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Cheers, F.

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A NOTE ON GALLOPING CONDUCTORS

REPORT NO. MT - 14

BY

F. CHEERS

DIVISION OF MECHANICAL ENGINEERING

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Submitted by: D. C. MacPhail
Section Head

Author: F. Cheers

Approved by: J. H. Parkin
Director

SUMMARY

The results of wind tunnel tests on a series of two-dimensional models simulating circular cables with various ice formations are presented. The stability of small oscillations of a transmission line in a steady wind normal to the span is considered, and the wind tunnel results are used to calculate the behaviour of a cable with D shaped fairings. The results of dynamic tests with an oscillating model show reasonable agreement with the theoretical predictions.

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A NOTE ON GALLOPING CONDUCTORS

1. INTRODUCTION

The study of the galloping of transmission lines is rendered extremely complex by the number of variables involved. These include a number which change from span to span but are approximately constant for a given span - length and size of conductor, end fixings, tension and the nature of the surrounding countryside - and others which vary from time to time - wind strength and direction, and the form of ice deposited. In these circumstances considerable simplification is necessary if any basic causes of galloping are to be disclosed.

Den Hartog[1)] considers a two-dimensional body suspended normal to a steady horizontal wind; he shows that simple vertical oscillations will tend to be unstable "if the negative slope of the lift curve is greater than the ordinate of the drag curve". Ruedy[2)] shows that a flat strip with its chord parallel to the wind will oscillate if it is free to twist, although Den Hartog's criterion is not satisfied. Niven[3)] suggests that galloping can be caused by changes of tension with the wind blowing along the span. Harris[4)] considers the rate of energy input to a D section oscillating in a steady normal wind; he also describes experiments showing that the section is more likely to gallop when the wind is not normal to the span.

In the present work, investigation is limited to a simple case which is known to cause galloping, that of a conductor with an ice coating in a steady wind approximately normal to the span. The sections tested represent formalized ice formations on circular cylinders, but it is thought that the range of shapes has been covered fairly adequately.

From field reports, it seems clear that galloping, when it does occur, starts usually from small oscillations and builds up slowly. It seems reasonable, therefore, that a study of the stability of small oscillations will be of use in determining the damping necessary to prevent some forms of galloping. This form of analysis does not take account of the increase in damping which may occur at large amplitudes, and some unstable small oscillations will not build up to an amplitude sufficient to cause damage; it should, however, show which cases are likely to be critical, and it avoids the necessity of making step-by-step calculations. Little attention has been given to particular cases; it seems better that a fairly general approach should be given, and that application to various test spans should wait until later.

2. WIND TUNNEL TESTS

Tests on a series of two-dimensional models (Figure 1) were carried out in the 21 in. by 30 in. horizontal wind tunnel of the National Research Council; the wind speed was 50 ft./sec. for all models except the D section, (No. 8, Figure 1), for which it was found necessary to reduce the speed. The models were suspended horizontally from a strain-gauge balance frame, and passed through the tunnel from side to side, the holes in the tunnel walls being kept small. The sections chosen were meant to represent a 1-in. cylinder with various ice formations; the shapes were based on reports on sections which had caused galloping, and on the sections observed to form on a cylinder in the icing wind tunnel of the National Research Council's Low Temperature Laboratories. The D section was included in view of its general use on test spans.

The fittings were made so that the angle of attack could be adjusted at intervals of 10 degrees. It was found later that closer spacing would have been useful over a few short ranges; it seems likely, however, that the results as presented will be quite adequate to give a guide to the forces likely to be met.

All models were tested normal to the wind; two were tested also at angles of yaw of 30 degrees, which was the maximum possible with the mounting used. The forces measured are shown in Figure 2, the lift and drag being measured along the vertical and horizontal normals to the model axis, and the incidence and pitching moment about that axis. The results are shown in Figures 3 - 12, giving forces and moments in pounds and pound-feet per foot length of model at a wind speed of 50 ft./sec.

The effect of small changes in shape is most marked. Models 3 and 4 (Figures 5 and 6) have approximately the same amount of ice, but the changes in wind forces are considerable. Yaw of 30 degrees was found to have little effect beyond a reduction of the forces approximately in proportion to the square of the normal wind velocity. (Figures 10 and 11, and Figures 6 and 12).

3. STABILITY OF A CABLE UNDER AERODYNAMIC LOADING

3.1 General Theory

The free oscillations of suspension chains have been studied by Routh[5] and, more recently, by Pugsley[6]. In both cases the mass distribution along the chain was assumed to be of a form which would make solution of the equations of motion possible. In the present note the form of oscillation is assumed, and end effects due to adjacent spans ignored, but the effects of torsional motion, and of aerodynamic and damping forces, are included. Oscillations in a horizontal plane are excluded, as it seems unlikely that these will affect the stability conditions to any great extent.

Let x, y be the coordinates of any point P on a cable when hanging in equilibrium, and let s be the distance along the arc to P from some convenient origin, and m the mass per unit length of the cable at P. When the cable is executing a small oscillation suppose P, at time t, to have coordinates $x + \xi$, $y + \eta$; then the equations of motion are

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{1}{m}\left\{\frac{\partial}{\partial s}\left(T\frac{\partial \xi}{\partial s} + U\frac{\partial x}{\partial s}\right) - EI_1\frac{\partial^4 \xi}{\partial s^4} + P_1 + F_1\right\},$$

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{1}{m}\left\{\frac{\partial}{\partial s}\left(T\frac{\partial \eta}{\partial s} + U\frac{\partial y}{\partial s}\right) - EI_1\frac{\partial^4 \eta}{\partial s^4} + P_2 + F_2\right\}, \quad (1)$$

where T is the equilibrium tension in the cable,

U is the increase in tension at time t when the cable is in motion,

$EI_1\frac{\partial^4 \xi}{\partial s^4}$, $EI_1\frac{\partial^4 \eta}{\partial s^4}$ are the forces due to the additional bending of the cable while in motion, if the curvature is small,

P_1, P_2 are damping forces, and

F_1, F_2 are the changes in aerodynamic forces due to displacement from the equilibrium position.

In the present investigation it is proposed to ignore changes in tension, both along the span and with time, and to consider only oscillations of the cable normal to its equilibrium position. This is equivalent to neglecting among other things the excitation from adjacent spans. Then, if we take the x axis tangential to the cable at P,

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{1}{m}\left\{T\frac{\partial^2 \eta}{\partial s^2} - EI_1\frac{\partial^4 \eta}{\partial s^4} + P_2 + F_2\right\}.$$

The damping term P_2 will contain components which are functions of different derivatives of y with respect to t, in addition to solid friction and terms due to local damping; for convenience, however, it is assumed that we can write $P_2 = -P \frac{\partial \eta}{\partial t}$, with P constant, so that the equation becomes

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{1}{m}\left\{T \frac{\partial^2 \eta}{\partial s^2} - EI_1 \frac{\partial^4 \eta}{\partial s^4} - P \frac{\partial \eta}{\partial t} + F_2\right\}. \qquad (2)$$

In considering torsional oscillations, it is assumed that the cable behaves as a straight rod. The equation of motion is

$$\frac{\partial^2 \vartheta}{\partial t^2} = \frac{1}{I_2}\left\{S \frac{\partial^2 \vartheta}{\partial s^2} - R \frac{\partial \vartheta}{\partial t} + M_a\right\}, \qquad (3)$$

where ϑ is the angular displacement from the equilibrium position at time t,
I_2 is the moment of inertia about the axis of rotation,
S is the torsional stiffness of the cable,
$R \frac{\partial \vartheta}{\partial t}$ is the damping couple, assuming damping of the same form as for vertical oscillations, with R constant; and
M_a is the aerodynamic torque.

Let L, D, and M be the lift, drag, and pitching moment per unit length of the cable at an angle of attack α to a steady wind of velocity V, α being zero when the cable is at rest in its equilibrium position. Then, when $\alpha = 0$, $L = L_0$ and $M = M_0$, L_0 and M_0 being balanced out by mechanical and gravitational forces in the static equation of equilibrium. At time t

$$\alpha = \vartheta - \frac{1}{V}\frac{\partial \eta}{\partial t} \qquad (4)$$

and if $\frac{\partial L}{\partial \alpha}$, D, and $\frac{\partial M}{\partial \alpha}$ are constant over the small range of α

required, and are not affected by derivatives of α with respect to t,

$$F_2 = (L - L_0) - D \frac{1}{V}\frac{\partial \eta}{\partial t}$$

$$= \frac{\partial L}{\partial \alpha}\vartheta - \left(\frac{\partial L}{\partial \alpha} + D\right)\frac{1}{V}\frac{\partial \eta}{\partial t} ; \qquad (5)$$

$$M_a = M - M_0$$

$$= \frac{\partial M}{\partial \alpha}\left(\vartheta - \frac{1}{V}\frac{\partial \eta}{\partial t}\right) ; \qquad (6)$$

so that the equations of motion become

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{1}{m}\left\{T\frac{\partial^2 \eta}{\partial s^2} - EI_1\frac{\partial^4 \eta}{\partial s^4} - P\frac{\partial \eta}{\partial t} + \frac{\partial L}{\partial \alpha}\vartheta - \left(\frac{\partial L}{\partial \alpha} + D\right)\frac{1}{V}\frac{\partial \eta}{\partial t}\right\} ,$$

$$\frac{\partial^2 \vartheta}{\partial t^2} = \frac{1}{I_2}\left\{S\frac{\partial^2 \vartheta}{\partial s^2} - R\frac{\partial \vartheta}{\partial t} - \frac{\partial M}{\partial \alpha}\left(\vartheta - \frac{1}{V}\frac{\partial \eta}{\partial t}\right)\right\} . \qquad (7)$$

If we assume that both vertical and torsional oscillations take place in a number of sinusoidal loops, we can write

$$\eta = \eta_0 \sin\frac{a\pi s}{l}$$

$$\vartheta = \vartheta_0 \sin\frac{b\pi s}{l}$$

when l is the length of the cable and s is measured from one end, and we have

$$\frac{\partial^2 \eta_0}{\partial t^2}\sin\frac{a\pi s}{l} = \frac{1}{m}\left\{\left[-T\eta_0\left(\frac{a\pi}{l}\right)^2 - EI_1\eta_0\left(\frac{a\pi}{l}\right)^4 - P\frac{\partial \eta_0}{\partial t} - \right.\right.$$

$$\left.\left. - \left(\frac{\partial L}{\partial \alpha} + D\right)\frac{1}{V}\frac{\partial \eta_0}{\partial t}\right]\sin\frac{a\pi s}{l} + \frac{\partial L}{\partial \alpha}\vartheta_0 \sin\frac{b\pi s}{l}\right\} ,$$

$$\frac{\partial^2 \vartheta_0}{\partial t^2}\sin\frac{b\pi s}{l} = \frac{1}{I_2}\left\{\left[-S\vartheta_0\left(\frac{b\pi}{l}\right)^2 - R\frac{\partial \vartheta_0}{\partial t} + \frac{\partial M}{\partial \alpha}\vartheta_0\right]\sin\frac{b\pi s}{l} - \right.$$

$$\left. - \frac{\partial M}{\partial \alpha}\frac{1}{V}\frac{\partial \eta_0}{\partial t}\sin\frac{a\pi s}{l}\right\} ;$$

or

$$\frac{\partial^2\eta}{\partial t^2} = \frac{1}{m}\left\{-\eta\left[T\left(\frac{a\pi}{l}\right)^2 + EI_1\left(\frac{a\pi}{l}\right)^4\right] - \left[P + \frac{1}{V}\left(\frac{\partial L}{\partial\alpha} + D\right)\right]\frac{\partial\eta}{\partial t} + \vartheta\,\frac{\partial L}{\partial\alpha}\right\},$$

$$\frac{\partial^2\vartheta}{\partial t^2} = \frac{1}{I_2}\left\{\left[-S\left(\frac{b\pi}{l}\right)^2 + \frac{\partial M}{\partial\alpha}\right]\vartheta - R\,\frac{\partial\vartheta}{\partial t} - \frac{1}{V}\,\frac{\partial M}{\partial\alpha}\,\frac{\partial\eta}{\partial t}\right\}. \tag{8}$$

