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A NOTE ON SKEMPTON'S A PARAMETER WITH ROTATION OF PRINCIPAL STRESSES

by K.T. Law and R.D. Holtz

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SOMMAIRE

Dans cet article, les auteurs proposent une période pour déterminer les augmentations majeures et mineures de contrainte principale dans l'équation de pression interstitielle de Skempton (1954). Cette méthode est de plus en plus utilisée pour l'analyse des données publiées qui semblent contradictoires. On indique aussi que la méthode permet d'interpréter, tant expérimentalement que théoriquement, les cas de contraintes triaxiales comportant une rotation de la contrainte principale. Les conséquences pratiques de l'étude de cette méthode paraissent aussi dans l'article.

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A note on Skempton's A parameter with rotation of principal stresses

K. T. LAW* and R. D. HOLTZ†

This Paper proposes a system of defining the major and minor principal stress increments in the Skempton's (1954) pore pressure equation. Using this system, a unified trend emerges for use in analysing published data that are seemingly conflicting. The system is also shown to give a consistent interpretation, both theoretically and experimentally, to triaxial stress states involving principal stress rotation. The practical implications resulting from the study of this system are examined.

Cette étude propose un système pour la détermination des augmentations de contraintes principales majeure et mineure, dans l'équation de pression interstitielle de Skempton (1954). En utilisant ce système, une tentative d'unification en découle pour être utilisée dans l'analyse des données publiées qui sont apparement contradictoires. Le système paraît aussi donner une interprétation satisfaisante théoriquement aussi bien qu'expérimentalement, aux états de contraintes triaxiales impliquant la rotation de la contrainte principale. Les implications pratiques résultant de l'étude de ce système, sont examinées.

NOTATION

A	Skempton pore pressure parameter
A_0, A_u	pore pressure parameters for removal of in situ shear stress in an extension test and during perfect sampling
K_0	coefficient of earth pressure at rest
p'	$(\sigma'_1+\sigma'_3)/2$
q	$(\sigma'_1-\sigma'_3)/2$
Δu	change of pore pressure
σ_1, σ_3	major and minor total principal stresses respectively
$\Delta\sigma_1$, $\Delta\sigma_2$, $\Delta\sigma_3$	major, intermediate and minor total principal stress increments respectively
a	axial
l	lateral
ac	axial compression
ae	axial extension
lc	lateral compression
le	lateral extension

Application of Skempton's pore pressure parameter, A, to cases involving rotation of principal stresses has generated some confusion in spite of its popular use in practice. In the original derivation by Skempton (1954), no consideration of rotation of principal stresses was made. This is where the confusion arises and modification is required therefore, in order that the parameter may be applied to field and laboratory situations where the principal stresses do rotate.

For the case of extension loading, Skempton (1961) revised his previous expression for A by introducing the absolute value of the principal stress difference. This expression was later

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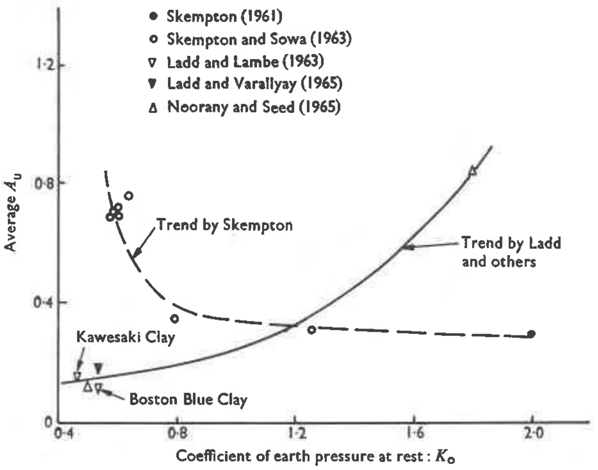


Fig. 1. Pore pressure parameters for perfect sampling computed by various authors

used by Skempton and Sowa (1963) in analysing pore pressure characteristics during 'perfect sampling'. This approach, however, still did not formally define the major and minor principal stress increments. Consequently, application of this approach to more general situations is limited.

