

Uncertainty evaluation of this comparison samples from Sample inhomogeneity

The raw GO are industrial graphene oxide products purchased from Graphenea Inc. The sample homogeneity is tested based on the rules: $m=20$ samples are measured, and for each sample, $n=3$ flakes in each sample are measured at the same condition by the same method. The average and the uncertainty from the sample homogeneity is calculated following the equations from (1) to (12), and the results shown in Table 1. As a result, the uncertainty evaluation of this comparison samples from Sample inhomogeneity S_H is 0.10 nm.

$$X_{11}, X_{12}, \dots, X_{1n_1}, \text{ the average } \bar{X}_1; \quad (1)$$

$$X_{21}, X_{22}, \dots, X_{2n_2}, \text{ the average } \bar{X}_2; \quad (2)$$

.....

$$X_{m1}, X_{m2}, \dots, X_{mn_m}, \text{ the average } \bar{X}_m; \quad (3)$$

$$\bar{X} = \frac{\sum_{i=1}^m \bar{X}_i}{m}$$

The average of all data (4)

$$N = \sum_{i=1}^m n_i$$

All measurement numbers: (5)

$$Q_1 = \sum_{i=1}^m n_i (\bar{x}_i - \bar{x})^2$$

Sum of Differences Between Groups: (6)

$$Q_2 = \sum_{i=1}^m \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2$$

Sum of variances within the group: (7)

$$v_1 = m - 1$$

Degrees of freedom between groups: (8)

$$v_2 = N - m = \sum_{i=1}^m n_i - m$$

Degrees of freedom within groups: (9)

$$F = \frac{s_1^2}{s_2^2} = \frac{\frac{Q_1}{v_1}}{\frac{Q_2}{v_2}}$$

F distribution variable quantity of the degree (v_1, v_2): (10)

$$s_H^2 = \frac{N(m-1)}{N^2 - \sum_{i=1}^m n_i^2} (s_1^2 - s_2^2)$$

When $s_1 > s_2$, the standard deviation of sample difference:

(11)

When $s_1 < s_2$, the standard deviation of sample difference:

$$s_H^2 = \frac{s_2^2}{n} \cdot \frac{\sqrt{2}}{\sqrt{\sum_{i=1}^m n_i - m}} \quad (12)$$

Table 1 the data analysis of the sample homogeneity in the leader laboratory (nm)

n \ m	1	2	3	\bar{x}_i	$\bar{\bar{x}}$	Q_1	Q_2	N	ν_1	ν_2
1	0.818	0.931	0.868	0.872	0.990	0.655	0.164	60	19	40
2	1.142	1.003	1.068	1.071						
3	1.175	1.179	0.996	1.117						
4	0.902	0.890	0.958	0.917						
5	1.098	1.181	1.109	1.129						
6	0.773	0.914	0.934	0.874						
7	1.136	1.130	1.142	1.136						
8	0.899	0.820	0.761	0.827						
9	0.964	0.893	0.939	0.932						
10	0.929	0.853	0.869	0.884						
11	1.016	0.931	0.907	0.951						
12	0.958	1.004	0.953	0.972						
13	1.136	1.130	1.075	1.1147						
14	1.027	0.987	0.927	0.980						
15	0.936	0.935	0.923	0.932						
16	1.236	1.128	1.045	1.136						
17	0.872	0.908	1.042	0.941						
18	0.890	0.952	0.886	0.909						
19	1.037	1.139	1.296	1.157						
20	0.937	0.928	0.977	0.947						
S_1^2	0.034				S_2^2	0.0041				
$S_H=0.10$										