Analytical model for calculating structure-borne sound transmission at junctions of semi-infinite plates: reference manual for NRCJ38/PREP38

Bosmans, I.
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Ivan Bosmans

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Introduction

NRCJ38/PREP38 represents an analytical calculation model predicting structure-borne sound transmission at a junction of semi-infinite plates. This software calculates the angle averaged transmission coefficient for a wide variety of plate junctions. In addition, the model calculates the coupling loss factor (CLF) and the modal density of the connected plates. Both the CLF and the modal density are two essential parameters required by Statistical Energy Analysis (SEA). The latter method represents a prediction tool to evaluate the energy distribution in complex vibrating structures, and is often applied to predict sound insulation in buildings.

The software package consists of two programs: NRCJ38 and PREP38. 'NRCJ38' stands for National Research Council Junction model revision 38, whereas 'PREP38' represents the PREProcessor for revision 38. The preprocessing program essentially prepares the calculations by creating and editing input data files. NRCJ38 is the actual implementation of the calculation model. This documentation serves as a reference manual to both programs, showing the users how to prepare and run the calculations.

This text is organized as follows. First, the theoretical background of the calculation model will be summarized and reference will be made to the relevant publications. Next, an overview is given of the various plate junctions which can be calculated using NRCJ38/PREP38. Next, the model of the plate junction, consisting of four basic plate and beam elements will be presented. Using these building blocks, it will be demonstrated how the various variants of the plate junction can be modelled. This section will deal with plate orientation, local and global coordinate systems and junction eccentricities. The following paragraphs will show how to prepare the calculations using the preprocessor PREP38, and how to run the calculations using NRCJ38. Finally, the accuracy of the model will be validated by comparison with data obtained using closed form expressions available in the literature.

The Fortran77 source codes of NRCJ38 and PREP38 are given in Appendix D. Since this report serves mainly as a reference manual, Appendix D is not included in this text.

Note: NRCJ38/PREP38 is primarily intended as a research tool for scientists and engineers who are familiar with the principles of SEA. Although part of the text deals with some of the assumptions inherent to the calculation model, the underlying theory of SEA will not be discussed. It is strongly advised that the reader would familiarize himself with SEA by consulting the literature.¹²
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1. Theory: wave approach for semi-infinite plates.

The calculation model implemented by NRCJ38 is based on the so-called wave approach for semi-infinite plates. In this method, vibrational energy flow between plates is assessed by modelling reflection and transmission of structure-borne sound waves at junctions of semi-infinite plates coupled along a common edge. The excitation of the plate assembly is taken as a plane wave incident upon the junction on one of the plates. The incident wave, with wavenumber $k_i$ and angle of incidence $\theta_i$, generates bending and in-plane waves propagating away from the junction on all plates, where the propagation direction of the waves is determined by Snell's law (See Figure 1). The forces and displacements at the plate edges are expressed in terms of the amplitudes of these propagating waves. The equilibrium and continuity conditions at the junction line lead to a set of linear equations, the solution of which yields the unknown wave amplitudes. These amplitudes determine the energy flow associated with each traveling wave. Structure-borne sound transmission is quantified by the transmission coefficient, $\tau(\theta_i)$, which is defined as the ratio of the transmitted intensity to the intensity carried by the incident wave. Based on the assumption of a diffuse wavefield on the source plate, the transmission coefficient is averaged over all angles of incidence. For isotropic plates with a uniform plate thickness, the angle averaged transmission coefficient is given by:

$$\tau = \int_0^{\pi/2} \tau(\theta_i) \cos(\theta_i) \, d\theta_i$$

(1)

The SEA coupling loss factor is expressed in terms of this angle-averaged transmission coefficient. A more detailed overview of the wave approach is given by Wöhle, Beckmann and Schreckenbach, and by Craven and Gibbs.

Figure 1. A junction of semi-infinite plates is excited by an incident wave, with wavenumber $k_i$ and angle of incidence $\theta_i$, generating bending (B) and in-plane (L,T) waves propagating away from the junction on all plates.

The theory which has been incorporated in NRCJ38, is based on a slightly different formulation of the boundary conditions compared to the previous work of Wöhle et al., and of Craven and Gibbs. In addition, as will be shown in the next section, the model has been extended to include a wider variety of plate junctions. The details of the calculation model have been published in the literature, and reference will be made to these publications throughout this text.
2. Overview of the implemented plate junctions.

Tables 1-4 give an overview of the various types of plate junctions implemented in the calculation model. Distinction has been made between the junction geometry, the coupling type, the continuity of the junction, and the elastic properties of the plates. The illustrations in the tables do not represent limiting cases, since the calculation model can deal with any combination of the shown configurations.

<table>
<thead>
<tr>
<th>Junction Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Junctions of 2-4 plates.</td>
</tr>
<tr>
<td><img src="image" alt="Junctions of 2-4 plates" /></td>
</tr>
<tr>
<td>Arbitrary coupling angles.</td>
</tr>
<tr>
<td><img src="image" alt="Arbitrary coupling angles" /></td>
</tr>
</tbody>
</table>

Table 1. Overview of the various junction types: junction geometry according to the number of plates and the coupling angles.
### Overview of the implemented plate junctions

<table>
<thead>
<tr>
<th>Coupling Type</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rigid connections.</strong>&lt;br&gt;The plates are connected directly to each other.</td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>Junctions with an elastic interlayer.</strong>&lt;br&gt;Plates can be connected using an elastic interlayer.</td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>Plate/beam junctions.</strong>&lt;br&gt;All plates are connected to a beam. The beam can be modelled as a thin or thick (Timoshenko) beam. The beam material can be isotropic or orthotropic.</td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>Junctions with a plate strip.</strong></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
</tbody>
</table>

**Table 2.** Overview of the various junction types: coupling type.
### Continuity of the junction

**Line connections.**
Line connections are continuous along the entire length of the common plate edge.

**Point connections.**
Point connections are modelled using a discontinuous elastic interlayer.

### Elastic properties of the connected plates and plate strips

#### Isotropic plates / plate strips

#### Orthotropic plates / plate strips.

Table 3. Overview of the various junction types: continuity of the junction (line versus point connections).

Table 4. Overview of the various junction types: elastic properties of the coupled plates/plate strips.
3. Modelling a plate junction.

In the calculation model, the plate junction is composed of a number of basic elements. Figure 2 identifies these four elements as: the junction beam, a semi-infinite plate, a plate strip and an elastic interlayer. An arbitrary plate junction basically consists of a specific combination of these building blocks. Each element plays a specific role in the model:

- **Junction beam**: This beam represents a coupling element connecting all the plates and plate strips together. In case of rigid junction, where all plates are coupled directly to each other, the junction beam acts as a common node and does not represent any physical part of the plate junction. In case of a plate/beam junction, the junction beam represents the physical beam, and is characterized by mass, stiffness and dimensions.

- **Semi-infinite plate**: These are the plate elements that support the reflected and transmitted structure-borne sound waves at the junction. These plates are of semi-infinite extent, the only boundary being the plate junction.

- **Plate strips**: Unlike a semi-infinite plate, plate strips have two plate edges parallel to the coupled plate edge. Plate strips are used as undamped elements which support standing waves and alter the dynamic behaviour at the junction.

- **Elastic interlayer**: Plates and plate strips are either connected directly to the junction beam (in case of a rigid junction) or indirectly using an elastic interlayer. This interlayer is purely a coupling element and is modelled as a spring. The spring stiffness is constant over the entire junction length in case of a continuous elastic interlayer. Point connections are modelled by a space dependent spring stiffness.

![Figure 2. An arbitrary plate junction is composed of four basic elements: junction beam, semi-infinite plate, plate strip and elastic interlayer. A global coordinate system is attached to the junction beam, and each plate (or plate strip) has a local coordinate system.](image-url)
Figure 2 demonstrates that each junction has two different sets of coordinate systems: one global coordinate system attached to the junction beam \((x_0,y_0,z_0)\), and one local coordinate system attached to each plate and plate strip \((x_p,y_p,z_p)\). The origin of the global coordinate system is located at the centre of gravity of the junction beam cross-section. The local coordinate system’s origin is located at the intersection of the coupled plate edge and the midplane of the plate (or plate strip). Figure 2 further shows the displacements \((\xi, \eta, \zeta, \alpha)\) and the forces \((F_x, F_y, F_z, M_z)\), which are considered in the calculation model when formulating the equilibrium and continuity conditions at the junction.

