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CCPR K3 key comparison of luminous intensity Gaertner, Arnold

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CCPR K3 Key Comparison of Luminous Intensity

Arnold Gaertner
NRC Metrology

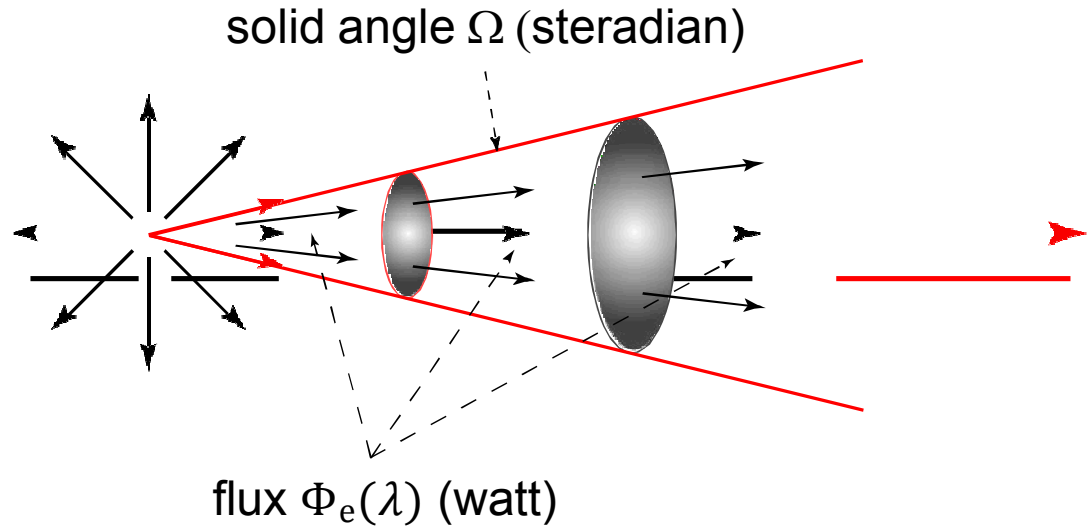
CORM 2019 Annual Technical Conference
2019-October-28

Intensity

Radiant Intensity

$$I_e(\lambda) = \frac{\Phi_e(\lambda)}{\Omega} = \frac{\text{radiant flux}}{\text{solid angle}}$$

unit = watt per steradian



Intensity

Luminous Intensity

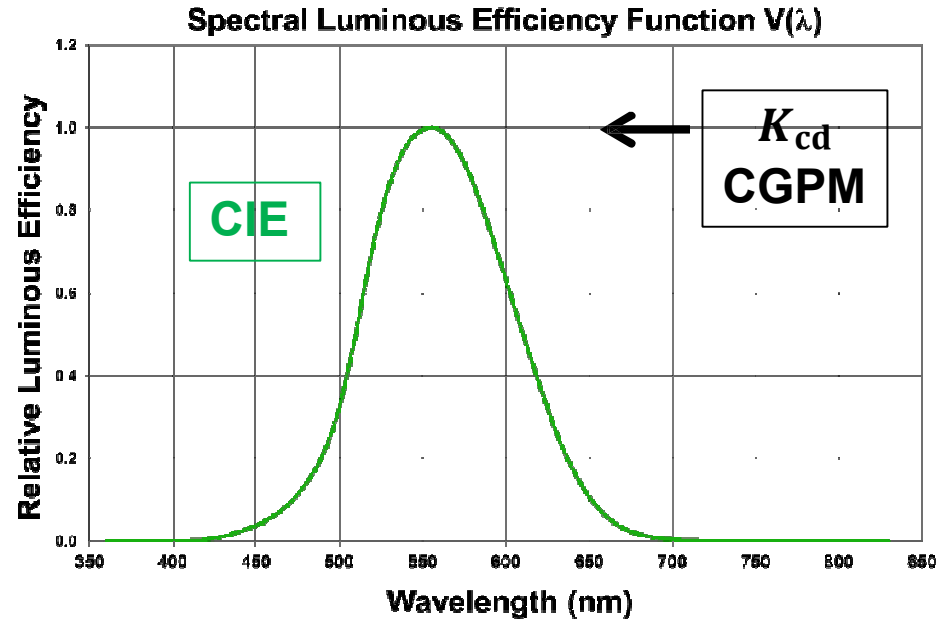
$$I_v = \frac{\Phi_v}{\Omega} = \frac{\text{luminous flux}}{\text{solid angle}}$$

unit = candela = lumen per steradian

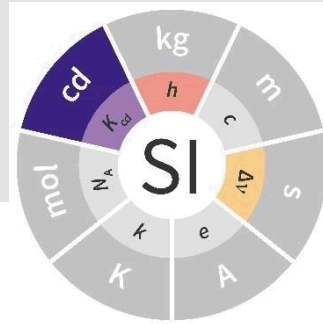
$$\Phi_v = K_{cd} \int_{360\text{ nm}}^{830\text{ nm}} V(\lambda) \cdot \Phi_e(\lambda) \cdot d\lambda$$

unit = lumen

$$K_{cd} = 683 \frac{\text{lumen}}{\text{watt}}$$

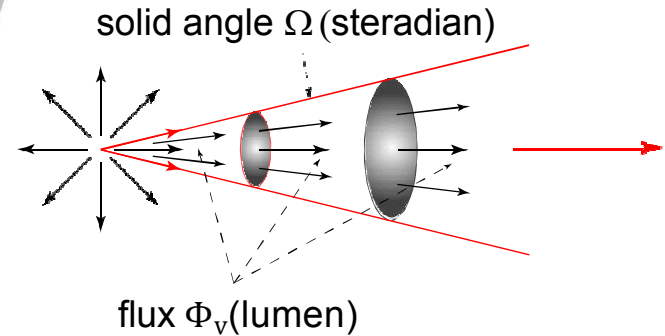


Intensity



Luminous Intensity, SI unit candela

$$I_v = \frac{\Phi_v}{\Omega} \quad \text{unit = lumen per steradian = candela}$$

