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SPURIOUS WAVES DURING GENERATION OF MULTI-CHROMATIC WAVES IN THE WAVE TANK IN SHALLOW WATER

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ABSTRACT

Accurate generation of the primary waves and the reproduction of the group-induced second-order low and high frequency waves have been considered essential for physical i.e. model test in the laboratory. In the laboratory when multi-chromatic primary waves are generated the required bounded waves will be generated naturally at the difference frequencies. In addition to that several unwanted free waves are also generated. The free waves, having the same frequencies of the bounded waves are reproduced due to mismatch of the boundary conditions at the wave paddle. The other two types of free waves are due to the wave paddle displacement and the local disturbances.

We carried out physical experiments to identify the second order spurious waves in shallow water in the Offshore Engineering Basin (OEB) at the Institute for Ocean Technology (IOT) of National Research Council (NRC) Canada. In the basin water depths in the range of 0.4m to 0.6m are used for the experiments. The peak wave periods also have varied from 1.133s to 2.145s. In the experiments multi-chromatic waves are used. The drive signals of the wave-makers are generated using first-order and second-order wave generation techniques. Total 14 wave probes are used to capture the data in the wave tank. A NRC-IOT code is used to isolate the primary waves, the bounded waves and the unwanted free waves from the measured data at each wave probe. The measured data are analyzed in this paper to illustrate the differences in the waves generated by two different generation techniques.

KEYWORD

Physical experiments, *multi-chromatic shallow water waves*, *second order waves, Bounded waves, second-order spurious waves*, *data comparisons*.

INTRODUCTION

During the generation of multi-chromatic primary waves in the wave basin the required bounded waves will also be generated naturally at the difference frequencies. On top of that several unwanted free waves are also generated. The second order free wave FW-1, having the same frequency of the bounded wave is reproduced due to mismatch of the boundary conditions at the

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wave paddle. The second order free wave FW-2 appears due to the displacement of the wave paddle. The second order free wave FW-3 is due to the local disturbances. The local disturbances usually disappear at some distance from the wave-makers and thus will not be discussed here. If the wave basin is not unconditionally flat then there might be some other types of unwanted free waves in the basin in addition to the aforesaid three categories.

Proper understanding of the effects of the wave-action and consequent loading pattern of the primary waves along with their bounded waves are very important factors to design, implementation and operation of any ocean structures, mooring system, floating vessels, harbor resonance, etc.. In an accurate physical model test in the laboratory it is crucial to choose the precise design parameters of such structures and/or vessels, etc. So it is essential to reduce or eliminate the unwanted free waves components for physical model tests and to ensure the reproduction of the group-induced second-order low frequency and high frequency components in the laboratory. Please see Zaman and Mak (2007) for high frequency second-order wave components and Zaman et al (2010) for mono- and bi-chromatic waves, not to be discussed here.

In the case of multi-chromatic waves the simplest unit contributing to the second order waves is a two-frequency group. In such a wave field the total contributions of any unwanted free waves is the summation of all the similar components due to interaction of any two wave components in the multi-frequency wave field. A two-frequency wave group is evolved due to the interaction of any two waves of frequencies f_1 and f_2, the group-induced second-order low and high frequency waves are generated along with other unwanted free waves. A low frequency wave or bounded wave will be produced due to the difference ($f_1 - f_2$) of the frequencies and a high frequency or short wave would be generated due to the summation ($f_1 + f_2$) of the frequencies. The profile of the bounded wave having frequency ($f_1 - f_2$) is generally termed as the set-down in the larger wave zone and the set-up in the smaller wave zone. These set-down and set-up phenomena were first investigated and reported by Longuet-Higgins and Stewart (1961, 1962, 1963, 1964 and 1977) in a series of papers. They introduced the radiation stress concept, which explained that in a wave group individual wave components exerted an internal compressive force in the direction of the wave propagation. To balance this force the mean water level goes down in the region of larger waves known as set-down and goes up in the region of smaller waves known as set-up. Bowen et al (1968) later explained the set-up and set-down phenomena with experimental data.

The theoretical and experimental descriptions of such natural high and low frequency waves along with their various unwanted free waves were given by many researchers. Hansen (1978), Sand (1982), Barthel et al (1983), Sand and Mansard (1986a, b), Mansard (1991), Mansard et al (1987), Schaffer (1993), Stansberg (2006), Zaman and Mak (2007), Zaman et al (2010), Spinneken and Swan (2009) are a few who described the methods to curtailing these unavoidable free waves from the resulting surface elevations in the wave basin.

In our present experiments the first-order wave generation (FOG) and the second-order wave generation (SOG) techniques for the multi-chromatic waves were used. In the experiment water depths are varied in the range of 0.4m to 0.6m. The wave periods also have varied from 1.133s to 2.145s. However in this paper several cases for multi-chromatic waves in the water depth of 0.4m are shown and discussed. Comparisons are made between data obtained from the first-order wave generation technique and the second-order wave generation technique.

