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Multi-phase Granular Flow in a Reactor

by R. Djebbar, S.B. Beale, M. Sayed

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MULTI-PHASE GRANULAR FLOW IN A REACTOR

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ABSTRACT

The paper describes a numerical model developed for calculations of gravity-driven two-phase solid/liquid flow in an industrial pressurized reactor. The Mohr-Coulomb criterion for granular solid flow is introduced into the equations of motion by means of a non-linear shear viscosity. The methodology is validated by consideration of incompressible single-phase granular flow in a 2-D bin. Subsequently, the methodology for two-phase solid-liquid flow is developed. Comparisons with previous models which did not account for friction in the solid phase, reveal the importance of including these terms in the model.

NOMENCLATURE

A, B	Constants in equation of state
b	Forchheimer's constant
C	Inter-phase slip coefficient
c	Cohesion term
D	Diameter or width
d_0	Threshold rate of deformation
d_I,d_{II}	Principle rates of deformation
$\vec{g}$	Acceleration due to gravity
H	Height
k	Permeability
m	Exponent in equation of state
P	Solid pressure
p	Fluid pressure
r_f, r_s	Fluid, solid volume fraction
t	Thickness
w	Aperture width
$\vec{u}_f$, $\vec{u}_s$	Fluid, solid velocity
x, y	Displacement
Δ	Rate of deformation
η	Solid shear viscosity
κ	Kappa number
μ	Fluid viscosity
ρ_f, ρ_s	Fluid density, solid density
σ	Normal stress
τ	Shear stress
ϕ	Angle of internal friction
ϕ_w	Angle of friction with wall

INTRODUCTION

Several chemical and process applications involve two-phase flow of granular materials, where particle-particle interaction and the flow of interstitial fluid are both significant. The present work concerns one of those applications, namely reactor-vessels used to process wood chips into pulp. In those reactors, wood chips and liquid material are introduced at the top, and removed at the bottom. The bulk material undergoes various heating and chemical treatments inside the reactor. Liquid is also injected at several places along the reactor walls.

Numerical simulations are needed in order to optimise the performance of those reactors and to prevent problems such as hanging of the chip column, or plugging of extraction screens from arising. The role of numerical simulation is particularly important in view of the obvious difficulties of monitoring and measuring flow conditions inside the reactors.

Two main classes of flow are described in the present work: Incompressible single-phase granular flow in a 2-D bin and two-phase viscous/Mohr-Coulomb flow of a liquid/solid mixture. The behaviour of the solid phase presents one of the major difficulties in the development of numerical models for the present problem. Available numerical models have tended to consider the granular material to behave in a fluid-like manner. The emphasis of the present work is on incorporating a detailed solid-phase model that captures the frictional properties of the granular material in a CFD formulation for two-phase flow. Although chemical reactions and heat transfer are present, these are considered beyond the scope of this paper. The following sections of this paper discuss the governing equations with emphasis on the solid-phase, and the numerical approach. Results of test cases of flowing frictional granular materials are then presented. Those tests were aimed at verifying the appropriate performance of the model. Finally, the results of two-phase flow in an industrial reactor, of typical geometry and boundary conditions, are given.

GRANULAR FLOW

Granular solids flowing at low rates of deformation are known to display a different motion from viscous fluids. The shear stress, τ, along the failure plane is usually considered to be proportional to the normal stresses, σ, according to the well-known Mohr-Coulomb criterion,

$$\tau = c + \sigma \sin\phi \tag{1}$$

where c is the cohesion term, and ϕ is the angle of internal friction. It should be noted that for relatively large rates of deformation, collisions between individual grains give rise to a different regime of flow [1,2]. The latter is not considered here, but may be of interest in future applications.

The Mohr-Coulomb criterion is introduced into a conventional computational fluid dynamics (CFD) code by defining a shear viscosity coefficient, η, as,

$$\eta = \frac{P\sin\phi}{\Delta} \tag{2}$$

where $\Delta = \max\left(\left|d_{\mathrm{I}} - d_{\mathrm{II}}\right|, d_0\right)$, d_{I} and d_{II} are principle values of the rate of deformation tensor, and d_0 is a threshold or minimum value: For $\Delta > d_0$, the flow is plastic, while for $\Delta \le d_0$, it is viscous. The continuity and momentum equation, with η prescribed as above, may readily be discretized and solved using any convenient method. The interested reader will find a more detailed account in [3].