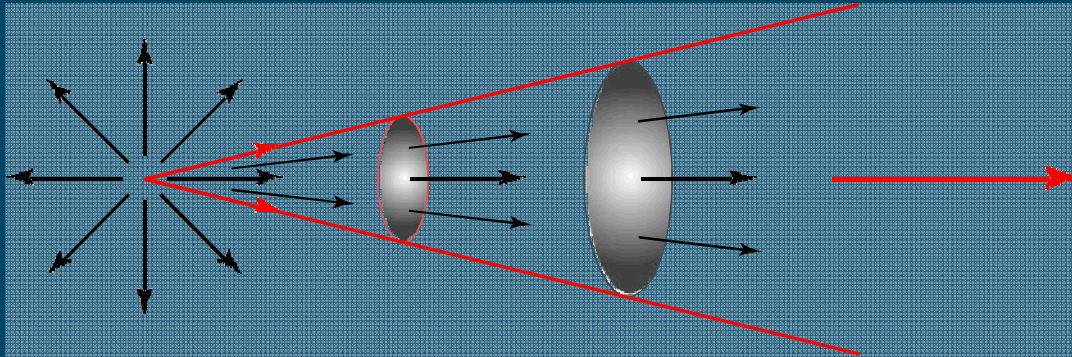


CGPM definition:

The candela, symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd} , to be 683 when expressed in the unit lm W^{-1} , which is equal to cd sr W^{-1} , or $\text{cd sr kg}^{-1} \text{m}^{-2} \text{s}^3$, where the kilogram, metre and second are defined in terms of h , c and $\Delta\nu_{Cs}$.

<https://www.bipm.org/en/measurement-units/base-units.html>

CCPR KEY COMPARISON CCPR-K3.2014



CCPR Key Comparison CCPR-K3.2014

Comparison Organisation

- Selection of participants, artifacts and protocol

Comparison Procedures

- Comparison measurements and measurement verification
- Data analysis and comparison of participant SI candela realisations
- Write the report

CCPR Key Comparison CCPR-K3.2014

Comparison Organisation

- Selection of NRC as pilot
- Selection of participants (12 max)
- Task Group
 - Selection of artifact
 - Lamp vs photometer: standards-quality incandescent lamps
 - Type of lamp: Incandescent (Osram Wi41/G and NPL/Polaron heavy current)
 - Type of comparison (star type: participant – pilot – participant)
 - Standard lamps are fragile and expensive
 - Draft the technical protocol (artifact transportation, measurement reporting, uncertainties, etc.)
- Register the comparison: CCPR-K3.2014

RMO Group	RMO Group Members	Maximum Number of Participants
Group 1	EURAMET+COOMET	6
Group 2	APMP+AFRIMETS	4
Group 3	SIM	2

CCPR Key Comparison CCPR-K3.2014

Comparison Organisation

- Selection of participants

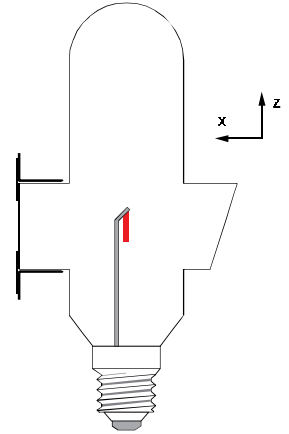
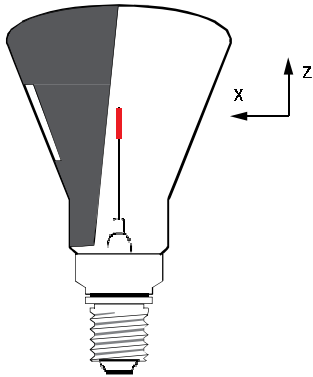
NMI	Country
NMISA	South Africa
NIM	China
NMIA	Australia
NMIJ	Japan
IO-CSIC	Spain
LNE-CNAM	France

NMI	Country
METAS	Switzerland
NPL	UK
PTB	Germany
VNIIOFI	Russia
NIST	USA
NRC	Canada

CCPR Key Comparison CCPR-K3.2014

Comparison Artifact

- Type of lamp: Incandescent (Osram Wi41/G and NPL/Polaron heavy current)