Figure 3 shows that the position and orientation of the plates and plate strips relative to the global coordinate system is determined by the fixation point coordinates \((x_{0p}, y_{0p})\) and the coupling angle \(\theta_p\). In case of a plate/beam junction, the fixation point coordinates locate the intersection between the junction beam cross-section and the plate’s midplane. The coupling angle represents the angle between the \((x,z)\)-plane of the plate and the \((x,z)\)-plane of the global coordinate system.

The following paragraphs show how to combine the basic elements in order to build a specific junction.

3.1 Rigid junction of semi-infinite plates.
In a rigid junction, the plates are connected directly to the junction beam, which acts as a common node. Since the junction beam does not represent any physical part of the junction, the plates may be considered as being directly connected to each other. The junction beam only plays a theoretical role, as it locates the global coordinate system of the junction. It should be noted that rigid junctions may still be characterized by an eccentricity, which can be modelled by taking the appropriate fixation point coordinates.

The model for rigid connections of thin orthotropic plates is discussed in Refs. 8 and 9.

3.2 Junction with a continuous elastic interlayer (line connection).
Elastically connected plates are coupled to the junction beam by a continuous elastic interlayer. In the model, the elastic interlayer is taken into account by transforming the forces and displacements at the plate edge to the corresponding quantities at the edge of the interlayer. This transformation depends on the
mathematical description of the interlayer, which, in the case of NRC38, is modelled as a spring. Figure 4 shows the dimensions of the elastic interlayer. The interlayer thickness is represented by $h_i$, and the width of the interlayer is equal to the plate thickness, $h_p$. Further, the elastic interlayer material is assumed to be homogeneous and isotropic, with Young's modulus $E_i$ and Poisson's ratio $\nu_i$.

In NRC38, the interlayer spring stiffness can be calculated in three different ways depending on the assumed stress condition (and the corresponding deformation) of the interlayer:

1. **One dimensional stress.** In this case, the interlayer is assumed to expand and contract freely in the $y$- and $z$-direction of the local coordinate system of the plate (due to the Poisson effect). The spring stiffnesses are calculated as:

\[
K_x = \frac{E_i h_p}{h_i} \quad K_y = K_z = \frac{E_i h_p}{2(1+\nu_i)h_i} \quad K_{xz} = \frac{E_i h_i^2}{12h_i}
\]

where $K_x$, $K_y$, and $K_z$ are the spring stiffness corresponding to translation in the $x$, $y$, and $z$-direction of the plate, and $K_{xz}$ represents the rotational spring stiffness around the plate's $z$-axis.

2. **Two dimensional stress.** In this plane stress condition, the interlayer is assumed to deform freely only in the $y$-direction of the local coordinate system. The spring stiffnesses are given by:

\[
K_x = \frac{E_i h_p}{h_i(1-\nu_i^2)} \quad K_y = \frac{E_i h_p}{2(1+\nu_i)h_i} \quad K_{xz} = \frac{E_i h_i^2}{12(1-\nu_i^2)h_i}
\]

![Elastically connected plate (θ_p = -90°)](image)

**Figure 4.** Dimensions at a junction with a continuous elastic interlayer. The fixation point is taken at the interface between the elastic interlayer and the junction beam.

3. **Three dimensional stress.** When the interlayer is prohibited from expanding and contracting in the $y$- and $z$-directions, the spring stiffnesses are given by:
NRJ38/PREP38  Modelling a plate junction

Since the interlayer is located between the plate edge and the junction beam, the fixation point coordinates are defined differently compared to a rigid connection. The fixation point is now assumed to be located at the interface between the elastic interlayer and the junction beam, as shown in Figure 4.

The calculation model for structure-borne sound transmission between elastically connected plates is treated in Refs. 9 and 10.

3.3 Plate/beam junction.
In this case, the junction beam is given a proper mass and stiffness, and is able to support wave propagation along the beam axis. As a result, the presence of the junction beam will significantly affect the dynamic behaviour of the junction. In the calculation model, the bending wave response of the beam can be described using thin (Euler) or thick (Timoshenko) beam theory. In addition, the beam can be modelled as isotropic or orthotropic. The beam is assumed to have a rectangular cross-section.

References 9 and 11 present two models for structure-borne sound transmission at plate/beam junctions.

3.4 Plate strip junction.
Plate strips are modelled very similarly to semi-infinite plates. They too are characterized by a coupling angle and fixation point coordinates, as well as by the same material properties. The main difference is that plate strips have a free edge parallel to the junction. The location of this free edge is determined by the depth of the plate strip, which is measured in the x-direction of the local coordinate system.

More information about the plate strip model is given in Ref. 11.

3.5 Junction with a spatially periodic elastic interlayer (point connections).
Junctions between point connected plates are modelled using an elastic interlayer with a spatially varying spring stiffness. Figure 5 shows that the junction is assumed to be periodic, with spatial period L. Several, arbitrarily spaced point connections can be located within this period. Each connection has a finite contact area, and therefore does not truly represent a point connection.

The model for point connected plates is discussed in Refs. 9 and 10.
Figure 5. Periodic elastic interlayer. The junction is determined by the spatial period $L$, the position $z$, and the width $w_i$ of the connections.
4. Assumptions and limitations of the calculation model.

The theory behind the calculation model is based on a number of assumptions. As a result, the model is not generally applicable to any plate junction, and its applicability depends on the appropriateness of the various assumptions in a given situation. The main assumptions of the calculation model are listed below:

- **Thin plate theory.** Bending wave response of the plates is modelled using thin plate theory. As a result, the shear deformation and rotary inertia is neglected. This assumption effectively implies an upper frequency limit for the applicability of the model. Cremer et al.\textsuperscript{3} proposed a (rather conservative) limit as the frequency at which \( \lambda_b > 6h_p \), where \( \lambda_b \) denotes the bending wavelength and \( h_p \) the plate thickness. The upper frequency limit can be lower for anisotropic plates than for isotropic plates, since shear deformation can occur at relatively low frequencies in the former case.

- **Homogeneous, linear elastic materials.** The plate and beam materials are assumed to be homogeneous and should exhibit a linear elastic behaviour. These assumptions may not be justified for some wood based materials commonly used in wood frame constructions.

- **Anisotropy of junction beam.** The principal directions in orthotropic junction beams are assumed to be parallel to the axes of the global coordinate system. Beams representing framing members in dimensional lumber, do not satisfy this assumption since they are characterized by a polar orthotropy. In a plane perpendicular to the grain, the principal directions are normal (radial) and parallel (tangential) to the annual growth rings of the wood.

- **Elastic interlayer modelled as a spring.** Modelling the elastic interlayer as a spring is only correct as long as the wavelength in the interlayer is much larger than the interlayer thickness. The spring model is therefore only a low frequency approximation of the physical reality. At higher frequencies, the interlayer response will be governed by standing waves. In this case, the elastic interlayer should be modelled as a wave supporting medium.\textsuperscript{12}

- **Idealized interlayer deformation.** The expressions for the spring stiffnesses corresponding to the four degrees of freedom, as given by Eqs. (2-4), correspond to an idealized case of interlayer deformation. The principal assumption here is that the deformation is constant over the thickness of the interlayer. Eqs. (2-4) either allow or prohibit Poisson contraction (or expansion) over the entire layer thickness. In reality however, the Poisson effect is always present and the corresponding deformation is not uniform over the interlayer thickness. Poisson contraction and expansion is more pronounced in the middle of the interlayer, than at the interface with the plate and the junction beam. Equations (2) and (4) represent two extreme cases, where Eq. (2) yields a lower limit and Eq. (4) the upper limit of the spring stiffness.

