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# PODDED PROPELLERS IN ICE: DYNAMOMETER CALIBRATIONS 

SR-2005-14

Matthew Prim

August 2005

### 0.1 SUMMARY

In order to interpret the electrical output signals from a dynamometer (dyno), precise calibration is required. Furthermore, when a system of multiple dynos is constructed, additional calibration must be done to account for interactional inaccuracies.

In order to properly calibrate a dyno system, a theoretical calculation of loads is performed with its results compared alongside the actual measured load values. This process was performed on the podded propeller setup as part of the "Podded Propellers in Ice" project.

The next, and most enlightening step is the construction of a calibration matrix using a linear least squares approximation. This matrix represents the transformation of measured load values to their adjusted accurate values. When applied to raw dyno data, the matrix yields properly adjusted data, accounting for inaccuracies due to multi-dyno interaction.

As expected, a certain degree of error is present in the matrix coefficients. This error is acceptable when expressed as a percentage of the dynamometer ranges.

The following information is currently being examined for the purposes of direct application in "Podded Propellers in Ice" test results.

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## 1 INTRODUCTION

Dynamometers (dynos), while very valuable scientific tools, require calibration in order to produce meaningful data output. Calibrations are performed by applying known forces in various configurations. The theoretical induced forces are calculated using statics methods and compared to the dyno readings. This process enables the construction of a transformation matrix. Contained in the matrix are the coefficients required to convert the dyno readings to meaningful force and moment values.

### 1.1 Factory Calibration

A dynamometer consists of a network of wires carrying an electrical current. As the wires are strained (due to a force/moment) the dimensions of the wire (cross-sectional area. length) are slightly altered, therefore changing the electrical signal by a small amount). This voltage change alone is of little use in a mechanical sense. But once we establish a calibration equation, this voltage reading becomes a direct indicator of applied force/moment. There exists an equation of the form

$$
\begin{aligned}
& \qquad F=C V+C_{O} \\
& \text { where } \mathrm{F}=\text { force applied } \\
& \mathrm{V}=\text { voltage output } \\
& \mathrm{C}=\text { calibration coefficient } \\
& \mathrm{Co}=\text { y-intercept (i.e. value of } \mathrm{F} \text { when } \mathrm{R}=0 \text { ) }
\end{aligned}
$$

This equation is determined by the manufacturer prior to distribution of the dyno for use in the field. For the purposes of the "Podded Propellers in Ice" study we require a system of three dynos, strategically located for optimum analysis. Once the dynos are arranged properly (section 2) a further calibration is required to account for affects due to the interaction of the dynos in a system (cross-talk etc.). This additional calibration is described in detail in the following sections.

## 2 SETUP

The propeller/shaft is fitted with 3 dynos - one attached to the blade, one on the forward end of the shaft, and one on the aft end of the shaft. As mentioned above each of these dynos were independently calibrated at the factory where they were fabricated.


Figure 1 - Single Dynamometer

After the system of dynos is put together to suit the needs of the project, further calibration is required in order to evaluate the behavior of the dynos as a system. Problems such as cross-talk are common and must be accounted for though calibration.

The components are combined to form the system seen below:


Figure 2 - Dynamometer system setup

During calibrations, the following frames of reference were used for each component:


Figure 3 - Frames of Reference for Dynos

It is essential to note that the propeller blade is rotated to various angles $(\theta)$ during the calibrations. Therefore the above blade reference frame is not fixed relative to the shaft; rather it is rotating about its own y-axis as the blade rotates.

## 3 STATICS CALCULATIONS

In order to calibrate the dyno system, the measured forces/moments must be compared to the theoretical (ideal) values. During calibration, known forces are applied at the center of the propeller, these forces must be transferred to the dyno locations using theoretical statics methods. The six principal equations of statics are:

| $\Sigma \mathrm{F}_{\mathrm{x}}=0$ | $\Sigma \mathrm{M}_{\mathrm{x}}=0$ |
| :--- | :--- |
| $\Sigma \mathrm{~F}_{\mathrm{y}}=0$ | $\Sigma \mathrm{M}_{\mathrm{y}}=0$ |
| $\Sigma \mathrm{~F}_{\mathrm{z}}=0$ | $\Sigma \mathrm{M}_{\mathrm{z}}=0$ |

Using these six relationships along with reasonable simplifying assumptions such as rigid body mechanics, the theoretical values can be found for the forward and aft dynos and compared to the actual measured loads.

