Coordinate conversions between astronomical systems using integer arithmetic to simulate a 24-bit computer

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COORDINATE CONVERSIONS BETWEEN ASTRONOMICAL SYSTEMS USING INTEGER ARITHMETIC TO SIMULATE A 24 - BIT COMPUTER

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OTTAWA
SEPTEMBER 1966

ABSTRACT

Astronomical-coordinate conversions are solved using integer arithmetic in Fortran IV to simulate fixed-point arithmetic. It is concluded that a 24-bit computer can be used to solve the conversions in single precision, with an accuracy greater than, or equal to that of the 20-bit output of shaft-angle encoders on the Algonquin Radio Observatory telescope system.

CONTENTS

FIGURES

COORDINATE CONVERSIONS BETWEEN ASTRONOMICAL SYSTEMS USING INTEGER ARITHMETIC TO SIMULATE A 24-BIT COMPUTER

- E.J. Messerli - *

INTRODUCTION

Computer control of the 150-foot radio telescope at Algonquin Radio Observatory (ARO) will allow observations in systems different from those in which the telescope may be directly controlled. For example, a galactic scan may be made, while control is in equatorial (or altazimuth) coordinates.

The laborious computations involved in obtaining coordinates in the controlling system must be carried out frequently by the computer, to obtain a sequence of coordinates defining the desired path. Interpolation between these coordinates gives a sequence of finer points which are compared with the actual pointing angle of the telescope. The difference, or pointing error, is then conditioned by an error subroutine and is applied through a digital-analog converter as the actual drive signal.

The accuracy of the converted coordinate pairs must be equal to, or greater than, the shaft-angle encoding accuracy of the telescope system — 20 bits for equatorial coordinates or 18 bits for altazimuth coordinates. Since it is most efficient to perform the conversion computations in single-precision arithmetic, it is desirable to know which machines will give 20-bit accuracy with single-precision arithmetic. Since the control and data-acquisition computers being considered for the ARO system have work lengths of 12, 16, 18, or 24 bits, obviously the 24-bit computer is the only one for which the use of single precision may be feasible.

To determine the feasibility of this, it is necessary to specify the type of arithmetic — fixed point or floating point — then, either code the solution for an available 24-bit machine, or simulate a 24-bit machine. This simulation, using machine language, is long but relatively straightforward. However, the choice of fixed-point arithmetic allows it to be written in Fortran IV, using the integer mode.

The results of this method of simulation, run on the NRC IBM S360/50 computer, are reported here. A program for conversions in decimal form has also been written, and all program listings are included, as Appendix B. Appendix A illustrates the coordinate systems, and gives a flow chart for solution.

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*NRC Summer Student, 1966

SIMULATION TECHNIQUE

Accurate simulation of computations on an N-bit machine may, in theory, be coded in machine language on a machine of any word length, providing the following rules are obeyed:

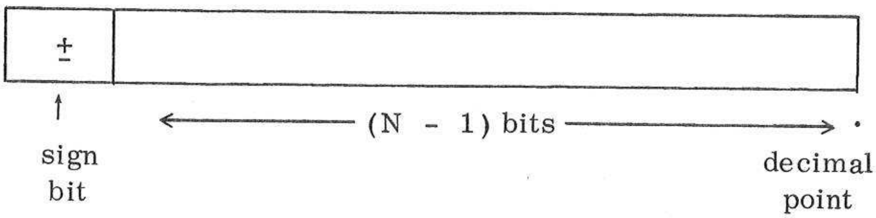
1) number representation (length, format) must be the same as for the N-bit machine

2) type of arithmetic must be the same (i.e., fixed- or floating-point)

3) arithmetic operations must be the same (i.e., adjustment rules or rounding/truncation procedures).

Moreover, if we restrict ourselves to fixed-point arithmetic, these rules can be, effectively, obeyed without resorting to machine language. This is done by coding in Fortran with operations done in integer mode. The machine used in the simulation must have a word length greater than or equal to N bits, or the number corresponding to all N bits having value "1" cannot be represented as an integer.

In using the integer mode in Fortran, the decimal point is considered to be located to the right of the least significant bit:



Numbers are scaled to lie in the range $|I| \leq 2^{(N-1)}$, where one bit is allowed for the sign, and stored in either two's-complement (most common) or sign-magnitude form. If X is a number such that $|X| \leq R$, the integer representation of X is:

$$IX = \frac{X}{R} * 2^{(N-1)}$$

Arithmetic operations may now be written, where $IS = 2^{(N-1)}$:

1) Addition and subtraction

$$IZ = IX \pm IY, \text{ where } |IZ| \leq IS$$

2) Multiplication of IX by IY

$$IZ = IX * IY/IS$$

3) Division of IX by IY

$$IZ = IX * IS/IY$$

To illustrate, let $IS = 2^3 = 8$, $|X|, |Y| \leq 1$.

Therefore X = 0.625 is scaled to 5 (≡ 0101)

Y = 0.5 is scaled to 4 (≡ 0100)

and XY = .3125 is scaled to 2 (≡ 0010)

(Note that .3125 × 8 is truncated to 2)

The machine operation is:

```
   0101
 * 0100

     Multiply → 0001 0100   (double-accumulator)
Shift Left One → 0010 1000
```

Take high-order word → 0010 (≡ 2). We arrive at the same answer simply by doing integer arithmetic; i.e.,

$$IZ = {}^{5 \times 4}/_{8} = {}^{20}/_{8} = 2$$

Note that it is assumed that the intermediate result IX * IY, which may be greater than IS in magnitude, is available. This is true for machines with only two operating registers and a compiler (Fortran) which results in the set of machine-language instructions:

Load operating register (M_1)
Multiply (M_2)
Divide (M_3)

where (M) denotes contents of location M, and M_1, M_2, and M_3 correspond to IX, IY, IS. Since products are normally stored in a double-length accumulator, and division is normally made into the contents of a double-length accumulator (even for single-precision arithmetic), the intermediate result is not lost.

For machines such as the IBM S360/50 with more than 2 operating registers, the intermediate result (IX * IY) is not available (i.e., IZ = IX * IY/IS is incorrect if $|IX * IY| > 2^{31}$, the maximum integer size allowed — no overflow is indicated if this occurs). In this case it is necessary to constrain computations to be done in real variable mode (preferably double precision), then truncate the results to an integer. This can be done by using the mixed-mode feature of the S360 Fortran IV, and multiplication will then appear as follows:

```
Double Precision X
X = 1.0
- - - - - - - - - - - -
IZ = X * IX * IY/IS
```

with a similar form for division.

(Note: IZ = IX * IY/IS * X gives incorrect results, since the compiler works from left to right and, hence, the first multiplication is still done in integer mode.)