SINGLE-PHASE INCOMPRESSIBLE GRANULAR FLOW

The present formulation of the solid-phase was tested by modelling problems in single-phase granular flow. The results presented below concern flow in a vertical two-dimensional bin. The computer program PHOENICS which is based on a SIMPLE [4,5] algorithm was used for this purpose, with the solid pressure, P, being solved-for as a dependent variable.

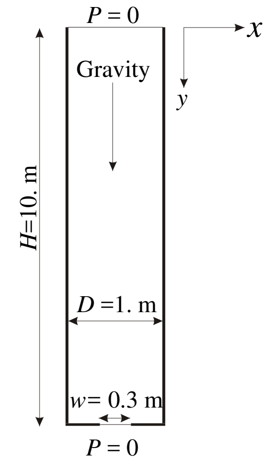

Figure 1 Schematic of a vertical two-dimensional bin, of unit thickness, t.

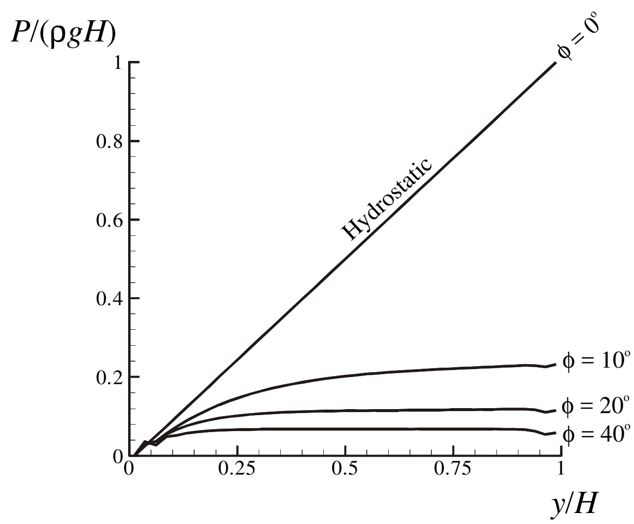

Figure 2. Dimensionless pressure, $P/\rho gH$ as function of dimensionless depth, y/H for flow in a bin.

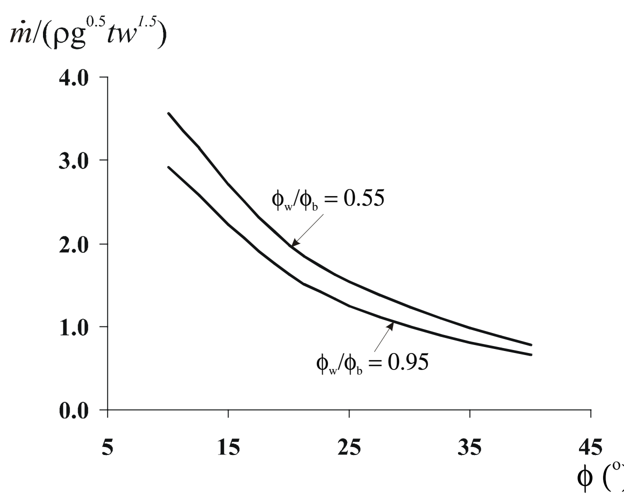

Figure 3. Dimensionless flow rates as a function of the bulk angle of internal friction.

The geometry of the vertical bin is shown in Fig. 1. No-slip was imposed at the walls with the angle of friction between the walls and the bulk material, ϕ_w, specified, and $0 \le \phi_w \le \phi$. The inlet velocity was not fixed, as is usual in fluid mechanics, rather zero pressure was prescribed at both inlet and outlet.

The resulting values of the normal stress on the walls of the vertical bin are shown in Fig. 2. After an increase with depth near the top, those stresses reach constant values that do not change with further increases of depth. This behaviour is a key feature of flowing granular material, and is often described by the well-known Janssen's formula, (for details see Brown and Richards [6]). The important corollary, namely that the mass efflux out of a vessel tends to a value independent of the vessel height, was also observed. It can also be seen from Fig. 2 that the solid pressure increases as ϕ decreases.