It will be convenient to write these equations in the form

$$\frac{\partial^2\eta}{\partial t^2} + A\,\frac{\partial\eta}{\partial t} + f_1\eta = \frac{1}{m}\,\frac{\partial L}{\partial\alpha}\,\vartheta, \tag{9}$$

$$\frac{\partial^2\vartheta}{\partial t^2} + B\,\frac{\partial\vartheta}{\partial t} + \left(f_2 - \frac{1}{I_2}\,\frac{\partial M}{\partial\alpha}\right)\vartheta = -\frac{1}{VI_2}\,\frac{\partial M}{\partial\alpha}\,\frac{\partial\eta}{\partial t}, \tag{10}$$

where $A = \frac{1}{m}\left\{P + \frac{1}{V}\left(\frac{\partial L}{\partial\alpha} + D\right)\right\} = A_1 + \frac{1}{mV}\left(\frac{\partial L}{\partial\alpha} + D\right),$

$B = \frac{R}{I_2},$

$f_1 = \frac{1}{m}\left\{T\left(\frac{a\pi}{l}\right)^2 + EI_1\left(\frac{a\pi}{l}\right)^4\right\},$

$f_2 = \frac{S}{I_2}\left(\frac{b\pi}{l}\right)^2$

and to consider oscillations in 3 cases:

(i) in the absence of wind forces;

(ii) oscillations under wind loading, but with no torsional motion;

(iii) oscillations under wind loading with both vertical and torsional motion.

3.2 Free Oscillations in the Absence of Wind Forces

In this case, the equations (9), (10) reduce to

$$\frac{\partial^2 \eta}{\partial t^2} + A_1 \frac{\partial \eta}{\partial t} + f_1 \eta = 0 ,$$

$$\frac{\partial^2 \vartheta}{\partial t^2} + B \frac{\partial \vartheta}{\partial t} + f_2 \vartheta = 0 ,$$

where $A_1 = P/m$. These are the equations of simple oscillations with damping; if the damping is small, the frequencies are $\frac{1}{2\pi}\sqrt{f_1}$, $\frac{1}{2\pi}\sqrt{f_2}$ for vertical and torsional modes respectively, and the times for oscillations to damp to half amplitude are $2 \log_e 2/A_1$, $2 \log_e 2/B$. This offers a convenient method of finding the constants f_1, f_2, A_1, B for a test span.

3.3 Oscillations with Wind Loading, but without Torsional Motion

In this case equation (10) vanishes, and equation (9) becomes

$$\frac{\partial^2 \eta}{\partial t^2} + A \frac{\partial \eta}{\partial t} + f_1 \eta = 0 .$$

The cable will therefore execute oscillations with frequency $\frac{1}{2\pi}\sqrt{f_1}$ if the damping term is small, the stability of the oscillation depending on $A = A_1 + \frac{1}{V}\left(\frac{\partial L}{\partial \alpha} + D\right)$. In the case of the D section with flat face normal to the wind $\left(\frac{\partial L}{\partial \alpha} + D\right)$ is small, and, within the accuracy of the present experiments, appears to be positive; Lanchester[7], in discussing the "aerial tourbillion" makes it clear that it was stable while at rest. As the section is rotated or moved so that the flat face is no longer normal to the wind the drag diminishes and $\frac{\partial L}{\partial \alpha}$ remains nearly constant up to $\alpha \approx 40°$, so that $\left(\frac{\partial L}{\partial \alpha} + D\right)$ becomes increasingly negative over this range. It seems unlikely, however, that this factor is of great importance in practical cases, in view of the marked instability introduced by permitting torsional movement.

3.4 Oscillations with Wind Loading and with Vertical and Torsional Motion

In this case the equations of motion are, as (9), (10),

$$\frac{\partial^2\eta}{\partial t^2} + A\,\frac{\partial\eta}{\partial t} + f_1\eta = \frac{1}{m}\frac{\partial L}{\partial\alpha}\,\vartheta\;,$$

$$\frac{\partial^2\vartheta}{\partial t^2} + B\,\frac{\partial\vartheta}{\partial t} + \left(f_2 - \frac{1}{I_2}\frac{\partial M}{\partial\alpha}\right)\vartheta = -\frac{1}{VI_2}\,\frac{\partial M}{\partial\alpha}\,\frac{\partial\eta}{\partial t}\;.$$

Eliminating ϑ, we have an equation

$$\frac{\partial^4\eta}{\partial t^4} + \lambda_1\,\frac{\partial^3\eta}{\partial t^3} + \lambda_2\,\frac{\partial^2\eta}{\partial t^2} + \lambda_3\,\frac{\partial\eta}{\partial t} + \lambda_4\eta = 0, \qquad (11)$$

where $\lambda_1 = A + B$,

$$\lambda_2 = f_1 + f_2 - \frac{1}{I_2}\frac{\partial M}{\partial\alpha} + AB,$$

$$\lambda_3 = f_1B + f_2A - \frac{1}{I_2}\,\frac{\partial M}{\partial\alpha}\left(A - \frac{1}{Vm}\,\frac{\partial L}{\partial\alpha}\right),$$

$$\lambda_4 = f_1\left(f_2 - \frac{1}{I_2}\,\frac{\partial M}{\partial\alpha}\right) \qquad (12)$$

The conditions for the stability of this motion are[8)] that λ_1, λ_2, λ_3, λ_4 and $\lambda_5 = \lambda_1\lambda_2\lambda_3 - \lambda_3^2 - \lambda_1^2\lambda_4$ should all be positive. Generally λ_1, λ_2 and λ_3 need not be considered separately, as if one of these vanishes λ_5 is usually already negative. The vanishing of λ_4 indicates the onset of a divergence; the negative aerodynamic torsional stiffness becomes greater numerically than the torsional stiffness of the cable. In this case it becomes necessary to consider second order terms to predict the behaviour of the cable. The vanishing of λ_5 indicates a critical speed, at which the oscillations have neutral stability; at this speed the frequency of oscillations is $\frac{1}{2\pi}\sqrt{\frac{\lambda_3}{\lambda_1}}$.

If λ_5 is expanded, we have

$$\lambda_5 = A^3B\left(f_2 - \frac{1}{I_2}\frac{\partial M}{\partial \alpha}\right) + A^2B\left\{\left(f_1 + f_2 - \frac{1}{I_2}\frac{\partial M}{\partial \alpha}\right)B + \frac{1}{VI_2m}\frac{\partial M}{\partial \alpha}\frac{\partial L}{\partial \alpha}\right\} +$$

$$+ A\left\{f_1B^3 + B\left(f_1 - f_2 + \frac{1}{I_2}\frac{\partial M}{\partial \alpha}\right) + \frac{1}{VI_2m}\frac{\partial M}{\partial \alpha}\frac{\partial L}{\partial \alpha}\left(f_1 - f_2 + \frac{1}{I_2}\frac{\partial M}{\partial \alpha} + B^2\right)\right\} -$$

$$- \frac{1}{VI_2m}\frac{\partial M}{\partial \alpha}\frac{\partial L}{\partial \alpha}\left\{B\left(f_1 - f_2 + \frac{1}{I_2}\frac{\partial M}{\partial \alpha}\right) + \frac{1}{VI_2m}\frac{\partial M}{\partial \alpha}\frac{\partial L}{\partial \alpha}\right\} \qquad (13)$$

In general, the damping terms are small in comparison with the stiffness terms, and we can write

$$\lambda_5 = \left\{A\left(f_1 - f_2 + \frac{1}{I_2}\frac{\partial M}{\partial \alpha}\right) - \frac{1}{VI_2m}\frac{\partial M}{\partial \alpha}\frac{\partial L}{\partial \alpha}\right\}\left\{B\left(f_1 - f_2 + \frac{1}{I_2}\frac{\partial M}{\partial \alpha}\right) + \frac{1}{VI_2m}\frac{\partial M}{\partial \alpha}\frac{\partial L}{\partial \alpha}\right\} \qquad (14)$$

The aerodynamic forces are stable when $\frac{\partial L}{\partial \alpha} > 0$ and $\frac{\partial M}{\partial \alpha} < 0$; but for any non-circular section there must be a range of angle of attack in which one or both of these terms is unstable, so that any section will have a tendency to gallop if placed at a suitable angle to the wind.

This analysis gives no direct indication of the mode in which dangerous galloping will begin. There is evidence from dynamic tests in the wind tunnel and from field reports on galloping lines that an unstable oscillation in one mode may feed energy into a different mode which may not be unstable. Results from test spans should be useful in solving this problem.

The application of the theory to a number of practical cases is discussed below.

4. APPLICATION OF THEORY

4.1 Test Span with D Fairing

The Hydro-Electric Power Commission of Ontario is using D section fairings on 336,400 ACSR test spans. The calculations outlined are based on figures supplied by them, although it has been necessary to make some assumptions in estimating the mechanical properties of the span. The torsional characteristics chosen are those given by the Hydro for the cable alone, the moment of inertia being calculated on the assumption that the cable is homogeneous, and the hysteresis loss has been taken at the more optimistic figure of 4 per cent per cycle. The vertical stiffness was calculated on the assumption that the length of cable behaved as a taut wire, having the mass and tension of the test span, but vibrating simply with ends fixed. The figures obtained are given in Table I.

TABLE I

PROPERTIES OF HYPOTHETICAL TEST SPAN

336,400 ACSR cable, 550 ft. span, m = 0.643 lb./ft. = 0.02 slug/ft.

$I = 291 \times 10^{-6}$ lb. ft.²/ft. $= 9.03 \times 10^{-6}$ slug ft.²/ft.