COMPUTATIONAL RESULTS

To re-emphasize, the simulation is for fixed-point arithmetic on a 24-bit machine. The integer trigonometric routines are derived from routines written for a 24-bit SDS-910, and hence, are not suitable for any other word lengths (since the number of terms carried in the power series depends on the word length). The sine-cosine routines were tested and errors were found to be equal to, or less than, two bits relative to a reference computation. This corresponds to an absolute error of less than 10^{-6} on a scale of 1.0.

The arctangent routine was found to be accurate to within one bit (corresponding to an absolute error of approximately 0.08 second of arc). Thus, these routines will not contribute significant errors to the results.

Appendix B gives numerical results for a conversion from galactic to equatorial 1950 coordinates. The first line of each result gives the angles in degrees of the galactic (L, B) and equatorial (ALPHA, DELTA) coordinates. This conversion is performed in double precision (representing 17 significant decimal places); hence, the decimal results serve as an accurate reference computation. (The double-precision form of CONVT is not included with the program listings.) The second line (IN DECIMAL) is the same equatorial result as the first; however, it is scaled to lie in the same range as the integer result (IN INTEGER).

The relative error between the decimal and integer results determines how many bits have been dropped by the integer routine, and may be interpreted according to Table I.

TABLE I

RELATIVE ERROR BETWEEN DECIMAL AND DIGITAL RESULTS

Relative error	Bits dropped	Resulting precision	Angular error (arc sec)
0-1	1	23	0.16
1-3	2	22	0.32
3-7	3	21	0.64
7-15	4	20	1.3
15-31	5	19	2.6
31-63	6	18	5.2

Inspection of the results indicates, as expected, that the largest errors occur near the pole (large declination); however, the relative error in declination is always less than 2. The relative error in right ascension only becomes greater than 15 for certain angles of almost 89° or greater, with the error in declination in this region being very much smaller.

The actual measure of error should be the arc displacement $\Delta\theta$, and it is easily shown that:

$$(\Delta\theta)^2 \cong (\Delta\delta)^2 + \cos^2\delta\,(\Delta\alpha)^2,$$

where δ represents declination

α represents right ascension

$\Delta\delta$, $\Delta\alpha$ represent error in δ and α, respectively

Rewriting the above equation gives:

$$\begin{aligned} |\Delta\theta|^2 &= |\Delta\delta|^2 + \cos^2\delta\,|\Delta\alpha|^2 \\ &\leq |\Delta\delta|^2 + 2\,|\Delta\delta|\,|\Delta\alpha|\cos\delta + \cos^2\delta\,|\Delta\alpha|^2 \\ &\leq (|\Delta\delta| + |\Delta\alpha|\cos\delta)^2 \end{aligned}$$

or

$$\boxed{|\Delta\theta| \leq |\Delta\delta| + |\Delta\alpha| \cos\delta}$$

$$(\cos\delta \geq 0 \text{ for } |\delta| \leq 90°)$$

Using this as a measure and δ as an independent variable, (because of dependence of $\Delta\alpha$ on δ), we are able to estimate the bounds on the errors given in Table II. (Results are referred to by numbers appearing on the left-hand side of result sheets).

TABLE II

RESULTS FOR 24-BIT COMPUTER

Declination (degrees)	Max. rel. error in α	Max. rel. error in δ	Max. (est.) arc error (arc sec)	Result No.
-28.4 to + 83.6	1	2	.32	1-20
88-89	35	2	.32	20-39
89.112	35	.1	.16	25

However, there does not appear to be any correlation between $|\Delta\theta|$ and δ . This occurs because of the compensating effect of the cos δ factor, and randomness introduced by truncation. For the results included, $|\Delta\theta|$ is distributed as shown in Table III.

TABLE III

DISTRIBUTION OF MAXIMUM ABSOLUTE ANGULAR ERROR (MAX. $|\Delta\theta|$)

PRODUCED BY A 24-BIT PROCESSOR

Max. $|\Delta\theta|$ (arc sec)	No. of results
0.00 - 0.08	0
0.08 - 0.11	6
0.11 - 0.16	11
0.16 - 0.23	9
0.23 - 0.32	7
0.32 - 0.45	6
> 0.45	0
Total of	39

Thus, the probability that $|\Delta\theta| > 1.28$ arc sec is very small.

These results indicate that, for coordinates farther than 1° from a pole, 24-bit single precision is adequate. Very likely it is adequate to within a fraction of a degree of the pole with the routine used. For coordinates closer to a pole, either extra precision must be carried, or , preferably, an alternative form of the solution should be used. Large errors in α will introduce large errors in δ , since α is used to solve for δ in the routines given, using either $\sin(\alpha - C_{EQ})$ if $|\alpha - C_{EQ}| \geq 45°$, or $\cos(\alpha - C_{EQ})$ if $|\alpha - C_{EQ}| < 45°$. This tends to minimize the effect of fairly large changes in the sine or cosine of some angles. Consider, for example, that $|\alpha - C_{EQ}| < 45°$. Then $\cos(\alpha - C_{EQ})$ is used and

$$\Delta \cos(\alpha - C_{EQ}) \cong - \sin(\alpha - C_{EQ})\Delta\alpha$$

Thus, for the same relative error ($\Delta\alpha$) the effect is much less near 0° than it is near 45°. However, very near a pole an alternate solution should be employed; for example, an arcsine routine for δ.

It is interesting to compare the results if one bit is dropped from all computations (23-bit computer). Equivalent values for this case (again referred to the same decimal computation) are given in Table IV. Although data are not included in the appendix, these results are for the same angles.

TABLE IV

RESULTS FOR 23-BIT COMPUTER

Declination (degrees)	Max. rel. error in α	Max. rel. error in δ	Max. (est.) arc error (arc sec)	Result No.
-28.4 to +83.6	10	4	1.3	1-20
88-89	224	6	1.3	21-38
89.112	134	1.1	.64	25

Note that the effect in the final answer is (in general) a loss of more than a single bit (2 bits) in accuracy, if these results are compared with those of Table II.

On the basis of these results, it may be concluded that a 24-bit computer can be used for coordinate conversions, with single-precision arithmetic. The results will be compatible with the output of 20-bit shaft encoders, except for a small area about a pole. It does not appear that a computer of word length less than 24 bits can achieve acceptable results with single-precision calculations, so that care must be taken in programing a 24-bit computer, if one is chosen for the task.

APPENDIX A

COORDINATE SYSTEMS

The astronomical coordinate systems of interest are of (or equivalent to) the form shown in Fig. 1 (a), where we may regard (the center of) the earth as the origin. For equatorial coordinates, A corresponds to right ascension, while B corresponds to declination.

