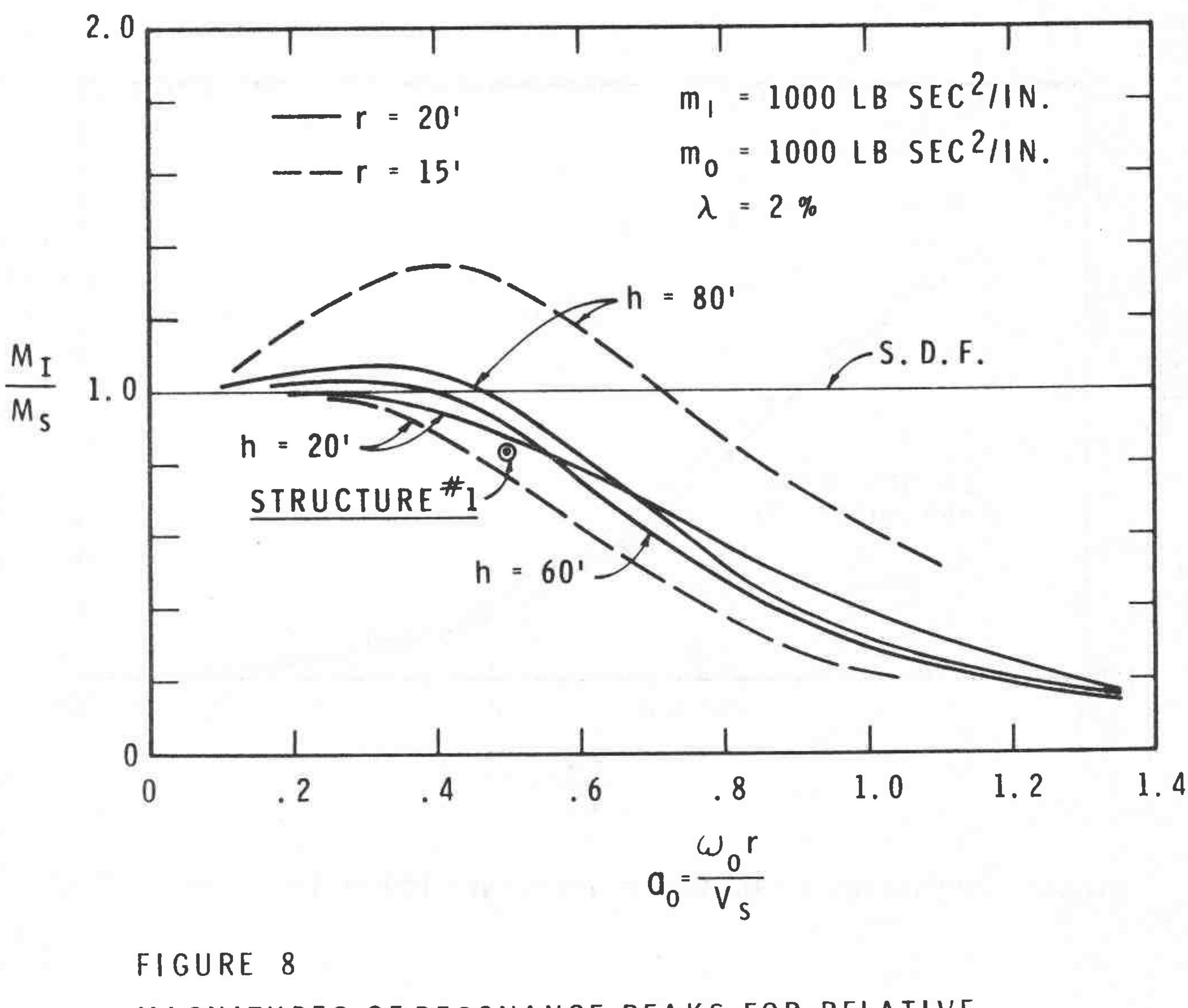

FIGURE 8

MAGNITUDES OF RESONANCE PEAKS FOR RELATIVE DISPLACEMENTS, m_1=1000 LB SEC^2/IN., BYCROFT FOUNDATION

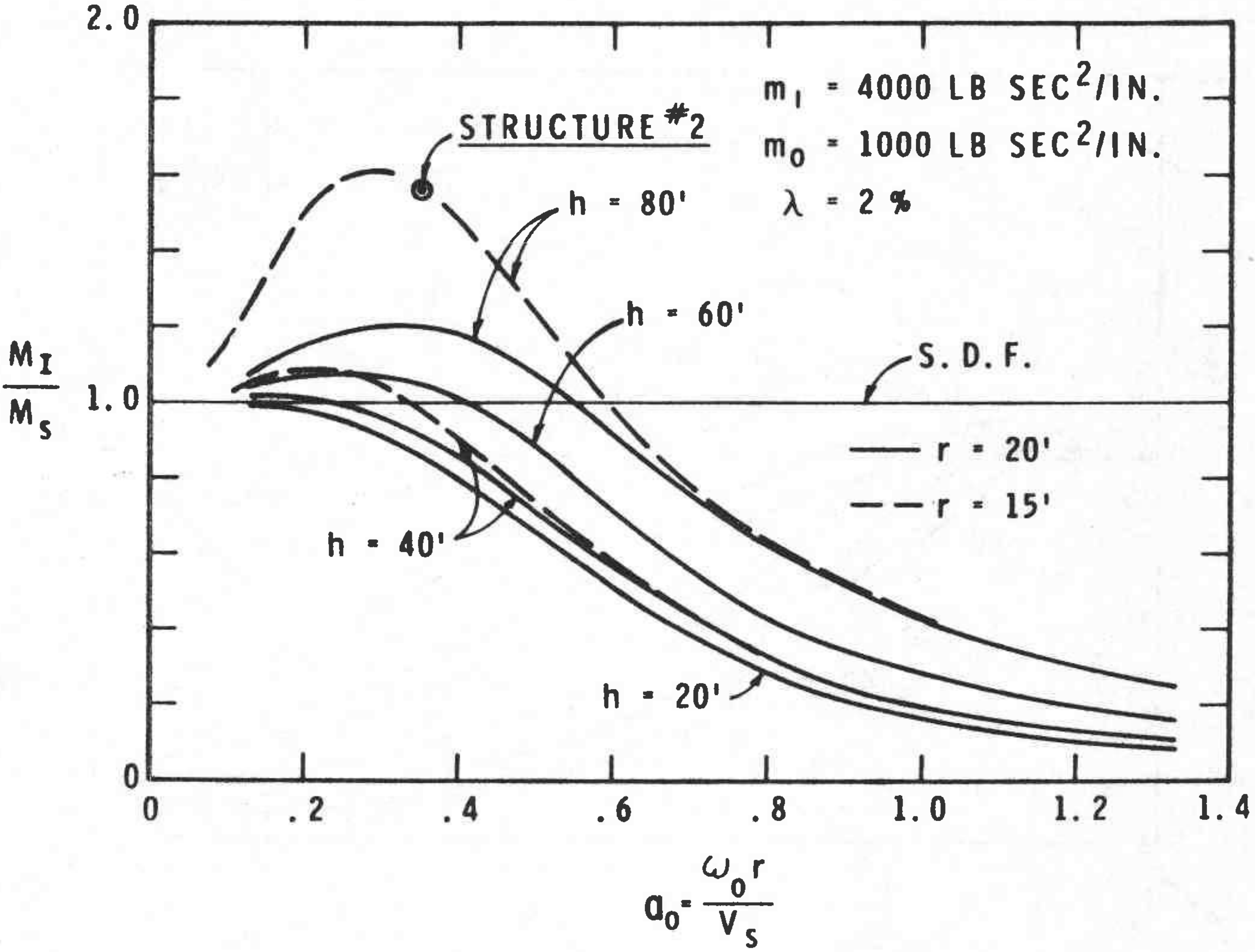

FIGURE 9

MAGNITUDES OF RESONANCE PEAKS FOR RELATIVE