Tension	2000 lb.	7000 lb.
f_1	3.28	11.39
f_2	266	466
B	0.107	0.140

The aerodynamic derivatives of the D section tested in the wind tunnel (Model 8) are shown in Figure 13. It is assumed that $\frac{\partial L}{\partial \alpha}$, $\frac{\partial M}{\partial \alpha}$ and D vary as V^2. As the angle of attack is changed from 0 degrees to 180 degrees the derivatives $\frac{\partial L}{\partial \alpha}$ and $\frac{\partial M}{\partial \alpha}$ become stable and unstable at intervals, while $\left(\frac{\partial L}{\partial \alpha} + D\right)$ is unstable (negative) only over a short range between 0 degrees and 40 degrees.

The stability limits of the span are shown in Figure 14. The damping is sufficiently small for equation (14) to be used throughout. Except for a short interval (α = 90° to 120°) where $\frac{\partial L}{\partial \alpha}$ is negative and $\frac{\partial M}{\partial \alpha}$ positive the vertical damping A_1 is unimportant. This is true only because $(f_1 - f_2)$ is negative; with $(f_1 - f_2) > 0$, as in the dynamic tests discussed below, the torsional damping would have no effect over most of the incidence range. Over the range 50 degrees to 120 degrees, $\frac{\partial M}{\partial \alpha}$ is positive and a divergence appears, but generally at a higher speed than the critical speed given by $\lambda_5 = 0$.

The effect of changing the torsional damping independently of the other parameters, and of reducing the tension from 7000 lb. to 2000 lb. which produces changes in most of the parameters, is shown in Table II, for the test span with D section fairing at $\alpha = 0°$. It will be seen that changes in the mechanical constants are likely to have relatively small effects on the critical speeds.

TABLE II

STABILITY OF TEST SPAN; D SECTION AT $\alpha = 0°$

Tension	B	f_1	f_2	Critical Speed
7000 lb.	0.140	11.39	466	14.2 ft./sec.
7000 lb.	0.280	11.39	466	20 ft./sec.
2000 lb.	0.107	3.28	266	10.8 ft./sec.

4.2 Dynamic Tests on D Section

The model used to measure the forces on the D section (No. 8) was mounted in the wind tunnel in such a way that it could oscillate vertically and in torsion under the spring loads and damping due to its supports. It was found initially that if two modes were possible the model would start small oscillations in one mode, and then the energy would be fed into the second, higher frequency, mode which was damped; the movement was therefore restricted to a single mode. The wind speed at which galloping started was then measured for various stiffnesses, and compared with that predicted by the theory, as shown in Table III. At the lower speeds agreement was good, but at higher speeds the theory gave critical speeds which were too low; it seems likely that this was caused by the increase in damping caused by the pull due to drag forces on the restraining lines.

With some improvement in the method of support, the model was tested over a range of angles from 0 degrees to 180 degrees. The observed galloping speeds, together with the theoretical critical speeds and divergence speeds, are shown in Figure 15. Agreement in the range $0° < \alpha < 40°$ is good; in the range around $\alpha = 90°$ the slope of the lift curve changes too rapidly for accurate estimation from the measurements taken, and this, together with the difficulty of getting precise readings of the true mean angle of attack of an oscillating model, may be responsible for the lack of agreement shown. No galloping was observed in the range $50° < \alpha < 80°$, the onset of the divergence being avoided by the model twisting to another angle of attack.

In all cases where galloping took place, the actual rotation of the model was small, and might not have been noticed by casual observation.

TABLE III

FREE OSCILLATIONS OF A 'D' SECTION ($\alpha = 0°$)

f_1	f_2	A_1	B	Galloping Speed Calc.	Galloping Speed Obs.
96.5	179	0.248	0.295	30	40
96.5	141	0.256	0.248	24	32
72	141	0.256	0.248	36.5	42
103	83	0.267	0.178	17.5	16
76.3	58	0.183	0.131	18.5	21

4.3 Flat Strip

Approximate lift and pitching moment curves of a flat plate are sketched in Figure 16. It will be seen that if the flat plate is nearly parallel to the wind stream $\frac{\partial M}{\partial \alpha}$ is positive, and at a suitable speed the section will become unstable as λ_4 becomes negative. This is the case considered by Ruedy[2]. However, as Den Hartog points out[10], as the angle of attack is increased $\frac{\partial M}{\partial \alpha}$ becomes negative, but at about 30 degrees to 40 degrees $\frac{\partial L}{\partial \alpha}$ also becomes negative, and the strip will have an instability similar to that of a D section with flat face approximately normal to the wind.

5. CONCLUSIONS

From the tests that have been made, it appears that the wind forces measured on stationary models can be used to calculate the stability of small oscillations of ice-laden conductors normal to a steady wind. Although the measurements taken are not wholly sufficient to give an accurate value of the derivatives at all angles of attack, they undoubtedly give a satisfactory idea of the possible regions of instability and the magnitude of the forces to be expected. The analysis described can be used to estimate the stability of a conductor vibrating in any mode, although for simplicity only the first has been considered.

If the hypothetical test span considered is a reasonable approximation to a true span in that the torsional frequency is higher than the flexural, improvements in galloping behaviour are most likely to be obtained by controlling the torsional motion. If this could be prevented altogether it seems certain that a large number of cases of galloping would be avoided. Increasing the damping in torsion will cause some improvement, but the most profitable course seems to be the increasing of the torsional frequency, or at least increasing the numerical difference between the two stiffnesses ($f_1 - f_2$). If stiff ties between phases at points along the span similar to those suggested by Mr. Henry[11] are practicable they would seem to offer a solution.

This case considers only the one case of an ice-loaded conductor in a steady normal wind. An oblique wind or one parallel to the span may have similar or greater tendency to cause galloping, but even in these cases it is thought that freedom in torsion, even if the torsional motion is small, may be of importance, and this, together with the aerodynamic "end effects" on models of finite length, should be watched carefully in making tests.

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11. Ibid. p. 34.

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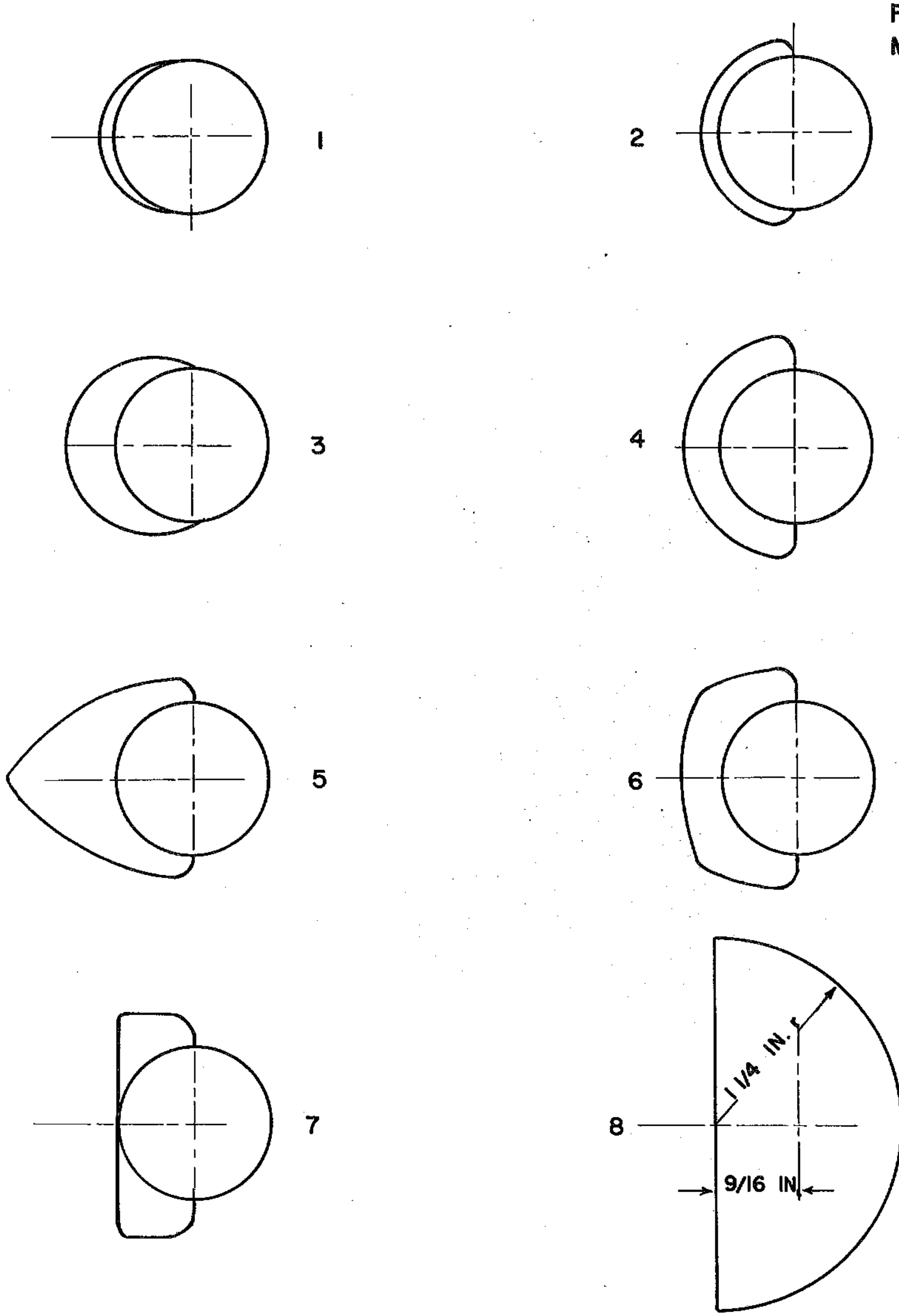
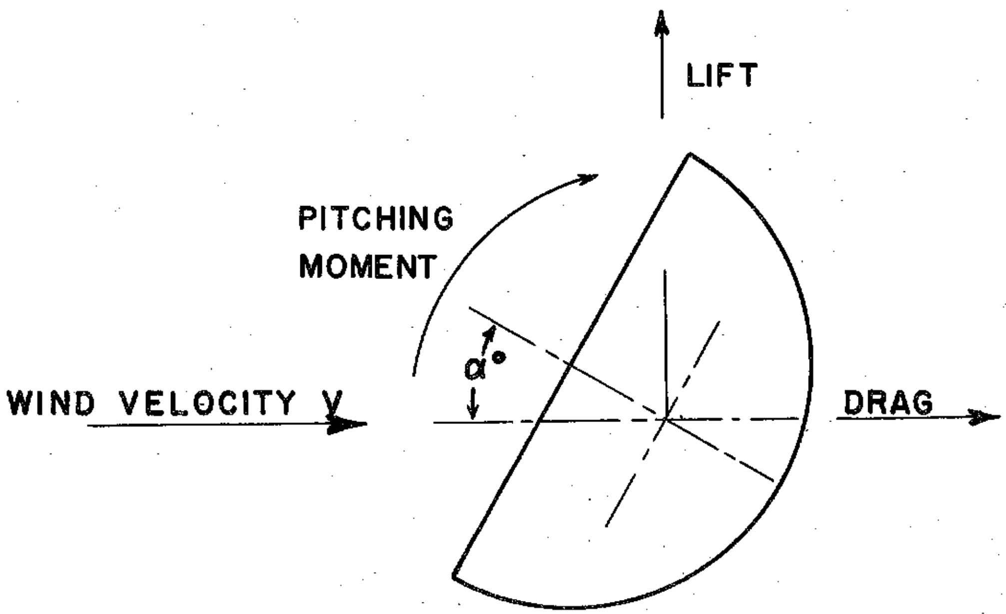
/mr