If we consider two such systems, the transformation from one to the other can be specified in terms of the angles C_1, ϕ, C_2, as indicated in Fig. 1 (b). The transformation from system 1 to system 2 is then defined by the following equations, which represent equivalent projections in two 3-dimensional Cartesian systems. The third dimension is fictitiously introduced by considering the unspecified radius of all great circles to have a fixed value (say 1). The equations are:

$$X = \cos(A_2 - C_2)\cos B_2 = \cos(A_1 - C_1)\cos B_1$$

$$Y = \sin(A_2 - C_2)\cos B_2 = \sin(A_1 - C_1)\cos B_1 \cos\phi - \sin B_1 \sin\phi$$

$$Z = \sin B_2 = \sin(A_1 - C_1)\cos B_1 \sin\phi + \sin B_1 \cos\phi$$

To convert from system 2 to system 1, A_1 and A_2, B_1 and B_2, C_1 and C_2 are interchanged and ϕ is replaced by $-\phi$.

A flow chart (Fig. 2) for direct solution of the equations is the basis for the solution program of Appendix B.

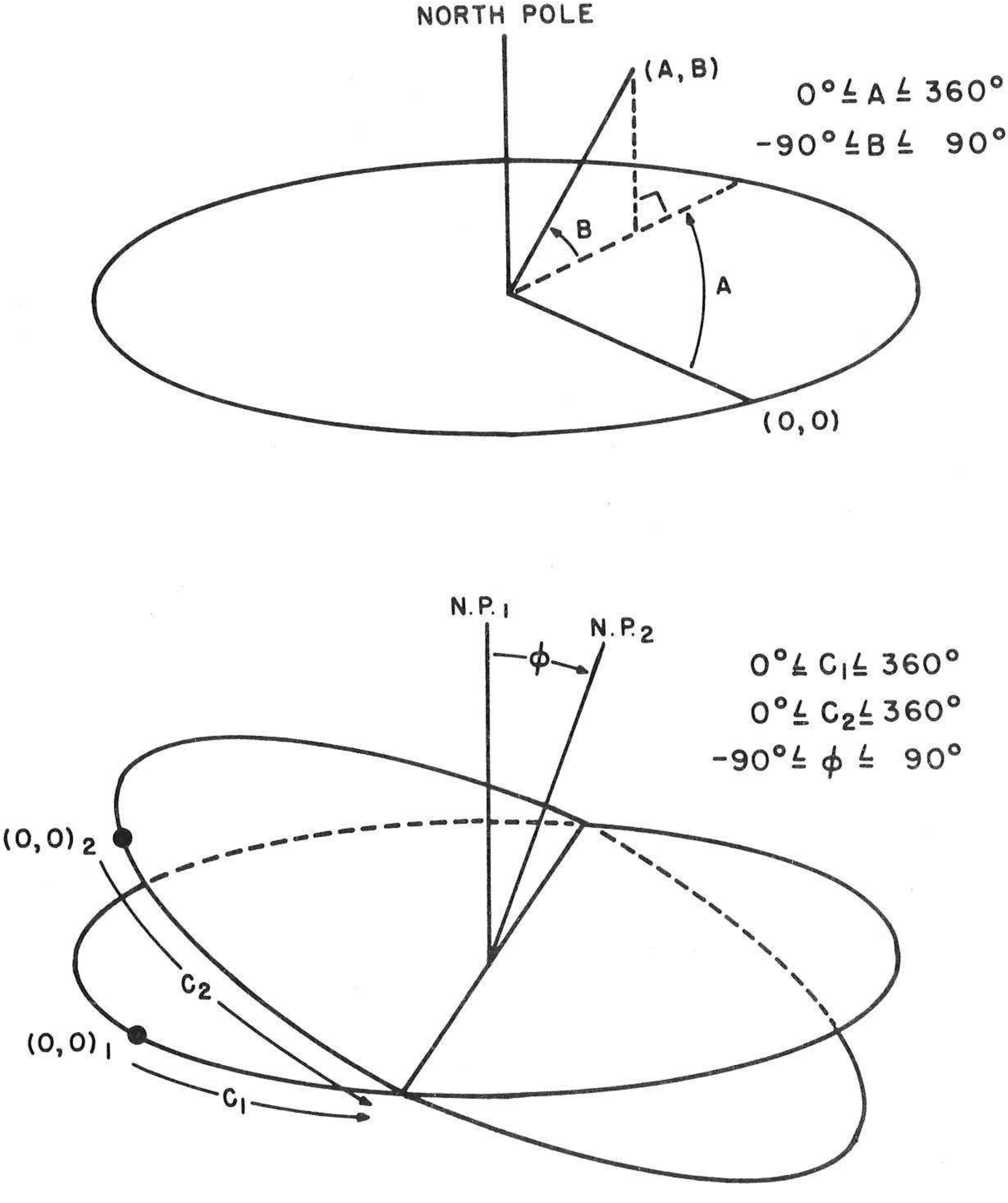


Fig. 1 Coordinate systems
(a) Above - single system
(b) Below - two oriented systems