The blade dyno presents a different challenge in that it is rotating, thus trigonometric relationships are employed to describe the forces and moments as functions of the angle of rotation ( $\theta$ ). See appendix $A$ for the comprehensive data/calculation spreadsheet. The results for the statics calculations are included in the following sections (Note that subscripts "в,F,А" represent the blade, forward, and aft dyno values respectively. A force with no subscript represents an applied load during calibration):

### 3.1 Blade Dyno

After calculation, the blade dyno transformation equations (equations which transfer loads from applied reference frame to blade reference frame) were found to be:
$\mathrm{Fx}_{\mathrm{B}}=-\mathrm{F}_{\mathrm{x}} \operatorname{Cos} \theta-\mathrm{F}_{\mathrm{y}} \operatorname{Sin} \theta$
$\mathrm{Fy}_{\mathrm{B}}=\mathrm{F}_{\mathrm{z}}$
$\mathrm{Fz}_{\mathrm{B}}=-\mathrm{F}_{\mathrm{y}} \operatorname{Cos} \theta+\mathrm{F}_{\mathrm{x}} \operatorname{Sin} \theta$
$\mathrm{Mx}_{\mathrm{B}}=\mathrm{M}_{\mathrm{y}} \operatorname{Cos} \theta+\mathrm{M}_{\mathrm{x}} \operatorname{Sin} \theta$
Mув $=\mathrm{M}_{\mathrm{z}}$
$\mathrm{Mz}_{\mathrm{B}}=\mathrm{M}_{\mathrm{y}} \operatorname{Cos} \theta-\mathrm{M}_{\mathrm{x}} \operatorname{Sin} \theta$

As expected, a certain degree of error is associated with these calculations when compared to the measured values. These errors are:

|  | Blade  <br> Fx Fy |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Averages Difference | 0.970 | 0.312 | 0.948 | 0.721 | 0.231 | 0.077 |
| Max Difference | 55.667 | 57.420 | 94.712 | 21.173 | 13.241 | 16.819 |
| Dyno Range | 2780.000 | 2540.000 | 4490.000 | 112.900 | 113.000 | 113.700 |
| Max Error \% | 2.002 | 2.261 | 2.109 | 18.754 | 11.718 | 14.792 |
| Average Error\% | 0.035 | 0.012 | 0.021 | 0.638 | 0.204 | 0.068 |

Several max error values create concern for accuracy in the test. Fortunately, the average error values seem to fall into an acceptable range.

### 3.2 Forward Dyno

The forward dyno transformation equations are:
$\mathrm{Fx}_{\mathrm{F}}=-1.2871 \mathrm{~F}_{\mathrm{x}}-2.215 \mathrm{M}_{\mathrm{y}}$
$\mathrm{Fy}_{\mathrm{F}}=2.218 \mathrm{M}_{\mathrm{x}}$
$\mathrm{Fz}_{\mathrm{F}} \approx-\mathrm{F}_{\mathrm{z}} \quad$ (approx.)
$\mathrm{Mx}_{\mathrm{F}}=0.107 \mathrm{M}_{\mathrm{x}}$
$\mathrm{MyF}_{\mathrm{F}}=0.0621 \mathrm{~F}_{\mathrm{x}}+0.107 \mathrm{M}_{\mathrm{y}}$
$\mathrm{Mz}_{\mathrm{A}}=0$ (shaft is free to rotate)
Notice that in this case a translation of forces is required as apposed to the rotation seen in section 3.1.

Forward dyno error:

|  | Forward |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  | Fx | Fy | Fz | Mx | My |
|  |  |  |  |  |  |
| Averages Difference | -3.754 | -0.965 | -10.295 | -0.049 | 0.465 |
| Max Difference | 209.113 | 45.965 | 79.921 | 3.229 | 11.117 |
| Dyno Range | 2690.0 | 2700.0 | 4490.0 | 121.4 | 105.9 |
| Max Error \% | 7.774 | 1.702 | 1.780 | 2.660 | 10.498 |
| Average Error\% | $\mathbf{0 . 1 4 0}$ | $\mathbf{0 . 0 3 6}$ | $\mathbf{0 . 2 2 9}$ | $\mathbf{0 . 0 4 1}$ | $\mathbf{0 . 4 3 9}$ |