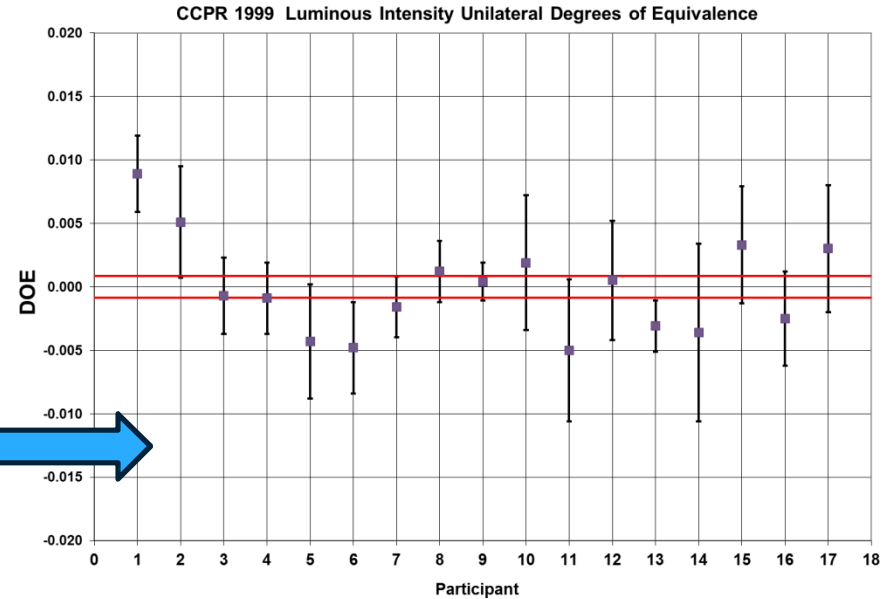
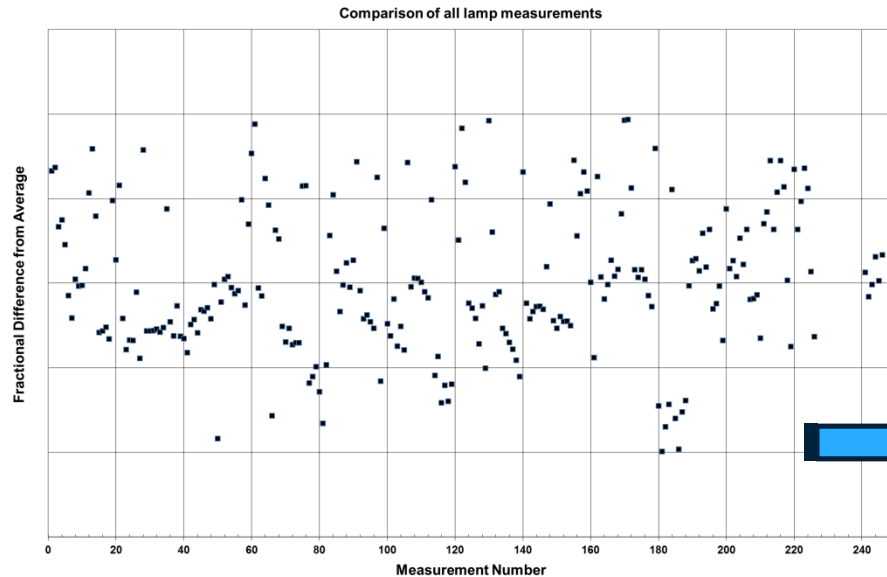
CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Measurements

- Comparison measurements
 - Each participant supplied their own calibrated (~6) lamps (ship or hand-carry)
 - NRC received and measured ~70 comparison lamps
 - Each participant re-measured their lamps
- Measurement verification and artifact certification
 - Each participant compares before and after shipment measurements
 - NRC provides relative data for all the artifacts of each participant (internal consistency check)
 - Removal of unstable artifacts => **final comparison artifacts**

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Data Analysis



CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

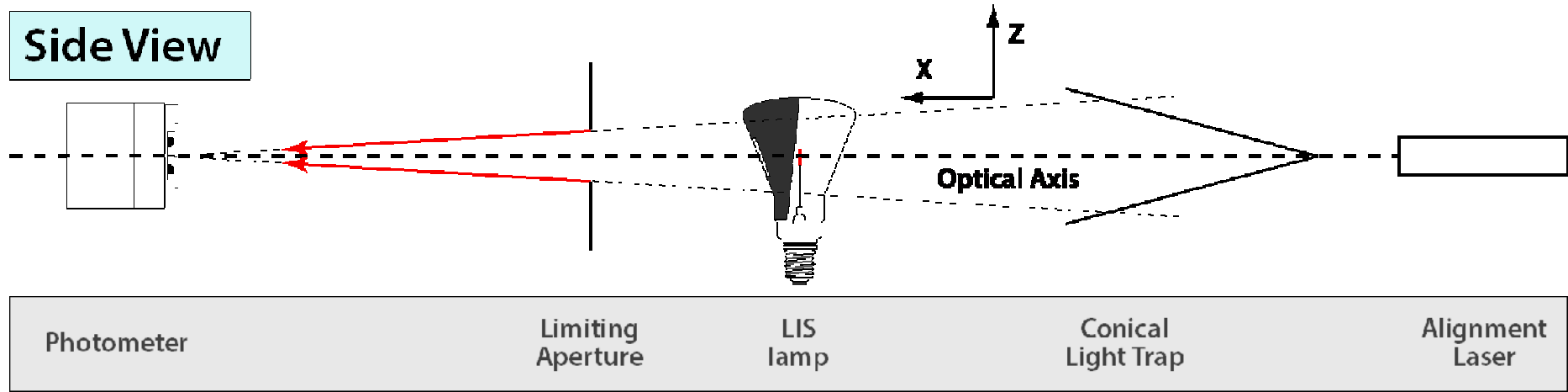
- Data analysis
 - Determine final NRC measurement value for each artifact
 - Determine final NRC measurement value for each participant
- Comparison of participant SI candela realisations
 - KCRV (Key Comparison Reference Value)
 - Weighted mean with cut-off
 - Identification of 'outliers': deviation from KCRV greater than 6 times their uncertainty
 - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$
- All this requires an uncertainty analysis (NRC and Participant measurements)

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Measurements at pilot (NRC)

- Comparison of all artifacts under identical measurement configuration

Side View

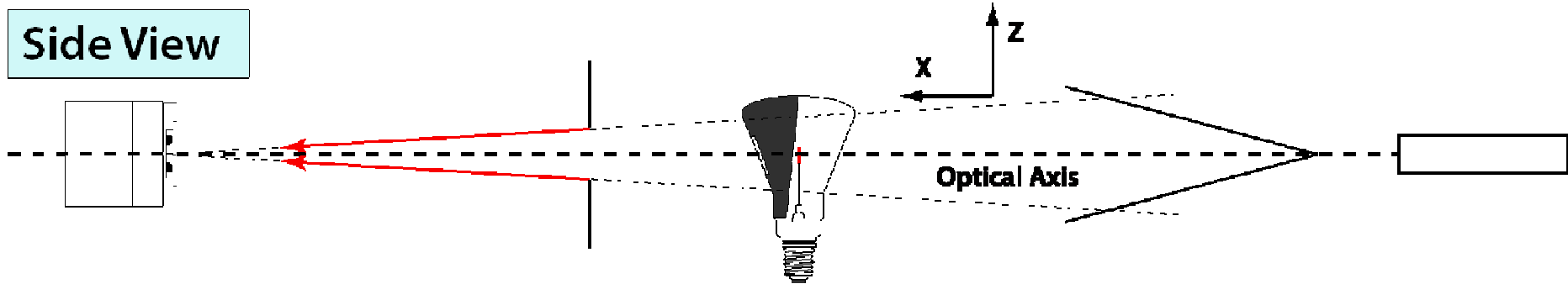


CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Measurements at pilot (NRC)