DISPLACEMENTS, m_l=4000 LB SEC^2/IN., BYCROFT FOUNDATION

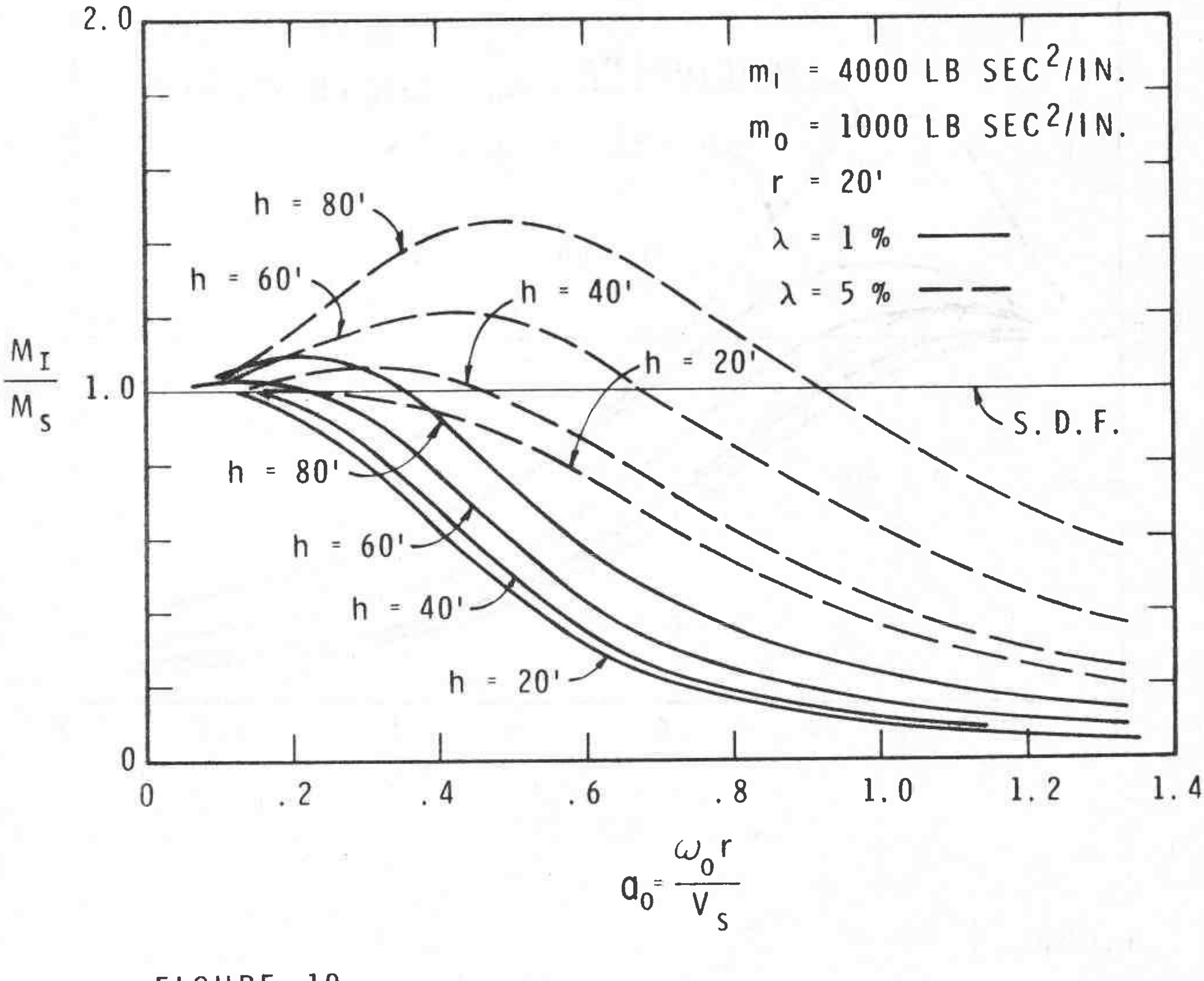

FIGURE 10

MAGNITUDES OF RESONANCE PEAKS FOR RELATIVE DISPLACEMENTS, m_1 = 4000 LB SEC^2/IN., λ = 1 PER CENT AND 5 PER CENT, BYCROFT FOUNDATION