### 3.3 Aft Dyno

Aft dyno transformation equations:
$\mathrm{Fx}_{\mathrm{A}}=0.2871 \mathrm{~F}_{\mathrm{x}}+2.215 \mathrm{M}_{\mathrm{y}}$
$\mathrm{Fy}_{\mathrm{A}}=2.218 \mathrm{M}_{\mathrm{x}}$
$\mathrm{Fz}_{\mathrm{A}}=$ bizarre readings (data not considered)
$\mathrm{Mx}_{\mathrm{A}}=0.1492 \mathrm{M}_{\mathrm{x}}$
$\mathrm{My}_{\mathrm{A}}=$ not functioning properly (data not considered)
$\mathrm{Mz}_{\mathrm{A}}=0$ (shaft is free to rotate)
With error:

$$
\begin{array}{lll}
\text { Aft } \\
\text { Fx } & \text { Fy } & M x
\end{array}
$$

| Averages Difference | 2.939 | 0.577 | 0.020 |
| :--- | ---: | ---: | ---: |
| Max Difference | 59.582 | 37.244 | 2.429 |
| Dyno Range | 2230.0 | 2240.0 | 56.4 |
| Max Error \% | 2.672 | 1.663 | 4.306 |
| Average Error\% | $\mathbf{0 . 1 3 2}$ | $\mathbf{0 . 0 2 6}$ | $\mathbf{0 . 0 3 6}$ |

### 3.4 Shaft Torque

Shaft Torque $=\mathrm{M}_{\mathrm{z}}$
The error here is also fairly reasonable, yet perhaps more inconsistent than the blade dyno values:

## Torque

Averages Difference 0.269
Max Difference $\quad 10.769$
Dyno Range $\quad 165.1$
Max Error \% 6.523
Average Error\% 0.163

## 4 CORRELATION GRAPHS

When theoretical values are graphed against measured values, a trend should appear indicating that the measured values are reasonable and valid. Ideally the graph would have a slope of 1.0 and a correlation factor of 1.0 as seen in the theoretical example below :


As expected, this perfect trend does not appear in "real world" data. Instead, less dramatic, yet significant trends are sought in order to measure the degree of resemblance
between measured and calculated (theoretical) data. A realistic trend may resemble the following:


See appendix $B$ for the blade dyno, forward dyno, aft dyno, and shaft torque graphs constructed from measured and calculated data.

## 5 THE CALIBRATION MATRIX

Denoting the number of calibration data as m, the calculated theoretical loads as P, and the measured loads as R, we are able to construct a calibration matrix. This matrix contains coefficients which can be used directly in the field to produce sensible, useful data output from the dynos.

### 5.1 Linear Least Squares Method

The linear least squares method was employed in the construction of the calibration matrix. Two slightly different forms were explored which will be discussed in the following two sections.

### 5.1.1 MatLab least squares function method

The first method used to calculate the calibration matrices involves the explicit least squares function in MatLab. This is an iterative process, ideal for computer use, in which a tolerance is specified. MatLab iterates the procedure until the required tolerance (accuracy) is achieved. In basic form the code reads:
X = LSQR(A,B, TOL, MAXIT)

Where the matrix algebra equation

$$
\mathrm{AX}=\mathrm{B}
$$

must be satisfied by solving for X . The tolerance is assigned to TOL and the maximum number of iterations is assigned to MAXIT. The finished code reads as follows:

```
P_loads_12ch
R_loads_12ch
F = P';
Amean = R';
N = size(F,1);
Imin = 1;
Imax = 920;
tol = 0.0001;
maxiter = 20;
for irow = 1:9
    x = lsqr(Amean(Imin:Imax,1:9),F(Imin:Imax,irow),tol,maxiter);
    CalMtx(irow,:) = x';
end
CalMtx = inv(CalMtx)
```

For an in-depth explanation of the MatLab function "LSQR", consult a MatLab manual. When applied to the blade dyno case with a 6-channel system where P represents calculated loads and R represents measured loads we have:

| $\mathrm{X}=$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{F}_{\mathbf{x}}$ | $\mathbf{F}_{\mathbf{y}}$ | $\mathbf{F}_{\mathbf{z}}$ | $\mathbf{M}_{\mathbf{x}}$ | $\mathbf{M}_{\mathbf{y}}$ | $\mathbf{M}_{\mathbf{z}}$ |
| 1.2739 | -0.0312 | 0.0509 | 0.7771 | 1.1262 | -0.0570 |
| -0.0580 | 1.0225 | -0.0005 | -0.2578 | -0.4933 | 0.0035 |
| -0.3046 | 0.0659 | 1.0945 | 0.8037 | -1.6174 | 0.5645 |
| 0.0122 | 0.0451 | 0.0151 | 1.7710 | -0.0694 | 0.0325 |
| 0.8529 | -0.0318 | 0.0254 | 0.2649 | 5.6040 | -0.0491 |
| 0.2490 | -0.0093 | 0.0071 | 0.0970 | 1.2804 | 0.6257 |