FIG. 1
MT -14

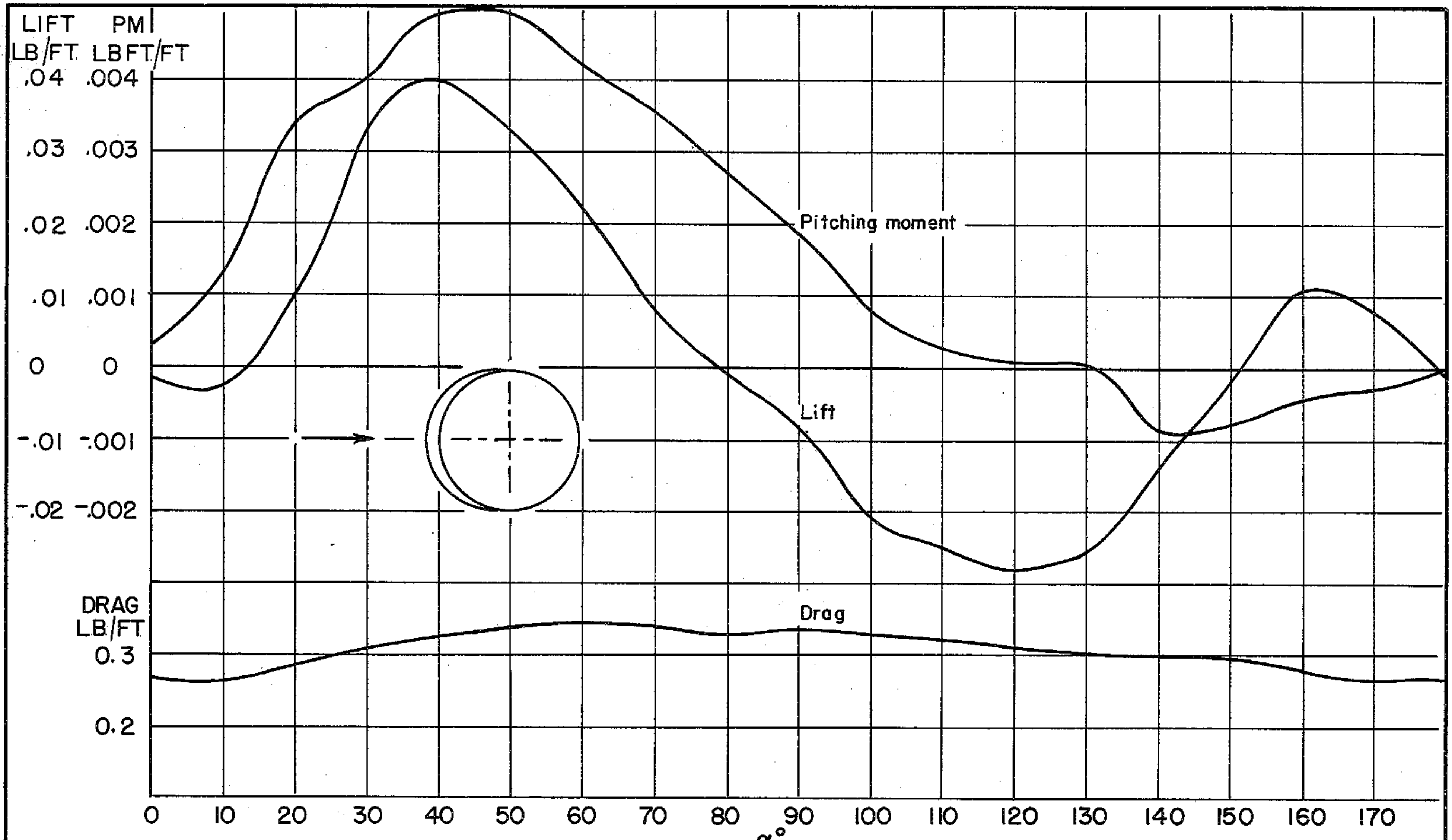


SECTIONS OF MODELS TESTED (FULL SIZE)

FIG. 2
MT - 14



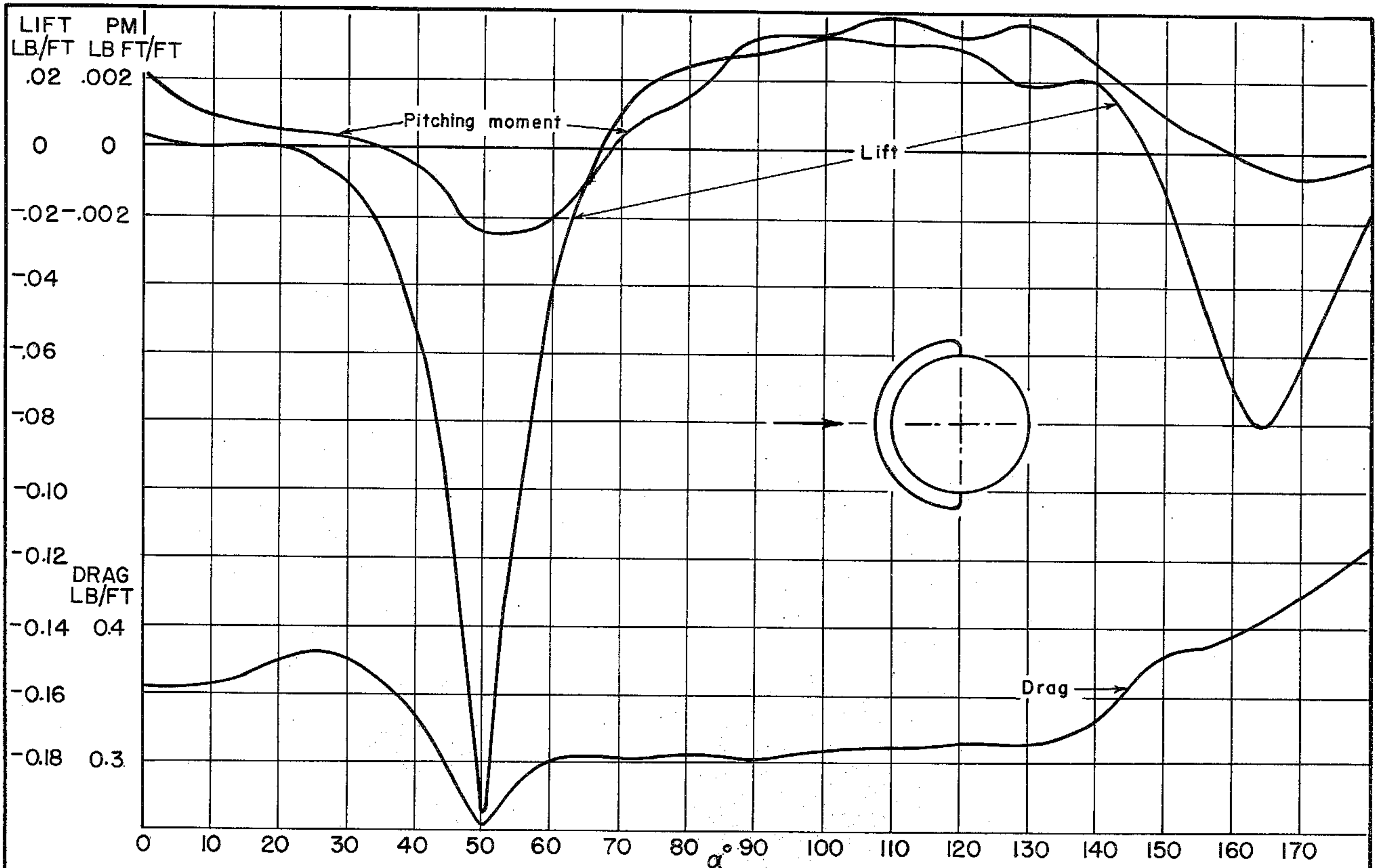
FORCES MEASURED IN TESTS



MODEL I

LIFT, DRAG AND PITCHING MOMENT AT 50 FT/SEC

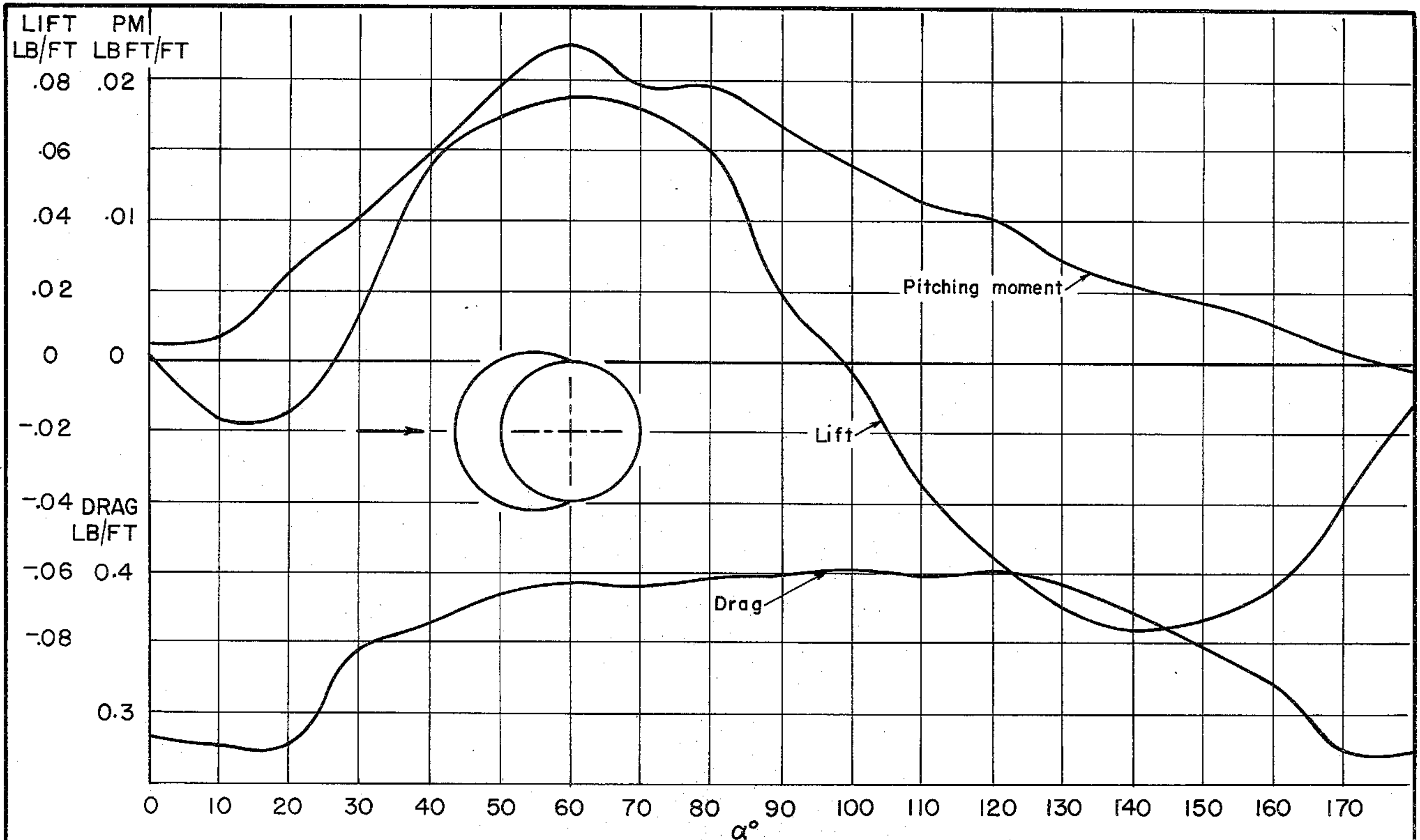
FIG. 3
MT-14



MODEL 2

LIFT, DRAG AND PITCHING MOMENT AT 50 FT/SEC.