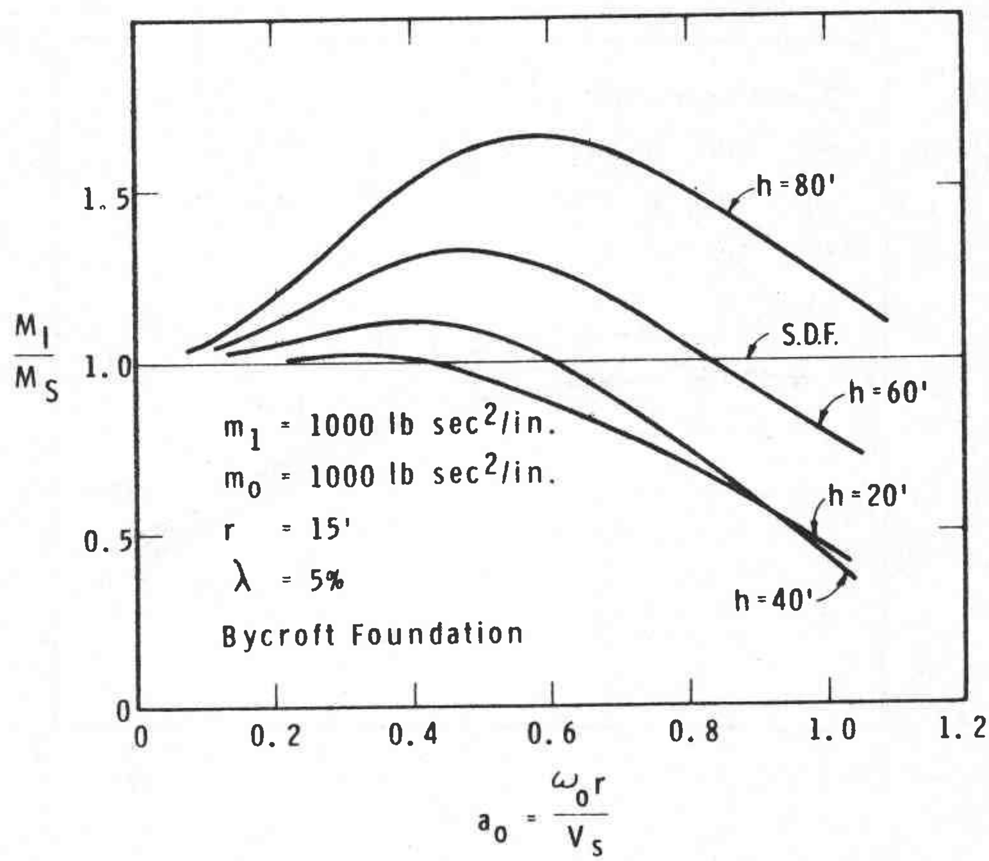

FIGURE 11

MAGNITUDES OF RESONANCE PEAKS FOR RELATIVE DISPLACEMENT, m_1 = 1000 LB SEC^2/IN., λ = 5%, r = 15 FT., BYCROFT FOUNDATION

GA 4557-4

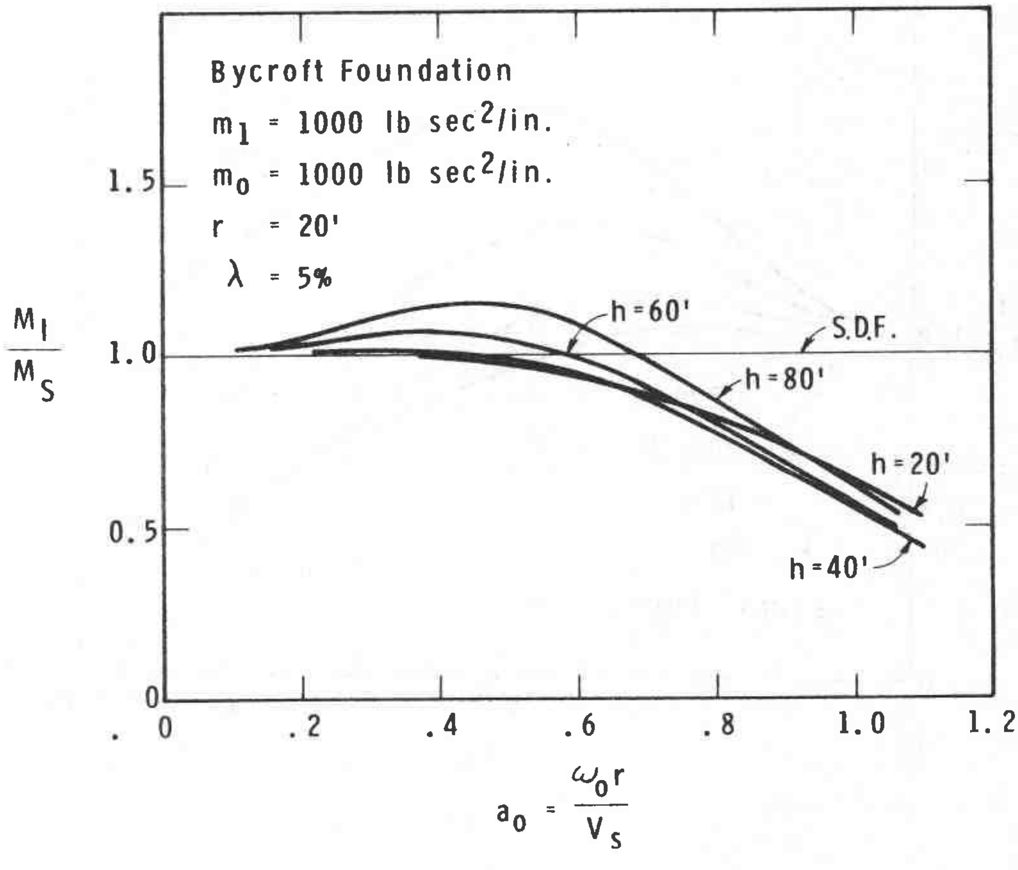

FIGURE 12

MAGNITUDES OF RESONANCE PEAKS FOR RELATIVE DISPLACEMENT, m_1 = 1000 LB SEC^2/IN., λ = 5%, r = 20 FT., BYCROFT FOUNDATION

8Q 4557-5

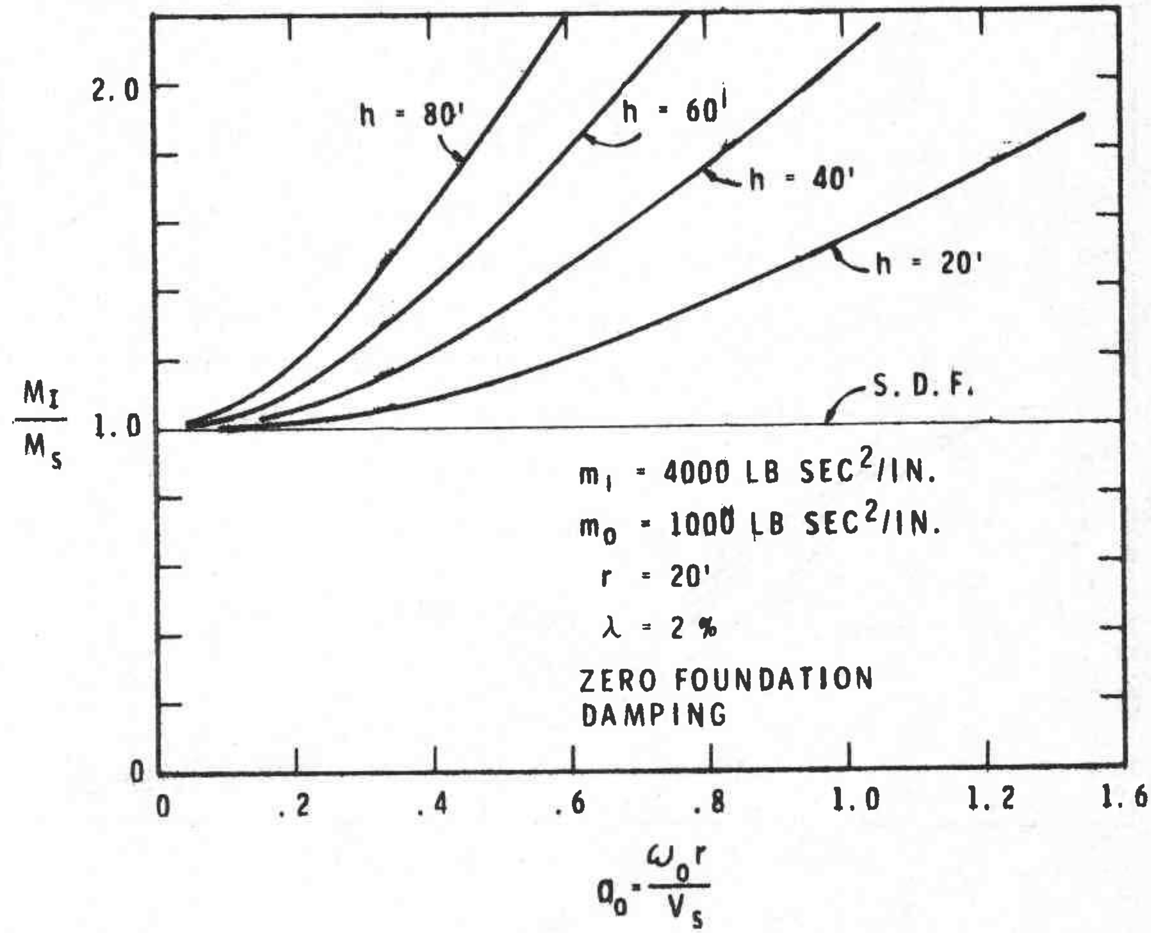

FIGURE 13

MAGNITUDES OF RESONANCE PEAKS FOR RELATIVE DISPLACEMENTS, m_1 = 4000 LB SEC^2/IN., BYCROFT FOUNDATION WITH ZERO FOUNDATION DAMPING