The 9-channel forward/aft/torque matrix appears as
$\mathrm{X}=$
1.0e+003 *

| Forward |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{F}_{\mathrm{x}}$ | $\mathrm{F}_{\mathrm{y}}$ |  |  |  | $\mathrm{F}_{\mathrm{x}}$ |  |  |  |
| $\mathbf{F}_{\mathbf{x}}-0.1054$ |  | 0.0001 |  |  | 0.000 |  |  |  |
| $\mathbf{F}_{\mathbf{y}}-0.0419$ | 0.0061 | 0.0000 | -0.1462 | -0.8677 | 0.0000 | -0.0080 | 0.0930 | 0.0003 |
| $\mathbf{F}_{\mathbf{z}} 0.3319$ | -0.0050 | 0.0009 | 0.3759 | 6.8807 | 0.0003 | 0.026 | -0.223 | . 0006 |
| $\mathbf{M}_{\mathbf{x}}-0.0026$ | 0.0003 | . 0000 | -0.0066 | -0.0530 | 0.0000 | -0.0004 | 0.0049 | 0.0000 |
| $\mathbf{M}_{\mathbf{y}}-0.0019$ | 0.000 | 0.0000 | -0.0216 | -0.0392 | -0.00 | -0.0012 | 0.0147 | 0.0000 |
| $\mathrm{F}_{\mathrm{x}} 0.1774$ | -0.0046 | -0.0000 | 1993 | .678 | 0.0004 | 0.013 | . 140 | 0.0005 |
| $\mathbf{F}_{\mathbf{y}} 0.0897$ | -0.0019 | 0.0000 | 0.1230 | 1.8577 | 0.0003 | 0.0082 | 060 | -0.0004 |
| $\mathbf{M}_{\mathbf{x}} 0.0090$ | -0.0005 | -0.0000 | 0.01 | 0.1859 | -0.0000 | 0.000 | 0082 | 0.0000 |
| Trq.-0.0167 | 0.0003 | -0.0000 | -0.0200 | -0.3 | -0.0000 | -0.00 | 0.126 | 0.00 |

This method produced error values which create concern. For this reason, we explore a second approach.

### 5.1.2 Least squares matrix algebra method

The second method is also a least squares approximation, but taken from first principles. The following matrix relationship is used to solve for matrix " C ":
$C_{6,6}=\left(P_{6, m}\right)\left(R_{m, 6}\right)^{T}\left[\left(R_{6, m}\right)\left(R_{m, 6}\right)^{T}\right]^{-1}$
This method was found to yield the most meaningful results. Therefore this method will be employed in full and discussed in detail. The above equation applies to a 6-channel system, which expands to:

$$
\left[\begin{array}{lll}
C_{1,1} & \ldots & C_{1,6} \\
C_{2,1} & \ldots & C_{2,6} \\
C_{3,1} & \ldots & C_{3,6} \\
C_{4,1} & \ldots & C_{4,6} \\
C_{5,1} & \ldots & C_{5,6} \\
C_{6,1} & \ldots & C_{6,6}
\end{array}\right]=\left[\begin{array}{lll}
F_{1,1} & \ldots & F_{1, m} \\
F_{2,1} & \ldots & F_{2, m} \\
F_{3,1} & \ldots & F_{3, m} \\
F_{4,1} & \ldots & F_{4, m} \\
F_{5,1} & \ldots & F_{5, m} \\
F_{6,1} & \ldots & F_{6, m}
\end{array}\right] \bullet\left[\begin{array}{lll}
R_{1,1} & \ldots & R_{1, m} \\
R_{2,1} & \ldots & R_{2, m} \\
R_{3,1} & \ldots & R_{3, m} \\
R_{4,1} & \ldots & R_{4, m} \\
R_{5,1} & \ldots & R_{5, m} \\
R_{6,1} & \ldots & R_{6, m}
\end{array}\right] \bullet\left[\left[\begin{array}{lll}
R_{1,1} & \ldots & R_{1, m} \\
R_{2,1} & \ldots & R_{2, m} \\
R_{3,1} & \ldots & R_{3, m} \\
R_{4,1} & \ldots & R_{4, m} \\
R_{5,1} & \ldots & R_{5, m} \\
R_{6,1} & \ldots & R_{6, m}
\end{array}\right] \bullet\left[\begin{array}{lll}
R_{1,1} & \ldots & R_{1, m} \\
R_{2,1} & \ldots & R_{2, m} \\
R_{3,1} & \ldots & R_{3, m} \\
R_{4,1} & \ldots & R_{4, m} \\
R_{5,1} & \ldots & R_{5, m} \\
R_{6,1} & \ldots & R_{6, m}
\end{array}\right]^{T}\right]^{T}
$$