- Comparison of all artifacts under identical measurement configuration

Side View



Photometer

$V_{i,j,m}(\text{volt})$

NRC

Limiting
Aperture

LIS
lamp

$I_{v(i,j)}(\text{cd})$

Participant

Conical
Light Trap

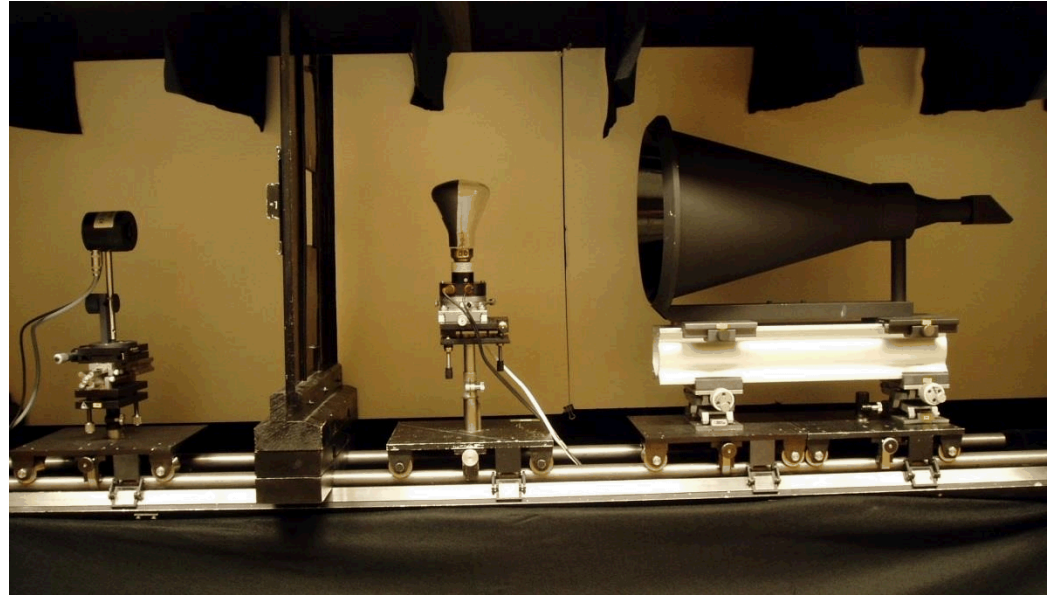
Alignment
Laser

$$R_m^{i,j} = \frac{I_{v(i,j)}}{V_{i,j,m}} \left(\frac{\text{cd}}{\text{volt}} \right)$$

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Measurements at pilot (NRC)

- Comparison of all artifacts under identical measurement configuration
- $d \sim 3.2 \text{ m}$
- 3 photometers
- $m \geq 2$ measurements/lamp
- ~ 250 measurements
- ~ 2 months



CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Measurements at pilot (NRC)

- **How accurate is the comparison?**
- Sources of Uncertainty • $u(V_{i,j,m})$ • (~15!)
 - NRC Optical Coordinate System (2)
 - NRC Photometer (5)
 - Participant Lamps
 - Electrical (4)
 - Optical (3)
 - Photometric (1)

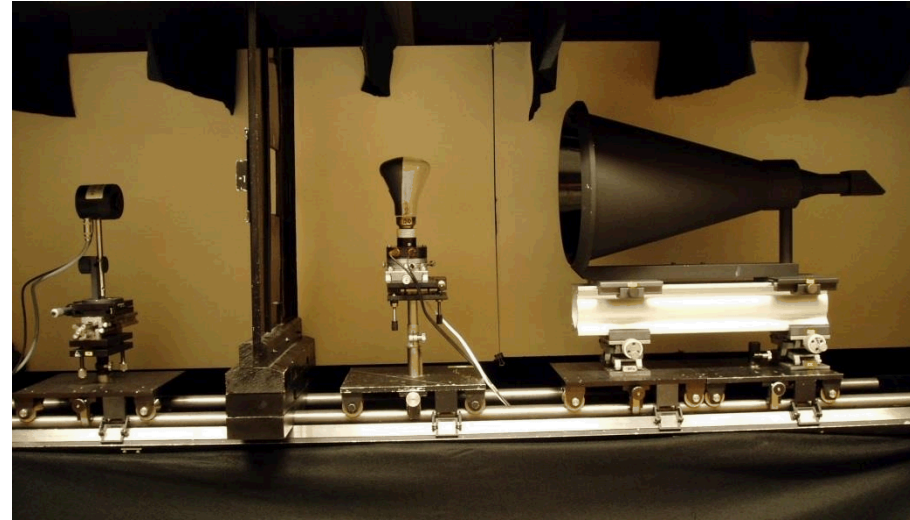
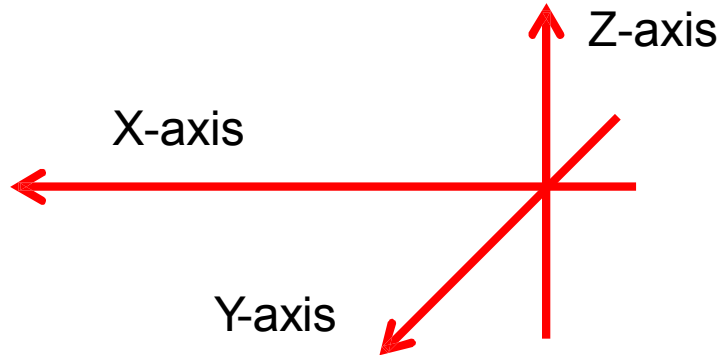
Consider:

- In 3D space there are 6 variables: 3 spatial and 3 angular
- Lamp output: % change $\approx$ 7 times % change in lamp current
 - Am I operating the lamp electricals to the same standards as the participant?
- How/with what do I ensure stability over 2 months of measurements?