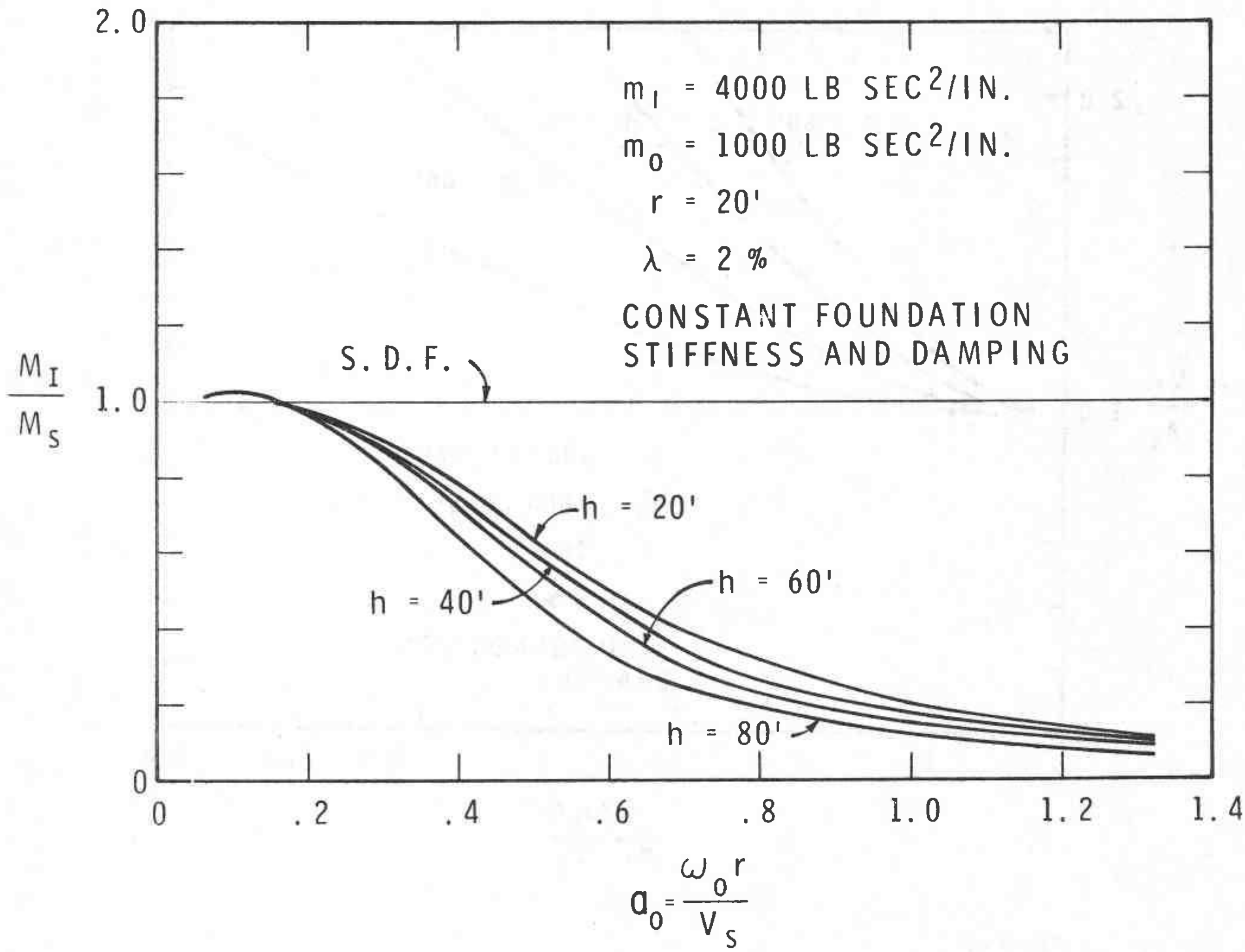

FIGURE 14

MAGNITUDES OF RESONANCE PEAKS FOR RELATIVE DISPLACEMENTS, m_1 = 4000 LB SEC^2/IN., CONSTANT S.D.F. FOUNDATION STIFFNESS AND DAMPING