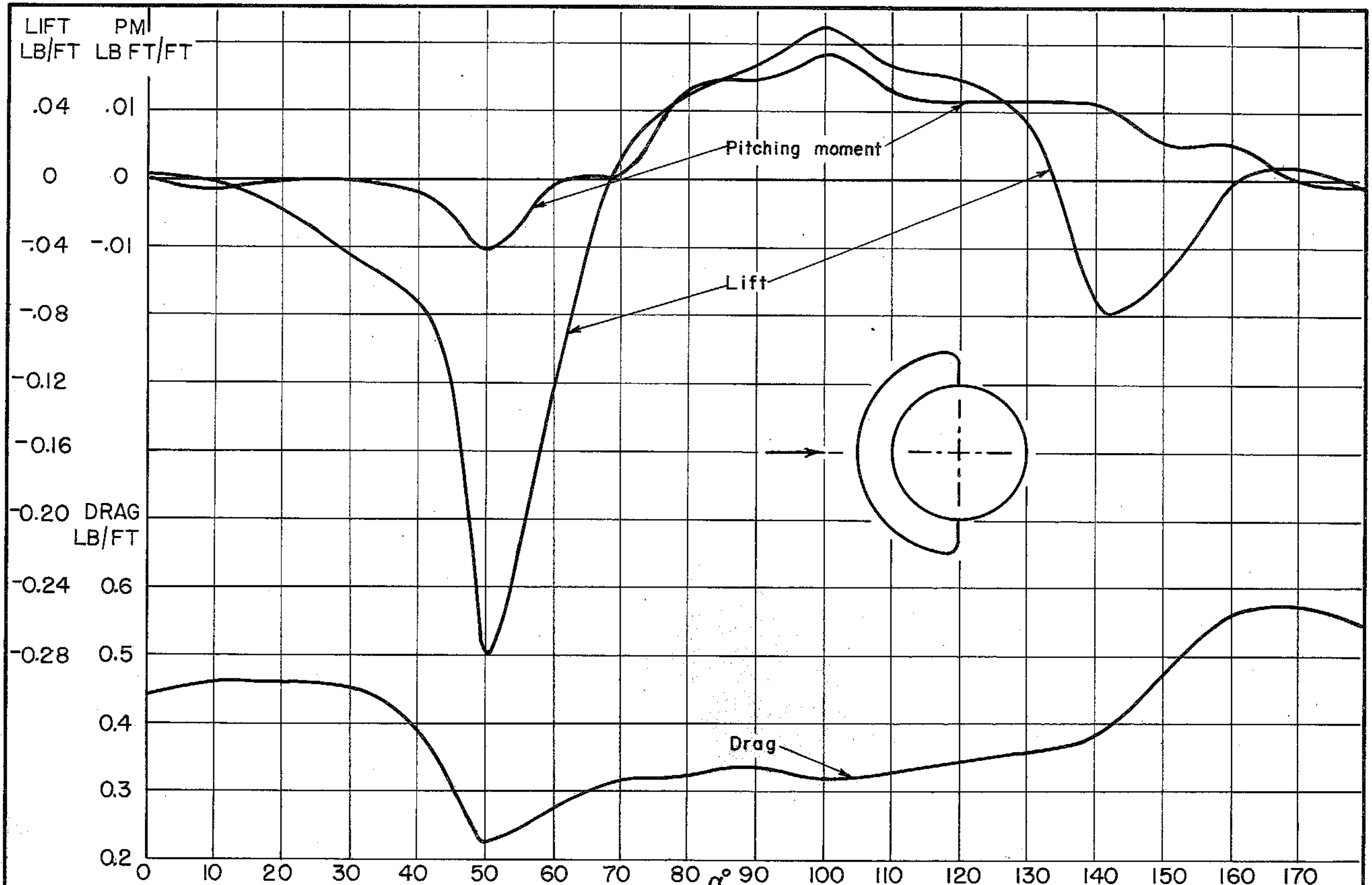
FIG. 4
MT-14



MODEL 3

LIFT, DRAG AND PITCHING MOMENT AT 50 FT/SEC.

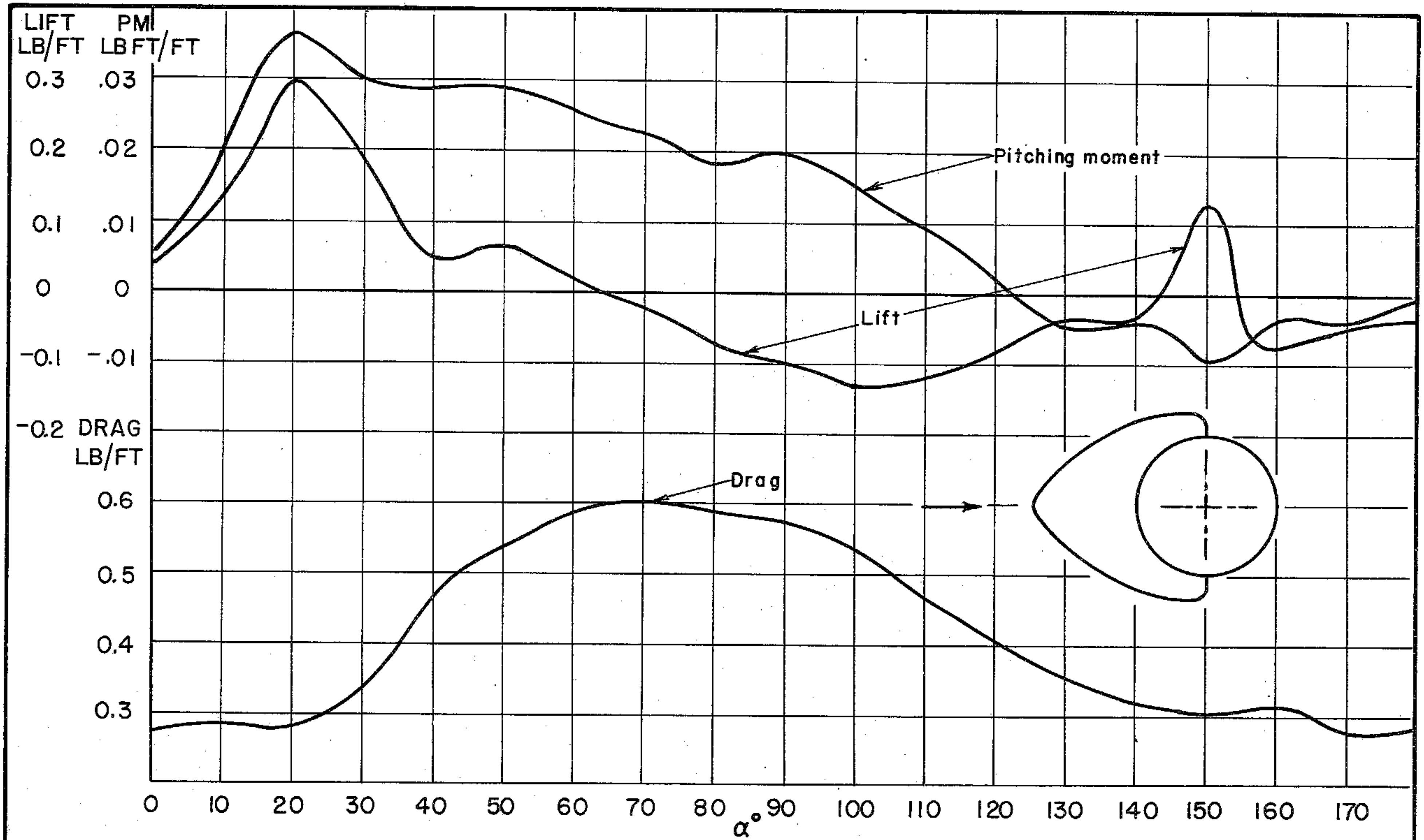
FIG. 5
MT-14



MODEL 4

LIFT, DRAG AND PITCHING MOMENT AT 50 FT/SEC.