A $6 \times 6$ matrix is therefore composed of calibration coefficients $\mathrm{c}_{\mathrm{i}, \mathrm{j}}$. These entries represent the relative contributions of each channel to one and other, thus:
$\mathrm{F}_{\mathrm{x}(\text { actual })}=\mathrm{F}_{\mathrm{x}} \mathrm{c}_{1,1}+\mathrm{F}_{\mathrm{y}} \mathrm{c}_{1,2}+\mathrm{F}_{\mathrm{z}} \mathrm{c}_{1,3}+\mathrm{M}_{\mathrm{x}} \mathrm{c}_{1,4}+\mathrm{M}_{\mathrm{y}} \mathrm{c}_{1,5}+\mathrm{M}_{\mathrm{z}} \mathrm{c}_{1,6}$

Ideally, the calibration matrix should resemble the 6x6 identity matrix

| 1 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 |

Due to various error sources and load cell cross talk, as well as unavoidable secondary stresses, this is not the case.

After the matrix is constructed, residual values are calculated. These residual values are obtained using the equation
Residual = P - (C* R)
as a measure of the accuracy of calibration matrix C.

It is important to note that at early stages of calculations, significant amounts of data appeared to be "outlying data" and created difficulty in the formation of calibration matrices. For this reason, some data has been disregarded to enable MatLab to properly construct the matrices. With this said, the remaining data represents the initial population fairly well and is indicative of the correct, accurate values. As mentioned earlier, abnormal readings are expected due to various error sources.

When blade dyno data is used, the above least squares expression produces the following matrix of coefficients:

| $\mathrm{F}_{\mathrm{x}}$ | $\mathrm{F}_{\mathrm{y}}$ | $\mathrm{F}_{4}$ | $\mathbf{M}_{\text {x }}$ | M ${ }_{\text {y }}$ | $\mathrm{M}_{\mathbf{z}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{F}_{\mathbf{x}} 0.9112$ | 0.0402 | -0.0327 | -0.3547 | -0.1931 | 0.1131 |
| $\mathrm{F}_{\mathrm{y}}$-0.0179 | 0.9743 | -0.0027 | 0.1375 | 0.0913 | -0.0045 |
| $\mathrm{F}_{\mathbf{z}} 0.0958$ | -0.0289 | 0.9113 | -0.4794 | 0.4081 | -0.7564 |
| $\mathbf{M}_{\mathbf{x}}-0.0106$ | -0.0248 | -0.0075 | 0.5688 | 0.0101 | -0.0229 |
| $\mathbf{M}_{\mathbf{y}}-0.1386$ | 0.0008 | 0.0012 | 0.0294 | 0.2069 | 0.0010 |
| $\mathbf{M}_{\mathbf{z}}-0.0767$ | 0.0013 | 0.0013 | -0.0014 | -0.3413 | 1.5634 |

Residual values for the blade dyno are:

```
Residual Minimum =
    \(\begin{array}{llllll}0.0019 & 0.0001 & 0.0045 & 0.0000 & 0.0000 & 0.0000\end{array}\)
Residual Maximum =
    \(\begin{array}{llllll}50.0289 & 57.7510 & 86.8366 & 12.6245 & 13.9002 & 13.1683\end{array}\)
Residual Mean =
    \(-0.6048 \quad 0.2696 \quad 1.2362 \quad 0.5620 \quad 0.1434 \quad-0.3020\)
(0.021\%) ( 0.0101\%) (0.013\%)(1.095\%)(0.127\%)(0.266\%)
Residual Standard Deviation =
    \(\begin{array}{llllll}8.1404 & 5.1960 & 8.9139 & 3.6143 & 1.6967 & 3.9505\end{array}\)
```