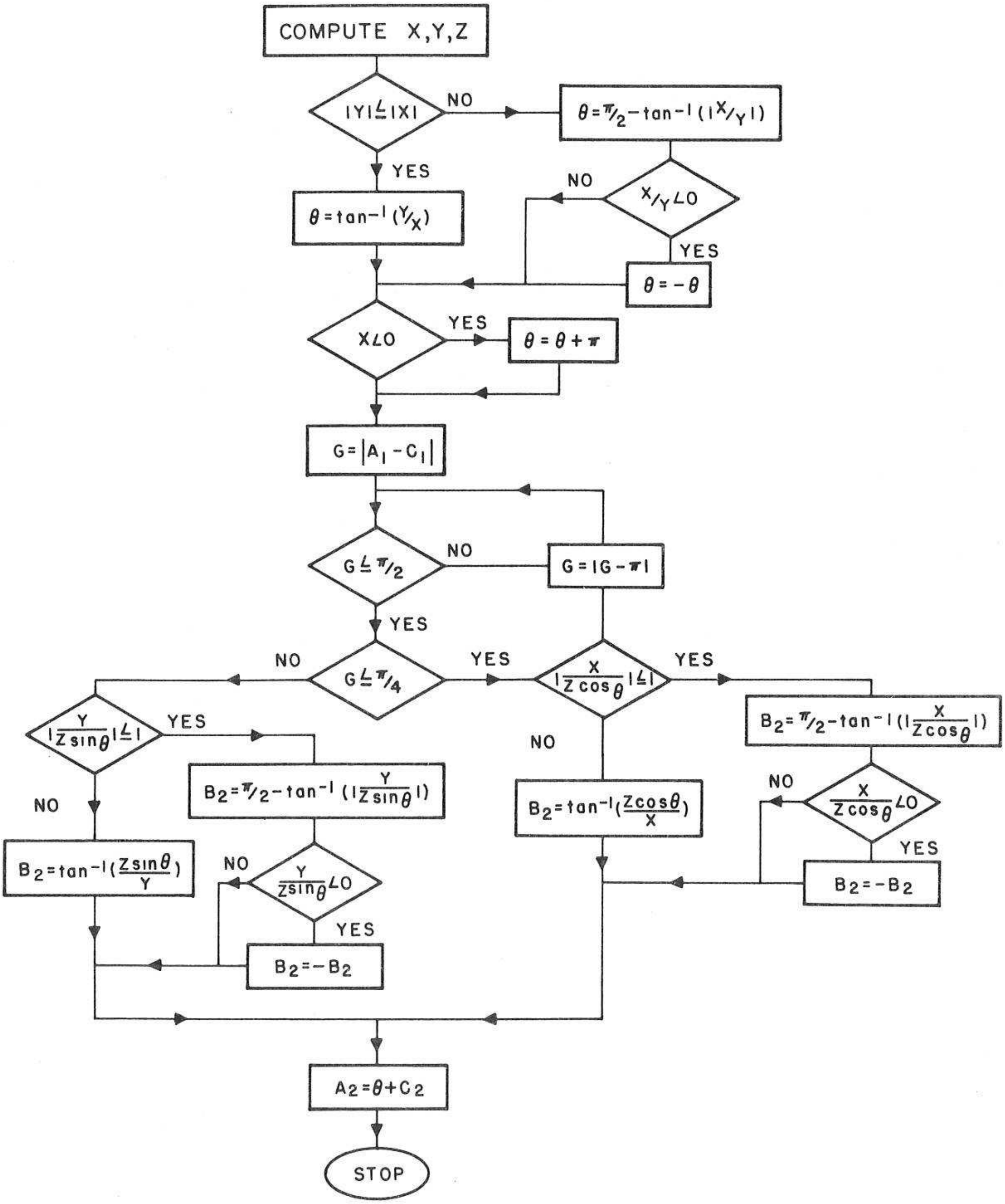


Fig. 2 Flow chart for coordinate conversion

APPENDIX B

PROGRAM LISTINGS

The following programs simulate the 24-bit computer:

1) Main Program

This program coordinates the use of function subprograms and subroutines and is basically the integer solution of the equations in Appendix A, following flow chart of Fig. 2. Solution in decimal is also given as a comparison — this solution uses normal Fortran subroutines and is carried out in floating point. Single-precision floating point on the IBM S360/50 has a 24-bit fraction and 7-bit characteristic, which is equivalent to seven significant decimals always maintained.

In all programs IS corresponds to the maximum integer size, and angles are scaled so that 180° => IS, and sine and cosine results are scaled so that 1.0 => IS. Hence IS has a value 2^{23} for a 24-bit machine, since 1 bit is allowed for the sign.

2) CONVT

This program (which may also be easily coded in double precision) takes coordinates and rates of change of coordinates in one system (expressed in degrees and any desired units for rate) and gives the equivalent coordinate and rate in the second system. For example, if we had coordinates defined as in Appendix A, to solve the equations given there, we would simply write:

```
Call CONVT (A1, B1, C1, 0., 0., PHI, A2, B2, C2, 0., 0.)
```

Page 15 indicates output from CONVT when it is used for conversion from galactic (L, B) to equatorial (ALPHA, DELTA) coordinates.

3) IATAN

This is the integer routine for the arctangent where the argument IX is scaled to have a maximum magnitude equal to IS (and hence corresponding to 1.0). It is strictly a power series evaluation which returns an answer in the range $-IS/4 \leq IATAN \leq IS/4$, corresponding to range -45° to +45°. Logic to decide whether IX lies in correct range (cotangent is solved otherwise), and in which quadrant the solution lies, is done outside the routine. X must have double-precision value 1.0 when this routine is used.

4) ISIN

Angles scaled in range ±IS are input to this routine, and this reduces the angle to the range ±IS/2 and evaluates a power series. The result ISIN is scaled so that sin 90° corresponds to IS. Again X must be a double-precision variable with value 1.0. (This could be eliminated from appearing in the calling statement by setting it equal to 1.0 inside each subroutine.)

5) ICOS

The ICOS routine evaluates the integer cosine by adding IS/2 to IX and using the same statements as ISIN.

6) SCALE

This subroutine takes an argument X in degrees, any range, and expresses it in the equivalent range $-IS \le IX \le IS$.