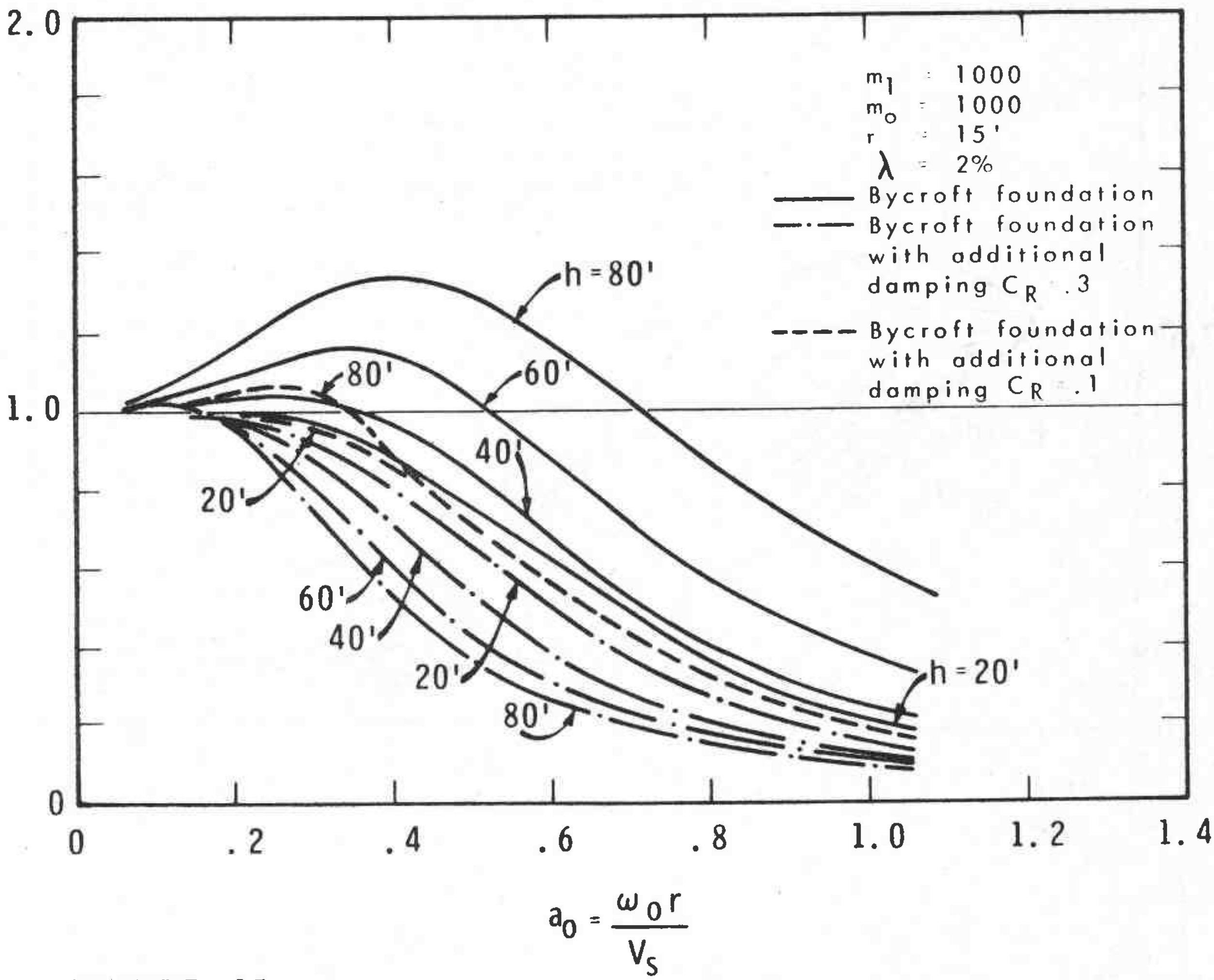

FIGURE 15

MAGNITUDES OF RESONANCE PEAKS FOR RELATIVE DISPLACEMENT WITH ADDITIONAL ROCKING DAMPING, m_1 = 1000 LB SEC^2/IN

GR 4729-2

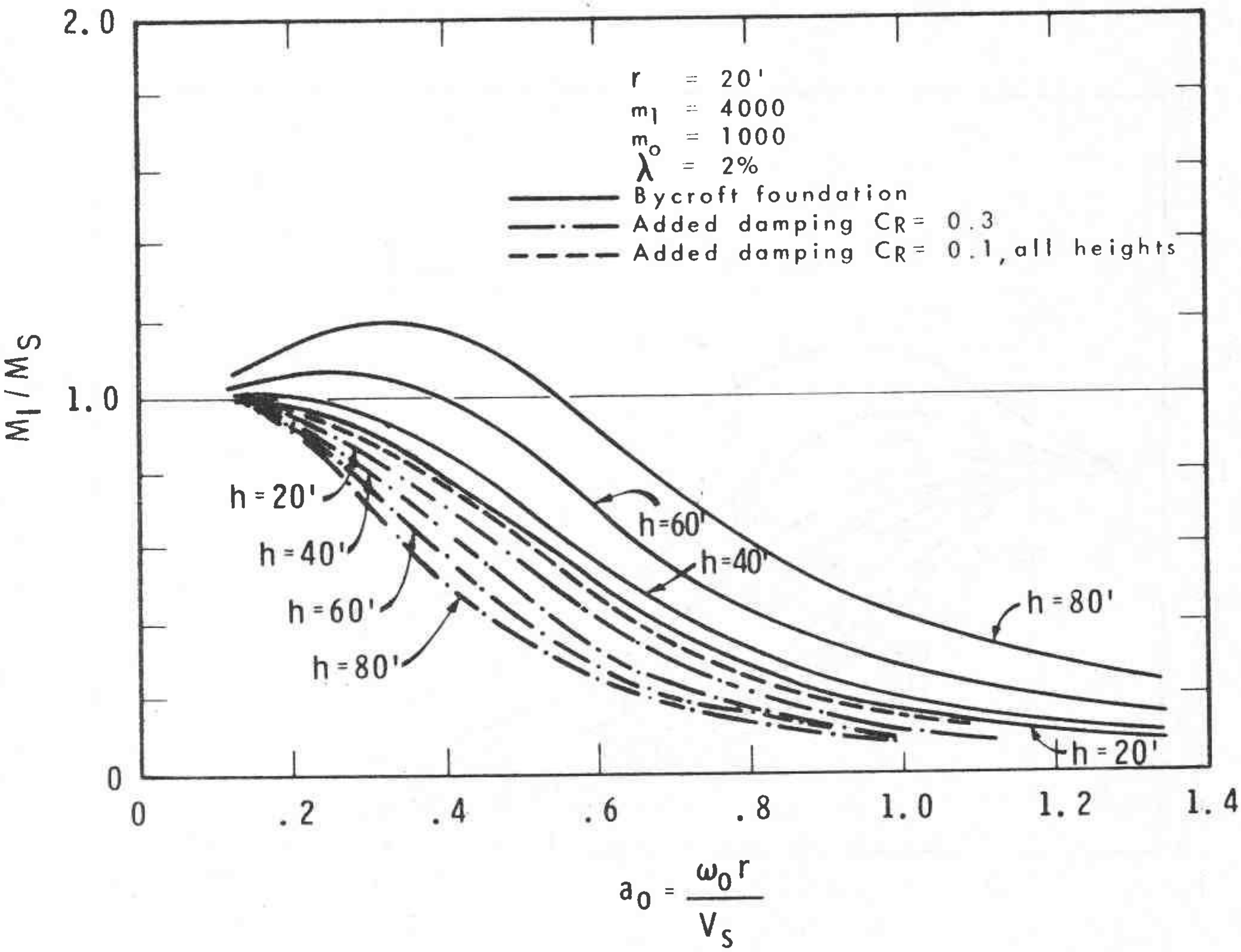

FIGURE 16