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Measurements at pilot (NRC)

- Sources of Uncertainty • $u(V_{i,j,m})$
 - NRC Optical Coordinate System (2)
 - Starting line is X-axis (laser beam)
 - Alignment of Y-axis to X-axis (laser)
 - Alignment of Z-axis to XY axes



CCPR Key Comparison CCPR-K3.2014

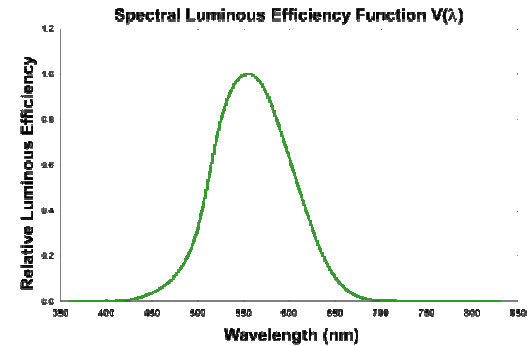
Comparison Procedures • Measurements at pilot (NRC)

- Sources of Uncertainty • $u(V_{i,j,m})$

- NRC Optical Coordinate System (2)
- NRC Photometer (5)
 - Spectral Mismatch Error

$$F^* = \frac{\int_{360\text{ nm}}^{830\text{ nm}} P_e^T(\lambda) \cdot V(\lambda) \cdot d\lambda}{\int_{\text{all wavelengths}} P_e^T(\lambda) \cdot R(\lambda) \cdot d\lambda} \cdot \frac{\int_{\text{all wavelengths}} P_e^S(\lambda) \cdot R(\lambda) \cdot d\lambda}{\int_{360\text{ nm}}^{830\text{ nm}} P_e^S(\lambda) \cdot V(\lambda) \cdot d\lambda}$$

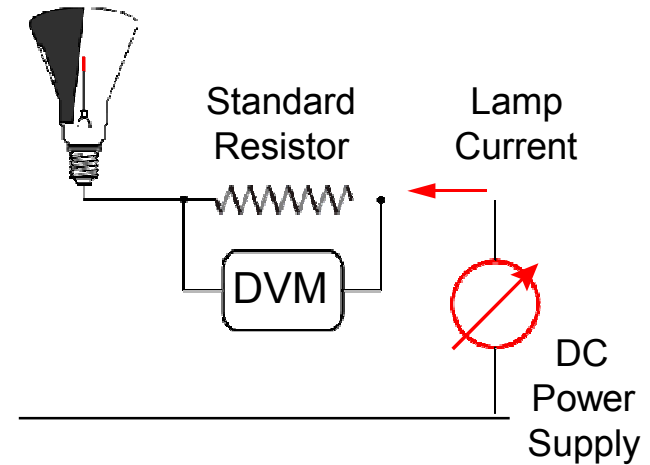
- Responsivity Drift (what is constant over the 2 months of measurements?)
- Signal Noise (fluctuations)
- Alignment to optical axis (Y-Z centre)
- Alignment to optical axis (Y-Z angular)



CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Measurements at pilot (NRC)

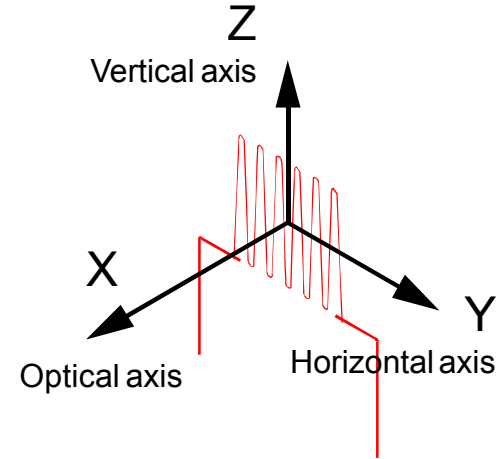
- Sources of Uncertainty • $u(V_{i,j,m})$
 - NRC Optical Coordinate System (2)
 - NRC Photometer (5)
 - Participant Lamps
 - Electrical (4)
 - Standard Resistor calibration (lamp current measurement)
 - DVM voltage calibration (lamp current measurement)
 - Lamp current setting
 - Lamp current fluctuations
 - % change in lamp output is approximately 7 times % change in lamp current
 - Optical (3)
 - Photometric (1)



CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Measurements at pilot (NRC)

- Sources of Uncertainty • $u(V_{i,j,m})$
 - NRC Optical Coordinate System (2)
 - NRC Photometer (5)
 - Participant Lamps
 - Electrical (4)
 - Optical (3)
 - Vertical filament plane (parallel to Z-axis, rotation about Y-axis)
 - Vertical filament plane (parallel to Y-axis, rotation about Z-axis)
 - Lamp to photometer distance (photometer signal $\propto \frac{1}{d^2}$)
 - Photometric (1)
 - Lamp output fluctuations



CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Measurements at pilot (NRC)

- Sources of Uncertainty • Summary • $u(V_{i,j,m})$
 - 4 predominant sources of uncertainty:

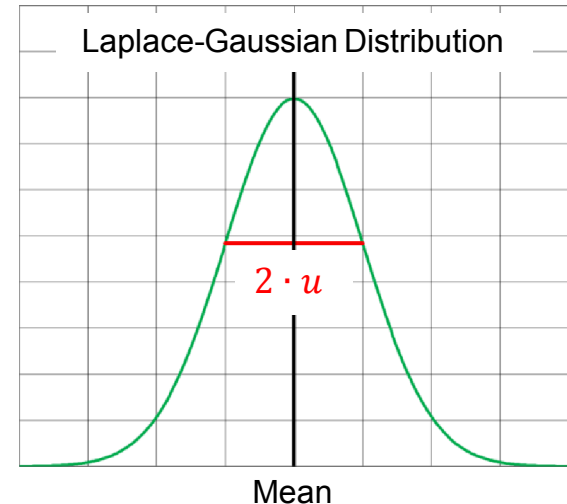
Source of Uncertainty	Type	Relative Standard Uncertainty
NRC Photometer		
Spectral Mismatch Error	B	0.01%
Responsivity Drift	A	0.05%
Participant Lamps (optical)		
Vertical Filament Plane	A	0.01%
Lamp-to-Photometer distance	A	0.03%