Graphs of residual values can be found in appendix C. The above values are quite reasonable and reflect the expected trend of dominant diagonal coefficients. Continuing with this principal, the forward dyno transformation matrix (excluding $\mathrm{M}_{\mathrm{z}}$ for accuracy \{see below\}) is:

| $\mathrm{C}=$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{F}_{\mathrm{x}}$ | $\mathrm{F}_{\mathbf{y}}$ | $\mathrm{F}_{\mathbf{z}}$ | $\mathbf{M}_{\text {x }}$ | M ${ }_{\text {y }}$ |
| $\mathrm{F}_{\mathrm{x}} 1.4433$ | 1.3890 | -0.0274 | -23.9980 | 7.7752 |
| $\mathbf{F}_{\mathbf{y}} \mathbf{-} 0.1305$ | 0.2085 | -0.0019 | 7.6337 | -3.3540 |
| $\mathrm{F}_{\mathbf{z}} 0.1444$ | -2.0040 | 1.1347 | 35.0696 | 2.8613 |
| $\mathbf{M}_{\mathbf{x}}-0.0047$ | 0.0048 | 0.0001 | 0.4329 | -0.1263 |
| $\mathbf{M}_{\mathbf{Y}} \mathbf{- 0 . 0 6 9 3}$ | -0.0669 | 0.0013 | 1.1547 | -0.3682 |

With residual values:

```
Residual Minimum =
    0.0249}00.0048 0.1163 0.0001 0.0004 
Residual Maximum =
    99.7688
Residual Mean =
    -0.1109 0.2802 -7.3671 0.0179 0.0047
Residual Standard Deviation =
    26.5143 8.3904 25.5028
```

It is clear from these values that the forward dyno readings show less consistent trends then their blade dyno counterparts. As stated above, the $\mathrm{M}_{\mathrm{z}}$ channel is not included in the matrix due to its inaccuracy. In theory, all $\mathrm{M}_{\mathrm{z}}$ values for both the forward and aft dynos have a magnitude of zero because the shaft is free to rotate within the dyno.
Unfortunately the measured values are non-zero and thus create difficulties in the matrix construction in MatLab. The inaccuracies involved with the forward and aft dynos are due to many factors which are unavoidable in the real world. These include, slight deflections in the material that is assumed rigid, internal residual stresses, and those error sources mentioned above. For this reason, a more in-depth analysis of force/moment relationships has been conducted yielding several calibration matrices in order to better understand their interdependence. If shaft torque is included in the above matrix, the coefficients become:

```
C=
\begin{tabular}{lrrrrrr} 
& \(\mathbf{F}_{\mathbf{x}}\) & \multicolumn{1}{c}{\(\mathbf{F}_{\mathbf{y}}\)} & \multicolumn{1}{c}{\(\mathbf{F}_{\mathbf{z}}\)} & \multicolumn{1}{c}{\(\mathbf{M}_{\mathbf{x}}\)} & \multicolumn{1}{c}{\(\mathbf{M}_{\mathbf{y}}\)} & \multicolumn{1}{c}{ Torque } \\
\(\mathbf{F}_{\mathbf{x}}\) & 1.4531 & 1.5401 & -0.0273 & -26.1735 & 7.9559 & -0.1806 \\
\(\mathbf{F}_{\mathbf{y}}\) & -0.1262 & 0.2749 & -0.0018 & 6.6783 & -3.2747 & -0.0793 \\
\(\mathbf{F}_{\mathbf{z}}\) & 0.1360 & -2.1355 & 1.1346 & 36.9626 & 2.7041 & 0.1572 \\
\(\mathbf{M}_{\mathbf{x}}\) & -0.0046 & 0.0069 & 0.0001 & 0.4033 & -0.1238 & -0.0025 \\
\(\mathbf{M}_{\mathbf{y}}\) & -0.0698 & -0.0743 & 0.0013 & 1.2599 & -0.3770 & 0.0087 \\
\(\mathbf{T r q}^{2}\) & 0.0052 & 0.0979 & -0.0000 & -1.4008 & 0.0998 & 1.0294
\end{tabular}
Residual Minimum =
    0.0178
Residual Maximum =
    99.5213 38.8239 74.2615 1.9155}111.0406 10.6234
```

| Residual Mean $=$ |
| :--- |
| $\quad-0.1354 \quad 0.2695$ |$\quad-7.3458 \quad 0.0175$ 0.0059 00.3534