FOMULATION OF THE FIRST-ORDER AND SECOND-ORDER GENERATION METHODS

A wave group would be generated with the presence of at least two frequencies. The difference of these two frequencies would generate a long period bounded wave with a period equal to the period of the wave group. This long wave is also known as 'set-up and set-down'. The description of the waves in nature is normally given by a wave spectrum. For irregular waves with a given spectral density, by Fourier series expansion, the first order wave profile (see Sand 1982) can be written as function of the coefficients a_n and b_n.

$$\eta_1(x,t)=\sum_{n=1}^{N} a_n \cos(\omega_n t - k_n x) + b_n \sin(\omega_n t - k_n x) \quad (1)$$

In which N is the total number of frequencies in the wave trains, ω_n is the cyclic frequency and k_n is the wave number, a_n and b_n are the Fourier coefficients, respectively.

For the piston type wave maker, the first-order control signal is given by the summation of contributions from all the frequencies in the spectrum [see also (Barthel et al (1983) and Sand and Mansard (1986b)].

$$X_1(t)=\sum_{n=1}^{N}\left[\begin{array}{c}\dfrac{\cosh(k_n h)\sinh(k_n h)+k_n h}{2\sinh^2(k_n h)}\times \\ (a_n \sin(\omega_n t)-b_n\cos(\omega_n t))\end{array}\right] \quad (2)$$

The simplest unit contributing to the second order waves is a two frequency group. The total second order wave elevation is then found as the summation of the contributions from all pairs of frequencies.

By means of Laplace equations, the second order contributions due to one pair of difference frequency can be given as follows:

$$\begin{aligned}\eta_2^-(x,t)= G_{mn}^-(a_m a_n - b_m b_n)\cos(\Delta\omega_{mn}^- t - \Delta k_{mn}^- x) + \\ G_{mn}^-(a_m b_n + a_n b_m)\sin(\Delta\omega_{mn}^- t - \Delta k_{mn}^- x)\end{aligned} \quad (3)$$

where G_{mn}^- is a second order quadratic transfer function, $\Delta\omega_{mn}^- = \omega_n - \omega_m$ and $\Delta k_{mn}^- = k_n - k_m$, n and m are the index of one pair of difference frequency.

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The quadratic transfer function is given by the following equation:

$$G^-_{mn} = \left(\frac{A^-_{nm}(\omega_n-\omega_m)}{2g} - \frac{k_m k_n g}{4\omega_m\omega_n}(1+\tanh k_m h \tanh k_n h) + \frac{1}{4}(k_m \tanh k_m h + k_n \tanh k_n h)\right)\delta \tag{4}$$

where,

$$A^-_{nm} = \frac{1}{2}\frac{B^-_{mn}+C^-_{mn}}{(\omega_n-\omega_m)^2-(k_n-k_m)g\tanh(k_n-k_m)h}g^2$$

$$B^-_{mn} = \frac{k_m^2}{\omega_m\cosh^2 k_m h} - \frac{k_n^2}{\omega_n\cosh^2 k_n h}$$

$$C^-_{mn} = \frac{2k_m k_n(\omega_n-\omega_m)(1+\tanh k_m h\tanh k_n h)}{\omega_m\omega_n}$$

where, h is the water depth and g is the acceleration due to gravity and

$$\delta = \begin{cases} 1 & n \neq m \\ 0.5 & n = m \end{cases}$$

The second order control signal for a correct reproduction of the wave train up to second order takes the following form:

$$X_2^-(t) = \sum_{n=f_0}^{N}\sum_{m=f_0}^{N-n} ((a_n b_m - a_m b_n)F_1^- + (a_n a_m + b_m b_n)F_{23}^-)\times \cos(\Delta\omega^-_{mn}t) + ((a_n a_m + b_m b_n)F_1^- + (a_m b_n - a_n b_m)F_{23}^-)\times \sin(\Delta\omega^-_{mn}t) \tag{5}$$

The function, F_1^- is written as:

$$F_1^- = F_{11}^- + F_{12}^- \tag{6}$$

$$F_{11}^- = \left[G^-_{mn}\Delta k_f^- h\begin{Bmatrix}(\Delta k^-_{mn}h - \Delta k_f^- h)\sinh(\Delta k^-_{mn}h + \Delta k_f^- h) + \\ (\Delta k^-_{mn}h + \Delta k_f^- h)\sinh(\Delta k^-_{mn}h - \Delta k_f^- h)\end{Bmatrix}\right] / \left[2((\Delta k^-_{mn}h)^2 - (\Delta k_f^- h)^2)\sinh(\Delta k^-_{mn}h)\sinh(\Delta k_f^- h)\right] \tag{7}$$

$$F_{12}^- = \frac{f_m \Delta k_f^- k_m h(1+H_n)[\delta k_m^- h\sinh(\delta k_m^+ h) + \delta k_m^+ h\sinh(\delta k_m^- h)]}{8(f_n - f_m)((k_m h)^2 - (\Delta k_f^- h)^2)\sinh(\Delta k_f^- h)\sinh(k_m h)\tanh(k_n h)} + \frac{f_n \Delta k_f^- k_n h(1+H_m)[\delta k_n^- h\sinh(\delta k_n^+ h) + \delta k_n^+ h\sinh(\delta k_n^- h)]}{8(f_n - f_m)((k_n h)^2 - (\Delta k_f^- h)^2)\sinh(\Delta k_f^- h)\sinh(k_n h)\tanh(k_m h)} \tag{8}$$