The corresponding non-dimensional flow rates are shown in Fig. 3. They display the expected trend of decreasing flow rate with increasing angle of internal friction, ϕ. The values of the non-dimensional flow rates are also in general agreement with available estimates such as the Beverloo equation (see Tardos [7]), which is based on substantial experimental data. It should be noted, however, that available observations do not normally specify values of ϕ and ϕ_w.

TWO PHASE GRANULAR/LIQUID FLOW

The present work addresses the subject of two-phase flow in industrial reactors used to process pulp. Härkönen [8,9] devised the first numerical transport model in this context, see Fig. 4. Fluid flow within the unconcolidated porous matrix of solid material was described by the Ergun-modified form of Darcy's law. The method was based on the premise that a solid pressure, P, distinct from the fluid pressure, p; was present; however no shear forces were permitted within the granular material, other than at the wall. Subsequent related material was presented by others [10,11,12,13]. The work described here differs from previous material in that; in addition to the granular pressure, P, a shear or frictional term, Eq. (1), is introduced. It is shown that this term has a profound effect upon the resulting solution. Additionally, the previously-neglected inertial terms are included in the analysis.

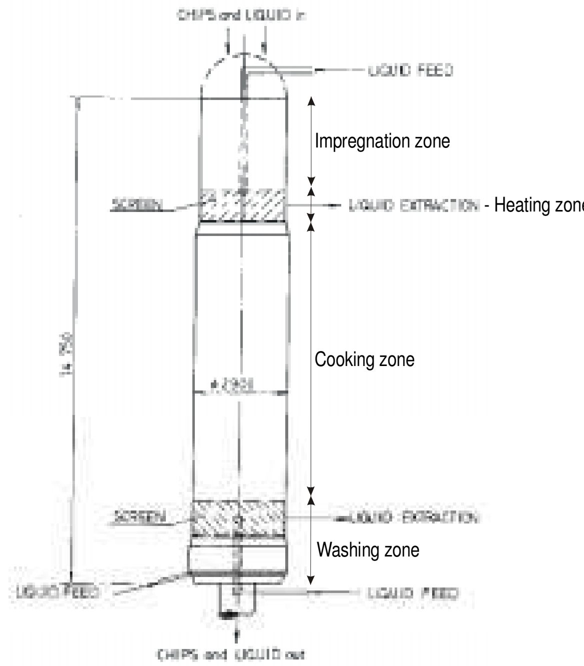

Figure 4 Schematic of an industrial reactor. Adapted from Härkonën [8].

For the problem of Fig. 4, wood chips and liquid enter by the top, and leave by the bottom of a pressure vessel, within which there are four main regions: (i) impregnation, (ii) heating, (iii) cooking, and (iv) washing zones. Additional liquid is added at three more locations in the apparatus; and extracted via screens. Smooth flow of the granular material within the vessel, under the action of gravity and fluid drag is critical to the successful operation of the apparatus.

The present work employed an IPSA [14,15] algorithm to obtain the simultaneous solution for solid and fluid phases, for which the transport equations are considered to be of the form,

$$\vec{\nabla} \cdot \left(r_f \rho_f \vec{u}_f \right) = 0 \tag{3}$$

$$\vec{\nabla} \cdot \left(r_s \rho_s \vec{u}_s \right) = 0 \tag{4}$$

$$\vec{\nabla} \cdot \left(r_f \rho_f \vec{u}_f ; \vec{u}_f \right) = r_f \rho_f \vec{g} - r_f \vec{\nabla} p + \vec{\nabla} \cdot \left(\vec{\nabla} r_f \mu \vec{u}_f \right) + C\left(\vec{u}_f - \vec{u}_s \right) \tag{5}$$

$$\vec{\nabla} \cdot \left(r_s \rho_s \vec{u}_s ; \vec{u}_s \right) = r_s \rho_s \vec{g} - r_s \vec{\nabla} p - \vec{\nabla}\left(r_s P \right) + \vec{\nabla} \cdot \left(\vec{\nabla} r_s \eta \vec{u}_s \right) + C\left(\vec{u}_s - \vec{u}_f \right) \tag{6}$$