MAGNITUDES OF RESONANCE PEAKS FOR RELATIVE DISPLACEMENT WITH ADDITIONAL ROCKING DAMPING, m_1 = 4000 LB SEC^2/IN

SR 4729 -1

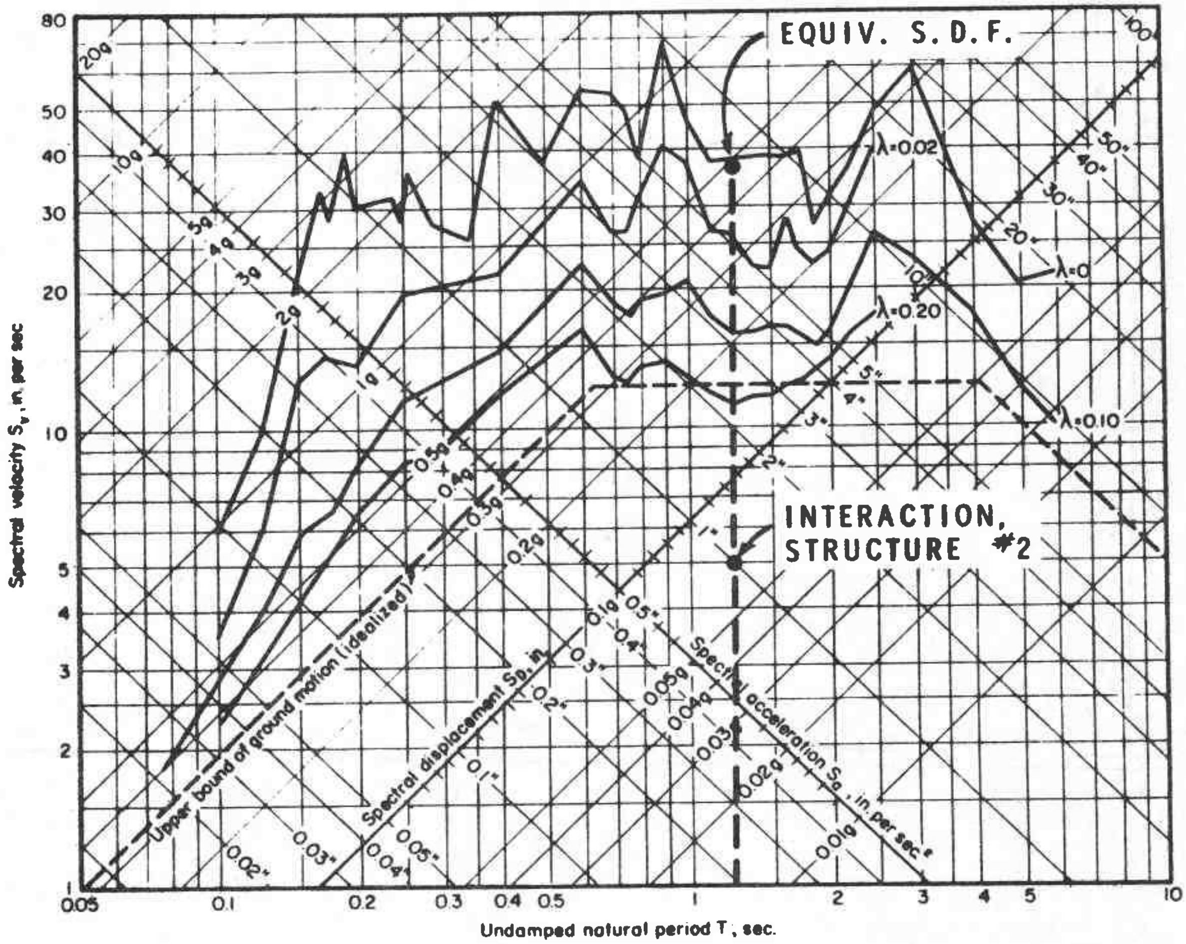

FIGURE 17

ELASTIC RESPONSE SPECTRA, 1940 EL CENTRO EARTHQUAKE, N-S COMPONENT (18)