The free long wave number, Δk_f^- is computed from the dispersion relation given as:

$$(\Delta\omega^-_{mn})^2 = g\Delta k_f^- \tanh(\Delta k_f^- h) \tag{9}$$

where,

$$\delta k_m^{\pm} = k_m \pm \Delta k_f^-$$

$$\delta k_n^{\pm} = k_n \pm \Delta k_f^-;\ H_m = \frac{2k_m h}{\sinh(2k_m h)};\ H_n = \frac{2k_n h}{\sinh(2k_n h)}$$

$$F_{23}^- = F_2^-(F_{3,m}^- - F_{3,n}^-) \tag{10}$$

$$F_2^- = \frac{\Delta k_f^-(1+H_m)(1+H_n)}{8\tanh(k_m h)\tanh(k_n h)} \tag{11}$$

$$\omega_n = 2\pi f_n \tag{12}$$

$$\omega_m = 2\pi f_m \tag{13}$$

$$F_{3,m}^- = \frac{f_m}{(f_n - f_m)} * \sum_{j=1}^{\infty}\frac{2k_j h\sin(k_j h)[k_j h\sin(k_j h)\coth(\Delta k_f^- h) + \Delta k_f^- h\cos(k_j h)]}{((k_j h)^2 + (\Delta k_f^- h)^2)(\sin(k_j h)\cos(k_j h) + k_j h)} \tag{14}$$

where, $k_j h$ is computed from the following expression:

$$\frac{\omega_m^2 h}{g} = -k_j h\tan(k_j h)\text{ ,with } (j-\tfrac{1}{2})\pi < k_j h < j\pi \tag{15}$$

In the above equation, f_n and f_m are the possible frequency components in the wave field. For example, for only two waves the above frequency components will be modified to f_1 and f_2, respectively. The functions F_{11}^- and F_{12}^- respectively, will reduce or eliminate the Free wave-1 and Free wave-2 from the wave field. The function F_{23}^- is to eliminate the second order free wave-3 due to local disturbances. This function is not considered here as the local disturbances disappeare after some distance from the wave maker.

DESCRIPTION OF THE EXPERIMENTAL SETUP

The experiment was carried out at the Offshore Engineering Basin of National Research Council Canada, Institute for Ocean Technology. The top view of the basin is shown in Fig. 1. The Offshore Engineering Basin is 75 m long x 32 m wide. 56 independently controlled segmented wave generators installed on the west wall generated the waves. Each segmented wave generator is 2 m high and 0.5 m wide. Passive absorbers, made of expanded metal sheets with varying porosities and spacing, are installed on the east wall. A solid metal wall is used to cover the north side of the basin. The water depths for the experiments are 0.4m, 0.5m and 0.6m. However, the cases presented and discussed in this paper are confined to the 0.4m water depth experiments. All other data will be published elsewhere later.

During the experiment, 14 wave probes installed as shown in Fig. 1 and Table-1 measured the location of the wave probes throughout the basin. All the wave probes are capacitance type. All the data was acquired using GDAC (GEDAP Data Acquisition and Control) client-server acquisition system, developed by National Research Council Canada, Institute for Ocean Technology.

Table 1 Location of the wave probes in the OEB

No of the probe	Distance from the east wave paddle (m)	Distance from the south wall (m)
1	26.891	13.475
2	27.221	13.475
3	27.731	13.475
4	27.731	12.955
5	27.731	12.635
6	27.731	14.825
7	27.731	18.365
8	29.081	13.475
9	32.621	13.475
10	41.621	13.475
11	2.0	12.635
12	2.0	13.475
13	2.0	18.365
14	10.744	13.475

RESULTS AND DISCUSSIONS

Two cases of the 0.4m water depth experiments are reported. Table 2 summarizes the incident wave parameters of the wave conditions examined in this paper.

Table 2 Incident wave parameters

	h (m)	T_p (s)	H_s (m)	h/L
Case-1	0.4	1.133	0.06	0.224
Case-2	0.4	1.705	0.06	0.130

We used JONSWAP and TMA spectrum systems for this experiment. However this paper deals with the JONSWAP spectrum only. For each case, there were two runs, one used first-order generation technique and the other one used the second-order wave generation technique. Fig. 2 shows the comparisons of the spectrums at 6 different locations in the wave basin between first-order and second-order wave generation techniques for Case-1. These 6 locations are at Probe-14, Probe-1, Probe-2, Probe-3, Probe-8 and Probe-9. These probes are on the same line, see Fig. 1. In Fig. 2 it may be observed that the low frequency components are not prominent in either generation technique.