- **Plate strip model.** Since a stiffening rib is modelled as an undamped plate strip, the net energy flow to the rib is assumed to be zero, even though the rib supports resonances and anti-resonances. As a result, the stiffening rib is not included in the corresponding SEA model. This approximation is only justified as long as the energy dissipation in the stiffening rib is negligible compared to the dissipation in the connected plates. This implies that the plate strip model should be applied in a frequency range where the rib supports only few modes. At high frequencies, the dissipation cannot be ignored and the rib should be modelled as a plate
In this case, the coupling loss factor should be calculated by modelling the stiffening rib as a semi-infinite plate in stead of a plate strip.

- **Junction eccentricity.** The eccentricity of the plate edge with respect to the origin of the global coordinate system is taken into account by the fixation point coordinates. Although the fixation point can be easily located for an elastic plate junction or a plate/beam junction, the situation is not as straightforward in case of rigid connections since the location of the plate edge at a rigid joint cannot be clearly identified. Unfortunately, an exact solution for this problem is not available, since the fixation point is purely an artifact of an approximate calculation model. Users should rely on their experience when choosing the fixation point coordinates for a specific joint. For rigid junctions between relatively thin plates, the fixation point can usually be taken at the origin of the global coordinate system. Eccentricities at rigid plate joints involving a plate strip are proposed in Ref. 11.
5. Preparing the calculations using PREP38.

The input data files of the calculation model are created and edited using the preprocessor PREP38. The data files contain information concerning the connected plates, the junction beam and the type of connection, as well as some specific settings for the calculations. These settings relate to the integration over all angles of incidence, the frequency resolution and the convergence of the algorithm for point connected plate junctions. The use of the preprocessor will be explained in the next section. The following paragraphs will list all the input variables required by the calculation model.

5.1 Using the preprocessor PREP38.

a. Starting a preprocessing session.
PREP38 is an interactive character mode executable which can be started using the following calling sequence:

```
> prep38 [filename]
```

The square brackets [] indicate optional command line parameters. The optional parameter “filename” is a string of maximum 8 characters, specifying the name of the input file which should be created or edited. The filename should not contain a file extension. If the parameter is not specified at startup, the user will be prompted to enter the filename. At the end of the session, the input data will be saved in “filename.dat”.

If the file “filename.dat” is not present in the working directory, a new data file will be created. The preprocessor will prompt the user to enter all the parameters necessary to run the calculations. For each parameter, a line will appear in the following format:

```
> Parameter description [Unit] ? _
```

The units in which the parameters are expressed are shown between square brackets. All input data for NRC38 are expressed in SI-units. The appropriate value should be entered behind the question mark. For example, the Young’s modulus in the x-direction of an orthotropic plate is requested by the following line:

```
> Young's modulus plate in the x-direction [Pa] ? _
```

Apart from entering numerical data, the user will be asked to supply additional information for which he has to enter a single non-numeric character. These questions appear in the following format:

```
> Question: y or n [] ? _
```

y and n are characters which represent the two possible answers to the question. These questions may be answered in lower or upper case, as PREP38 is case insensitive. The square brackets remain empty when creating a new file. For example: the following question will appear before entering the elastic properties of a plate:
> Isotropic(i) or orthotropic(o) plate [] ? 

The response to this question should be either 'i' or 'o'.

When all the data are entered, a menu will appear which allows the user to review and correct the entered data. This is discussed in the next paragraph.

c. Editing or reviewing a data file.

Data files are reviewed using the following menu, in which the data are organized into different categories:

> (1).....Change number of plates
> (2).....Change plate data
> (3).....Change beam data
> (4).....Change integration/angle data
> (5).....Change frequency data
> (6).....Change convergence data
> (7).....Change all 
>
> (x).....Save and exit
> (p).....Save all permutations and exit
> (q).....Quit without saving
>
> Enter your choice (1-7,x,p,q): _

This menu will appear at startup, when the data file "filename.dat" already exists in the working directory. The menu will also show up when a new file is created, just after all the data have been entered. This allows the user to review and correct the information before saving the file.

The desired category can be chosen by entering the corresponding number. For reviewing and editing all data, enter '7'. For each parameter to be reviewed, a line will appear in the following format:

> Parameter description [Old value  Unit] ? _

*Old value* represents the numeric value of the parameter, like it was saved in the existing file or previously entered during the current session. For accepting the old value, simply press enter. For changing it, enter the new value. For example, reviewing the elastic properties of a plate might bring up the following line:

> Young's modulus plate in the x-direction  [ 0.370000D+10 Pa] ? _

Similarly, the questions requiring non-numerical input also show the existing or previously entered response:

> Question: y or n [Old response] ? _
Also in this case, the *Old response* can be accepted by pressing enter, or changed by entering the new response. For example, the following questions might appear when reviewing the parameters of an isotropic plate:

> Isotropic(i) or orthotropic(o) plate [i] ? _

After reviewing the data in a specific category, the menu will reappear. The next session will show how to terminate a session.

d. *Terminating a preprocessing session.*
The session is terminated by entering ‘x’, ‘p’ or ‘q’ when the menu is displayed. While the first two options should be used to save the data, the last option allows the user to end the session without saving.

- **Save and exit.** Enter ‘x’ for saving the data file before terminating the program. If "filename.dat" is already present in the working directory, the user will be asked whether or not he wishes to overwrite the existing file. When the answer is negative, the user will be prompted to enter a new filename.

  If calculations had been carried out previously for an overwritten data file, the user will be asked whether or not he wishes to delete the existing result file. If the results may be overwritten too, the user should answer affirmative. If not, the user should rename either the data file, or the result file. The calculation model NRC38 does not start when the result file for that particular data file already exists in the working directory. This precaution is taken to avoid repeating a calculation unnecessarily or losing valuable calculation results.

- **Save all permutations and exit.** In order to calculate all coupling loss factors of a plate junction, all plates should be treated as a source plate. Since the first plate in filename.dat is taken as the source plate, the calculations should be repeated after reordering the plates in the input data file. The number of permutations required is equal to the number of semi-infinite plates. After entering ‘p’, PREP38 will estimate the number of data files necessary to calculate all coupling loss factors and will determine the appropriate plate order for each file. The first file will be saved as filename.dat, in which the plate order will be stored as entered by the user. The user will be prompted to enter the filenames for the additional data files. For overwriting existing files, the same rules apply as explained in the previous paragraph.

- **Quit without saving.** For quitting without saving the data, enter ‘q’. This option should be chosen when no changes have been made to an existing data file.

5.2 *Required input parameters.*
The parameters required by the calculation model are listed in detail below. The parameters have been organized in the same categories as in the menu of PREP38.

a. **Number of plates.**
   This category contains only a single item:
- Number of plates: NumPlates [-] (minimum 2, maximum 4)
  The number of plates includes the number of semi-infinite plates and plate strips.

![Diagram of plate system](image)

**Figure 6.** Dimensions at the junction.

**b. Plate data.**
This is by far the biggest category, as it contains all the material properties and dimensions of the plates and plate strips, as well as the data referring to the plate position in the global coordinate system and the type of connection to the junction beam. The dimensions at the plate junction are illustrated in Figure 6.

- Young's modulus: \( E \) [Pa] (only for isotropic plates)
- Poisson's ratio: \( v \) [-] (only for isotropic plates)
- Young's modulus in x-direction: \( E_x \) [Pa] (only for orthotropic plates)
- Young's modulus in z-direction: \( E_z \) [Pa] (only for orthotropic plates)
- Poisson's ratio in x-z plane: \( v_{xz} \) [-] (only for orthotropic plates)
- Shear constant in x-z plane: \( G_{xz} \) [Pa] (only for orthotropic plates)
- Density of plate material: \( \rho \) [kg/m\(^3\)]
- Plate thickness: \( h_p \) [m]
- Plate surface area: \( S_p \) [m\(^2\)]
- Coupling angle: \( \theta_p \) [°] (minimum -180°, maximum +180°, see Figure 3)
- Fixation point x-coordinate: \( x_{0p} \) [m] (See Figure 3)
- Fixation point y-coordinate: \( y_{0p} \) [m] (See Figure 3)

In case of plate strip, an additional parameter should specified:
- Plate strip depth: \( L_x \) [m] (only for plate strips)

In case of an continuous elastic connection, the properties of the elastic layer should be specified:
Preparing the calculations using PREP38

- Interlayer Young’s modulus: \( E_i \) [Pa]
- Interlayer Poisson’s ratio: \( v_i \) [-]
- Interlayer thickness: \( h_i \) [m]
- Interlayer stress condition: \( \text{StrCond} \) [-] (1, 2 or 3 see Eqs. (2-4))

In case of a periodic elastic interlayer, the following additional parameters should be specified (See Figure 5).
- Spatial period: \( L \) [m]
- Number of point connections within this period: \( \text{NumCon} \) [-] (minimum 1, maximum 50)
- Position of each point connection \( i \): \( z_i \) [m] (minimum 0, maximum \( L \))
- Width of each point connection \( i \): \( w_i \) [m] (maximum \( L \))

**Note 1:** It is assumed that the principal directions of orthotropic plates are parallel to the local coordinate system.