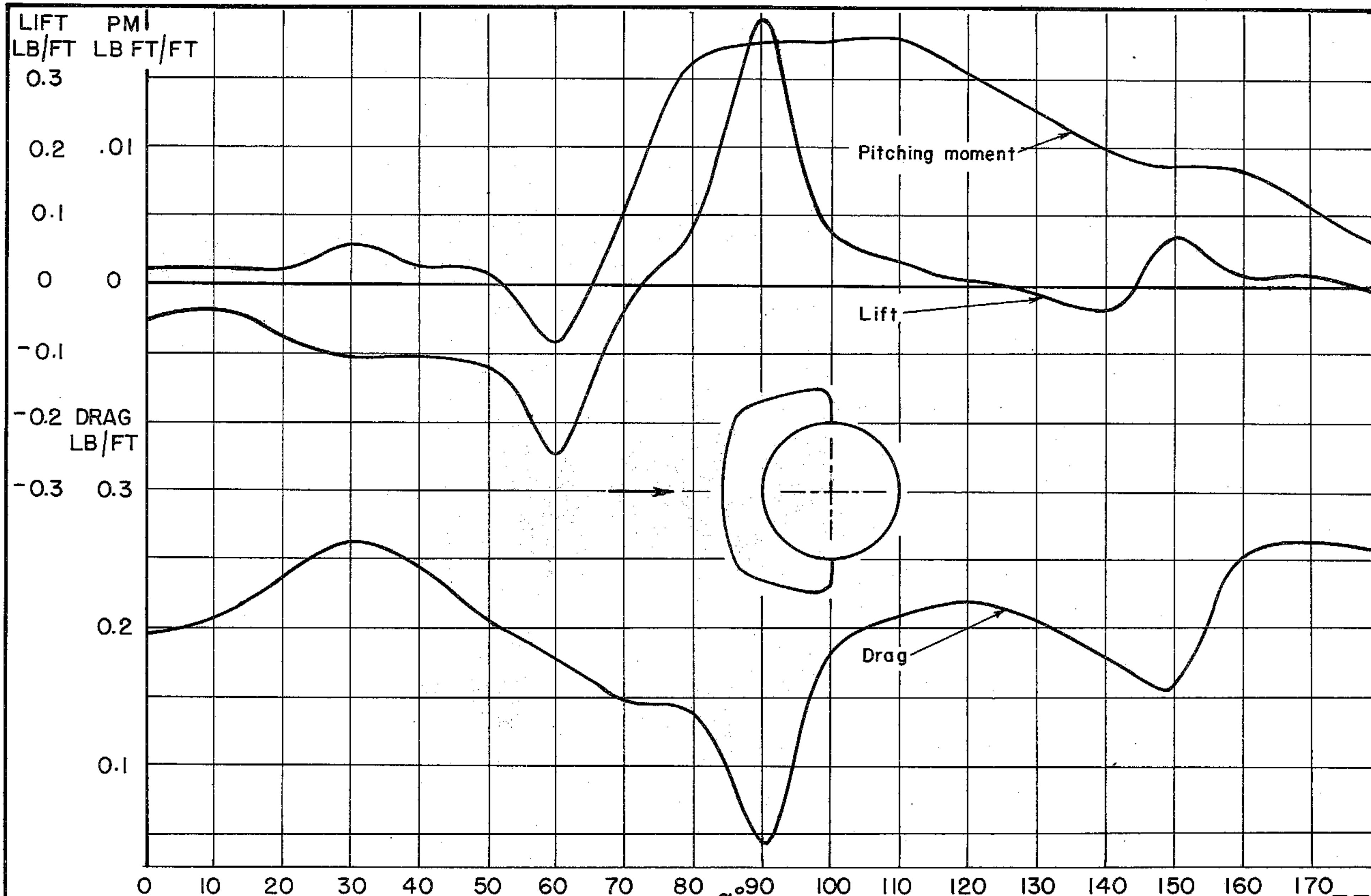
FIG. 6
MT-14



MODEL 5

LIFT, DRAG AND PITCHING MOMENT AT 50 FT/SEC

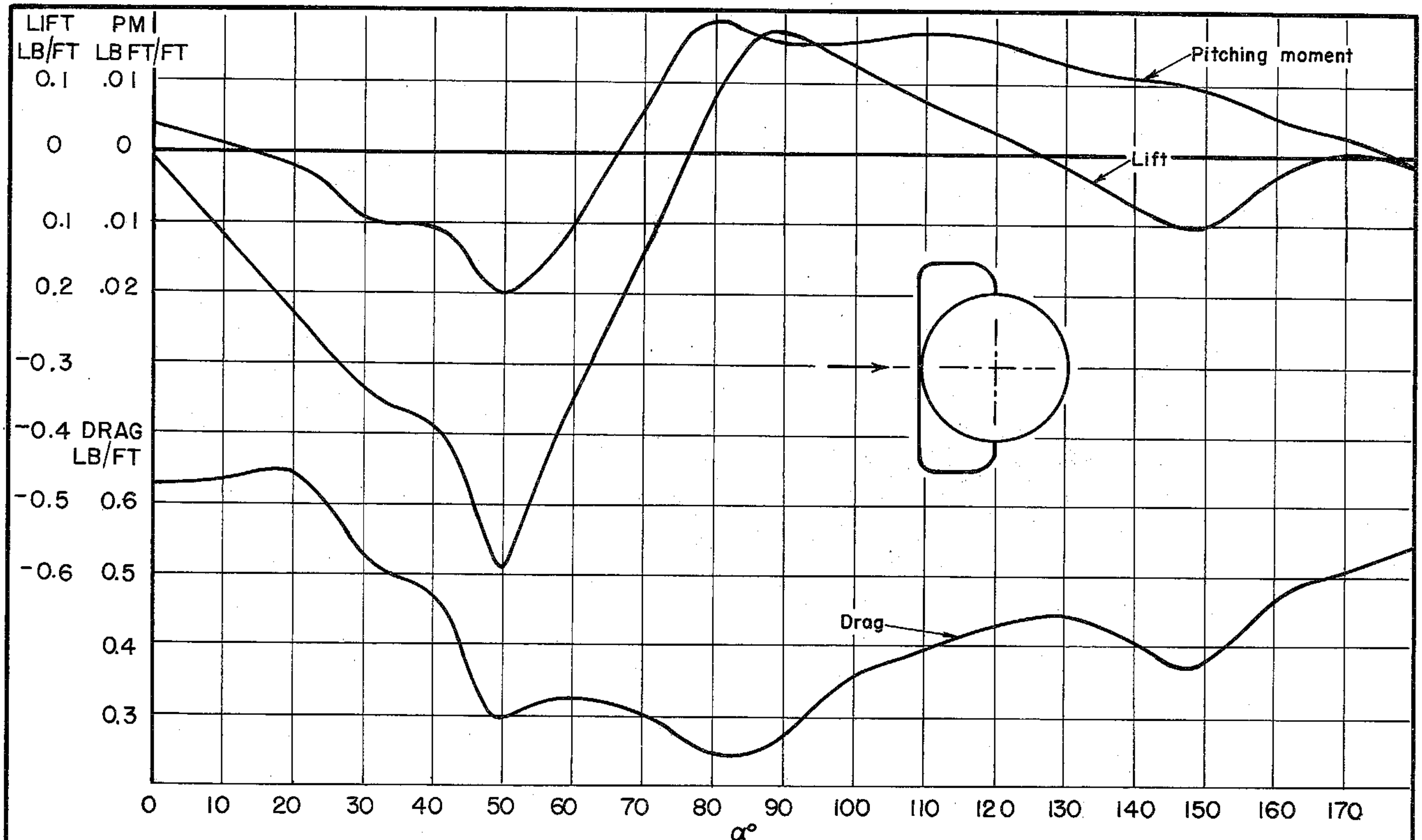
FIG. 7
MT-14



MODEL 6

LIFT, DRAG AND PITCHING MOMENT AT 50 FT/SEC

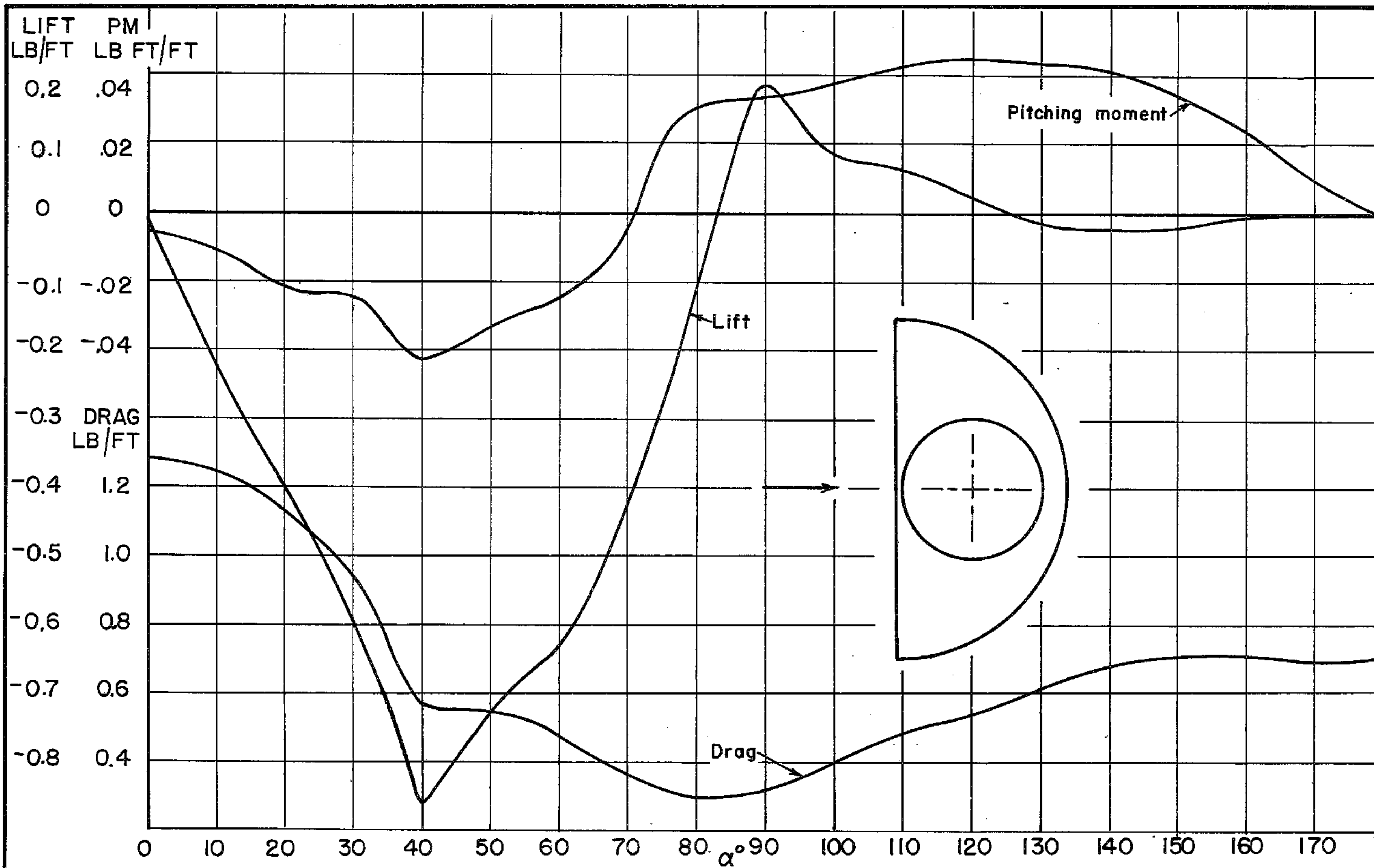
FIG. 8 MT-14



MODEL 7

LIFT, DRAG AND PITCHING MOMENT AT 50 FT/SEC

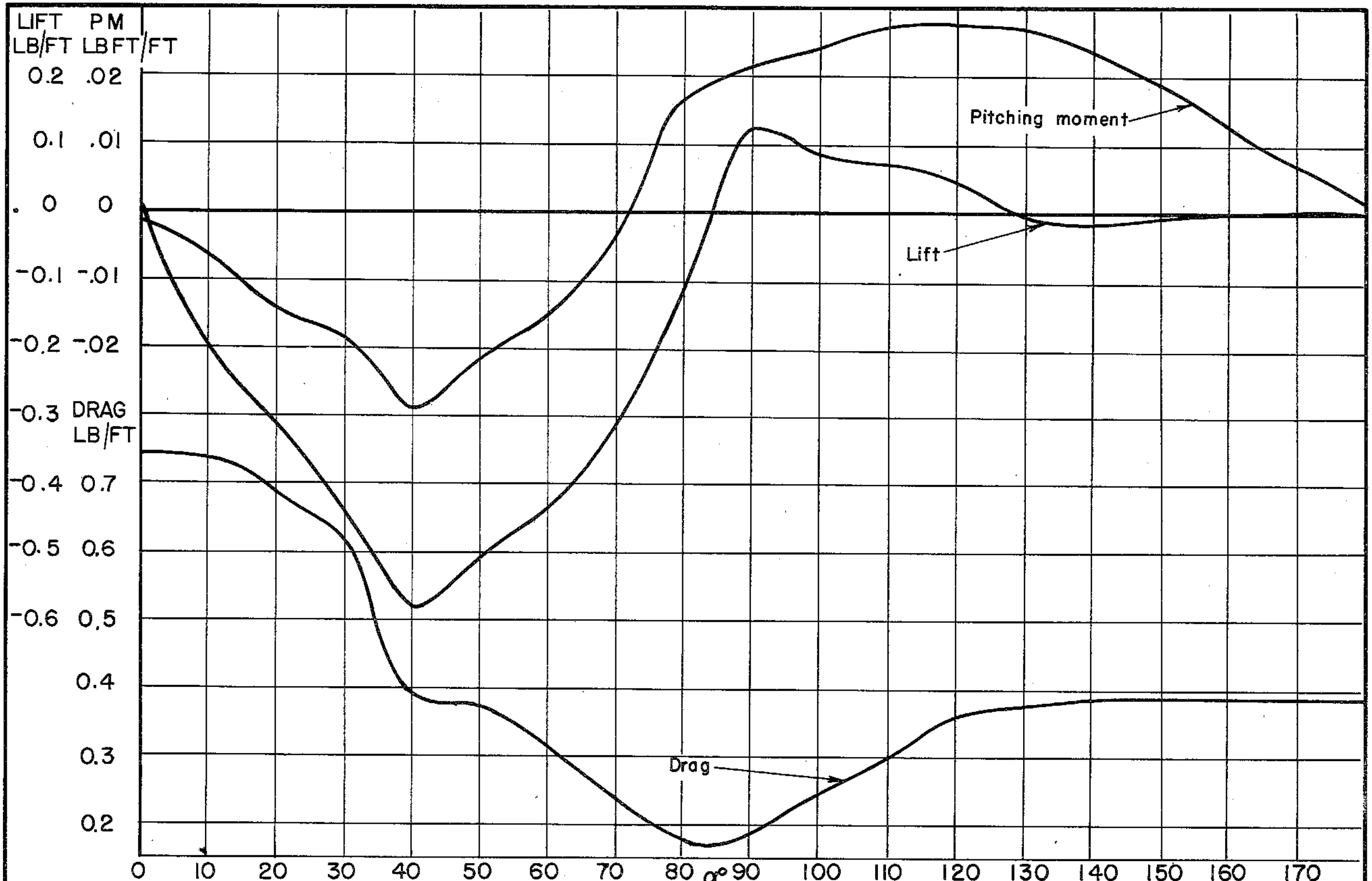
