# Flow vectors around an escort tug at a large yaw angle

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# Institute Report

## Resistance and static yaw experiments on the underwater vehicle Phoenix: modelling and analysis, utilizing statistical design of experiments methodology

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# Resistance and Static Yaw Experiments on the Underwater Vehicle "Phoenix"; Modeling and Analysis, Utilizing Statistical Design of Experiments Methodology


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***Abstract*- This research is focused on the hydrodynamic behaviour of a series of hull forms for an underwater vehicle, which includes a range of length-to-diameter ratios. Experimental data were gathered for several standard manoeuvring experiments and a reverse Design Of Experiment (DOE) was applied to the available data. From the DOE point of view, the effects of the main factors in each type of experiment were studied and an appropriate Response Surface Model (RSM) was fitted to the data. The developed empirical model is very useful in predicting the non-dimensional hydrodynamic force and moment coefficients, which is a major step in simulating the motion of an underwater vehicle.**


## I. INTRODUCTION

Statistical design of experiment (DOE) methodology was developed to make experimentation more efficient in terms of time and budget. First started in the agricultural sciences in the 1920s, DOE has gone through at least three industrial and academic eras and is now increasingly used in research and industry [1].

Basically, DOE is a methodology for systematically applying statistics to experimentation. DOE lets experimenters develop a mathematical model that predicts how input variables interact to create output variables or responses in a process or system. It has been recognized for many years that the factorial-based DOE is the correct and the most efficient method of doing multi-factored experiments. This method allows a large number of factors to be investigated in few experimental runs and it was further developed to include fractional factorial designs, orthogonal arrays, and response surface methodology. Regular factorial design includes the following steps:

a) Select the factors, i.e. decide which input variables are going to be studied;
b) Determine the factor levels;
c) Identify the responses; what do we measure as the output?
d) Perform the experiment with various combinations of factor levels to obtain the responses (outputs);
e) Estimate the factor effects, i.e. perform the ANOVA (Analysis of Variance);
f) Develop the model using important effects;
g) Check if the model fits the responses well and if the assumptions of regression are valid;
h) Analyze and interpret the results; and
i) Use the model for prediction.

From the results, we can also determine if we should add or drop factors, change factor levels, redefine the responses, etc. until a suitable model of the process will be obtained.

A major engineering application of the DOE is in manufacturing science and industry and other fields are becoming aware of its potential effectiveness. Many articles on the application of DOE in manufacturing, chemical and food science and technology can be found in [2]. Among the few, [3] and [4] can be mentioned as application of DOE in respectively aerodynamics and hydrodynamics. According to the highly non-linear manoeuvring mechanics, both [3] and [4] utilize the concept of subspaces and D-optimal design to model the responses through the whole range of definition of the factors. Reference [5] illustrates the vital need to have a well-designed experiment so as to reduce the number of runs.

As part of an underwater vehicle study at the Institute for Ocean Technology, National Research Council of Canada, the bare hull of an underwater vehicle named "Phoenix" was tested in the open water 90 m Ice Tank. The experiments were planned in May 2005 and performed in November 2005. The tests totalled 197 experimental runs. These test runs were conducted without the use of DOE methodology. Experimental data were obtained for simple resistance, static yaw, zigzag, pure sway and arc-of-a-circle maneuvers. From the experiments the overall maneuvering characteristics of the underwater vehicle can be predicted [6]. In this study, we will focus on the data from the resistance and static yaw tests only.

A reverse design of experiment will be applied using the available test data i.e. the assumption is that only some of these data would have been obtained through a statistically designed series of experiments, and a response surface model will be fitted to that portion of these data. It is desired to obtain answers for the following questions:

1. Is it possible to combine the results of two sets of experiments, namely resistance and static yaw, and develop a model for the responses versus the important factors: velocity, length-to-diameter ratio and drift angle, as in
$$(Fx, Fy, Mz) = g(l/d, V, \beta)? \quad (1)$$
2. According to the performed experiments and available data, how would we design an experiment that in the future will conserve time and cost?