**Note 2:** The first plate should not be a plate strip.

c. **Beam data.**

For plate/beam junctions, the junction beam is characterized by the following parameters (See Figure 6):
- Young’s modulus: \( E \) [Pa] (only for an isotropic beam)
- Poisson’s ratio: \( v \) [-] (only for an isotropic beam)
- Beam Young’s modulus in z-direction: \( E_z \) [Pa] (only for an orthotropic beam)
- Beam shear constant in y-z plane: \( G_{yz} \) [Pa] (only for an orthotropic beam)
- Beam shear constant in x-z plane: \( G_{xz} \) [Pa] (only for an orthotropic beam)
- Density of beam material: \( \rho \) [kg/m³]
- Beam width in x-direction: \( h_x \) [m]
- Beam width in y-direction: \( h_y \) [m]

**Note:** It should be noted that the subscripts x, y and z refer to the global coordinate system (See Figure 2).

d. **Integration/angle data.**

The calculations can be carried out in two different ways depending on the desired calculation result. If the angle-averaged transmission coefficient and the SEA coupling loss factors are required, the transmission coefficient should be integrated over all angles of incidence. In this case, the following parameters are required:
- Lower integration limit: \( \theta_{lo} \) [rad] (minimum 0, maximum \( \pi/2 \))
- Upper integration limit: \( \theta_{up} \) [rad] (minimum 0, maximum \( \pi/2 \))
- Number of function evaluations: \( \text{NumEval} \) [-] (minimum 1, maximum 500)
- Junction length: \( L_z \) [m]

The numerical integration is carried out from \( \theta_{lo} \) to \( \theta_{up} \). The number of function evaluations determines how often \( \tau(\theta_i) \) is calculated by the numerical integration routine. \( L_z \) is the length of the common plate edge. For point connected plates, \( L_z \) should be a whole multiple of the spatial period \( L \). If \( \theta_{lo} = \theta_{up} = \theta_i \) no integration is carried out and the transmission coefficient is calculated for a single angle of incidence \( \theta_i \).
If the transmission coefficient is required as a function of the angle of incidence, the angular range as well as the number of angles should be specified:

- Lower angle limit: \( \theta_{\text{min}} \) [rad] (minimum 0, maximum \( \pi/2 \))
- Upper angle limit: \( \theta_{\text{max}} \) [rad] (minimum 0, maximum \( \pi/2 \))
- Number of angles: \( \text{NumAngle} \) [-] (minimum 1, maximum 100)

e. Frequency data.
The data in this category determine the frequency range and resolution for the calculations. One can choose between a logarithmic and a linear frequency scale. In the first case, the calculations are carried out or averaged over frequency bands, with a fixed number of frequency points per band. In the second case, the calculations are carried out at a specified number of equally spaced frequency points.

For a logarithmic frequency scale, the required parameters are:

- Lower frequency limit: \( F_{\text{min}} \) [Hz]
- Upper frequency limit: \( F_{\text{max}} \) [Hz]
- Frequency resolution (3\(^\text{rd}\) octave): \( r \) [-] (integer number, minimum 0)

If \( F_{\text{min}} \) and \( F_{\text{max}} \) do not correspond to an octave or one-third-octave band centre frequency, the nearest band centre frequency higher than \( F_{\text{min}} \) and \( F_{\text{max}} \) will be taken. Valid octave band centre frequencies range from 31.5 Hz to 8000 Hz, and valid one-third-octave band centre frequencies from 25 Hz to 10000 Hz.

The frequency resolution \( r \) works as follows:
- \( r=0 \): the calculations are carried out only at octave band centre frequencies.
- \( r=1 \): the calculations are carried out only at one-third-octave band centre frequencies.
- \( r>1 \): the calculations are carried out at narrow band centre frequencies, with a frequency spacing of 3\(^\text{rd}\) of an octave. These narrow band results are then averaged over each one-third-octave band.

The maximum allowable number of narrow band frequencies is 567. The maximum allowable number of frequency bands is 27.

For a linear frequency scale, the following parameters are required:

- Lower frequency limit: \( F_{\text{min}} \) [Hz]
- Upper frequency limit: \( F_{\text{max}} \) [Hz]
- Number of frequencies: \( \text{NumFreq} \) [-] (minimum 1, maximum 567)

f. Convergence data.
Figure 1 illustrates that the plate response at a line junction is described by a single plane wave per wave type included in the analysis. The discontinuous character of point connected plate junctions, however, gives rise to a scattered wave field, which in theory is composed of an infinite number of plane waves. In practice, reasonably accurate predictions can be obtained by considering only a limited number of wave components.\(^9\)\(^,\)\(^10\) The number of wave components required by a specific junction should be determined based on a convergence test (See section 1). Based on the results of the test, one can decide to use the same number of wave components at all frequencies, or a frequency dependent value.

In case of a frequency independent number of wave components, only a single parameter is required:

- Number of wave components: \( \text{NumWaves} \) [-] (minimum 0, maximum see Note 1)
If the convergence of the model varies with frequency, a frequency dependent number of wave components should be specified. In this case, the number of wave components is set at a specified number of frequency points. The following data are required:

- Number of frequency points: \( \text{NumCvgFreq} \) [-] (minimum 2, maximum 27)
- Frequency point: \( \text{CvgFreq} \) [Hz]
- Number of wave components: \( \text{NumWaves} \) [-] (minimum 0, maximum see Note 1)

**Note 1:** The maximum allowable number of wave components, MaxWaves, is determined by the number of plates, NumPlates, and the maximum size of the set of equations, MaxSize:

\[
\text{MaxWaves} = \left( \frac{\text{MaxSize}}{4(\text{NumPlates}+1)} - 1 \right)/2
\]

The default value for MaxSize is 2400, but can be changed at startup of NRC38.

**Note 2:** For line connections, no convergence data are required and option 6 in PREP38’s menu will be disabled.
6. Running the calculation model NRCJ38.

The wave approach for semi-infinite plates is implemented in NRCJ38. This program uses the input data file created by PREP38 and calculates the SEA coupling loss factors and the transmission coefficients of the junction, as well as the modal densities of the connected plates. Three wave types are taken into account in the calculation model: bending waves and two types of in-plane waves. In isotropic plates the in-plane waves are quasi-longitudinal and shear waves. In-plane wave propagation in orthotropic plates is described by a slow and a fast in-plane wave, each characterized by a mixture of longitudinal and shear wave motion.

For orthotropic plates, the expression for the angle-averaged transmission coefficient differs from Eq. (1) for isotropic plates, since the probability of the propagation direction in a diffuse wavefield has to be taken into account. In addition, a new expression for the coupling loss factor and the modal density for orthotropic plates must be derived. In NRCJ38, the angle-averaged transmission coefficient, coupling loss factor and modal density are calculated according to the expressions presented in Refs. 8 and 9.

In the calculation model, the first plate entered in the data file is assumed to be the source plate. On this plate, the junction will be excited consecutively by an incident bending wave and by each of both in-plane waves. For each incident wave, the transmission coefficient at a given angle of incidence is calculated for all three wave types on all plate. As a result, nine transmission coefficients are calculated for each receiver plate. This procedure is repeated a number of times to calculate the angle-averaged transmission coefficient, or the transmission coefficient as a function of the angle of incidence (see section 5.2d).

The following sections will show how to run the calculation model, and will discuss the application of the model to line and point connected plate junctions. The format of the various results files is presented in the final section of this chapter.