The following 9x9 matrix shows the relationship between forward dyno, aft dyno and torque dyno readings, along with shaft torque:
$\mathrm{C}=$

| $\mathbf{F}_{\mathbf{x}}$ | $\mathbf{F}_{\mathbf{y}}$ | $\mathbf{F}_{\mathbf{z}}$ | $\mathbf{M}_{\mathrm{x}}$ | $\mathbf{M}_{\mathbf{y}}$ | $\mathbf{F}_{\mathrm{x}}$ | $\mathbf{F}_{\mathbf{y}}$ | $\mathbf{M}_{\mathrm{x}}$ | Torque |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

$\begin{array}{llllllllll}\mathbf{F}_{\mathbf{x}} & 1.4857 & 1.6034 & 0.0059 & -32.9238 & 5.0791 & 0.8946 & -3.1520 & 29.4884 & -0.2444\end{array}$
$\begin{array}{lllllllll}\mathbf{F}_{\mathbf{y}} & -0.1233 & 0.3440 & 0.0000 & 6.6181 & -3.1690 & 0.0132 & -0.0145 & 2.3200\end{array}-0.0720$
$\begin{array}{llllllllll}\mathbf{F}_{\mathbf{z}} & 0.1330 & -3.1018 & 1.1256 & 32.2797 & 3.9780 & -0.9278 & -1.5089 & -12.9292 & -0.2891\end{array}$
$\begin{array}{lllllllll}\mathbf{M}_{\mathbf{x}} & -0.0043 & 0.0095 & 0.0002 & 0.3810 & -0.1074 & -0.0024 & -0.0079 & 0.1785\end{array}-0.0032$
$\mathbf{M}_{\mathbf{y}}-0.0714 \quad-0.0772-0.00031 .5832-0.2402$-0.0426 $\begin{array}{lllllll}0.1512 & -1.4156 & 0.0118\end{array}$
$\begin{array}{lllllllll}\mathbf{F}_{\mathbf{x}} & -0.2813 & -2.6136 & -0.0249 & 43.3719 & -0.4739 & 0.1497 & 1.6107 & -22.1049\end{array} 0.3081$
$\begin{array}{lllllllll}\mathbf{F}_{\mathbf{y}} & -0.0079 & -0.0721 & -0.0080 & -6.5431 & 0.9888 & -0.2324 & 0.6787 & -3.5605\end{array} 0.1184$
$\begin{array}{lllllllll}\mathbf{M}_{\mathbf{x}} & 0.0036 & 0.0068 & -0.0005 & -0.5646 & 0.1670 & -0.0184 & 0.0199 & 0.2083\end{array} \quad 0.0061$
$\begin{array}{lllllllll}\text { Trq } 0.0025 & 0.1623 & 0.0001 & -1.6592 & -0.1217 & 0.0526 & 0.0714 & 0.1574 & 1.0211\end{array}$


Excluding moments, the above matrix appears as

| Forward |  |  | Aft |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{F}_{\mathrm{x}}$ | $\mathrm{F}_{\mathrm{y}}$ | $\mathrm{F}_{\mathrm{z}}$ | $\mathrm{F}_{\mathrm{x}}$ | $\mathrm{F}_{\mathrm{y}}$ |
| $\mathrm{F}_{\mathrm{x}}$ | 1.2917 | -0.4410 | -0.0066 | 1.1054 | -1.4603 |
| $\mathrm{F}_{\mathrm{y}}$ | 0.0142 | 0.7231 | -0.0087 | -0.0210 | 0.1497 |
| $\mathrm{F}_{\mathrm{z}}$ | 0.0135 | -1.0781 | 1.1299 | -0.6266 | -2.1908 |
| $\mathrm{F}_{\mathrm{x}}$ | -0.2321 | -0.0290 | -0.0167 | 0.1638 | 0.1017 |
| $\mathrm{F}_{\mathrm{y}}$ | -0.0569 | -0.4585 | -0.0002 | -0.2570 | 0.4237 |

```
Residual Minimum \(=\)
    \(\begin{array}{lllll}0.0006 & 0.0009 & 0.0326 & 0.0016 & 0.0008\end{array}\)
Residual Maximum =
        \(80.6085 \quad 38.9721 \quad 67.2584 \quad 64.5211 \quad 37.8689\)
Residual Mean =
    \(-0.8733 \quad 0.6096-8.0169-1.9048 \quad-0.7813\)
Residual Standard Deviation =
    \(\begin{array}{lllll}25.4299 & 8.5219 & 25.0765 & 16.0090 & 8.9337\end{array}\)
```