Because of this lack of a clear definition, Ladd and Lambe (1963) used a somewhat different approach for unloading from in situ stress conditions, such as might occur during sampling. With the coefficient of earth pressure at rest (K_0) less than one, they used the following relationship:

$$\Delta\sigma_1 = \Delta\sigma_v \quad \text{and} \quad \Delta\sigma_3 = \Delta\sigma_h \qquad (1)$$

where $\Delta\sigma_v$, $\Delta\sigma_h$, $\Delta\sigma_1$, and $\Delta\sigma_3$ are the changes of vertical, horizontal, major principal, and minor principal stresses respectively. Dealing with the same problem, Noorany and Seed (1965) derived the same expressions for $K_0 < 1$. They extended, however, the approach to the case of $K_0 > 1$, in which they defined

$$\Delta\sigma_1 = \Delta\sigma_h \quad \text{and} \quad \Delta\sigma_3 = \Delta\sigma_v \qquad (2)$$

The two approaches generally give conflicting values of A_u during perfect sampling as shown in Fig. 1. In Skempton's approach, A_u decreases with increasing value of K_0, while the opposite is true with the other approach. This contradiction is caused not by a difference in soil behaviour but by a lack of consistent definition of stress increment, as will be shown later.

Because of a growing appreciation of the practical significance of strength anisotropy in soft clay, there is an increasing interest in conducting controlled stress path tests in which the principal stresses may rotate. Thus the need for a clear definition of stress increments, free of any ambiguity in its application, becomes more apparent, and such a system is proposed here.

DEFINITION OF $\Delta\sigma_1$ AND $\Delta\sigma_3$

$\Delta\sigma_1$ and $\Delta\sigma_3$ are defined as the major and minor principal stress increments respectively. A principal stress increment is defined here as the maximum or minimum normal stress vector imposed on a given stress system with the conventional sign of positive for compression and negative for tension. $\Delta\sigma_1$ is the algebraically largest normal component of a given system of stress increments while $\Delta\sigma_3$ is the algebraically smallest normal component of that system.

The advantage of this system is that the stress increment is dissociated from the original stress. Thus the direction of $\Delta\sigma_1$ is independent of the direction of the original or final σ_1, and so is $\Delta\sigma_3$. This point is further illustrated in Table 1, which shows some combinations of $\Delta\sigma_1$ and $\Delta\sigma_3$ being applied to typical existing stress systems represented by σ_1 and σ_3.

RE-ANALYSIS OF PUBLISHED DATA ON A FOR PERFECT SAMPLING

Using the foregoing definition of $\Delta\sigma_1$ and $\Delta\sigma_3$, the same set of data shown in Fig. 1 has been re-analysed, and the results are shown in Fig. 2. The much more consistent trend of A_u with respect to K_0 is obvious.

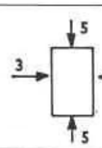
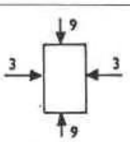
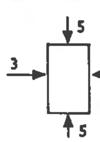
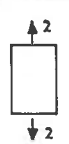
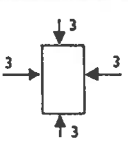
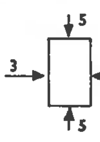
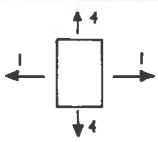
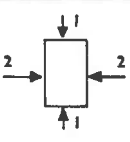
Two observations can be made from Fig. 2 that strongly support the proposed definition. Firstly, at higher values of K_0 corresponding to higher overconsolidation ratios, a lower pore pressure parameter is obtained. This conforms to existing experience with heavily overconsolidated clays in ordinary triaxial compression tests. Secondly, the pore pressure parameter A_u, obtained during perfect sampling, is approximately equal to the pore pressure parameter A_0 obtained from a triaxial extension test anisotropically (non-hydrostatically) consolidated to the in situ effective stresses at the stage in the test where the initial shear stress completely vanishes. For example, tests by Ladd and Varallyay (1965) on normally consolidated Boston Blue clay showed $A_u = 0{\cdot}82$ (average of 4 tests) and $A_0 = 0{\cdot}91$ (average of 4 tests). These two values are reasonably close to each other, whereas by using Ladd and Lambe's (1963) system, $A_u = 0{\cdot}18$, which is substantially different from A_0. The proposed definition therefore gives a consistent interpretation for the analysis of pore pressure response during perfect sampling. The applicability to more general triaxial stress states is examined in the following.