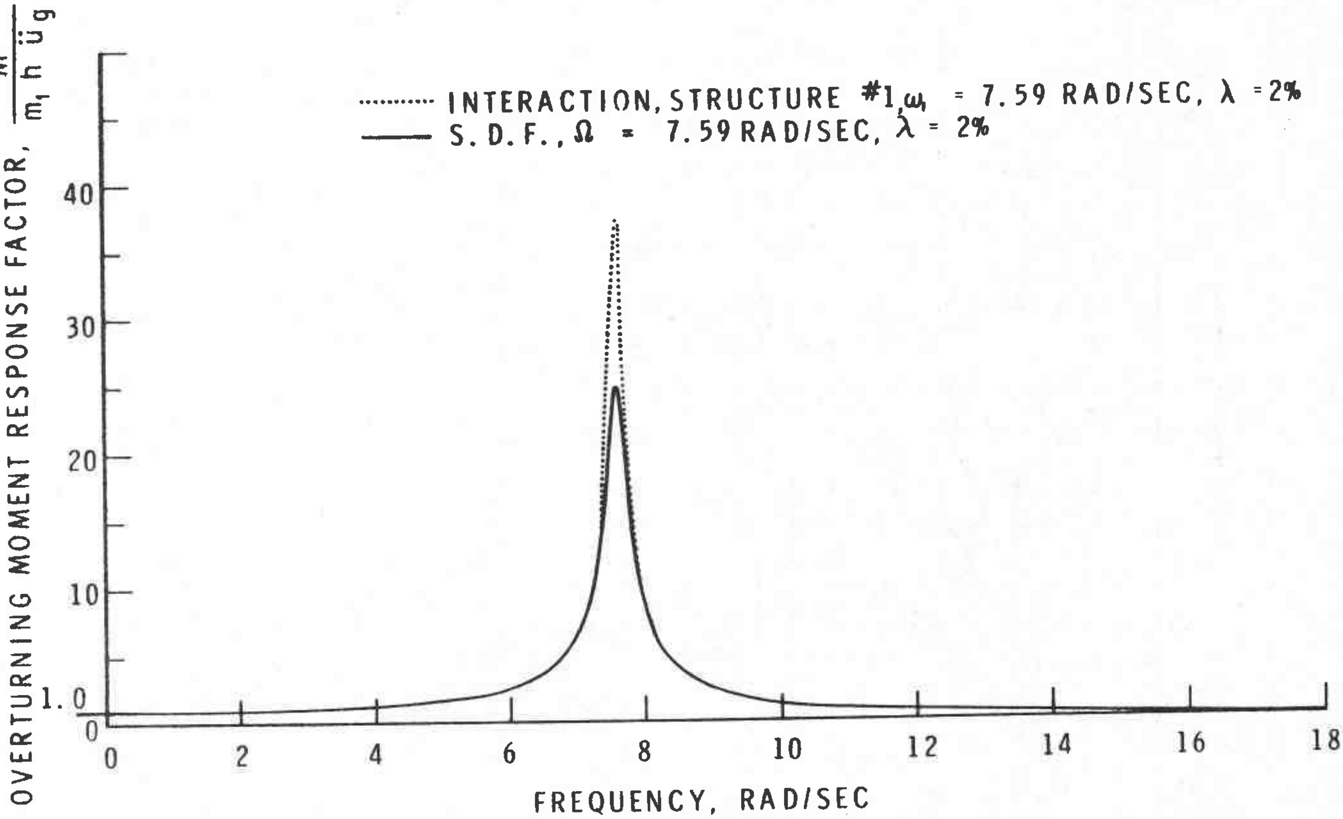

FIGURE 18

FREQUENCY RESPONSE CURVES FOR STRUCTURE NO. 1, OVERTURNING MOMENT

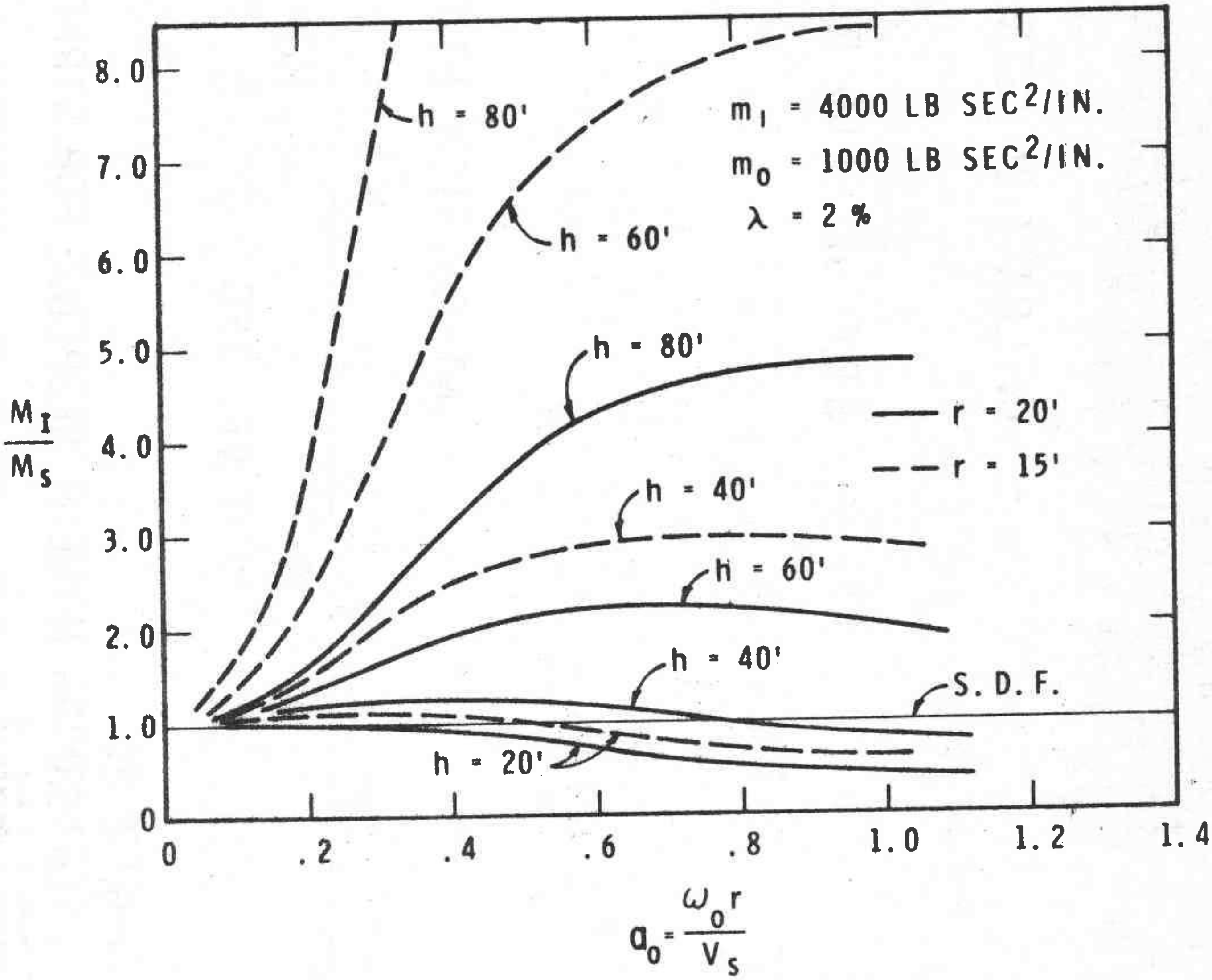

FIGURE 19