Fig. 3 shows the same comparisons for Case-2. In Fig. 3 one can perceive the differences in low frequency second order wave components between two different generation techniques. So from now on we will concentrate on Case-2 only to identify the spurious components. A NRC-IOT computer code that can split a surface elevation data set into its component waves is used to isolate the primary waves, bounded second order waves and unwanted free waves from the raw measured data at every probe location.

Figs. 4a and 4b show comparisons of various separated wave components for Case-2 at Probe-1. Fig. 4a shows the comparisons of the measured primary waves of first order generation with unwanted free waves of first order generation and second order generation techniques. Fig. 4b, on the other hand, shows the comparisons of the primary wave with bounded waves of first and second order generation techniques at Probe-1. Figs. 5a and 5b show similar comparisons at Probe-2 for Case-2.

From Figs. 4a and 5a it may be observed that the unwanted free waves obtained from both generation techniques are not significantly large with respect to their primary wave components.

Table 3 shows the comparisons of the bounded waves and unwanted free waves at two probe locations for both first-order and second-order generation techniques. The comparisons [%= (FW1 / PW)*100] or [%= (FW2 / PW)*100] are done with respect to the measured significant primary wave height, PW = 0.0446m at Probe-1 and PW = 0.0496m at Probe-2 obtained in the first-order wave generation method, respectively.

Table 3 Comparisons of Free waves and Bounded waves

	FW1 (FOG) %	FW1 (SOG) %	FW2 (FOG) %	FW2 (SOG) %	BW (FOG) %	BW (SOG) %
Maximum wave heights						
P-1	4.03	4.08	1.85	1.49	4.93	4.66
P-2	3.63	3.97	1.17	1.59	4.47	4.30
Average wave heights						
P-1	1.65	1.55	0.88	0.75	2.14	2.08
P-2	2.03	1.93	1.03	0.88	2.62	2.12
Significant wave heights						
P-1	3.01	2.71	1.51	1.13	4.02	3.30
P-2	3.37	3.32	1.63	1.29	4.18	3.44

P-1: Probe-1 and P-2 : Probe-2
FW1: Free wave-1, FW2: Free wave-2 and BW: Bounded wave

CONCLUSION

For the case of multi-chromatic wave generation, first-order and second-order wave generation techniques are used to study the propagation of the primary waves, bounded waves and unwanted free waves in the Offshore Engineering Basin of NRC-IOT. It is observed that for moderately shallow water for Case-1 and Case-2 the existence of the unwanted low frequency free wave components are not significant compare to the primary wave components. Comparing first-order and second-order wave generation techniques, it is observed that the

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magnitudes of the unwanted free waves are very similar in both generation techniques.

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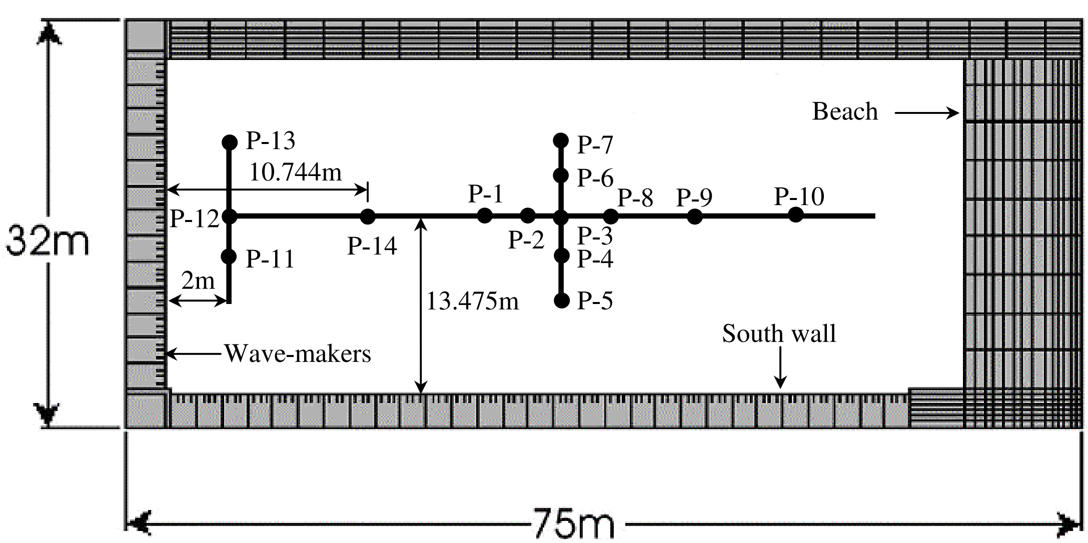