Table 1. Boundary conditions and properties for solid and fluid phases

Parameter	Value
Main inlet solid volume fraction, r_s	0.42
Main inlet solid pressure, P	5 000 (Pa)
Main inlet solid velocity, v_s	$2.4\text{x}10^{-3}$ (m/s)
Main inlet fluid velocity, v_f	$2.4\text{x}10^{-3}$ (m/s)
Main outlet fluid pressure, p	0 (Pa)
Solid density, ρ_s	1132.4 (kg/m^3)
Fluid density, ρ_f	1 000 (kg/m^3)
Fluid permeability, k	$\frac{\mu}{4.6x10^3}\frac{r_f^2}{r_s^2}$ (m^2)
Forchheimer's constant, b (Inertial permability term)	$\frac{3.9x10^6}{\rho_f}\frac{r_s}{r_f^2}$ (m^{-1})

Fluid flow in porous media is formulated by introducing Darcy's law via the inter-phase slip term in Eqs. (5) and (6), with $C = r_f\mu/k$, where k is the permeability (suitably modified for inertial effects using Ergun's equation [16,17]). For compressible granular flow, an equation of state is also required,

$$r_s = A + BP^m \tag{7}$$

This relationship is inverted to obtain the pressure gradient term in Eq. (6), as

$$P = \frac{10^4}{0.831 - 0.139\ln\kappa}\left(r_s - 0.356\right)^{1.695} \tag{8}$$

with κ = 195 [8,12]. Boundary conditions, transport properties used in the present work are given Table 1. The angle of internal friction is 45°.

Figure 5 shows contours of solid pressure, P in the reactor vessel. Three different cases are illustrated (a) A conventional viscous-viscous formulation with a kinematic viscosity of 10^{-3} (m^2/s). (b) Results reproduced from Härkönen [8] i.e., $\eta = 0$ within the bulk solid. (c) The viscous/Mohr-Coulomb formulation described above. In all three cases, the same equation-of-state is employed. It can be seen that the results of Fig. 5 (a,b) are broadly similar with some local differences: In both cases, a significant vertical solids pressure gradient is readily apparent. In Fig. 5(c) the pressure for Mohr-Coulomb granular flow is displayed. Here P increases only very slightly from top to bottom, i.e. it is essentially independent of height. The minor increases in each of the four zones are due to changes in cross-sectional area, along the length; a feature not normally associated with hydrostatic pressure.

Figure 6 shows the solid pressure, P, midway across the reactor vessel for two-phase Mohr-Coulomb solid flow. P increases only very slightly from $5\text{x}10^3$ Pa at the top, to $5.6\text{x}10^3$ Pa at the bottom. Also shown are results where both phases were treated as being viscous. It is apparent that the two approaches generate radically different profiles: For the viscous/viscous approach, the solid pressure increases from inlet to outlet (from zero to $1.8\text{x}10^4$ Pa), in a relatively linear fashion, with depth.

The effect of the angle of internal friction, on the solid pressure distribution, midway across the vessel, is shown in Fig. 7, for $\phi = 10°, 20°, 40°$. P is inversely proportional to ϕ, consistent with the results of Fig. 2. For values of $\phi > 20°$, the solid pressure in the impregnation zone is close to the inlet value of $5\text{x}10^3$ Pa. For $\phi = 10°$, gravitational forces predominate over shear forces, and the solid material is more closely-packed, leading to higher solid pressures.

CONCLUSIONS

A new model of two-phase flow of granular material within a viscous fluid has been developed. A CFD code based on the IPSA algorithm was modified to perform the calculations. The new model contains the inter-particle frictional properties of the granular material, in addition to gravity, Darcian inter-phase drag, pressure gradient and convection terms. The solid-phase formulation was tested by simulating an idealised case of granular flow in a vertical bin. The resulting pressures exhibited the distinctive profiles known to occur for granular flow.