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Data Analysis

- Sources of Uncertainty
 - 3 sources:
 - Participant LI values
 - NRC comparison measurements
 - Artifact repeatability at NRC
 - Kinds of uncertainties:
 - Type A
 - Type B
 - Uncorrelated
 - Correlated

$$R_m^{i,j} = \frac{I_{v(i,j)}}{V_{i,j}} \left(\frac{\text{cd}}{\text{volt}} \right)$$



CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Data Analysis

• Sources of Uncertainty • combination of uncertainties*

• Kinds of Uncertainties:

- Type A
- Type B
- Uncorrelated (u_c)
- Correlated (u_c)

$$Q = f(x_i)$$

$$u_{uc}^2(Q) = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 \cdot u_{uc}^2(x_i)$$

$$u_c^2(Q) = \left[\sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right) \cdot u_c(x_i) \right]^2$$

$$u_{tot}^2(Q) = u_{uc}^2(Q) + u_c^2(Q)$$

- *GUM, *Guides to the expression of uncertainty in measurement*, JCGM 100:2008, etc. www.bipm.org

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Data Analysis

- Sources of Uncertainty • combination of uncertainties • weighted mean

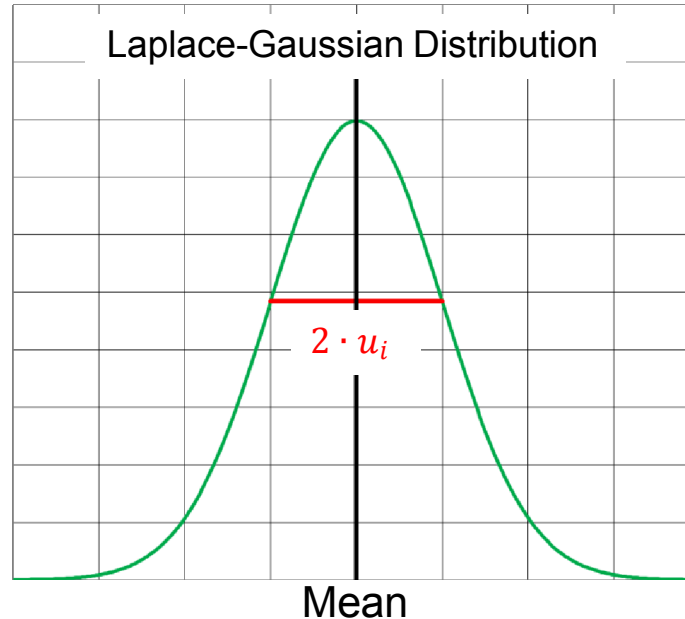
- Weights $W_i = \frac{1}{u_i^2}$

- Normalised $w_i = \frac{W_i}{\sum_{i=1}^n W_i}$

n

$$Q = \sum_{i=1}^n w_i \cdot x_i$$

$$\frac{\partial Q}{\partial x_i} = w_i$$



CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Data Analysis

- Sources of Uncertainty • combination of uncertainties • weighted mean

- Type A
- Type B
- Uncorrelated
- Correlated

Measured Quantity	Uncertainty		
	Type A	Type B	Combined
V_1	$u_A(V_1)$	$u_B(V_1)$	$u_A^2(V_1) + u_B^2(V_1)$
V_2	$u_A(V_2)$	$u_B(V_2)$	$u_A^2(V_2) + u_B^2(V_2)$
....			
V_n	$u_A(V_n)$	$u_B(V_n)$	$u_A^2(V_n) + u_B^2(V_n)$
$f(V_i) = \sum_{i=1}^n w_i V_i$	$u_A^2(f) = \sum_{i=1}^n w_i^2 u_A^2(V_i)$	$u_B^2(f) = \sum_{i=1}^n w_i^2 u_B^2(V_i)$	$u_A^2(f) + u_B^2(f)$
Weighted mean	Uncorrelated	Correlated	Combined

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

$$R_{i,j,m} = \frac{I_v(i,j)}{V_{i,j,m}} \left(\frac{\text{cd}}{\text{volt}} \right)$$

- Data analysis
 - Determine final NRC measurement value for each artifact: $R_{i,j} = R_{\{j,m\}_m} \sim 12 \times 6 = 72$ values
 - $u(R_{i,j})$ is a combination of NRC measurements (u_A and u_B), Participant (u_A and u_B) and lamp u_A
 - Determine final NRC measurement value for each participant: $R_i = R_{\{j,j\}_j} = 12$ values
 - $u(R_i)$ is a combination of the (u_A and u_B) components of $u(R_{i,j})$
- Comparison of participant SI candela realisations
 - KCRV (Key Comparison Reference Value)
 - Weighted mean with cut-off
 - Identification of 'outliers': deviation from KCRV greater than 6 times their uncertainty
 - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

- Comparison of participant SI candela realisations
 - KCRV (Key Comparison Reference Value)
 - Weighted mean with cut-off

median

$$u_{cut-off} = average(u_7 \text{ to } u_{12})$$

Participant Luminous Intensity uncertainty Relative standard values (ordered highest to lowest)	
unadjusted	adjusted $u_{adj}(NMI)$
u_1	u_1
u_2	u_2
....	
u_6	u_6
u_7	u_7
....	
u_j	u_j
u_k	u_{cutoff}
....	u_{cutoff}
u_{11}	u_{cutoff}
u_{12}	u_{cutoff}