## II. The Experiment Introduction And Definitions

All experiments were performed in the horizontal x-y plane for five different models. To perform these experiments an additional mechanism called PMM (Planar Motion Mechanism) was installed on the towing carriage. The towing carriage with one degree of freedom (DOF) tows the model in the longitudinal direction and is capable of speed up to 4 m/s. The PMM adds two DOF: linear in the y direction (sway) and angular about the z-axis (yaw).

The original bare hull of the underwater vehicle "Phoenix", shown in Fig. 1, has an overall length of 1.641 metre and a diameter of 0.203 metre. For the original vehicle the length-to-diameter ratio, l/d, is almost eight. The other versions are obtained by adding a part to the parallel middle body [6]. For the resistance tests the two factors are: towing velocity, V, and model dimensions, l/d; for the static yaw test the factors involved are: yaw (drift) angle, $\beta$, and l/d. Tables I and II define the factors and their treatment levels for the two types of experiments. Shown in Tables I and II, the resistance and static yaw tests respectively contribute 4*4=16 and 5*11=55 runs of the previously mentioned total number of runs.

The experiments resulted in three responses: axial force, Fx, sway force, Fy, and yaw moment, Mz. It should be noted that:

- The variable l/d is common for both types of experiments.
- For the resistance test, the desired response is the axial force, Fx, and the two other responses (Fy and Mz) are expected to be zero.
- All treatment levels of the static yaw tests have been performed with the same forward velocity of 2 m/s.

TABLE I.
Resistance Tests: Variables and Factor Levels

| Factors | Levels |
|---|---|
| l/d | 9, 10, 11 and 12 |
| V (m/s) | 1, 2, 3 and 4 |

TABLE II.
Static Yaw Tests: Variables and Factor Levels

| Factors | Levels |
|---|---|
| l/d | 8, 9, 10, 11 and 12 |
| $\beta$ (deg) | -2 to 20 with step 2° |

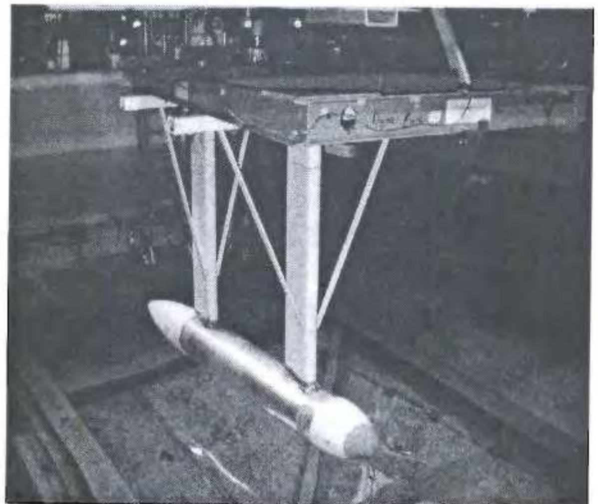

Figure 1. The bare-hull of the underwater vehicle and experimental set up

In addition to the factors and responses (the main concern of the experimenter), there are several constraints that dominate the experiment design. The constraints are due to the facilities, environmental conditions and the experimenters. For instance, randomization is a basic requirement in the theory of the experiment design so as to cancel out the steady errors caused by unknown variables, however, installing and aligning the model on carriage is a lengthy task, therefore, randomization over the variable, l/d, is practically impossible. It worth to mention that the randomization problem due to the presence of hard-to-change factors, can be solved using split-plot designs ([1], [7] and [8]), but the data here will be analyzed as if they were gathered randomly.

## III. Analysis of Data

The software "Design Expert™6.0.3" by Stat-Ease, Inc. was used to analyze the data. The ANOVA shows that in the resistance test, the velocity (V) is highly significant at the 10% level, whereas in the static yaw test, the yaw angle ($\beta$) is highly significant at the 10% level. None of the experiments result in a significant interaction effect between the factors.