MAIN PROGRAM

```
      DOUBLE PRECISION X
      REAL L
      PI=3.14159265359
      C1=33.0
      C2=282.25
      PHI=62.6
      X=1.0
      DB=0.
      DL=0.
      IS=  8388608
      IS6= SIN(PHI*PI/180.)*IS
      IC6= COS(PHI*PI/180.0)*IS
      READ(1,150) NDATA
150   FORMAT(I10)
      DO 100 K=1,NDATA
      WRITE(3,12)
12    FORMAT('                           L(DEG.)                          B
     1                           ALPHA                           DELTA',///)
      READ(1,11) L,B
11    FORMAT(2F10.5)
      CALL CONVT(L,B,C1,DL,DB,PHI,ALPHA,DELTA,C2,DA,DD)
      WRITE(3,140) L,B,ALPHA,DELTA
140   FORMAT(' ', 4F30.3 )
      IF(ABS(ALPHA)-180.0)  78,79,79
78    ALPHA=X*ALPHA/180.0*IS
      DELTA=X* DELTA/180.0*IS
      WRITE(3,17)  ALPHA,DELTA
17    FORMAT('0     IN DECIMAL
     1      ',2F30.5)
      GO TO 80
79    IF(ALPHA) 81,81,82
81    ALPHA = ALPHA&360.0
      GO TO 78
82    ALPHA= ALPHA -360.0
      GO TO 78
80    CALL SCALE(L-C1,IL,IS)
      CALL SCALE(B,IB,IS)
201   ICB=ICOS(X,IB,IS)
202   ISB=ISIN(X,IB,IS)
203   ICL=ICOS(X,IL,IS)
204   ISL=ISIN(X,IL,IS)
205   IX=X*ICB*ICL/IS
      IY= X*ICB*ISL/IS
      IY=X*IY*IC6/IS
      IY1=X*ISB*IS6/IS
      IY=IY-IY1
      IZ=X*ICB*ISL/IS
      IZ=X*IZ*IS6/IS
      IZ1=X*ISB*IC6/IS
      IZ=IZ&IZ1
208   IF(IABS(IX)-IABS(IY)) 20,19,21
19    ITHETA=IX/IY*IS/4
209   GO TO 22
21    IQ=X*IY*IS/IX
210   ITHETA=IATAN(X,IQ,IS)
22    IF(IX) 23,24,24
23    IF(ITHETA) 15,14,14
15    ITHETA= ITHETA&IS
211   GO TO 24
```

```
 14      ITHETA=ITHETA-IS
 212     GO TO 24
 20      IF(IX) 3,4,3
 4       IF(IY) 5,6,6
 5       ITHETA=-IS/2
 213     GO TO 24
 6       ITHETA= IS/2
 214     GO TO 24
 3       IQ=X*IX*IS/IY
 215     ITHETA=IS/2-IATAN(X,IABS(IQ),IS)
 216     IF(IQ) 28,28,22
 28      ITHETA=-ITHETA
 217     GO TO 22
 24      IST=ISIN(X,ITHETA,IS)
 218     ICT=ICOS(X,ITHETA,IS)
         I=IABS(ITHETA)
 220     IF(I-IS/2) 141,141,142
 142     I=IABS(I-IS)
 141     IF(I-IS/4)  29,29,30
 29      IF(DABS(X*IX*IS/ICT)-DABS(X*IZ)) 31,33,32
 32      IQ=X*IZ*ICT/IX
 221     IDEL=IATAN(X,IQ,IS)
 222     GO TO 40
 33      IDEL=(X*IX*IS/ICT)/(X*IZ)*IS/4
 223     GO TO 40
 31      IQ=X*IX*IS/IZ
         IQ=DABS(X*IQ*IS/ICT)
 224     IDEL= IS/2 - IATAN(X,IQ,IS)
 225     IF(X*IX*IS/IZ*IS/ICT) 37,40,40
 37      IDEL=-IDEL
 226     GO TO 40
 30      IF(DABS(X*IY*IS/IST)-DABS(X*IZ)) 41,43,42
 42      IQ=X*IZ*IST/IY
 227     IDEL= IATAN(X,IQ,IS)
 228     GO TO 40
 43      IDEL =(X*IY*IS/IST)/(X*IZ)*IS/4
 229     GO TO 40
 41      IQ=X*IY*IS/IZ
         IQ=DABS(X*IQ*IS/IST)
 230     IDEL=IS/2-IATAN(X,IQ,IS)
 231     IF(IQ) 47,40,40
 47      IDEL=-IDEL
 40      CALL SCALE(C2,IC2,IS)
 232     IF(IABS(ITHETA&IC2)-IS) 51,52,52
 51      IALPHA= ITHETA&IC2
 233     GO TO 99
 52      IF(ITHETA&IC2) 53,54,54
 53      IALPHA=ITHETA&IC2&2*IS
 234     GO TO 99
 54      IALPHA= ITHETA&IC2-2*IS
 99      WRITE(3,16)  IALPHA,IDEL
 16      FORMAT('0     IN INTEGER
        12I30,///)
 235     WRITE(3,18)
 18      FORMAT('0           ICB          ISB           ICL            ISL
        1IX            IY           IZ          ITHETA           IST          ICT')
         WRITE(3,13) ICB,ISB,ICL,ISL,IX,IY,IZ,ITHETA,IST,ICT
 13      FORMAT(' ',10I12,//////)
 100     CONTINUE
         CALL EXIT
         END
```

```
      SUBROUTINE CONVT (GL,B,C1,DL,DB,PHI,ALPHA,DEL,C2,DALPHA,DDEL)
      PI=3.14159265359
      BB=B*PI/180.0
      G=(GL-C1)*PI/180.0
      C6=  COS(PHI*PI/180.0)
      S6=  SIN(PHI*PI/180.)
      CB= COS(BB)
      SB=  SIN(BB)
      CL= COS(G)
      SL=  SIN(G)
      X=CB*CL
      Y=CB*SL*C6-SB*S6
      Z= CB*SL*S6 &SB*C6
      DX= -CB*SL* DL - SB*CL* DB
      DY=X*C6* DL - SB*SL*C6* DB-CB*S6* DB
      DZ=-SB*SL*S6* DB&X*S6* DL&CB*C6* DB
      IF(ABS(X)-ABS(Y))  20,20,21
21    THETA=  ATAN(Y/X)
22    IF(X)  23,24,24
23    THETA=THETA & PI
      GO TO 24
20    THETA= PI/2.0 - ATAN( ABS(X/Y))
      IF(X/Y)  25,22,22
25    THETA=-THETA
      GO TO 22
24    DALPHA=(X*DY-Y*DX)/(X*X&Y*Y)
      ST=SIN(THETA)
      CT=COS(THETA)
      G=ABS(THETA)
140   IF(G-PI/2.)  141,141,142
142   G=ABS(G-PI)
      GO TO 140
141   IF(G-PI/4.)  29,29,30
29    IF(ABS(X/CT) - ABS(Z))  31,31,32
32    DEL = ATAN(Z*CT/X)
      GO TO 40
31    DEL=PI/2.0 - ATAN(ABS(X/(Z*CT)))
      IF(X/(Z*CT))  37,40,40
37    DEL=-DEL
40    DDEL=(X*CT*DZ-X*Z*ST*DALPHA-Z*CT*DX)/(X*X&Z*Z*CT*CT)
      GO TO 60
30    IF(ABS(Y/ST)-ABS(Z))  41,41,42
42    DEL = ATAN(Z*ST/Y)
      GO TO 50
41    DEL= PI/2.0 - ATAN(ABS(Y/(Z*ST)))
      IF(Y/(Z*ST))  47,50,50
47    DEL=-DEL
50    DDEL=(Y*ST*DZ&Y*Z*CT*DALPHA- Z*ST*DY)/(Y*Y&Z*Z*ST*ST)
60    IF(THETA) 61,61,62
61    ALPHA=THETA&C2*PI/180.0
      GO TO 65
62    IF(THETA&C2*PI/180.0-2.0*PI)  61,63,63
63    ALPHA= THETA& C2*PI/180. - 2.0*PI
65    ALPHA = ALPHA*180.0/PI
      DEL= DEL*180.0/PI
      RETURN
      END
```

```
      SUBROUTINE SCALE(X,IX,IS)
      Z=X/180.
      I=Z
      J=2*(I/2)
      IF(J-I)  1,2,3
1     IX=(Z-I-1)*IS
      RETURN
2      IX=(Z-I)*IS
      RETURN
3     IX=(Z-I&1)*IS
      RETURN
      END
```

```
      FUNCTION IATAN(X,IZ,IS)
      DOUBLE PRECISION X
      IC1= X*0.00108423*512*IS
      IC3=-0.00534860*128*IS
      IC5=X*0.01267292*64*IS
      IC7=-X*0.02106178*32*IS
      IC9=X*0.03152520*16*IS
      IC11=-X*0.05302625*16*IS
      IC13=X*0.15915433*4*IS
      IATAN=X*IC1*IZ/IS
      IATAN=X*IATAN*IZ/IS
      I=X*IC3 *IZ/IS
      IATAN=X*IATAN*IZ/IS/4
      IATAN= IATAN&I
      IATAN=X*IATAN*IZ/IS
      I=X*IC5 *IZ/IS
      IATAN=X*IATAN*IZ/IS/2
      IATAN= IATAN&I
      IATAN=X*IATAN*IZ/IS
      I=X*IC7 *IZ/IS
      IATAN=X*IATAN*IZ/IS/2
      IATAN= IATAN&I
      IATAN=X*IATAN*IZ/IS
      I=X*IC9 *IZ/IS
      IATAN=X*IATAN*IZ/IS/2
      IATAN= IATAN&I
      IATAN=X*IATAN*IZ/IS
      I=X*IC11*IZ/IS
      IATAN=X*IATAN*IZ/IS
      IATAN= IATAN&I
      IATAN=X*IATAN*IZ/IS
      I=X*IC13*IZ/IS
      IATAN=X*IATAN*IZ/IS/4
      IATAN=(IATAN&I)/2
      RETURN
      END
```

```
      FUNCTION ISIN(X,IX,IS)
      DOUBLE PRECISION X
      IC1=X*0.00015148*2**12*IS
      IC3=X*0.00467377*64*IS
      IC5=X*0.07968968*8*IS
      IC7=X*0.64596371*IS
      IC9=X*.5707963*IS
      IC91= IS
13    IF(IABS(IX)-IS/2) 1,1,2
    1 IZ=2*IX
      GO TO 5
2     IF(IX) 3,4,4
3     IZ=2*(-IX-IS)
      GO TO 5
4     IZ=2*(IS-IX)
5     ISIN=X*IC1*IZ/IS
      ISIN=X*ISIN*IZ/IS
      ISIN=X*ISIN*IZ/IS/64
       I=-X*IC3 *IZ/IS
      ISIN=ISIN&I
      ISIN=X*ISIN*IZ/IS
      ISIN=X*ISIN*IZ/IS/8
       I= X*IC5 *IZ/IS
      ISIN=ISIN&I
      ISIN=X*ISIN*IZ/IS
      ISIN=X*ISIN*IZ/IS/8
       I=-X*IC7 *IZ/IS
      ISIN=ISIN&I
      ISIN=X*ISIN*IZ/IS
      ISIN=X*ISIN*IZ/IS
       I= X*IC9 *IZ/IS
      ISIN=ISIN&I
       I= X*IC91*IZ/IS
      ISIN=ISIN&I
      RETURN
      END
```

```
      FUNCTION ICOS(X,IX,IS)
      DOUBLE PRECISION X
      IC1=X*0.00015148*2**12*IS
      IC3=X*0.00467377*64*IS
      IC5=X*0.07968968*8*IS
      IC7=X*0.64596371*IS
      IC9=X*.5707963*IS
      IC91= IS
      IF(IX-IS/2) 7,8,8
7     IY=IX&IS/2
      GO TO 13
8      IY=IX-IS-IS/2
13    IF(IABS(IY)-IS/2)  1,1,2
1     IZ=2*IY
      GO TO 5
2     IF(IY)  3,4,4
3     IZ=2*(-IY-IS)
      GO TO 5
4     IZ=2*(IS-IY)
5     ISIN=X*IC1*IZ/IS
      ISIN=X*ISIN*IZ/IS
      ISIN=X*ISIN*IZ/IS/64
       I=-X*IC3 *IZ/IS
      ISIN=ISIN&I
      ISIN=X*ISIN*IZ/IS
      ISIN=X*ISIN*IZ/IS/8
       I= X*IC5 *IZ/IS
      ISIN=ISIN&I
      ISIN=X*ISIN*IZ/IS
      ISIN=X*ISIN*IZ/IS/8
       I=-X*IC7 *IZ/IS
      ISIN=ISIN&I
      ISIN=X*ISIN*IZ/IS
      ISIN=X*ISIN*IZ/IS
       I= X*IC9 *IZ/IS
      ISIN=ISIN&I
       I= X*IC91*IZ/IS
      ISIN=ISIN&I
      ICOS=ISIN
      RETURN
      END
```