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

- Comparison of participant SI candela realisations
 - KCRV (Key Comparison Reference Value)
 - Weighted mean with cut-off

$$u_{ad}^2(R_i) = u_{ad}^2(NMI) + u_{transfer}^2(R_i)$$

$$\text{weights } w_{i,adj} = \frac{1}{u_{ad}^2(R_i)}$$

$$\text{normalised } w_{i,adj} = \frac{w_{i,adj}^i}{\sum_j w_{i,adj}^j}$$

median

$u_{cut-off}$

Participant Luminous Intensity uncertainty Relative standard values (ordered highest to lowest)	
unadjusted	adjusted $u_{adj}(NMI)$
u_1	u_1
u_2	u_2
....	
u_6	u_6
u_7	u_7
....	
u_j	u_j
u_k	u_{cutoff}
....	u_{cutoff}
u_{11}	u_{cutoff}
u_{12}	u_{cutoff}

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

- Comparison of participant SI candela realisations
 - KCRV (Key Comparison Reference Value)
 - Weighted mean with cut-off

$$R_{KCRV} = \sum_{i=1}^n w_{i,adj} \cdot R_i \quad \left(\frac{\text{cd}}{\text{volt}} \right)$$

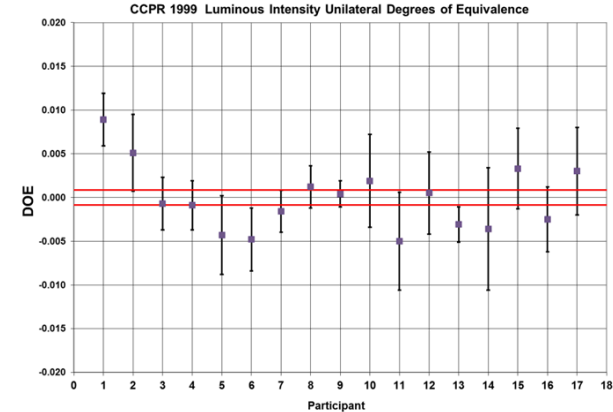
$$u^2(R_{KCRV}) = \sum_{i=1}^n w_{i,adj}^2 \cdot u^2(R_i) \quad (\text{uncorrelated})$$

Participant Luminous Intensity uncertainty Relative standard values (ordered highest to lowest)	
unadjusted	adjusted $u_{adj}(NMI)$
u_1	u_1
u_2	u_2
....	
u_6	u_6
u_7	u_7
....	
u_j	u_j
u_k	u_{cutoff}
....	u_{cutoff}
u_{11}	u_{cutoff}
u_{12}	u_{cutoff}

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

- Comparison of participant SI candela realisations
 - KCRV (Key Comparison Reference Value)
 - Weighted mean with cut-off
 - Identification of 'outliers': deviation from KCRV greater than 6 times their (k=1) uncertainty
 - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$



$$\chi^2_{observed} = \sum_{i=1}^n \frac{(R_i - R_{KCRV})^2}{u_{ad}^2(R_i)}$$

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

- Comparison of participant SI candela realisations
 - KCRV (Key Comparison Reference Value)
 - Weighted mean with cut-off
 - Identification of 'outliers': deviation from KCRV greater than 6 times their uncertainty
 - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$

IF $\chi^2_{observed} > \chi^2_{0.05}(\nu)$ (**inconsistent!**)

THEN add Mandel-Paule adjustment uncertainty s

$$u_{adj}^2(R_i) = u_{adj}^2(NMI) + u_{transfer}^2(R_i) + s^2$$

And REPEAT calculations with various s until 'consistent'

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

- Comparison of participant SI candela realisations
 - KCRV (Key Comparison Reference Value)
 - Weighted mean with cut-off
 - Identification of 'outliers': deviation from KCRV greater than 6 times their uncertainty
 - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$
 - Calculate the Unilateral Degrees of Equivalence (DOE): D_i

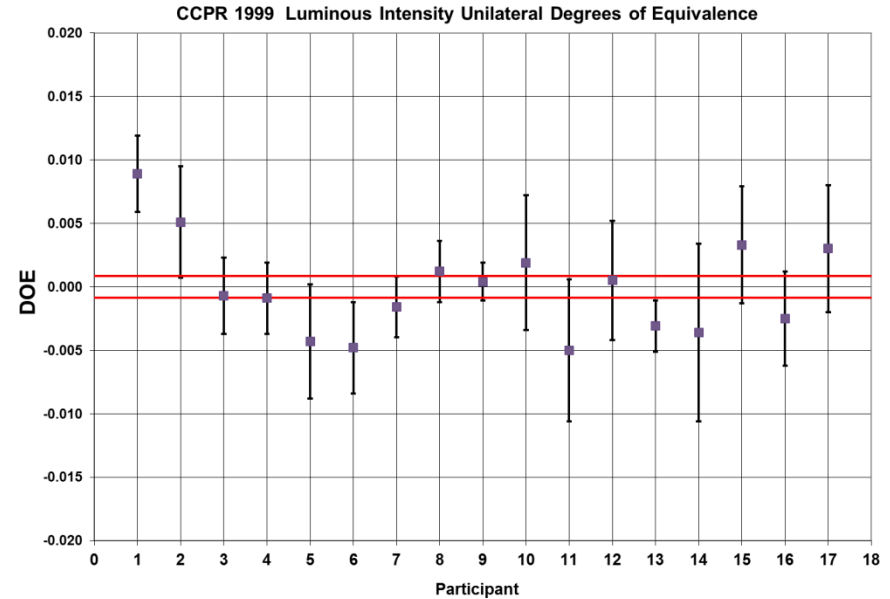
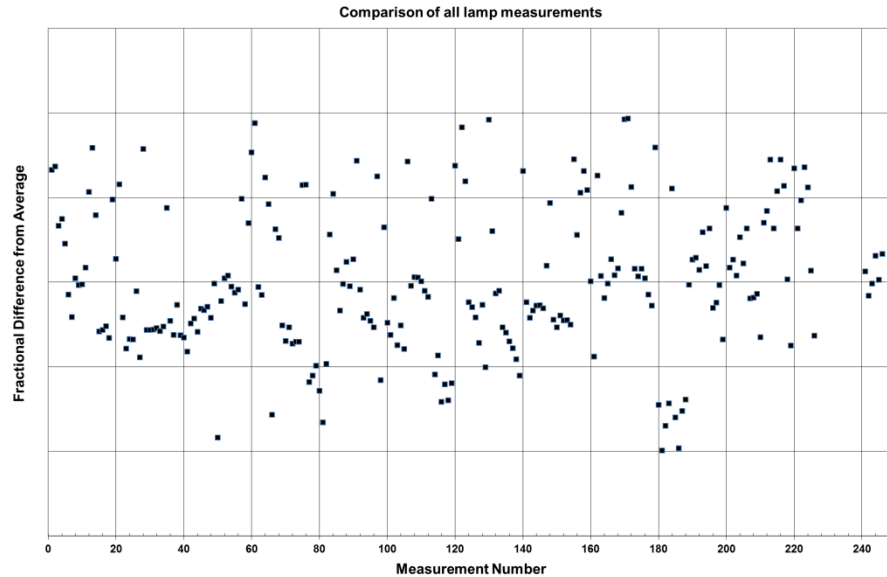
$$D_i = \frac{R_i - R_{KCRV}}{R_{KCRV}}$$