Fig. 1: Layout of the experimental tank (not to scale)

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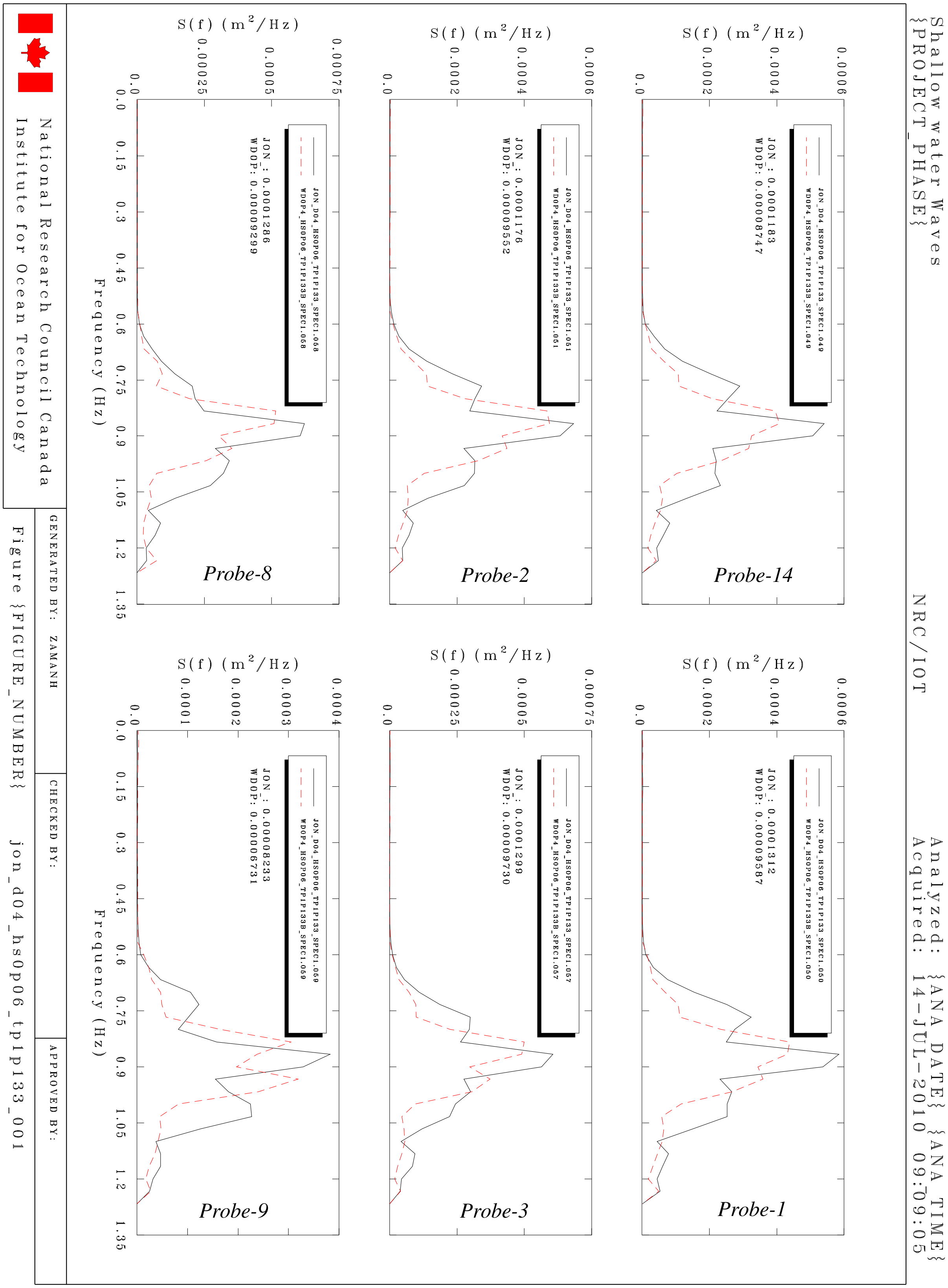

Fig. 2: Comparisons between FOG and SOG spectrums at Probe-14, Probe-1, Probe-2, Probe-3, Probe-8 and Probe-9 ($h=0.4m$, $T_p=1.133s$, and $H_s=0.06m$) . Black line is Ist-order and Red line is 2nd order generation method. (Case-1)