6.1 Starting the calculations.

NRCJ38 is a character mode executable, which can be started using the following calling sequence:

```plaintext
> nrcj38 [filename] [switch] [switch] ...
```

Optional command line parameters are indicated between square brackets. The optional parameter "filename" is a string of maximum 8 characters, specifying the name of the input data file which was created using PREP38. The filename should not contain a file extension. If this parameter is not specified at startup, the default filename 'nrcj38' assumed.

Certain aspects of the calculations may be modified at startup by specifying an appropriate switch. A switch is composed of a slash (/) and a character, which may optionally be followed by real or integer value. Several switches may be specified at startup, and all switches and numeric values should be separated by a space. The following switches are available:

```
/b This switch restricts the calculations to incident bending waves only. Transmitted in-plane waves will still be considered.
```
This switch is only applicable to point connected plate junctions, and starts a convergence test to determine the required number of wave components, NumWaves. Convergence is evaluated based on the sensitivity of the coupling loss factor (CLF) from bending on plate 1 to bending on plate 2. On exit, a number of wave components, N, is proposed which satisfies: $\text{CLF(MaxWaves)} - \text{CLF}(N) < \text{MaxErr}$ (in dB). MaxErr is an optional parameter to the /c switch. The default value for MaxErr is 0.5 dB. MaxWaves depends on the size of the set of equations, MaxSize, as shown in Eq. (5).

This switch sets MaxSize to $\text{dim}$. The default value for MaxSize is 2400, which is also the maximum allowable value for dim. This setting is used to limit the duration of the convergence test by reducing the maximum number of wave components, MaxWaves (See Eq. (5)).

This switch is used when only a simple bending wave calculation is required. In this model, only rotation of the junction beam is allowed (pinned joint). As a result, no in-plane waves will be considered.

This switch prevents the screen from scrolling during the calculations, by limiting the output to the screen ('quiet' calculation).

After specifying this switch, the sum of transmission coefficients will be calculated at each angle of incidence. If the data file is set to integration, the sum of transmission coefficients will be calculated only at normal incidence. No other calculations will be performed and the results will only be shown on screen. This switch is intended to quickly check the validity of the model.
Running the calculation model NRC38

After starting the model, the information shown in Figure 7 will be printed to the screen (regardless of the /q switch).

Next, a series of status lines will be printed to show the progress of the calculations. The format of each line depends on the settings for the calculations, but the information usually includes the current frequency point and the incident wave type. The latter is identified using the variable \textit{Iwave}, which is equal to 1, 2 and 3 for bending, fast and slow in-plane waves, respectively. Depending on the operating system on which the model is executed, the screen may scroll down for each printed line. If the /q switch is specified, no status lines will be printed to the screen.

Upon termination of the calculations, a number of result files will be written to disk. The formats of these files are discussed in section 6.6.

\textbf{Note:} In order to avoid losing valuable calculation results or unnecessarily repeating a calculation, NRC38 will not start when the result files corresponding to the input file are found in the working directory. In order to run the model, one should rename or delete the old result files or rename the input data file \textit{fikmrne.dat}.

6.2 \textbf{Calculating a line connected plate junction.}

Running the calculations for a line connected plate junction is rather straightforward. The procedure only requires two steps: a preprocessing step to create the data file using PREP38, and a calculation step using NRC38. No convergence test is needed and the switches /c and /d are irrelevant.

6.3 \textbf{Calculating a point connected plate junction.}

Calculating structure-borne sound transmission at point connected plate junctions requires a significantly stronger computational effort compared to line junctions. The response of point connected plates is described by a scattered wave field which is modelled as a superposition of a number of plane waves. Each wave component adds an unknown wave amplitude to the problem, and therefore increases the dimension of the set of equations built up from the continuity and equilibrium conditions. As a result, calculation times are significantly longer for point connected plate junctions compared to line junctions. In addition, the number of wave components required for an accurate description of the plate response is not known \textit{a priori}, and should be determined based on a convergence test. Consequently, running the calculation model for point connected plates requires four steps:

- Preparing a preliminary data file (PREP38).
- Running a convergence test (NRC38).
- Editing the data file to enter the convergence data obtained from the previous step (PREP38).
- Running the calculations (NRC38).

The next section explains how to perform a convergence test.

6.4 \textbf{Running a convergence test.}

A convergence test is started using the /c switch. During the test, the coupling loss factor is calculated repeatedly while increasing the number of wave components. The convergence test starts at NumWaves = 0 and stops at NumWaves = MaxWaves. The latter is calculated based on Eq. (5) and depends on the
number of plates and the maximum allowable dimension of the set of equations, MaxSize. Convergence is evaluated by studying the sensitivity of the coupling loss factor from bending on plate 1 to bending on plate 2, $n_{1b12}$, with respect to the number of wave components. On exit, an appropriate number of wave components $N$ is suggested, which satisfies:

$$10\log(n_{1b12}(\text{MaxWaves})) - 10\log(n_{1b12}(N)) < \text{MaxErr} \quad (6)$$

The default value of MaxErr is 0.5 dB, but may be changed at startup using the switch /c. It should be noted that satisfaction of Eq. (6) does not guarantee convergence. The user should check the convergence data more in detail to make sure that the proposed number of wave components is acceptable. For this purpose, the result file of the convergence test contains the coupling loss factors as a function of NumWaves, at all frequencies.

Since the convergence test is computationally expensive, it is strongly recommended to reduce the calculation time by limiting the frequency and integration points, and by using the appropriate switch. For instance, to evaluate the convergence as a function of frequency, it may be sufficient to carry out the calculations only at octave band centre frequencies. Further, the numerical integration over all angles of incidence should not use more than 100 function evaluations. The most substantial reduction in calculation time can be achieved by reducing the maximum allowable dimension MaxSize of the set of equations using the /d switch. The default value for MaxSize is 2400 and it is recommended to reduce it to 1200. This leads to the following calling sequence for the convergence test:

```
> nrcj38 filename /c /d 1200
```

It should be noted that MaxSize = 1200 may not be sufficient to obtain convergence. If the convergence results indicate that this is the case, the test should be repeated for an increased value of MaxSize.

**Note:** NRCJ38 saves temporary data in the working directory during the convergence test. As a result, only one convergence test should run in a directory at a given point in time.

### 6.5 Interrupting the calculations.
Calculations for point connected plate junctions may be interrupted by pressing CTRL-C at any time during the computations. Since the program saves data in temporary files at regular intervals, the calculations will resume after restarting the model with limited loss of data.

### 6.6 Overview and format of the result files.
NRCJ38 creates an number of ASCII files containing the calculation results. The file type and format depend on the settings during the calculations. The names of these files are composed of “filename”, which is the first command line parameter of NRCJ38, and a file extension which identifies a particular file type. Table 5 gives an overview of the various files created by NRCJ38. The table shows the filename, a short description of the file contents and the calculation settings for which the file is created.

The format of the result files is discussed in detail in the following paragraphs.
Running the calculation model NRCJ38

Table 5. Overview of the various ASCII files created by NRCJ38.

<table>
<thead>
<tr>
<th>Filename</th>
<th>Description of file contents</th>
<th>Calculation mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>*.log</td>
<td>Date and time at beginning and end of calculations. Switches applied at startup.</td>
<td>Integration, angles of incidence, convergence test, sum of transmission coefficients.</td>
</tr>
<tr>
<td>*.res</td>
<td>Angle-averaged transmission coefficient as a function of frequency.</td>
<td>Integration.</td>
</tr>
<tr>
<td>*.rsb</td>
<td>Angle-averaged transmission coefficient as a function of frequency, averaged over one-third octave bands.</td>
<td>Integration with band averaging.</td>
</tr>
<tr>
<td>*.dn</td>
<td>Transmission loss as a function of frequency.</td>
<td>Integration.</td>
</tr>
<tr>
<td>*.dnb</td>
<td>Transmission loss as a function of frequency, averaged over one-third octave bands.</td>
<td>Integration with band averaging.</td>
</tr>
<tr>
<td>*.clf</td>
<td>Coupling loss factor as a function of frequency.</td>
<td>Integration.</td>
</tr>
<tr>
<td>*.mod</td>
<td>Modal density as a function of frequency.</td>
<td>Integration.</td>
</tr>
<tr>
<td>*.fn</td>
<td>Transmission coefficient as a function of the angle of incidence, at frequency point n.</td>
<td>Angles of incidence.</td>
</tr>
<tr>
<td>*.cvg</td>
<td>Convergence data: coupling loss factor $\eta_{lmb}$ as a function of the number of wave components.</td>
<td>Convergence test.</td>
</tr>
</tbody>
</table>

a. **Files created when integrating over all angles of incidence.**

When an integration over all angles of incidence is requested in the data file filename.dat, four different quantities will be calculated to characterize the plate junction: the angle-averaged transmission coefficient, the transmission loss, the coupling loss factor and the modal density.