APPLICATION TO TRIAXIAL TESTS

Theoretical development

For a saturated soil in which the skeleton behaves as an elastic, isotropic material, the pore pressure change (Δu) generated under conditions of no volume change, is related to the

Table 1. Examples using the proposed new definition of principal stress increments (units of stress are arbitrary, and axisymmetry in stress system is assumed)

Initial stress system	Stress increment	Final stress state	$\Delta\sigma_1$ Magnitude	$\Delta\sigma_1$ Direction	$\Delta\sigma_3$ Magnitude	$\Delta\sigma_3$ Direction
 +	 =		4	V*	0	H*
 +	 =		0	H	−2	V
 +	 =		−1	H	−4	V

* V = Vertical; H = Horizontal.

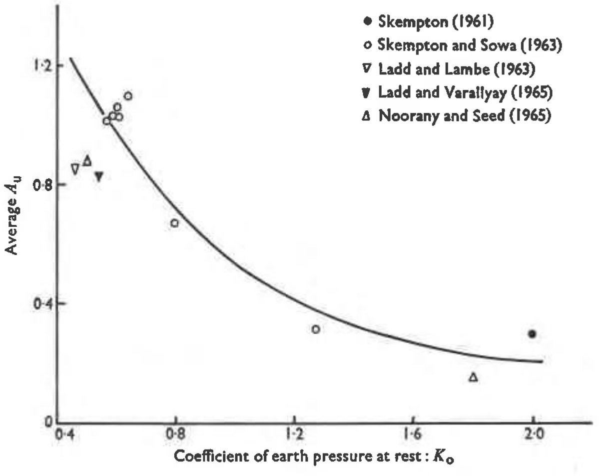


Fig. 2. Pore pressure parameters for perfect sampling computed using the proposed definition of $\Delta\sigma_1$ and $\Delta\sigma_3$

principal stress increments by

$$\Delta u = \tfrac{1}{3}\,(\Delta\sigma_1+\Delta\sigma_2+\Delta\sigma_3) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

where the increase of pore pressure is positive.

Consider two cases of axisymmetry:

$$(a)\ \Delta\sigma_2 = \Delta\sigma_3$$
$$(b)\ \Delta\sigma_2 = \Delta\sigma_1$$

which correspond to triaxial compression and extension tests respectively. For case (*a*), equation (3) can be rewritten as

$$\Delta u = \Delta\sigma_3+\tfrac{1}{3}\,(\Delta\sigma_1-\Delta\sigma_3) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

and for case (*b*)

$$\Delta u = \Delta\sigma_3+\tfrac{2}{3}\,(\Delta\sigma_1-\Delta\sigma_3) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

To account for inelastic behaviour, the constants in the two foregoing equations can be replaced by a parameter A, so that

$$\Delta u = \Delta\sigma_3+A\,(\Delta\sigma_1-\Delta\sigma_3) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

which is the well-known expression obtained by Skempton (1954).

If the constants in equations (4) and (5) are compared with A in equation (6), it is apparent that, for elastic behaviour, the pore pressure parameter derived from an extension test is twice that from a compression test.

Table 2. Definition of principal stress increments and formulae for pore pressure parameters for various types of triaxial tests

Test type	$\Delta\sigma_1$	$\Delta\sigma_2$	$\Delta\sigma_3$	Formula for A
Compression test				
Axial compression, ac	$\Delta\sigma_a$	0	0	$A_{ac} = \Delta u/\Delta\sigma_a$
Lateral extension, *l*e	0	$\Delta\sigma_l$	$\Delta\sigma_l$	$A_{le} = 1-\Delta u/\Delta\sigma_l$
Extension test				
Axial extension, ae	0	0	$\Delta\sigma_a$	$A_{ae} = 1-\Delta u/\Delta\sigma_a$
Lateral compression, *l*c	$\Delta\sigma_l$	$\Delta\sigma_l$	0	$A_{lc} = \Delta u/\Delta\sigma_l$

For inelastic response, the appropriate expressions for A for various triaxial loading conditions can be derived. This work is summarized in Table 2 in which the four different stress paths for compression and extension tests are included. As an illustration, consider the case of axial extension, in which the lateral (cell) pressure remains constant while the axial pressure decreases. The designation of the principal stress increments according to the proposed definition is represented by

$$\Delta\sigma_1 = \Delta\sigma_2 = 0; \qquad \Delta\sigma_3 = \Delta\sigma_a \quad \text{(negative, since it decreases)} \qquad (7)$$

Substituting (7) into (6) gives

$$A_{ae} = 1 - \Delta u/\Delta\sigma_a \qquad (8)$$

which is the same as shown in Table 2.