FIG. 9
MT-14



MODEL 8

LIFT, DRAG AND PITCHING MOMENT AT 50 FT/SEC

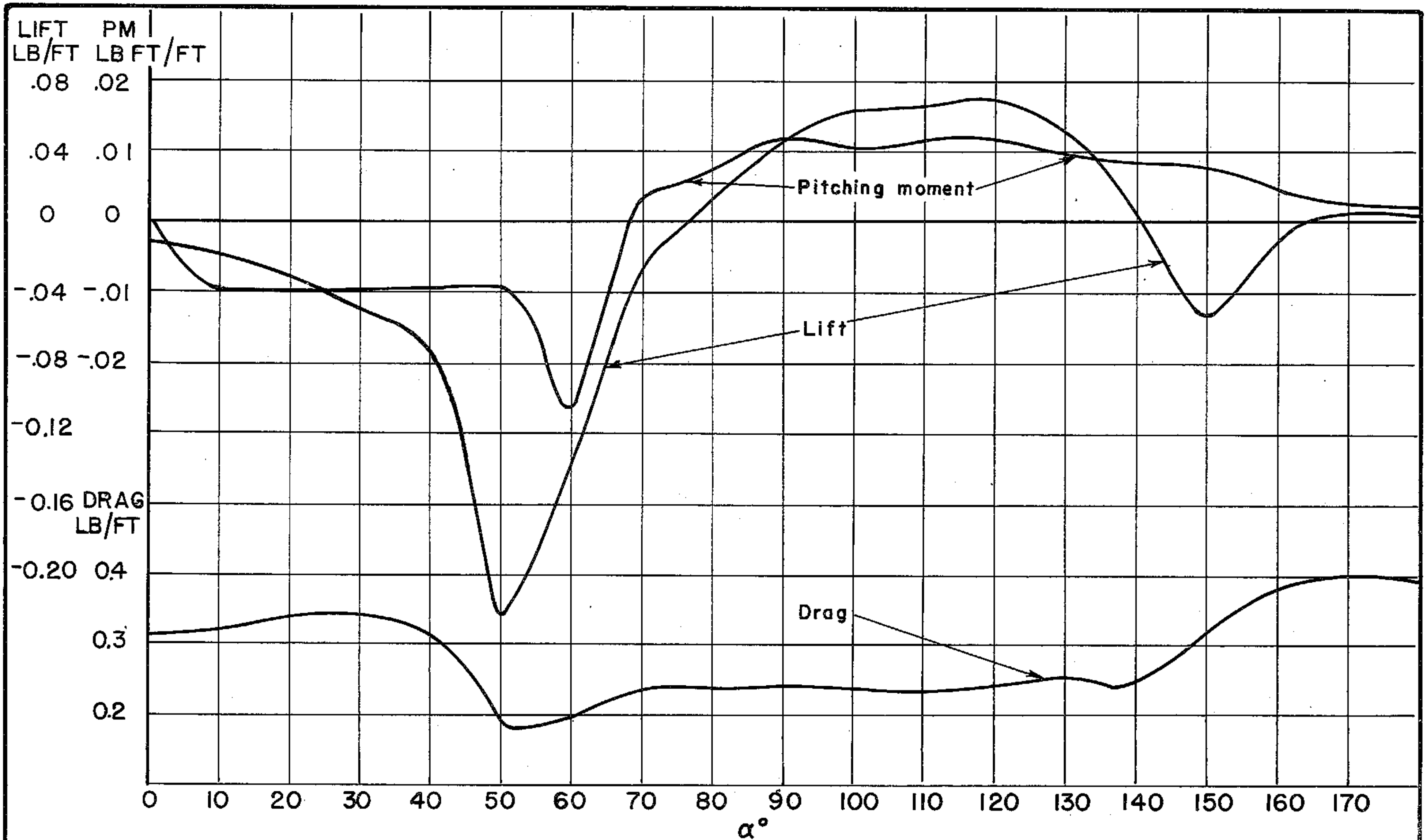
FIG. 10
MT-14



MODEL 8 – 30° YAW

LIFT, DRAG AND PITCHING MOMENT AT 50 FT/SEC

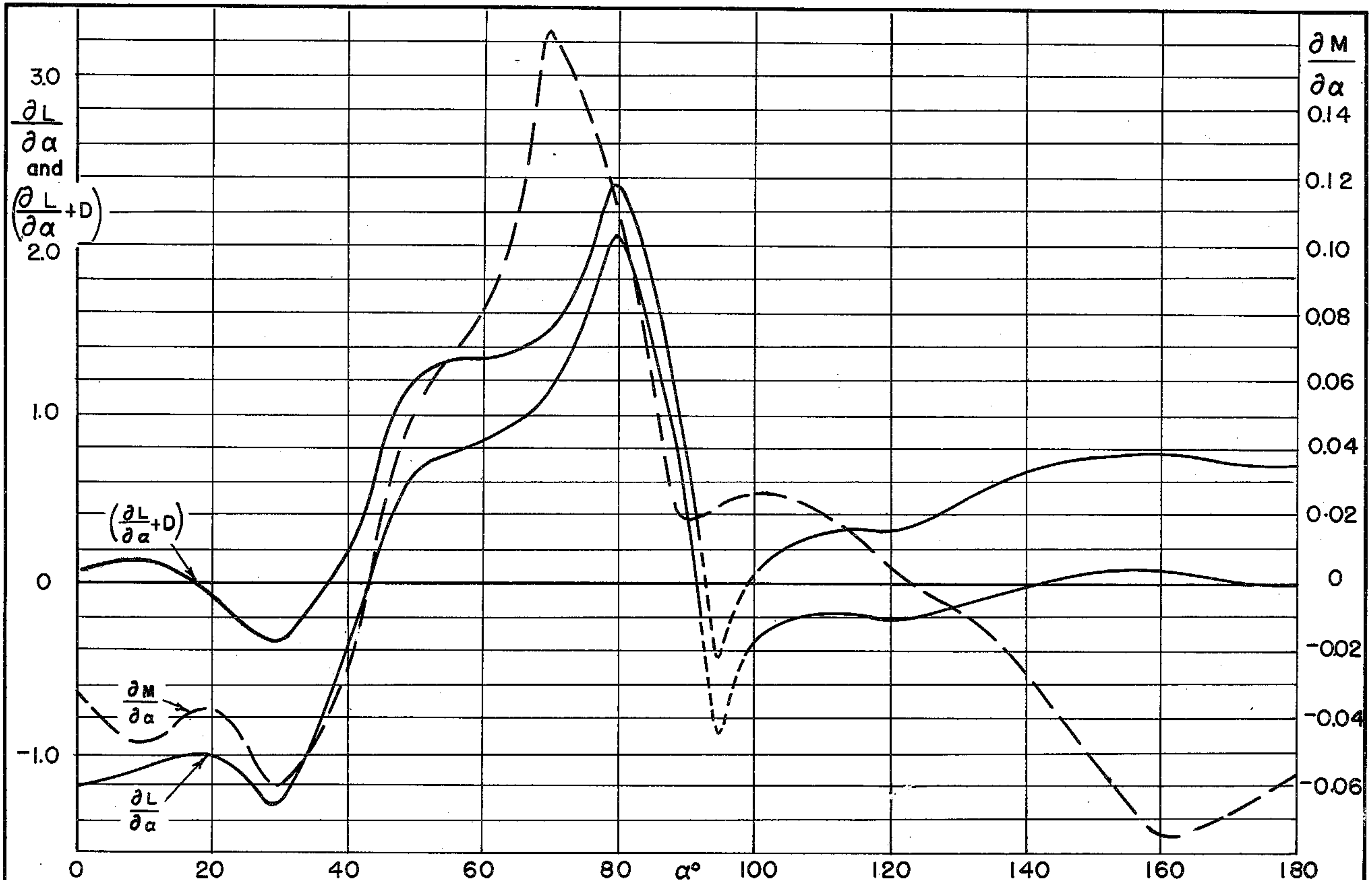
FIG. 11 MT-14



MODEL 4 – 30° YAW

LIFT, DRAG AND PITCHING MOMENT AT 50 FT/SEC

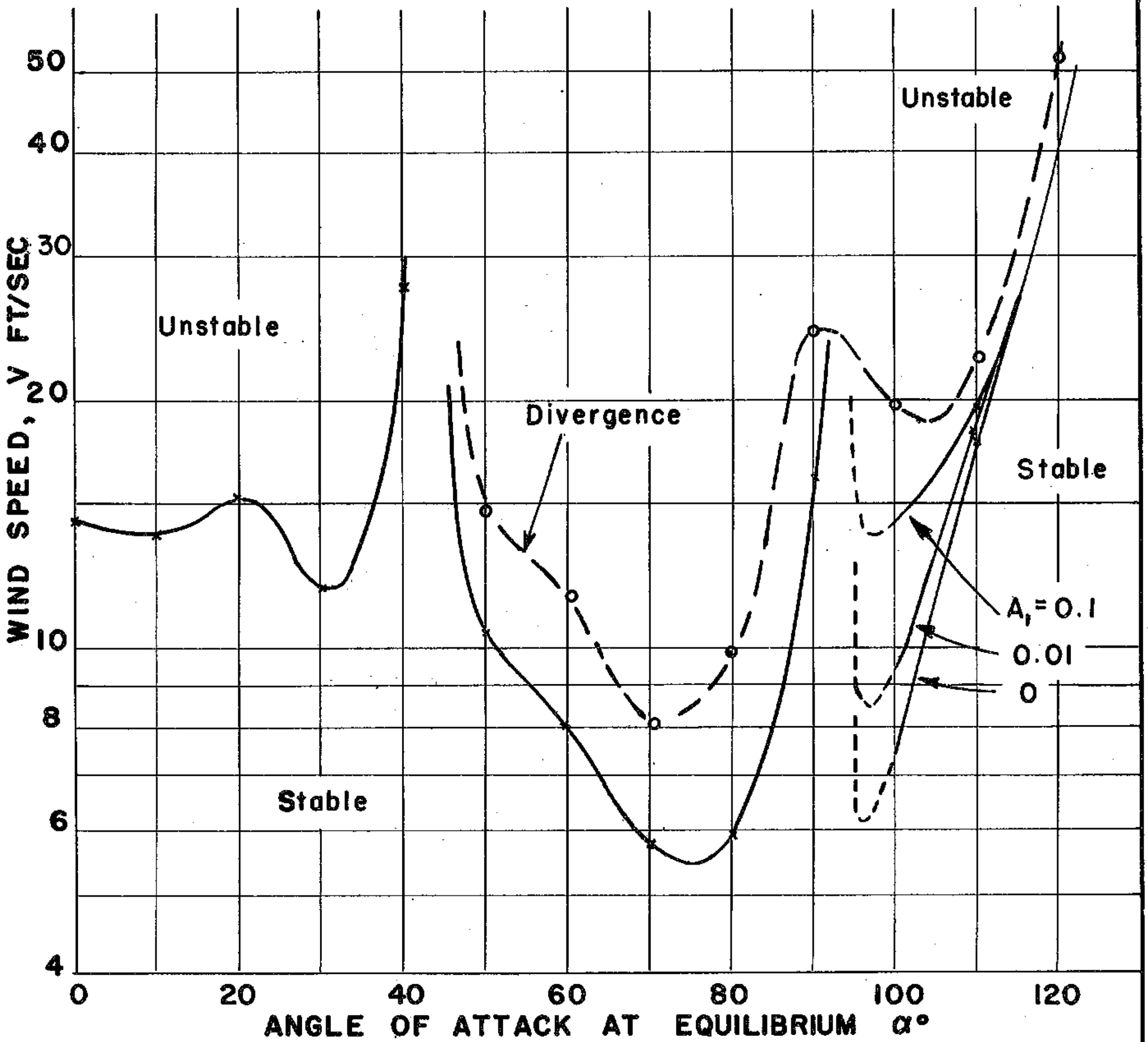
FIG. 12