Predicted flow rates also agreed with expected values. Those results were considered to be an adequate basis to proceed to the study of two-phase flow. We note, however, that many complex phenomena associated with single-phase granular flow were not addressed here. Such phenomena may include, for example, the occurrence of funnel flow, stress and velocity discontinuities, and the transition from slow to fast collision-dominated flow regime.

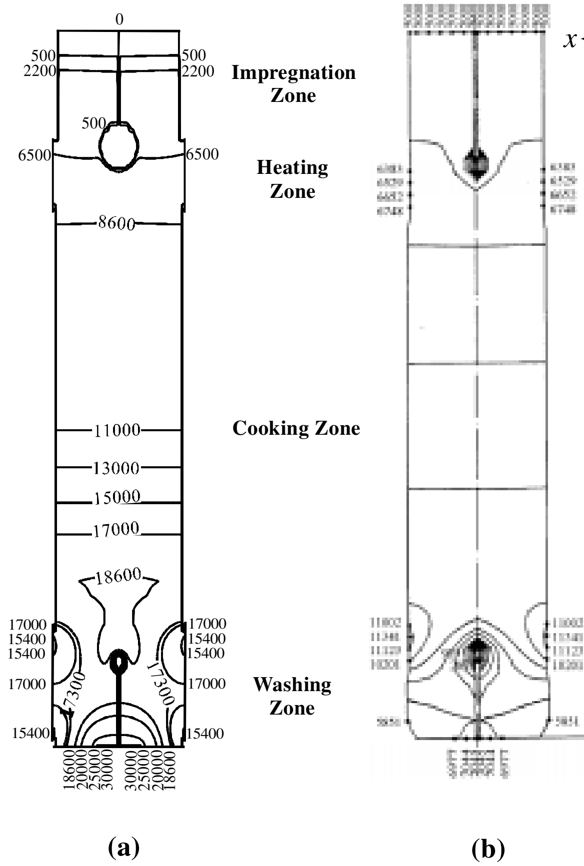

Figure 5 Solid pressure contours, P (Pa), for two-phase solid-fluid flow. (a) Viscous formulation, (b) From Härkönen [8], (c) Mohr/Coulomb formulation.

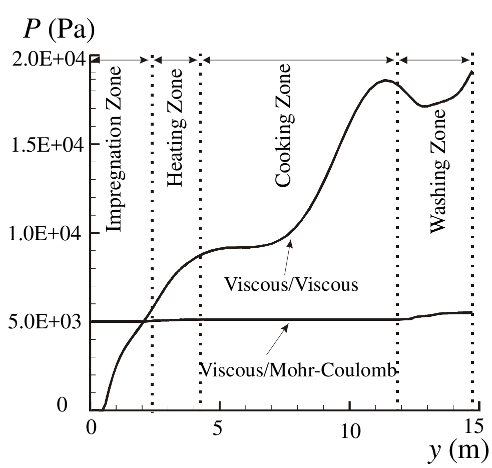

Figure 6. Solid phase pressure, P (Pa)

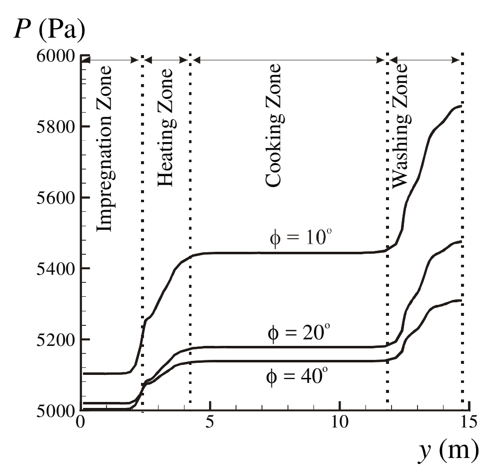

Figure 7. Effect of angle of friction, ϕ, (°).

The model was then used to examine the flow of a solid liquid mixture in an industrial application. The calculations provided solid and fluid pressure distributions and flow rates. Predicted solid pressure profiles depart significantly from previous model studies. The present results indicate that changes in solid pressure with depth are small. This conclusion is consistent with the well-known behaviour of granular flows.

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