One way to test the consistency of the new formulation is to examine if one would obtain the same value for A from, say, two compression tests having different total stress paths. This is shown as follows.

Defining $p' = (\sigma'_1 + \sigma'_3)/2$, as the average of the major and minor effective pressures and $q = (\sigma_1 - \sigma_3)/2$, half the principal stress difference, we can express the slope at any point on the effective stress path in a $p' - q$ diagram as

$$\left(\frac{dq}{dp'}\right) = \frac{d(\sigma_1 - \sigma_3)}{d(\sigma_1 + \sigma_3 - 2u)} \qquad (9)$$

For the axial compression case $d\sigma_1 = d\sigma_a$ and $d\sigma_3 = 0$. Hence

$$\left(\frac{dq}{dp'}\right)_{ac} = \frac{1}{1 - 2A_{ac}} \qquad (10)$$

For the lateral extension case $d\sigma_1 = 0$ and $d\sigma_3 = d\sigma_l$. Hence

$$\left(\frac{dq}{dp'}\right)_{le} = \frac{-1}{1 - 2(1 - A_{le})} \qquad (11)$$

$$= \frac{1}{1 - 2A_{le}}$$

Since both tests yield the same effective stress path, then

$$\left(\frac{dq}{dp'}\right)_{ac} = \left(\frac{dq}{dp'}\right)_{le}$$

Hence

$$A_{ac} = A_{le}$$

Similarly one can show that

$$A_{ae} = A_{lc}$$

Experimental study

Triaxial compression and extension tests were carried out using a modified Geonor cell equipped with a rotating bushing. The soil specimens were trimmed from either 54 mm dia. NGI piston samples (Bjerrum, 1954) or the 127 mm dia. Osterberg (1952) samples. Prior to undrained shearing, the specimens were consolidated to the estimated in situ effective stresses. Clays from Kars and Gloucester, two sites near Ottawa, Canada, were tested. The clays are typical of the soft Leda clays, and detailed geotechnical descriptions have previously been reported by Eden and Poorooshasb (1968) and Bozozuk and (1972) Leonards.

A summary of the test results is presented in Table 3 and Figs 3 and 4 for the Kars clay. The Gloucester clay behaves similarly to the Kars clay and the following observations apply to both soils.

Table 3. Results of the experimental study

Specimen	K_0	Test type	w_n, %	w_f, %	$\left(\frac{\sigma_1-\sigma_3}{2}\right)$ max kN/m²	A_f*
Kars clay						
195–22–5	0·75	ac	71·5	70·4	51·2	0·39
195–22–7	0·75	*l*c	73·5	72·0	34·9	0·73
195–22–3	0·75	ae	71·5	70·3	34·5	0·73
Gloucester clay						
198–5–4A	0·80	ac	90·5	88·9	47·9	0·40
198–5–6B	0·80	*l*c	85·4	83·4	34·5	0·80
198–5–6C	0·80	ae	86·2	83·8	35·0	0·80

* A_f is the pore pressure parameter at failure based on expressions in Table 2.

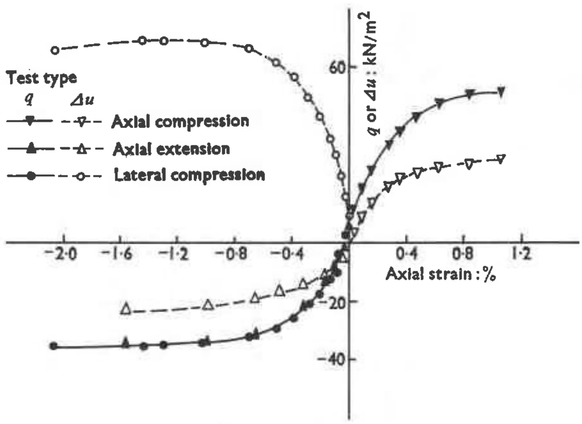


Fig. 3. Pore pressure–strain and stress–strain characteristics from triaxial compression and extension tests on Kars clay

Figure 3 shows the stress–strain and pore pressure–strain characteristics of the clay during undrained shear. The stress–strain curves for the axial extension and lateral compression tests are identical as are the effective stress paths (Fig. 4) for both tests. Similar effective stress paths were found by Bishop and Wesley (1975). The absolute pore pressure response of these two tests differs greatly (Fig. 3). However, if the appropriate formulae for the *A* parameters in Table 2 are used, one obtains practically the same values (Table 3).