EXAMPLE OF USE OF CONVT IN SOLVING TRANSFORMATION FROM GALACTIC TO EQUATORIAL 1950 COORDINATES

L	DL/DT	B	DB/DT	ALPHA	DALPHA/DT	DELTA	DDELTA/DT
0.0	1.000	10.000	0.0	256.352	0.628	-23.346	0.799
15.000	1.000	10.000	0.0	264.811	0.512	-10.968	0.847
30.000	1.000	10.000	0.0	271.997	0.455	1.957	0.874
45.000	1.000	10.000	0.0	278.689	0.445	15.171	0.886
60.000	1.000	10.000	0.0	285.614	0.488	28.480	0.886
75.000	1.000	10.000	0.0	293.764	0.618	41.679	0.870
90.000	1.000	10.000	0.0	305.091	0.942	54.408	0.818
105.000	1.000	10.000	0.0	324.544	1.784	65.706	0.657
120.000	1.000	10.000	0.0	2.451	3.213	72.372	0.151
135.000	1.000	10.000	0.0	47.554	2.373	69.250	-0.513
150.000	1.000	10.000	0.0	73.072	1.186	59.198	-0.775
165.000	1.000	10.000	0.0	86.759	0.713	46.858	-0.856
180.000	1.000	10.000	0.0	95.848	0.526	33.787	-0.882
195.000	1.000	10.000	0.0	103.113	0.455	20.493	-0.888
210.000	1.000	10.000	0.0	109.805	0.445	7.220	-0.880
225.000	1.000	10.000	0.0	116.709	0.483	-5.847	-0.860
240.000	1.000	10.000	0.0	124.552	0.573	-18.483	-0.821
255.000	1.000	10.000	0.0	134.256	0.736	-30.340	-0.753
270.000	1.000	10.000	0.0	147.137	1.001	-40.795	-0.629
285.000	1.000	10.000	0.0	164.774	1.358	-48.731	-0.410
300.000	1.000	10.000	0.0	187.394	1.613	-52.487	-0.075
315.000	1.000	10.000	0.0	211.166	1.491	-50.833	0.288
330.000	1.000	10.000	0.0	230.954	1.138	-44.356	0.555
345.000	1.000	10.000	0.0	245.562	0.828	-34.739	0.712