### A. The $2^k$ factorial design

The $2^k$ factorial design is the special case of the general factorial design. In this case, k factors all at two levels, usually called low level and high level, make the combinations. As mentioned, ANOVA is used to test for the statistical significance. A factor that has a greater effect on the response is statistically more significant. The factor effect is defined as the change in the mean response when the factor is changed from low level to high level. For instance, if A and B are two factors in an experiment, effect of A is evaluated as:

TermA1 = Estimate of effect of A at high B = $a_1b_1 - a_0b_1$
TermA2 = Estimate of effect of A at low B = $a_1b_0 - a_0b_0$ (2)
[A] = Estimate of the effect of A over all B
= (TermA1 + TermA2) /2

Effect of B is evaluated in the same way. [A] and [B] are the main effects. Indices '0' and '1' consequently indicate the low and high level for each factor, e.g. $a_1b_1$ is the response at the treatment combination in which both factors are in high level. There is also an interaction effect between the two factors, which is named [AB]. Interaction is actually a form of curvature and describes the dependence of the effect of one factor on the level of the other factor. The interaction effect is calculated as:

$$[AB] = \text{Estimate of effect of B on the effect of A} = (\text{TermA1} - \text{TermA2})/2 \quad (3)$$

It should be noted that in the presence of large interaction effects, the main effects might not be meaningful.

*B. Resistance Tests*

In the resistance tests the model with zero heading (drift angle) is towed through the tank. In each run, the towing velocity goes from a stationary zero value to a constant value and then again backs to zero. Having the maximum acceleration is important so as to perform constant velocity towing through a longer distance.

The variable l/d is assigned as factor A, and the variable V is factor B. The interaction effect between the two variables is named AB. Conventionally, it is a good practice to design a two-level factorial experiment to study the general trend of the main effects and interaction effects with a minimum number of runs.

The available data, as mentioned in Table I, included four levels for each factor in the resistance test. Hence, several 2-level factorial designs with different definitions of high and low levels for each factor can be extracted from the data. Two sets of 2-level designs, one with a narrow range of levels- Table III (a)- and the other with a wide range of levels- Table III (b)- were tried. Note that the actual values for the factors are shown in Table III, but for each factor the low and high levels will be respectively coded as minus and plus one, and the analysis is performed on the coded factors.

The effects and contribution percentage for the different terms of the $2^2$ factorial models are calculated as shown in Table IV. As mentioned, Fx is the desired response in the resistance tests and we are developing regression models for that response. The following conclusions were made:

(a) Disregarding the range of variability, forward velocity is a highly significant factor for the axial force.

(b) Length-to-diameter ratio and its interaction with the velocity do not affect the responses significantly.

(c) For a broad low to high-level interval the effect of all terms increase, which is physically obvious.

(d) If the ratio of the effects for two cases is defined as:

$$\text{Effect ratio} = \frac{\text{Case 2 (broad interval)}}{\text{Case 1 (narrow interval)}} \quad (4)$$

Then we have:

$$\text{Effect ratio}[A] = 5.18,\ \text{Effect ratio}[B] = 2.62,\ \text{Effect ratio}[AB] = 13.93$$

TABLE III.
$2^2$ FACTORIAL-DESIGN WITH:

(a) NARROW RANGE OF LEVELS

| Factor | Low | High |
|---|---|---|
| l/d | 10 | 11 |
| V | 2 | 3 |

(b) WIDE RANGE OF LEVELS

| Factor | Low | High |
|---|---|---|
| l/d | 9 | 11 |
| V | 2 | 3 |

TABLE IV.
EFFECTS AND CONTRIBUTION PERCENTAGE FOR THE DIFFERENT TERMS

| | Narrow Range | | Wide Range | |
|---|---|---|---|---|
| Term | Effect | % Contribution | Effect | % Contribution |
| [A] | 0.95 | 0.51 | 4.92 | 1.95 |
| [B] | 13.24 | 99.44 | 34.65 | 96.65 |
| [AB] | 0.30 | 0.05 | 4.18 | 1.40 |

It can be concluded that for a wide range of variability, factor A and interaction of two factors show their effect more significantly. In other words A and AB are more sensitive than B to the definition of low and high values. The reason should be sought in the fluid regime and its dependency on geometry and velocity of the body.