Finally, the inclusion of Shaft Torque yields

| Forward | Aft |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{F}_{\mathrm{x}}$ | $\mathrm{F}_{\mathrm{y}}$ | $\mathrm{F}_{\mathbf{z}}$ | $\mathrm{F}_{\mathrm{x}}$ | $\mathrm{F}_{\mathrm{y}}$ | Torque |
| $\mathbf{F}_{\mathbf{x}} 1.2957$ | -0.4145 | -0.0075 | 1.1502 | -1.3914 | 0.1232 |
| $\mathrm{F}_{\mathbf{y}} 0.0096$ | 0.6926 | -0.0076 | -0.0728 | 0.0701 | -0.1424 |
| $\mathrm{F}_{\mathbf{z}}-0.0021$ | -1.1825 | 1.1335 | -0.8038 | -2.4628 | -0.4865 |
| $\mathrm{F}_{\mathrm{x}}$-0.2337 | -0.0401 | -0.0163 | 0.1449 | 0.0728 | -0.0517 |
| $\mathrm{F}_{\mathrm{y}}$-0.0514 | -0.4216 | -0.0015 | -0.1944 | 0.5199 | 0.1720 |
| Trq0.0017 | 0.0712 | 0.0002 | 0.0355 | 0.1122 | 1.0569 |

Residual Minimum = $\begin{array}{llllll}0.0190 & 0.0021 & 0.0296 & 0.0120 & 0.0007 & 0.0011\end{array}$
Residual Maximum = $\begin{array}{lllllll}79.9189 & 39.3595 & 66.0330 & 64.6480 & 37.7489 & 10.4638\end{array}$
Residual Mean $=$
$\begin{array}{llllll}-0.8476 & 0.5799 & -8.1185 & -1.9156 & -0.7454 & 0.3621\end{array}$
Residual Standard Deviation =
$\begin{array}{llllll}25.4213 & 8.4860 & 24.8928 & 16.0051 & 8.8839 & 2.7049\end{array}$

## 6 CONCLUSIONS

The blade dyno, as expected, displays the strongest trends between calculated and measured values. The transformation matrix method of section 5.1.2 produces reasonable coefficients which will likely be used directly in determining podded propeller ice loadings in ice tank tests. The MatLab "LSQR" function method of section 5.1.1 appears to be more sensitive to outlying data and therefore reflects less consistent coefficients. As stated above the reasonable accuracy of these blade dyno values are expected because minimal force transformation is required from the location of applied force to the blade dyno reference frame. It is simply a mater of rotation with respect to $\theta$.

The forward and aft dynos have presented a considerably larger task in establishing a suitable trend. As mentioned above, this is largely due to unavoidable inaccuracies, as well as several simplifying assumptions involved in the statics calculations. While many considerable trends exist, the calibration matrix contains several coefficients of large values, which suggests that others of smaller value can be neglected. This issue is currently under investigation and a closer study of the MatLab code itself will uncover the significance of this range of values.

There are still noticeable error values and poor trends in certain areas of the data collection. If greater precision is required in the study, it is suggested that additional work be performed in the hopes of establishing an even more accurate data set for the future.

## APPENDIX A - COMPLETE STATICS CALCULATIONS

See podcalibrations2.xls [DataCals2 and DataBlade]

## APPENDIX B - CORRELATION GRAPHS

See podcalibrations2.xls [Graphs2 and BladeGraphs]







APPENDIX C - RESIDUALS

Blade Dyno - 6 Channel


Av:1.2362 Std:8.9139 Ratio:721.0723 \% Av:0.56202 Std:3.6143 Ratio:643.0838 \%



Av:0.14337 Std:1.6967 Ratio:1183.4646 \%Av-0.30199 Std:3.9505 Ratio:-1308.1563 \%



Forward Dyno - 5 Channel

Av-0.11092 Std:26.5143 Ratio:-23904.0229 \% $2 \mathrm{k}: 0.28024$ Std:8.3904 Ratio:2993.9841 \%



Av-7.3671 Std:25.5028 Ratio:-346.1731 \%Av:0.01788 Std:0.43072 Ratio:2408.8717 \%


Av:0.004682 Std:1.2777 Ratio:27289.1218 \%


Forward/Aft/Shaft Torque - 9 Channel

Av-0.49872 Std:26.8538 Ratio:-5384.5618 \%Av:0.27839 Std:8.329 Ratio:2991.8696 \%


Av-7.4005 Std:24.4951 Ratio:-330.9952 \%Av:0.01851 Std:0.42749 Ratio:2309.5559 \%


Av:0.023156 Std:1.2952 Ratio:5593.2211 \%