$$u_i^2 = u^2(R_i) + u^2(R_{KCRV}) - 2(w_i \cdot u^2(R_i))$$

R_i and R_{KCRV} are correlated

CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Data Analysis



CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

- Comparison of participant SI candela realisations
 - KCRV (Key Comparison Reference Value)
 - Weighted mean with cut-off
 - Identification of 'outliers': deviation from KCRV greater than 6 times their uncertainty
 - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$
 - Calculate the Unilateral Degrees of Equivalence (DOE)
 - Calculate the Bilateral Degrees of Equivalence

$$D_{i,j} = \frac{R_i - R_j}{R_{KCRV}}$$

$$u_{i,j}^2 = u^2(R_i) + u^2(R_j)$$

(R_i and R_j uncorrelated)

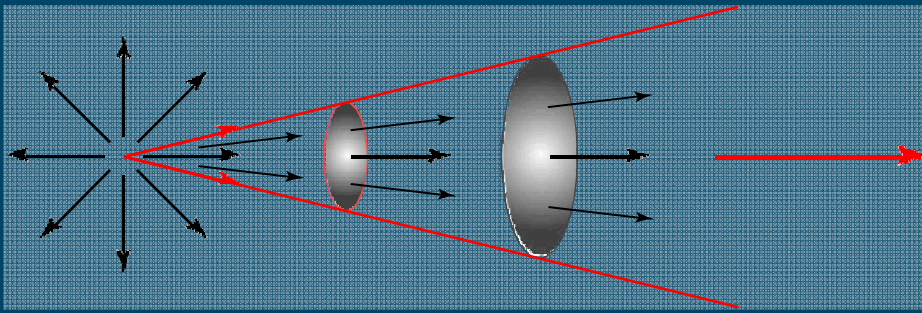
CCPR Key Comparison CCPR-K3.2014

Comparison Organisation

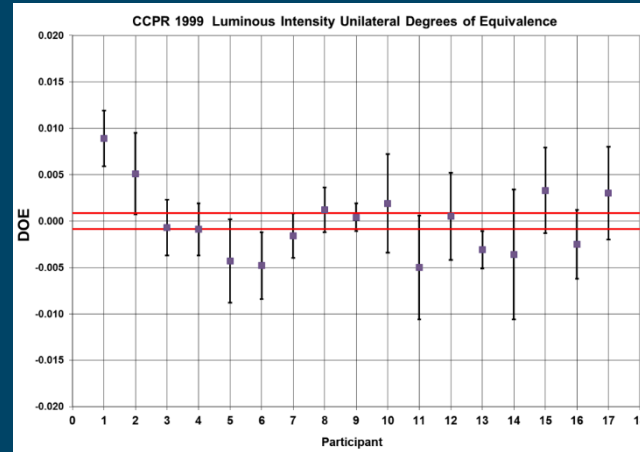
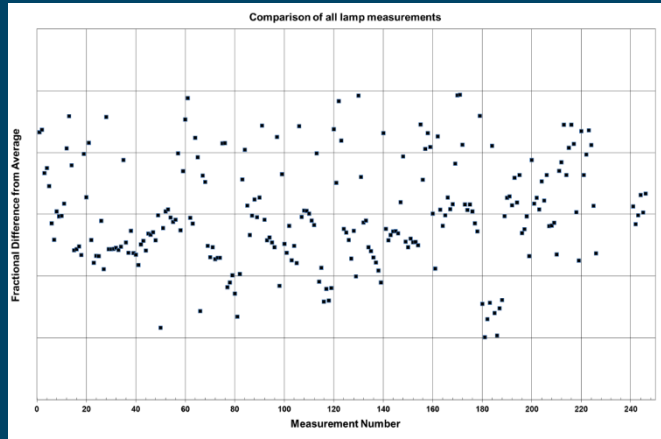
- Selection of participants, artifacts and protocol

Comparison Procedures

- Comparison measurements and measurement verification
- Data analysis and comparison of participant SI candela realisations
- Write the report
 - Draft A and any revisions, confidential to participants
 - Draft B to CCPR WG-KC for approval (and/or any revisions)
 - Approved Draft B to CCPR for approval
 - Final Report



CCPR KEY COMPARISON CCPR-K3.2014



ACKNOWLEDGEMENTS

W.S. Neil

R.J. Douglas

Éric Côté

J.C. Zwinkels

12 NMI participants

THANK YOU

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