For a better insight, it is beneficial to analyze all the available data and fit a model to them. Table V shows the sum of squares and contribution of the factors A, B and AB. Note that the sum of squares for any effect is directly proportional to the effect squared. Eliminating the interaction term AB and doing ANOVA for the factors A and B resulted in a significant model as shown in Table VI. The significance level used was 10%. Though from Table V, factor A appears to be statistically significant, it contributes less than 2% to the model.

Figs. 2 and 3 show the model interaction graphs. In Fig. 2, factor B (velocity) is the x-axis and different curves are drawn for different length-to-diameter ratios. In Fig. 3, factor A (length-to-diameter ratio) is the x-axis. For the resistance tests, as well as the static yaw tests, all regression assumptions were acceptable.

TABLE V.
SUM OF SQUARES AND CONTRIBUTION OF TERMS FOR THE SELECTED MODEL

| Term | Sum Sqr. | % Contribution |
|---|---|---|
| A | 40.80 | 1.40 |
| B | 2840 | 97.80 |
| AB | 23.25 | 0.80 |

TABLE VI.
ANOVA FOR THE SELECTED MODEL

| Source | Sum of Squares | DF | Mean Square | F-value | Prob > F | |
|---|---|---|---|---|---|---|
| Model | 2881 | 6 | 480.30 | 185.9 | < 0.0001 | Significant |
| A=l/d | 40.80 | 3 | 13.60 | 5.26 | 0.0227 | |
| B=V | 2840 | 3 | 947.0 | 366.6 | < 0.0001 | |
| Residual | 23.25 | 9 | 2.58 | | | |
| Correlation Total | 2905 | 15 | | | | |

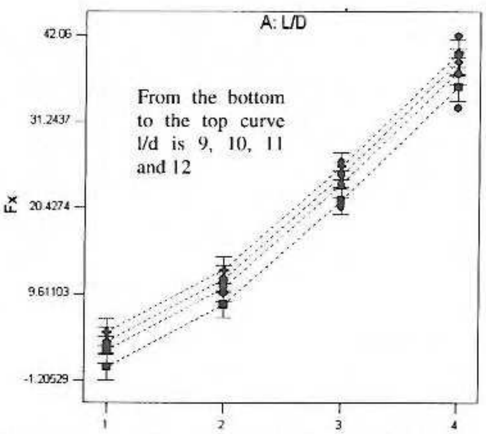

Figure 2. Interaction graph for axial force; velocity on x-axis

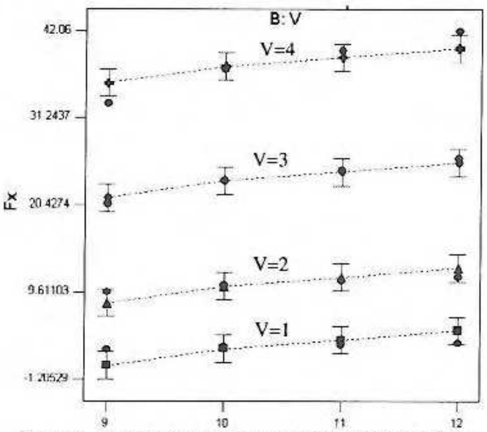

Figure 3. Interaction graph for axial force; l/d on x-axis

*C. Response Surface Model*

A regression model for a response, which depends on two factors, is a surface in the 3D space. The response surface may be represented graphically using a contour plot or a 3-D plot. This concept is used for more than two factors as well.

The Response Surface Model (RSM) can be a first-order model if the response is a linear function of the factors. If the response has curvature, then a higher order polynomial should be used. A second-order (quadratic) model is often able to capture the curvature. However, the linear or quadratic models may not cover all range of definition of the factors, and several RSMs may be required to model the system.