MT-14



DERIVATIVES OF D SECTION
(MODEL 8) AT 50 FT/SEC

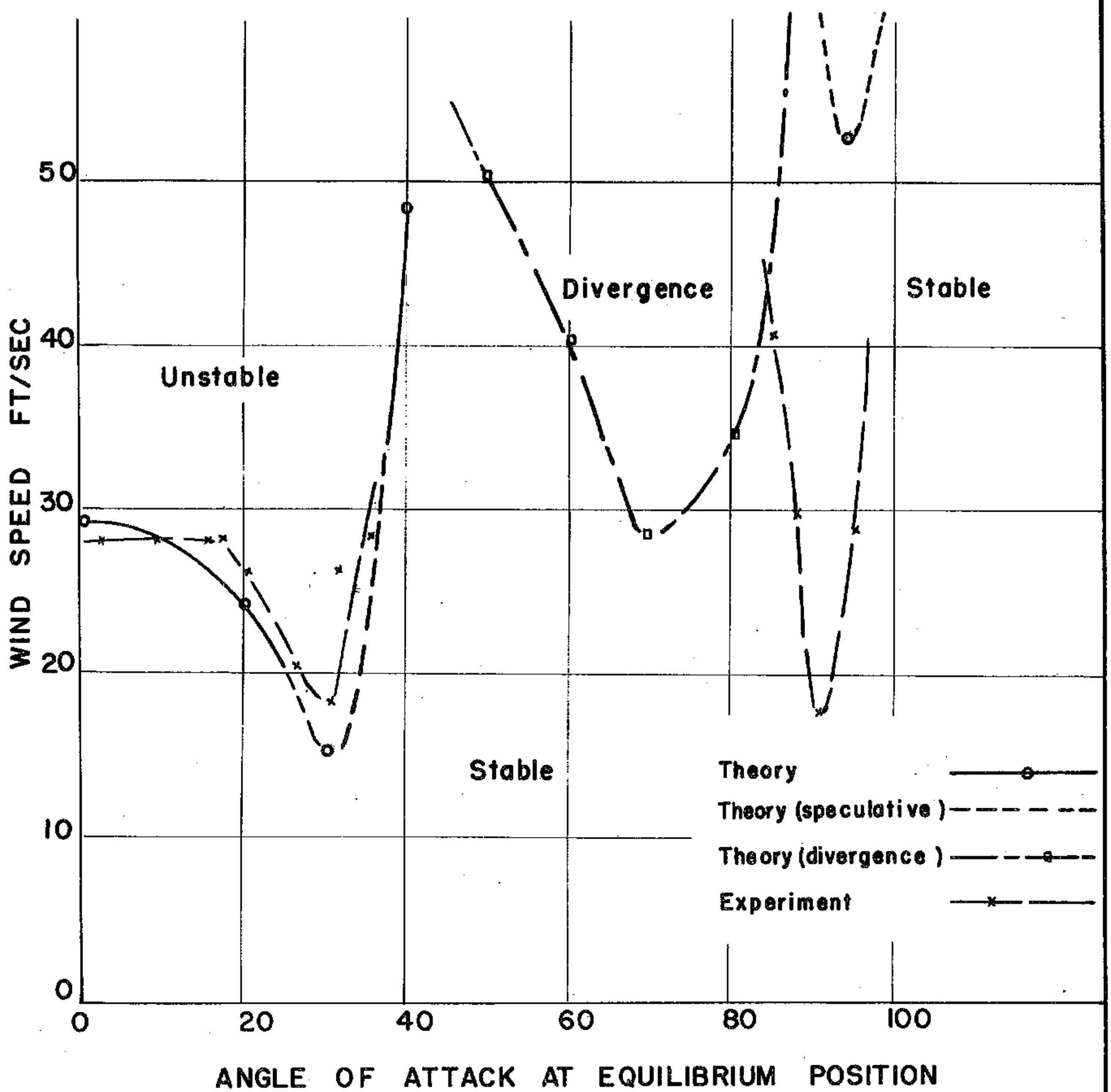
FIG. 13
MT-14

FIG. 14
MT-14



STABILITY BOUNDARIES OF D SECTION

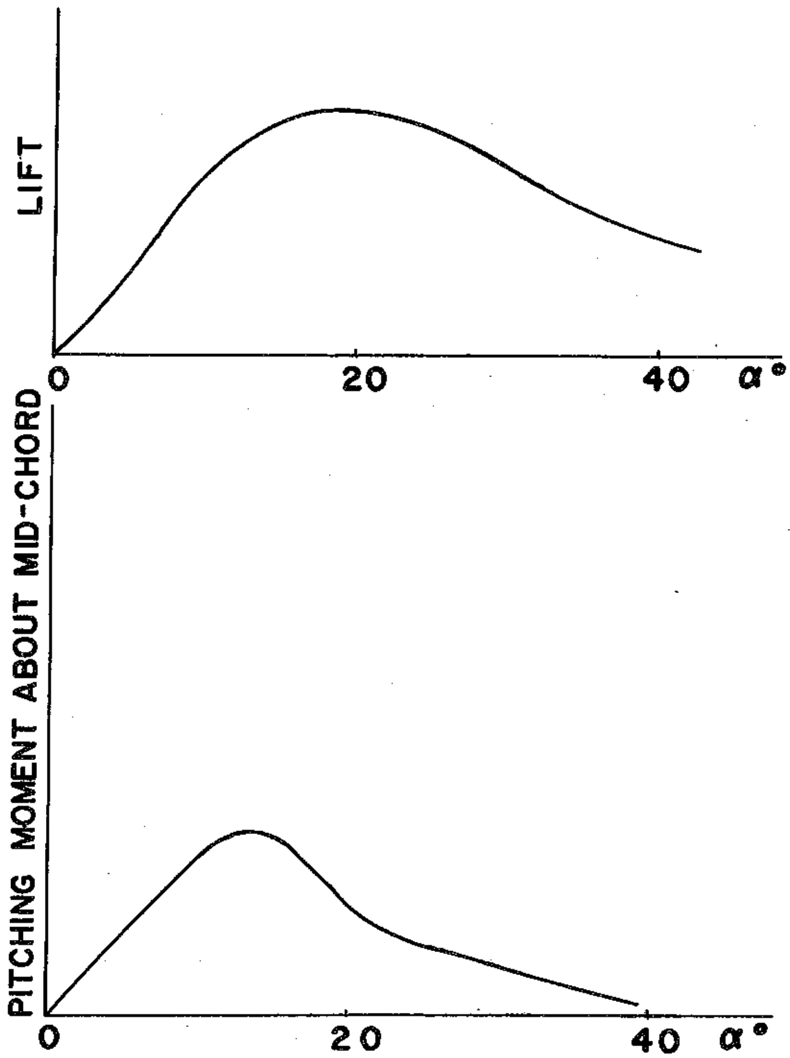
ON TEST SPAN



STABILITY BOUNDARIES OF D SECTION AT

DIFFERENT ANGLES OF ATTACK

FIG. 16
MT - 14



APPROXIMATE LIFT AND PITCHING MOMENT CURVES

FOR A FLAT PLATE