MAGNITUDES OF RESONANCE PEAKS FOR OVERTURNING MOMENTS, m_1 = 4000 LB SEC^2/IN., BYCROFT FOUNDATION

DA.4981-6

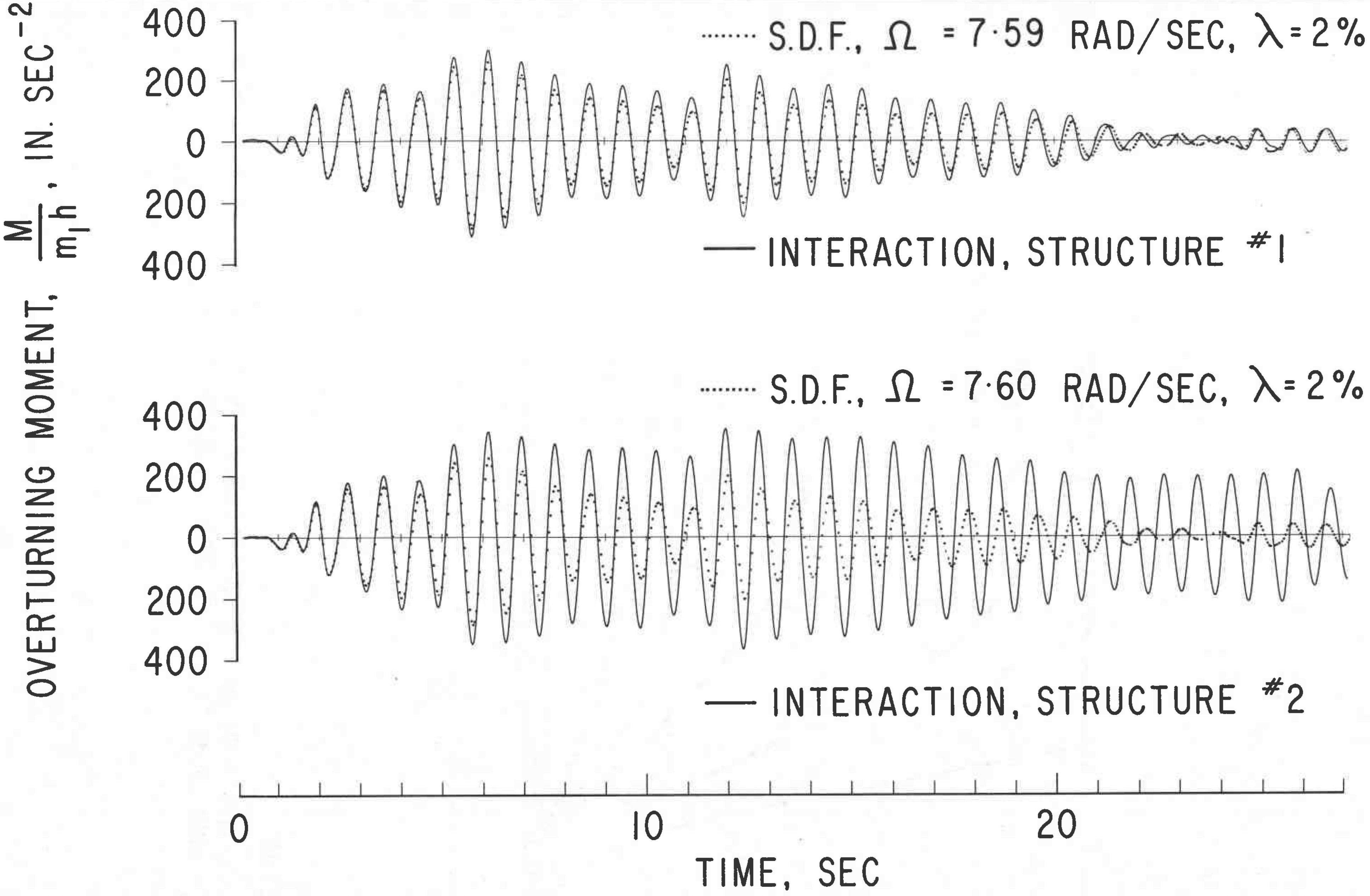

FIGURE 20
OVERTURNING MOMENT RESPONSE FOR STRUCTURES
NO. 1 AND 2