The result files for the first three quantities use exactly the same format. The first column contains the frequencies at which the calculations are carried out. The following columns are grouped according to the incident wave type, as shown in Table 6. For each incident wave type, 3*m columns of data are saved, where m represents the number of plates. The table refers to quasi-longitudinal and in-plane transverse waves, which should be generalized to fast and slow in-plane waves in case of orthotropic plates.

- The **angle-averaged transmission coefficient** $\tau$ is calculated according to Ref. 9, and saved in filename.res using the format illustrated in Table 6. If the calculations were carried out on a logarithmic frequency scale with band averaging ($r>1$, see section 5.2e), filename.res contains the narrow band results. The angle-averaged transmission coefficient, averaged over each one-third octave band is saved in filename.rsb, using the same format.
Table 6. General format for frequency dependent results: illustration for a junction between m plates. The first column contains the frequencies \( f \) [Hz]. This format applies to *.res, *.rsb, *.dn, *.dnb and *.clf files. Depending on the file type, X represents the angle-averaged transmission coefficient, transmission loss or coupling loss factor. The plate numbers refer to the receiving plates. The shaded parts of the table are not saved in the files.
- The transmission loss $R$ is expressed in terms of the angle-averaged transmission coefficient as:

$$R = -10\log(F)$$

The transmission loss is saved in filename.dn using the format illustrated in Table 6. If the calculations were carried out on a logarithmic frequency scale with band averaging ($r>1$, see section 5.2e), filename.dn contains the narrow band results. The angle-averaged transmission coefficient, averaged over each one-third octave band is saved in filename.dnb, using the same format.

- The coupling loss factor, $\eta$, is calculated using the expressions derived in Refs. 8 and 9, and is expressed in dB according to the following equation:

$$CLF = 10\log(\eta) + 120$$

CLF is saved in filename.clf using the format illustrated in Table 6. If the calculations were carried out on a logarithmic frequency scale with band averaging ($r>1$, see section 5.2e), filename.clf contains the coupling loss factors averaged over each one-third octave band. No narrow band coupling loss factors are saved.

- The modal density of the connected plates is calculated according to the expressions given in Ref. 9. The definition used for the calculations is usually symbolized by $n(\omega)$, where $\omega$ denotes the radian frequency. The modal density is saved in filename.mod. While the first column of the file contains the frequency, the following columns as illustrated in Table 7. If the calculations were carried out on a logarithmic frequency scale with band averaging ($r>1$, see section 5.2e), filename.mod contains the modal densities calculated at each one-third octave band centre frequency.

<table>
<thead>
<tr>
<th>Freq</th>
<th>Bending waves (m columns)</th>
<th>Quasi-longitudinal waves (m columns)</th>
<th>In-plane transverse waves (m columns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>$n_1$ $n_2$ $\ldots$ $n_m$</td>
<td>$n_1$ $n_2$ $\ldots$ $n_m$</td>
<td>$n_1$ $n_2$ $\ldots$ $n_m$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$ $\vdots$ $\vdots$ $\vdots$</td>
<td>$\vdots$ $\vdots$ $\vdots$ $\vdots$</td>
<td>$\vdots$ $\vdots$ $\vdots$ $\vdots$</td>
</tr>
<tr>
<td>$f$</td>
<td>$n_1$ $n_2$ $\ldots$ $n_m$</td>
<td>$n_1$ $n_2$ $\ldots$ $n_m$</td>
<td>$n_1$ $n_2$ $\ldots$ $n_m$</td>
</tr>
</tbody>
</table>

Table 7. Format for *.mod files: illustration for a junction between $m$ plates. The first column contains the frequencies $f$ [Hz]. In the next columns, $n_p$ denotes the modal density, where $p$ is the plate number.
Table 8. Format for angle dependent results: illustration for a junction between \( m \) plates. The first column contains the angles of incidence \( \theta_i \) [rad]. This format applies to *.fn files, where \( n \) represents the frequency number. \( X \) denotes the angle dependent transmission coefficient. The plate numbers refer to the receiving plates. The shaded parts of the table are not saved in the files.

<table>
<thead>
<tr>
<th>Ang/Plate</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>( m )</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>( m )</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>( m )</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>( m )</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_i )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
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<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td></td>
</tr>
<tr>
<td>( \theta_i )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
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<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td>( X )</td>
<td></td>
</tr>
</tbody>
</table>
b. **Files created when calculating angle dependent transmission coefficients.**

In this case, the transmission coefficient is calculated as a function of the angle of incidence at every frequency. In view of the large amount of data involved, the results are saved in several files, one file per frequency. The result files are called *filename.fn*, where *n* refers to the frequency point. For example, if the angle dependent transmission coefficient is calculated at 27 frequencies, the results will be saved in 27 files with file extension *.fn* where *n* in ranges from 1 to 27.

The format of each file is similar to that of the angle-averaged transmission coefficient. The first column contains the angles of incidence. The next columns list the transmission coefficients which are saved to disk as illustrated in Table 8 for a junction between m plates.

c. **Files created when running a convergence test.**

The results for the convergence test for point connected plates are saved in *filename.cvg*. The format of this file is illustrated in Table 9.

<table>
<thead>
<tr>
<th>F₁</th>
<th>F₂</th>
<th>...</th>
<th>Fₙ</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>N</td>
<td>...</td>
<td>N</td>
</tr>
<tr>
<td>0</td>
<td>CLF</td>
<td>CLF</td>
<td>...</td>
</tr>
<tr>
<td>1</td>
<td>CLF</td>
<td>CLF</td>
<td>...</td>
</tr>
</tbody>
</table>
| ...| ...| ... | ...| ...

**Table 9.** Format for convergence data file *.cvg*. The first row contains the frequencies, *Fᵢ [Hz]*, for which the convergence test was performed. The second row lists the proposed number of wave components, *N*, which satisfy Eq. (6). The following rows show the coupling loss factor for bending on plate 1 to bending on plate 2 [dB re. 10⁻¹²], as a function of the number of wave components which are shown in the first column.
7. Validation.

The validity of calculation model has been evaluated by experimental verification, the results of which are reported in the literature.\textsuperscript{9,10,11} Even before comparing measured and predicted data, one should make sure that the theoretical formulation is consistent with the fundamental laws of physics. Since NRC38 does not include any damping of the plates and beams, the energy carried towards the junction by the incident wave, should be equal to the total energy carried away from the junction by the transmitted waves. In other words, the sum of all transmission coefficients (including contributions from waves 'transmitted' on the source plate) should be equal to one. It can be easily verified (using the /$s$ switch) that NRC38 obeys the principle of energy conservation for all possible types of junctions. However, the transmission coefficients summing up to unity does not prove of the validity of the model, even though it is a necessary condition.

In this section, the validity of the calculation model is further demonstrated by comparison with literature data. Cremer \textit{et al.}\textsuperscript{3} derived closed form expressions for the transmission coefficient for a variety of relatively simple plate and beam junctions. In order to obtain these expressions, the authors had to simplify the problem significantly. Nevertheless, the theory is based on the same assumption of thin plate and beam theory as implemented in NRC38 and both formulations should give the same predictions.

The following paragraphs show comparisons between Cremer’s predictions and the results obtained using NRC38 for four different plate junctions: a rigid corner junction, a rigid cross junction, a junction with an elastic interlayer and a plate/beam junction. The equations given in Ref. 3 differ slightly from the ones used for the validation, mostly because they were derived for junctions between beams. To avoid any misunderstanding, Cremer’s revised formula’s are included in Appendices A, B and C.