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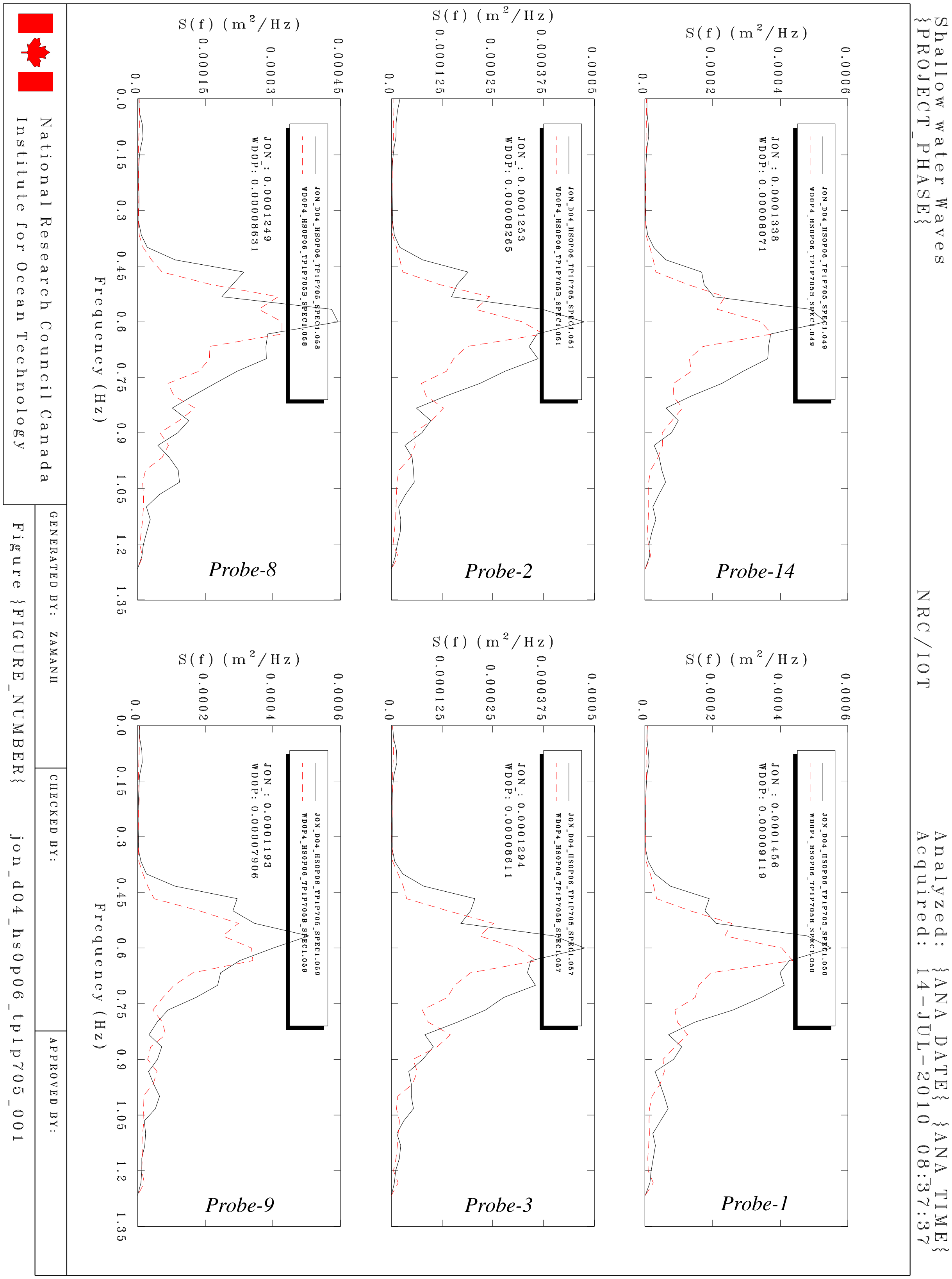

Fig. 3: Comparisons between FOG and SOG spectrums at Probe-14, Probe-1, Probe-2, Probe-3, Probe-8 and Probe-9 ($h=0.4m$, $T_p=1.705s$, and $H_s=0.06m$) . Black line is Ist-order and Red line is 2nd order generation method. (Case-2)