The pore pressure parameter at failure for both clays in the axial extension and lateral compression tests is about twice that from the axial compression tests. This indicates that the pore pressure response for these two soft sensitive clays is not far from being elastic. A similar observation was also made by Bozozuk (1972) and Law (1974).

PRACTICAL IMPLICATIONS

The preceding theoretical and experimental study indicates that the higher pore pressure response determined from a triaxial extension test stems from the fact that the intermediate principal stress increment is equal to the major principal stress increment. In many practical operations such as embankment construction, this condition is not realistic. Here, the prevalent behaviour is characterized by a plane strain condition, in which $\Delta\sigma_3 < \Delta\sigma_2 < \Delta\sigma_1$.

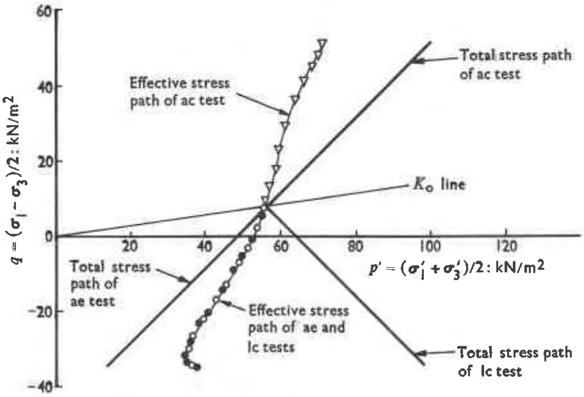


Fig. 4. Stress paths of the various types of triaxial tests on Kars clay

Generation of excess pore pressure is thus lower than that estimated from extension tests. If the soil involved fails in accordance with the Mohr Coulomb failure criterion, then the undrained strength of the soil will be underestimated. Further aggravating this underestimation, one may note that some soils possess higher strength in plane strain loading than in axisymmetric loading (Henkel and Wade, 1966). Therefore, undrained triaxial extension tests should not be indiscriminately recommended for estimating strength anisotropy for embankment stability analyses, as was advocated, for example, by Bjerrum (1972) and (1973).

Rotation of principal stresses in real situations generally ranges continuously from 0° to 90°. At any given angle of rotation, ambiguities are also found in the literature in calculating $\Delta\sigma_1$ and $\Delta\sigma_3$, since stresses are really tensor quantities, and as such, two directions and magnitude are required to completely characterize a particular state of stress. The dissociation of the stress increments from the initial stress system as proposed here provides a straightforward procedure to eliminate these ambiguities. Again referring to the case of embankment construction, one may simply calculate the imposed stress increments resulting from the loading and then estimate the excess pore pressure. Depending on the nature of the problem, this may be carried out using simple elastic analyses or sophisticated finite element techniques featuring non-linearity and work-softening (Desai, 1974; Law and Lo, 1976).

SUMMARY AND CONCLUSION

A simple definition of $\Delta\sigma_1$ and $\Delta\sigma_3$ is proposed for extending the Skempton (1954) pore pressure equation (which was originally derived for the case with no rotation of principal stresses) to stress paths where rotation in fact occurs. The proposed definition dissociates stress increments from the initial stress system, and the resulting flexibility allows any rotation of principal stresses to be easily and consistently treated. Advantages of the new formulation include:

(*a*) The problem of conflicting A values for perfect sampling recorded in the literature is resolved and a consistent trend emerges;
(*b*) The formulation is theoretically sound and consistent;
(*c*) The formulation enables a proper interpretation of triaxial test data from two soft sensitive clays to be obtained.

In conclusion, triaxial extension tests probably do not provide compatible strength data for soils stressed under plane strain conditions.

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