7.1 Corner junction.

Predicted results were compared for the corner junction between two identical plates shown in Figure 8. The plates were assumed to be rigidly connected, without any eccentricity. Cremer’s model and NRC38 were used to calculate the normal incidence transmission and reflection coefficients in case of an incident bending wave. The equations used for Cremer’s predictions are given in Appendix A.

![Figure 8](image)

\textbf{Figure 8.} Corner junction between two identical plates. The material properties of both plates are: Young’s modulus $2.1 \times 10^{11}$ Pa, Poisson’s ratio 0.3 and density 7800 kg/m$^3$. The plate thickness is 0.02 m.

Figure 9 shows a cumulative diagram of the predicted transmission ($\tau$) and reflection ($\rho$) coefficients. A corner junction typically involves wave conversion from bending (B) to quasi-longitudinal waves (L). It is clearly shown that the sum of transmission and reflection coefficients is equal to unity for both
models. The closed form expressions by Cremer and the more generic approach implemented in NRCJ38 essentially produce the same results.

Figure 9. Predicted normal incidence transmission (τ) and reflection (ρ) coefficients for the corner junction of Figure 8. Incident bending wave (B), transmitted and reflected bending (B) and quasi-longitudinal (L) waves.

Figure 10. Symmetric cross-junction. The material properties of the plates are: Young’s modulus $2.1 \times 10^{11}$ Pa, Poisson’s ratio 0.3 and density 7800 kg/m$^3$. In the NRCJ38 calculations, $h_1$ and $h_3$ were kept equal to 1 m, while $h_2$ and $h_4$ were varied to obtain a ratio $\sigma=h_2/h_1$ ranging from 1/64 to 64 in steps of a factor of four.

7.2 Cross junction.
Another comparison between predictions is made for the symmetric cross-junction shown in Figure 10. Cremer simplified the analysis considerably by allowing the junction only to rotate. As a result, no in-
plane waves will be generated at the junction, and the normal incidence transmission loss for incident and transmitted bending waves solely depends on the ratio of the plate thicknesses $\sigma = h_2/h_1$:

$$\begin{align*}
R_{12} &= R_{14} = 10\log\left(2\left(\sigma^{5/4} + \sigma^{-5/4}\right)^2\right) \\
R_{13} &= 10\log\left(2\left(1 + \sigma^{5/2}\right)^2\right)
\end{align*}$$

The comparison between predictions using Eqs. (9) and (10), and calculated data from NRCJ38 are shown in Figure 11. It is clearly demonstrated that both sets of data are identical.

**Figure 11.** Predicted normal incidence transmission loss as a function of $\sigma$ for the symmetric cross-junction shown in Figure 10. Line graphs: Eqs. (9) and (10). Scatter graphs: NRCJ38 predictions. Since the cross-junction is assumed to be pinned, the NRCJ38 calculations were carried out using the /p switch.

**Figure 12.** Junction between two identical plates coupled by an elastic interlayer. The material properties of the plates are: Young's modulus $3 \cdot 10^{10}$ Pa, Poisson's ratio 0.17 and density 2500 kg/m$^3$. The plate thickness $h_1$ is 0.1 m. The properties of the elastic interlayer are: Young's modulus $3 \cdot 10^7$ Pa, Poisson's ratio 0.4 and thickness $h_2$ 0.02 m.
7.3 Junction using an elastic interlayer.

The third junction used for the validation involves an elastic interlayer connecting two identical plates as illustrated in Figure 12. Cremer derived expressions for the normal incidence transmission loss in case of incident and transmitted bending waves, and incident and transmitted the quasi-longitudinal waves. The elastic interlayer is modelled as a spring, and no eccentricities were taken into account. The equations derived from Ref. 3 are given in Appendix B.

Predicted data obtained using Cremer's closed form expressions and NRU38 are compared in Figure 13. Also in this case, there is perfect agreement between the predictions of both models.

![Figure 13](image1.png)

**Figure 13.** Predicted bending and quasi-longitudinal wave transmission loss at normal incidence for the elastic junction shown in Figure 12. Line graphs: Cremer predictions. Scatter graphs: NRU38 predictions. For the NRU38 results, the spring constants for the elastic interlayer were calculated according to Eq. (2).

7.4 Plate/beam junction.

The final comparison is carried out for the plate/beam junction shown in Figure 14. The junction consists of two identical plates coupled by a beam which is located symmetrically with respect to the plates. In this case, an incident bending wave will not generate any in-plane waves. Cremer developed a closed form expression for the bending wave transmission coefficient as a function of the angle of incidence. The beam was modelled using thin beam theory, and no eccentricities were taken into account. The expressions used for Cremer's predictions are given in Appendix C.

Figure 15 shows the bending wave transmission coefficient as a function of the angle of incidence for 6 different frequencies. Also for this junction, the prediction using Cremer's theory and NRCJ38 are in excellent agreement.
Figure 14. Plate/beam junction involving two identical plates and a symmetrically positioned beam. The material properties of the beam and the plates: Young's modulus $2.1 \times 10^{11}$ Pa, Poisson's ratio 0.3 and density 7800 kg/m$^3$. The plate thickness is 0.02 m. Dimensions of the beam cross-section: $b = 0.037$ m and $h = 0.23$ m.
Figure 15. Predicted bending wave transmission coefficient as a function of the angle of incidence for the plate/beam junction shown in Figure 14. Results presented at 6 frequencies. Line graphs: Cremer predictions. Scatter graphs: NRCJ38 predictions.
8. References

Appendix A: Structure-borne sound transmission at a corner junction according to Cremer, Heckl and Ungar.\(^3\)

A.1 Introduction.
Cremer et al.\(^3\) derived closed form expressions predicting structure-borne sound transmission at a right-angled joint. The formula's published in Ref. 3 strictly apply to junctions of beams, but may be easily adapted to be applicable to plate junctions at normal incidence. Therefore, the formulas used in the validations may differ slightly from the published ones. The equations presented in the following paragraph are based on thin plate theory and require isotropic and homogeneous plates. The model incorporates bending and quasi-longitudinal waves.

A.2 Theory.
The normal incidence reflection and transmission coefficient from bending to bending are given by:

\[
\rho_{BB} = \left| r \right|^2, \quad \tau_{BB} = \chi \psi \left| t \right|^2
\]  

(A1)

The normal incidence reflection and transmission coefficient from bending to quasi-longitudinal waves are given by:

\[
\rho_{BL} = \frac{c_{ll}}{2c_{bl}} \left| t + t_j \right|^2
\]

(A3)

\[
\tau_{BL} = \frac{1}{2\beta_2} \left| 1 + r + r_j \right|^2
\]

(A4)

In these equations, the complex parameters \( r, r_j, t \) and \( t_j \) are given by:

\[
r = \frac{[\psi(1 - 2\beta_2, -\beta_1\beta_2) + \chi(1 + 2\beta_1, -\beta_1\beta_2)] + j[\psi(1 + \beta_1, -\beta_1\beta_2) + \chi(-1 + \beta_2, +\beta_1\beta_2)]}{\psi(-1 - \beta_1, -2\beta_2, -\beta_1\beta_2) + \chi(-1 - 2\beta_2, -\beta_1\beta_2) + j(\psi + \chi)(1 - \beta_1\beta_2)}
\]

(A5)

\[
r_j = \frac{-1 + \beta_1 - r(1 + \beta_2)}{1 + j\beta_2}
\]

(A6)

\[
t = \frac{2(\beta_1, +\beta_2) - j2(1 - \beta_1\beta_2)}{\psi(-1 - \beta_1, -2\beta_2, -\beta_1\beta_2) + \chi(-1 - 2\beta_1, -\beta_2, -\beta_1\beta_2) + j(\psi + \chi)(1 - \beta_1\beta_2)}
\]

(A7)

\[
t_j = \frac{1 + \beta_1}{-1 - j\beta_1}
\]

(A8)

The parameters \( \chi \) and \( \psi \) stand for:

\[
\chi = \frac{\sqrt{m_2 B_1}}{m_1 B_2}, \quad \psi = \frac{\sqrt{m_1 B_2}}{m_1 B_1}
\]

(A9)
where $m_i$ and $B_i$ represent the surface mass and the bending stiffness per unit length of plate $i$, and are calculated as:

$$B_i = \frac{E_i h_i^3}{12 (1 - \nu_i^2)} \quad \text{(A11)}$$

$$m_i = \rho_i h_i \quad \text{(A12)}$$

In Eqs. (A11) and (A12), $E$ and $\nu$ denote the Young's modulus and Poisson's ratio, $h$ represents the plate thickness and $\rho$ the density of the plate material.