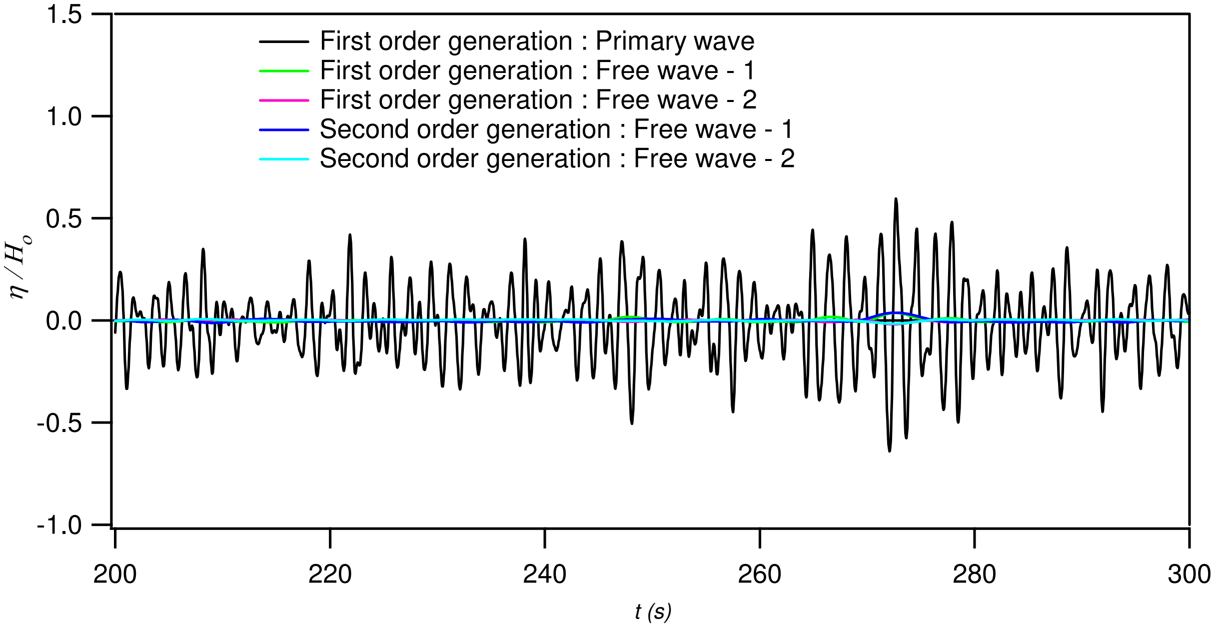

Fig. 4a: Comparisons of the measured amplitudes of Primary wave, Free wave-1 and Free wave-2 at Probe-1
(h=0.4m, T_p=1.705s and H_s=0.06m)
(Case-2)

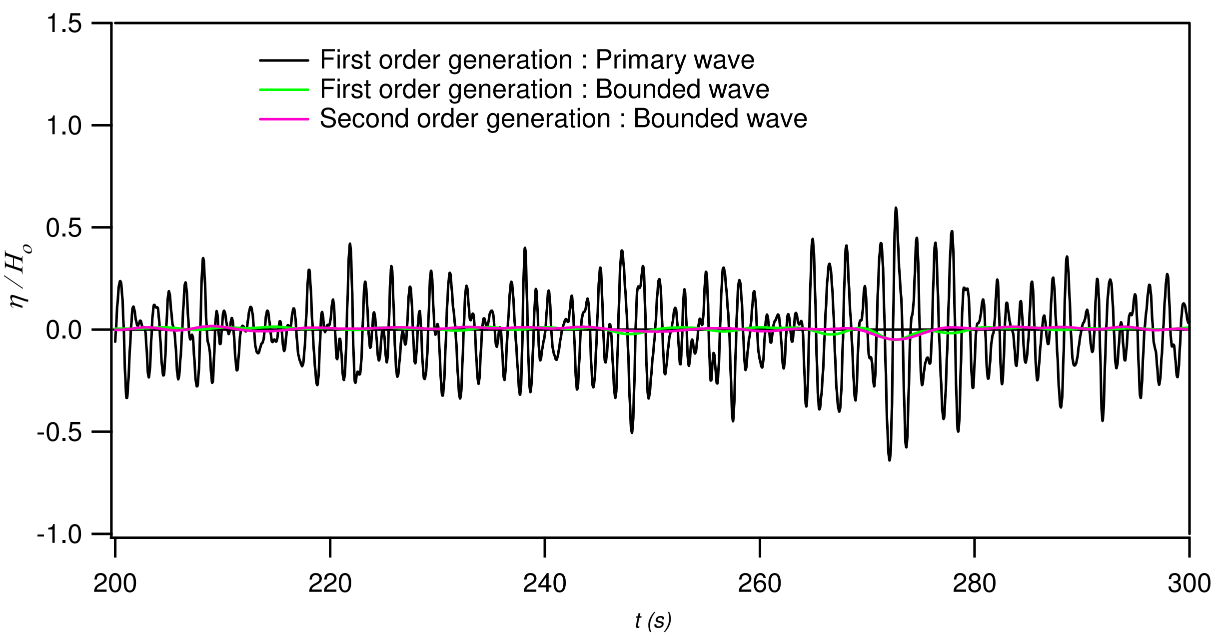

Fig. 4b: Comparisons of the measured amplitudes of Primary and Bounded wave's amplitudes at Probe-1
(h=0.4m, T_p=1.705s and H_s=0.06m)
(Case-2)