*D. Static Yaw Tests*

The second type of experiments analyzed was the static yaw tests. The vehicle, inclined with a yaw angle, was towed along the tank-x-axis. The yaw angle β was gradually increased through several runs. The variable l/d is assigned as factor A, and the variable β is factor B. the interaction effect between the two variables is named AB.

The available data, according to Table II, included five levels for factor A and 12 levels for factor B. Some of the available data can be used to develop a RSM. Central Composite Design (CCD) is a popular design to fit a response surface to the data [1]. A CCD will be tried in order to capture the static yaw experiment results. Fig. 4 shows the general scheme of design. The design points are shown as pairs of (l/d, β) values; factor A, namely l/d, takes values of eight, ten and twelve horizontally and factor B, β, takes values of zero, ten and twenty degrees vertically. The data shown in Table VII are used for this purpose. In Fig. 4 center-point has coordinates (l/d, β) of (10,10) and axial-runs are the runs augmented in between the square two-level design; they have coordinates (8,10), (12,10), (10,0) and (10,20).

The process of fitting a RSM for sway force, axial force, and yaw moment is similar. The linear model was suggested; however the quadratic terms were in the boundary of significance. The interaction term was negligible. Checking additional statistics for a second-order model revealed that including the quadratic terms will result in a more accurate but not redundant model. Table VIII shows the ANOVA for the quadratic model. The model is significant but the interaction term AB and quadratic term $A^2$ can be omitted.

Note that the p-values for AB and $A^2$ are evidently larger than 10%. For A and $B^2$ they are near 10% and they were included in the final model. The model equation, written for the actual factors, after omitting the terms AB and $A^2$ is as follows:

$$Fy = -54.51 + 5.86(l/d) + 1.25(\beta) + 0.28\,(\beta)^2 \qquad (5)$$

One can check if (5) fits the design data of Table VII. If the response surface captures the data with an acceptable accuracy, then the other available data can be used to check for the predictive capability of the model.

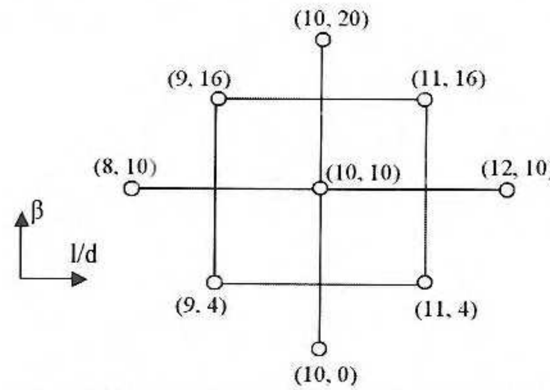

Figure 4. The general scheme of the Central Composite Design

TABLE VII.
DESIGN DATA FOR THE CENTRAL COMPOSITE DESIGN

| Run | A=l/d | B=β (deg) | Fx (N) | Fy (N) | Mz (N.m) |
|---|---|---|---|---|---|
| 1 | 9 | 4 | 10.60 | 9.45 | 7.16 |
| 2 | 11 | 4 | 11.66 | 15.00 | 12.78 |
| 3 | 9 | 16 | 13.81 | 79.57 | 27.77 |
| 4 | 11 | 16 | 14.52 | 98.56 | 38.26 |
| 5 | 10 | 0 | 10.48 | 3.62 | 4.29 |
| 6 | 10 | 20 | 15.26 | 150.10 | 37.48 |
| 7 | 8 | 10 | 11.85 | 35.07 | 17.10 |
| 8 | 12 | 10 | 14.27 | 57.99 | 32.30 |
| 9 | 10 | 10 | 13.29 | 54.00 | 26.34 |