L	DL/DT	B	DB/DT	ALPHA	DALPHA/DT	DELTA	DDELTA/DT
0.0	1.000	20.000	0.0	247.872	0.616	-17.276	0.733
15.000	1.000	20.000	0.0	256.177	0.500	-5.763	0.797
30.000	1.000	20.000	0.0	263.077	0.427	6.531	0.839
45.000	1.000	20.000	0.0	269.159	0.390	19.321	0.865
60.000	1.000	20.000	0.0	274.945	0.389	32.422	0.881
75.000	1.000	20.000	0.0	281.078	0.442	45.695	0.888
90.000	1.000	20.000	0.0	288.829	0.629	58.990	0.882
105.000	1.000	20.000	0.0	302.587	1.408	71.960	0.832
120.000	1.000	20.000	0.0	351.262	6.440	82.107	0.318
135.000	1.000	20.000	0.0	70.830	2.428	76.765	-0.758
150.000	1.000	20.000	0.0	91.413	0.806	64.256	-0.872
165.000	1.000	20.000	0.0	100.636	0.491	51.022	-0.887
180.000	1.000	20.000	0.0	107.190	0.401	37.718	-0.885
195.000	1.000	20.000	0.0	113.021	0.384	24.532	-0.872
210.000	1.000	20.000	0.0	118.921	0.408	11.599	-0.850
225.000	1.000	20.000	0.0	125.429	0.466	-0.920	-0.816
240.000	1.000	20.000	0.0	133.093	0.564	-12.788	-0.762
255.000	1.000	20.000	0.0	142.587	0.712	-23.631	-0.677
270.000	1.000	20.000	0.0	154.723	0.915	-32.839	-0.541
285.000	1.000	20.000	0.0	170.144	1.138	-39.497	-0.334
300.000	1.000	20.000	0.0	188.424	1.272	-42.511	-0.059
315.000	1.000	20.000	0.0	207.299	1.211	-41.197	0.231
330.000	1.000	20.000	0.0	224.016	1.006	-35.870	0.467
345.000	1.000	20.000	0.0	237.421	0.787	-27.554	0.630

SIMULATION RESULTS

	L(DEG.)	B	ALPHA	DELTA
①	0.0	0.0	265.61084	-28.91679
IN DECIMAL			-4398853.71957	-1347619.98255
IN INTEGER			-4398853	-1347619

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
8388607	0	7035280	-4568762	7035279	-2102543	-4056215	-775441	-2402021	8037353

	L(DEG.)	B	ALPHA	DELTA
②	60.00000	0.0	295.44651	23.76960
IN DECIMAL			-3008410.79779	1107743.46404
IN INTEGER			-3008410	1107742

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
8388607	0	7474304	3808348	7474303	1752600	3381108	615002	1915049	8167088

	L(DEG.)	B	ALPHA	DELTA
③	120.00000	0.0	5.75312	62.44889
IN DECIMAL			268114.79208	2910329.26130
IN INTEGER			268115	2910330

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
8388607	0	439029	8377111	439028	3855144	7437325	3891527	8334736	949164

④

	L(DEG.)	B	ALPHA	DELTA
	180.00000	0.0	85.61084	28.91679
IN DECIMAL			3989754.28043	1347619.98255
IN INTEGER			3989755	1347619

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
8388607	0	-7035278	4568764	-7035277	2102543	4056217	7613167	2402021	-8037353

⑤

	L(DEG.)	B	ALPHA	DELTA
	33.00000	20.00000	264.34237	9.05588
IN DECIMAL			-4457968.86179	422034.52059
IN INTEGER			-4457968	422033

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7882713	2869071	8388607	0	7882712	-2547204	1320345	-834556	-2579357	7982209

⑥

	L(DEG.)	B	ALPHA	DELTA
	93.00000	20.00000	290.82631	61.63027
IN DECIMAL			-3223727.40163	2872178.52633
IN INTEGER			-3223727	2872178

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7882713	2869071	4194305	7264747	3941357	594409	7381130	399685	1250964	8294807

	L(DEG.)	B	ALPHA	DELTA
	153.00000	20.00000	93.67369	61.63027
(7) IN DECIMAL			4365510.15718	2872178.52633
IN INTEGER			4365510	2872180

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7882713	2869071	-4194303	7264748	-3941355	594409	7381131	7988922	1250967	-8294807

	L(DEG.)	B	ALPHA	DELTA
	213.00000	20.00000	120.15763	9.05588
(8) IN DECIMAL			5599751.61734	422034.52059
IN INTEGER			5599752	422033

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7882713	2869071	-8388607	0	-7882712	-2547204	1320345	-7554052	-2579357	-7982209

	L(DEG.)	B	ALPHA	DELTA
	123.00000	-89.00000	12.25000	-26.40001
(9) IN DECIMAL			570891.37778	-1230329.60000
IN INTEGER			570892	-1230328

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
146403	-8387330	0	8388607	0	7513773	-3729870	4194304	8388607	0

(10)

	L(DEG.)	B	ALPHA	DELTA
	123.00000	-79.00000	12.25000	-16.40001
IN DECIMAL			570891.37778	-764295.82222
IN INTEGER			570892	-764296

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
1600624	-8234485	0	8388607	0	8047307	-2368451	4194304	8388607	0

(11)

	L(DEG.)	B	ALPHA	DELTA
	123.00000	-69.00000	12.25000	-6.40001
IN DECIMAL			570891.37778	-298262.04444
IN INTEGER			570892	-298261

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
3006209	-7831440	0	8388607	0	8336327	-935070	4194304	8388607	0

(12)

	L(DEG.)	B	ALPHA	DELTA
	123.00000	-59.00000	12.25000	3.59999
IN DECIMAL			570891.37778	167771.73333
IN INTEGER			570892	167771

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
4320453	-7190440	0	8388607	0	8372053	526724	4194304	8388607	0

(13)

	L(DEG.)	B	ALPHA	DELTA
	123.00000	-49.00000	12.25000	13.59999
IN DECIMAL			570891.37778	633805.51111
IN INTEGER			570892	633806

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
5503423	-6330962	0	8388607	0	8153397	1972514	4194304	8388607	0

(14)

	L(DEG.)	B	ALPHA	DELTA
	123.00000	-39.00000	12.25000	23.59999
IN DECIMAL			570891.37778	1099839.28889
IN INTEGER			570892	1099839

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
6519174	-5279120	0	8388607	0	7687005	3358371	4194304	8388607	0

(15)

	L(DEG.)	B	ALPHA	DELTA
	123.00000	-29.00000	12.25000	33.59999
IN DECIMAL			570891.37778	1565873.06667
IN INTEGER			570892	1565875

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7336844	-4066875	0	8388607	0	6987046	4642186	4194304	8388607	0

(16)

	L(DEG.)	B	ALPHA	DELTA
	123.00000	-19.00000	12.25000	43.59999
IN DECIMAL			570891.37778	2031906.84444
IN INTEGER			570892	2031907

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7931584	-2731063	0	8388607	0	6074792	5784946	4194304	8388607	0

(17)

	L(DEG.)	B	ALPHA	DELTA
	123.00000	-9.00000	12.25000	53.59999
IN DECIMAL			570891.37778	2497940.62222
IN INTEGER			570892	2497942

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
8285330	-1312267	0	8388607	0	4977957	6751936	4194304	8388607	0

(18)

	L(DEG.)	B	ALPHA	DELTA
	123.00000	1.00000	12.25000	63.59999
IN DECIMAL			570891.37778	2963974.40000
IN INTEGER			570892	2963975