The quasi-longitudinal and bending wave speeds in Eq. (A3), $c_u$ and $c_{bi}$, are given by:

$$c_u = \sqrt{\frac{E_i}{\rho_i (1 - \nu_i^2)}} \quad \text{(A13)}$$

$$c_{bi} = \sqrt{\omega \beta_i \frac{B_i}{m_i}} \quad \text{(A14)}$$

where $\omega$ represents the radian frequency.

Finally, the parameters $\beta_1$ and $\beta_2$ in Eqs. (A4-8) are calculated as:

$$\beta_1 = \frac{c_{bi} m_2}{c_u m_1} \quad \text{(A15)}$$

$$\beta_2 = \frac{c_{bi} m_1}{c_u m_2} \quad \text{(A16)}$$
Appendix B: Structure-borne sound transmission between elastically connected plates according to Cremer, Heckl and Ungar.\textsuperscript{3}

B.1 Introduction.
Cremer \textit{et al}.\textsuperscript{3} studied bending and quasi-longitudinal wave transmission at an in-line junction of two identical beams coupled by an elastic interlayer (see Figure 12). In Ref. 3, the interlayer was modelled as a spring. Since the closed form expressions were derived for beam junctions, they were slightly changed to be applicable to plate junctions at normal incidence. The equations presented in the following paragraph are based on thin plate theory and require isotropic and homogeneous plates.

B.2 Quasi-longitudinal wave transmission.
The normal incidence transmission loss $R$ for incident and transmitted quasi-longitudinal waves is calculated as:

$$R = 10 \log \left( 1 + \left( \frac{\omega Z_1}{2s_2} \right)^2 \right)$$  \hspace{1cm} (B1)

where $\omega$ represents the radian frequency, $Z_1$ denotes the quasi-longitudinal wave impedance of the plates, and is calculated as:

$$Z_1 = h_1 \sqrt{E_1 \rho_1}$$  \hspace{1cm} (B2)

where $h_1$, $E_1$ and $\rho_1$ are the plate thickness, the Young’s modulus and the density of the plates. The spring constant $s_2$ in Eq. (B1) is given by:

$$s_2 = \frac{E_2 h_1}{h_2}$$  \hspace{1cm} (B3)

where $E_2$ and $h_2$ are the Young’s modulus of the thickness of the interlayer. It should be noted that Eq. (B3) is identical to $K_x$ in Eq. (2).

B.3 Bending wave transmission.
The normal incidence bending wave transmission loss for incident and transmitted bending waves is given by:

$$R = 10 \log \left( \frac{\left( \frac{\nu + \epsilon^2 \nu^3}{4} + \frac{4 + \nu - \epsilon^2 \nu^3 - \epsilon^2 \nu^4}{2} \right)^2}{4 + \nu - \epsilon^2 \nu^3} \right)$$  \hspace{1cm} (B4)

where the parameters $\nu$ and $\epsilon$ are given by:

$$\nu = \frac{2 \pi B_1}{\lambda_1 C}$$  \hspace{1cm} (B5)

$$\epsilon = \frac{C}{\sqrt{K}}$$  \hspace{1cm} (B6)

$B_1$ represents the bending stiffness of the plates per unit length, and is calculated according to Eq. (A11). The bending wave length $\lambda_1$ is expressed in terms of the bending wave speed Eq. (A14):

$$\lambda_1 = \frac{B_1}{f}$$  \hspace{1cm} (B7)

where $f$ denotes the frequency. $C$ and $K$ represent the spring constants for bending and shear, and are calculated as:
where $E_2$ and $G_2$ are the Young's and shear modulus of the elastic interlayer. $C$ and $K$ are identical to $K_y$ and $K_z$ in Eq. (2).
Appendix C: Structure-borne sound transmission at a plate junction with reinforcing beam according to Cremer, Heckl and Ungar.3

C.1 Introduction.
Cremer et al.3 developed a closed form expression for the angle-dependent bending wave transmission coefficient at an in-line junction between two identical plates coupled by a beam (See Figure 14). Since the beam is mounted symmetrically with respect to the junction, wave conversion does not occur. The equations presented in the following paragraph are based on thin plate and beam theory and require isotropic and homogeneous materials.

C.2 Theory.
The bending wave transmission coefficient for an angle of incidence $\theta$ is calculated as:

$$\tau_{\alpha\beta} = |t|^2$$  \hspace{1cm} (C1)

where the complex parameter $t$ is given by:

$$t = \frac{j \alpha(4 + \gamma - \beta)}{j \alpha \left(4 + \gamma - \beta \left(1 - \frac{1}{2}\beta \gamma \left(\alpha^2 - 1\right)\right)\right)}$$  \hspace{1cm} (C2)

In Eq. (C2), $\alpha$ depends on the angle of incidence $\theta$ and is calculated as:

$$\alpha = \frac{\cos \theta}{\sqrt{1 + \sin^2 \theta}}$$  \hspace{1cm} (C3)

The remaining parameters in Eq. (C2) are given by:

$$\beta = \xi^2 \mu^3 \sqrt{1 + \sin^2 \theta} \left(1 - \left(\frac{c_{\Pi} \sin \theta}{c}\right)^2\right)$$  \hspace{1cm} (C4)

$$\gamma = \mu \left(1 - \left(\frac{c_{\alpha} \sin \theta}{c}\right)^2\right)$$  \hspace{1cm} (C5)

In Eq. (C4), $c_{\Pi}$ denotes the torsional wave speed on the beam, and is given by:

$$c_{\Pi} = \sqrt{\frac{T}{\Theta'}}$$  \hspace{1cm} (C6)

In the latter equation, $T$ is the torsional stiffness of the beam and $\Theta'$ the mass moment of inertia per unit beam length. For a beam with a rectangular cross-section with dimensions $b$ and $h$ ($b < h$), these parameters are calculated as:

$$T = kGb^3h$$  \hspace{1cm} (C7)

$$\Theta' = \frac{b(h^3 + h b^3)}{12}$$  \hspace{1cm} (C8)

where $G$ and $\rho$ represent the shear modulus and density of the beam material. The coefficient $k$ depends on the ratio of $h/b$ and is determined by interpolation of the data in the Table C1.

In Eq. (C4), $c$ represents the bending wave speed on the plate (Eq. (A14)), and $\mu$ and $\xi$ are given by:

$$\mu = \frac{2\pi m'}{\lambda m''}$$  \hspace{1cm} (C9)
In these equations, \( m' \) represents the mass per unit length of the beam, and \( m'' \) the mass per unit surface of the plates. \( \lambda \) denotes the bending wave length in the plate, which is given by Eq. (B7).

\[
\zeta = \frac{m'}{m'} \sqrt{\frac{E}{m'}}
\]  
(C10).

In these equations, \( m' \) represents the mass per unit length of the beam, and \( m'' \) the mass per unit surface of the plates. \( \lambda \) denotes the bending wave length in the plate, which is given by Eq. (B7).

<table>
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<th>2</th>
<th>3</th>
<th>6</th>
<th>10</th>
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<td>0.263</td>
<td>0.298</td>
<td>0.312</td>
<td>0.333</td>
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</table>

**Table C1.** Torsional coefficient \( k \) as a function of \( h/b \) according to Ref. 3.

In Eq. (C5), \( c_0 \) is the bending wave velocity in the beam and is calculated as:

\[
c_0 = \sqrt{\frac{E h^3}{12 m'}}
\]  
(C11)

where \( E \) is the beam's Young's modulus.