TABLE VIII.
ANOVA FOR THE QUADRATIC MODEL FOR SWAY FORCE

| Source | Sum of Squares | DF | Mean Square | F-value | Prob > F | |
|---|---|---|---|---|---|---|
| Model | 17997 | 5 | 3599 | 41.39 | 0.0057 | Significant |
| A | 412.6 | 1 | 412.6 | 4.75 | 0.1175 | |
| B | 16561 | 1 | 16561 | 190.45 | 0.0008 | |
| $A^2$ | 3.90 | 1 | 3.90 | 0.04 | 0.8459 | |
| $B^2$ | 514.9 | 1 | 514.9 | 5.92 | 0.0930 | |
| AB | 45.09 | 1 | 45.09 | 0.52 | 0.5235 | |
| Residual | 260.9 | 3 | 87 | | | |
| Correlation Total | 18258 | 8 | | | | |

Fig. 5, showing Fy versus yaw angle for l/d equal to 8 and 12, is plotted to assess the predictive capability of the model. The asterisk and circle signs represent the experimental data, which are available for the yaw angle from -2 to 20 degrees, in steps of two degrees (Table II). The solid and dashed lines are fitted to the RSM generated data with the same step-size. The model prediction is fairly acceptable, though there is a gap in some regions (e.g. at higher angles for the l/d=8 vehicle or lower angles for l/d=12). The RSM 3-D demonstration for sway force, Fy, is shown in Fig. 6. Plot of contours of the sway force model is shown in Fig. 7. In fact, Fig.7 is the bottom face of Fig. 6.

As mentioned, the same procedure can be applied to the axial force and yaw moment. The models for the sway force and yaw moment include the quadratic term $\beta^2$, but the axial force model is a simple linear model. The model equation, written for the coded factors are:

$$Fx = 12.58 + 0.55(A) + 1.47(B) \quad (6)$$

$$Fy = 45.05 + 5.86(A) + 41.63(B) + 10.25(B)^2 \quad (7)$$

$$Mz = 24.22 + 3.88(A) + 10.61(B) - 1.51(B)^2 \quad (8)$$

Notice that (7) corresponds to (5); the former is written for the actual factors and the latter for the coded factors. With the coded factors one can exactly see which factor has a larger effect on the response.