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
8387329	146400	0	8388607	0	3729870	7513770	4194304	8388607	0

	L(DEG.)	B	ALPHA	DELTA
	123.00000	11.00000	12.25000	73.59999
(19) IN DECIMAL			570891.37778	3430008.17778
IN INTEGER			570892	3430008

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
8234486	1600621	0	8388607	0	2368453	8047306	4194304	8388607	0

	L(DEG.)	B	ALPHA	DELTA
	123.00000	21.00000	12.25000	83.59999
(20) IN DECIMAL			570891.37778	3896041.95556
IN INTEGER			570892	3896042

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7831441	3006206	0	8388607	0	935072	8336326	4194304	8388607	0

	L(DEG.)	B	ALPHA	DELTA
	122.00000	25.40000	347.90061	87.80861
(21) IN DECIMAL			-563872.30844	4092177.82122
IN INTEGER			-563876	4092178

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7577726	3598166	146403	8387330	132251	292230	8382470	3059536	7642421	3458634

(22)

	L(DEG.)	B	ALPHA	DELTA
	122.00000	25.90000	341.23624	88.25393
IN DECIMAL			-874454.77095	4112931.30223
IN INTEGER			-874451	4112932

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7546039	3664157	146403	8387330	131697	219062	8384711	2748961	7189408	4322171

(23)

	L(DEG.)	B	ALPHA	DELTA
	122.00000	26.40000	330.29645	88.66012
IN DECIMAL			-1384285.91595	4131861.01828
IN INTEGER			-1384292	4131862

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7513775	3729870	146403	8387330	131134	145876	8386312	2239120	6238485	5608035

(24)

	L(DEG.)	B	ALPHA	DELTA
	122.00000	26.90000	311.35385	88.97933
IN DECIMAL			-2267074.87411	4146737.34701
IN INTEGER			-2267041	4146738

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7480941	3795295	146403	8387330	130561	72683	8387275	1356371	4080260	7329408

	L(DEG.)	B	ALPHA	DELTA
	122.00000	27.40000	282.02049	89.11219
(25) IN DECIMAL			-3634108.73614	4152928.91714
IN INTEGER			-3634073	4152929

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7447536	3860433	146403	8387330	129978	-519	8387598	-10661	-33492	8388542

	L(DEG.)	B	ALPHA	DELTA
	122.00000	27.90000	252.57602	88.98284
(26) IN DECIMAL			-5006320.20911	4146900.84626
IN INTEGER			-5006319	4146901

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7413563	3925279	146403	8387330	129385	-73722	8387282	-1382907	-4152897	7288497

	L(DEG.)	B	ALPHA	DELTA
	122.00000	28.40000	233.48651	88.66547
(27) IN DECIMAL			-5895956.12696	4132110.18213
IN INTEGER			-5895922	4132112

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7379028	3989822	146403	8387330	128783	-146915	8386329	-2272510	-6308123	5529587

	L(DEG.)	B	ALPHA	DELTA
	124.00000	25.40000	36.59939	87.80861
(28) IN DECIMAL			1705655.06401	4092177.82122
IN INTEGER			1705637	4092178

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7577726	3598166	-146400	8387329	-132248	292230	8382469	5329049	7642450	-3458568

	L(DEG.)	B	ALPHA	DELTA
	124.00000	25.90000	43.26376	88.25393
(29) IN DECIMAL			2016237.52652	4112931.30223
IN INTEGER			2016217	4112932

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7546039	3664157	-146400	8387329	-131695	219062	8384710	5639629	7189436	-4322123

	L(DEG.)	B	ALPHA	DELTA
	124.00000	26.40000	54.20355	88.66012
(30) IN DECIMAL			2526068.67152	4131861.01828
IN INTEGER			2526065	4131862

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7513775	3729870	-146400	8387329	-131132	145875	8386311	6149477	6238508	-5608009

	L(DEG.)	B	ALPHA	DELTA
	124.00000	26.90000	73.14615	88.97933
(31) IN DECIMAL			3408857.62968	4146737.34701
IN INTEGER			3408823	4146739

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7480941	3795295	-146400	8387329	-130559	72682	8387274	7032235	4080267	-7329404

	L(DEG.)	B	ALPHA	DELTA
	124.00000	27.40000	102.47951	89.11219
(32) IN DECIMAL			4775891.49170	4152928.91714
IN INTEGER			4775858	4152930

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7447536	3860433	-146400	8387329	-129976	-519	8387597	-8377946	-33495	-8388541

	L(DEG.)	B	ALPHA	DELTA
	124.00000	27.90000	131.92398	88.98284
(33) IN DECIMAL			6148102.96465	4146900.84626
IN INTEGER			6148136	4146902

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7413563	3925279	-146400	8387329	-129383	-73723	8387282	-7005668	-4152987	-7288446

	L(DEG.)	B	ALPHA	DELTA
	124.00000	28.40000	151.01349	88.66547
(34) IN DECIMAL			7037738.88250	4132110.18213
IN INTEGER			7037738	4132112

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7379028	3989822	-146400	8387329	-128780	-146915	8386328	-6116066	-6308188	-5529512

	L(DEG.)	B	ALPHA	DELTA
	123.00000	26.40000	12.25000	88.99999
(35) IN DECIMAL			570891.37779	4147700.19556
IN INTEGER			570892	4147701

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7513775	3729870	0	8388607	0	146402	8387328	4194304	8388607	0

	L(DEG.)	B	ALPHA	DELTA
	123.00000	26.90000	12.25000	89.49999
(36) IN DECIMAL			570891.37781	4171001.88444
IN INTEGER			570892	4171002

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7480941	3795295	0	8388607	0	73207	8388286	4194304	8388607	0

(37)

	L(DEG.)	B	ALPHA	DELTA
	123.00000	27.90000	192.25000	89.50001
IN DECIMAL			-7817716.62225	4171002.73778
IN INTEGER			-7817716	4171003

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7413563	3925279	0	8388607	0	-73203	8388285	-4194304	-8388607	0

(38)

	L(DEG.)	B	ALPHA	DELTA
	123.00000	28.40000	192.25000	89.00001
IN DECIMAL			-7817716.62224	4147701.04889
IN INTEGER			-7817716	4147702

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7379028	3989822	0	8388607	0	-146398	8387327	-4194304	-8388607	0

(39)

	L(DEG.)	B	ALPHA	DELTA
	122.50000	28.40000	215.97342	88.90673
IN DECIMAL			-6712125.10111	4143354.06573
IN INTEGER			-6712099	4143356

ICB	ISB	ICL	ISL	IX	IY	IZ	ITHETA	IST	ICT
7379028	3989822	73205	8388287	64394	-146527	8387077	-3088687	-7679723	3374995