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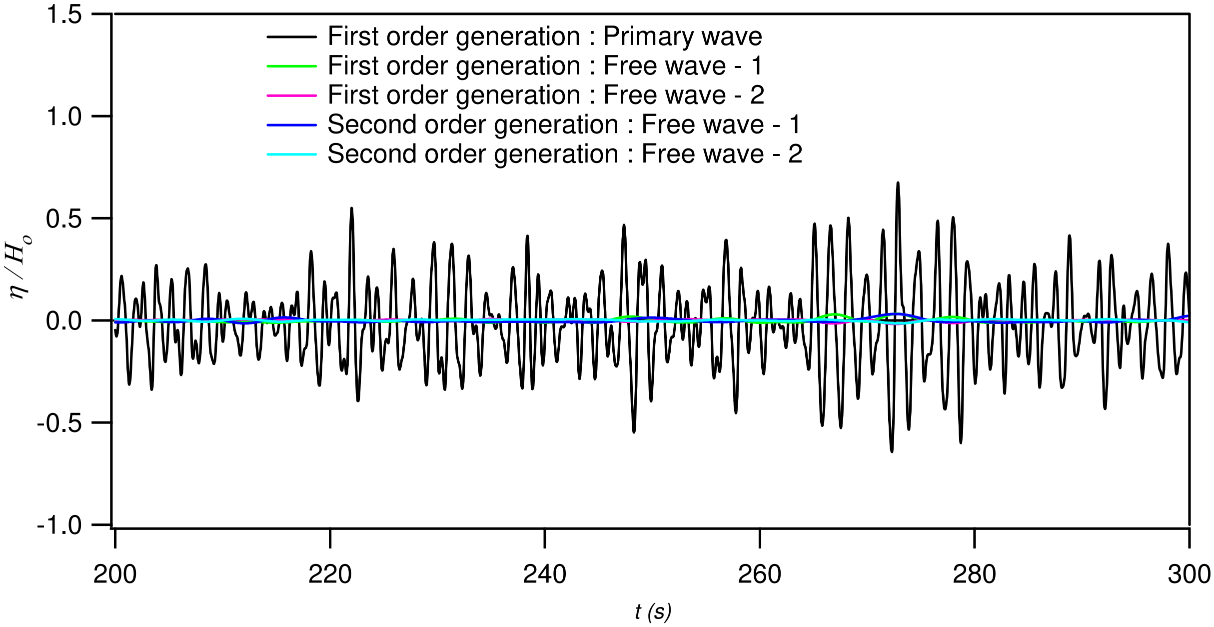

Fig. 5a: Comparisons of the measured amplitudes of primary wave, Free wave-1 and Free wave-2 amplitudes at Probe-2
(*h=0.4m,* T_p*=1.705s and* H_s*=0.06m*)
(Case-2)

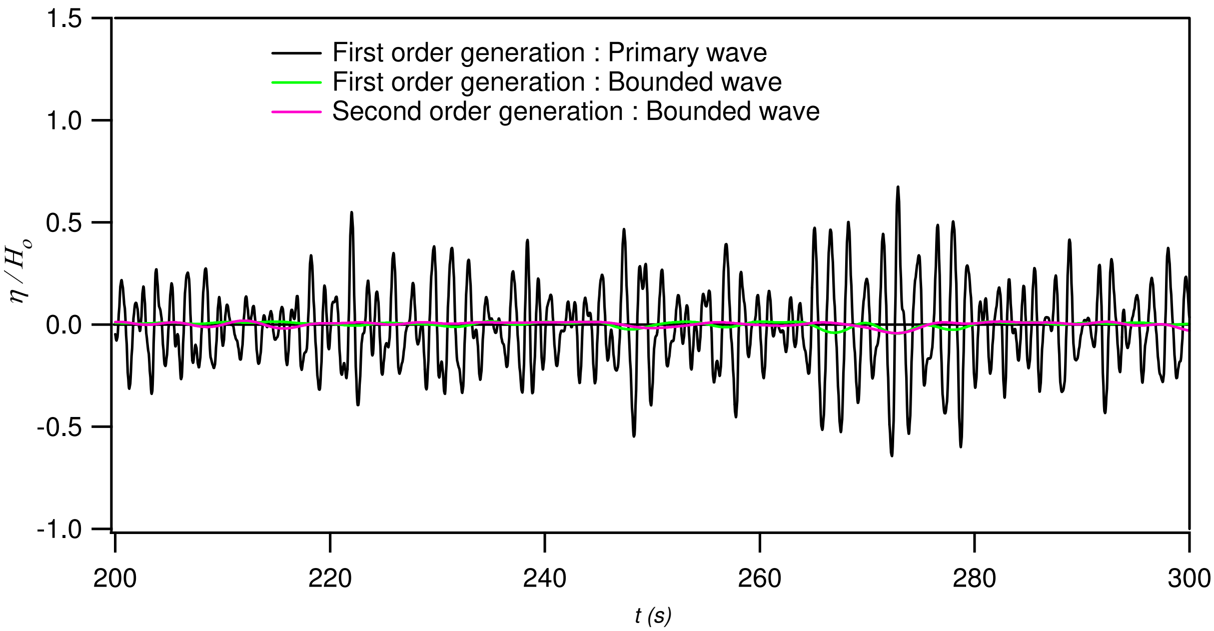

Fig. 5b: Comparisons of the measured amplitudes of primary and bounded wave's amplitudes at Probe-2
(*h=0.4m,* T_p*=1.705s and* H_s*=0.06m*)
(Case-2)