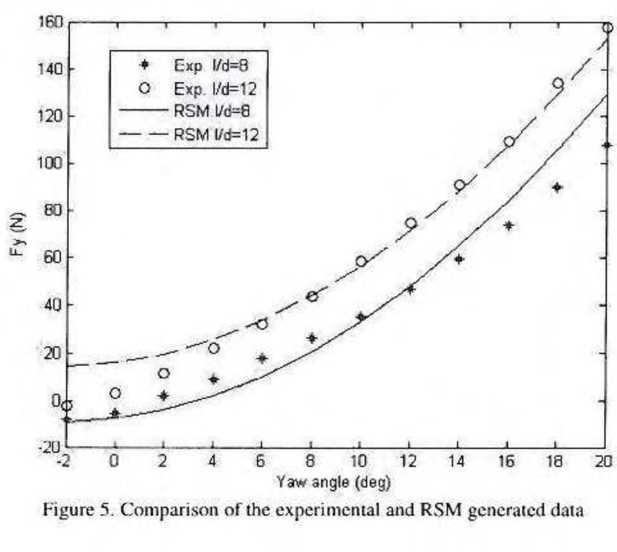

Figure 5. Comparison of the experimental and RSM generated data

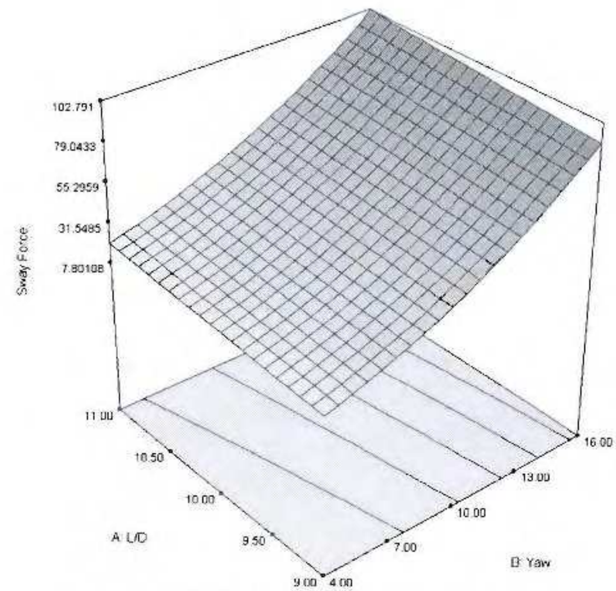

Figure 6. Response Surface Model for Sway Force

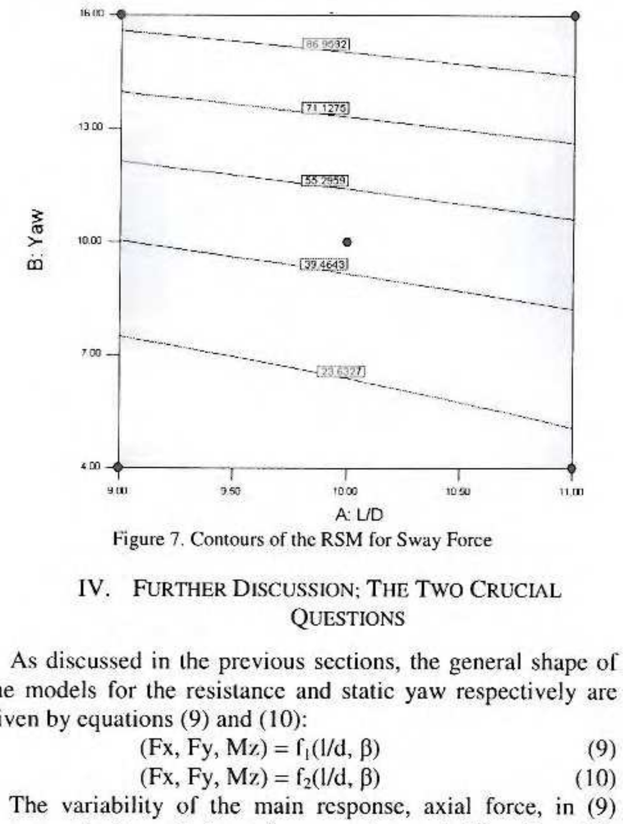

Figure 7. Contours of the RSM for Sway Force

## IV. FURTHER DISCUSSION; THE TWO CRUCIAL QUESTIONS

As discussed in the previous sections, the general shape of the models for the resistance and static yaw respectively are given by equations (9) and (10):

$$(Fx, Fy, Mz) = f_1(l/d, \beta) \quad (9)$$

$$(Fx, Fy, Mz) = f_2(l/d, \beta) \quad (10)$$

The variability of the main response, axial force, in (9) versus velocity and dimension was shown in Figs. 2 and 3, and (10) is the compact expression for (6) to (8), after re-writing them for the actual factors. It should be noted again that:

a- The variable l/d is common for both experiments.

b- For the resistance test, the desired response is the axial force and two other responses (Fy and Mz) are expected to be zero.

c- In the resistance test, factor B, velocity, is completely dominant. The magnitude of its effect and its contribution in the model is significantly higher than factor A, l/d.

d- In the static yaw test, factor B (yaw angle) is dominant.

e- None of the experiments result in either a statistically (small F-value) or practically significant (large contribution) interaction effect.

f- From these experiments nothing can be concluded about the interaction of V and β, because the static yaw tests were performed with the same forward velocity of 2 m/s.

Therefore we have already derived $f_1$ and $f_2$, in (9) and (10), and we have some clues to answer the first crucial question that we put in the introduction section. Equation (1) is repeated below:

$$(Fx, Fy, Mz) = g\ (l/d, V, \beta) \qquad (11)$$

The objective function is g. In other words, we are looking for a response surface model in the four-dimensional space. If the three factors l/d, V and β are named consequently A, B and C, then the first-order (linear) regression equation for the objective function is of the form:

$$g = \alpha_0 + \alpha_1 A + \alpha_2 B + \alpha_{12} A*B + \alpha_3 C + \alpha_{13} A*C + \alpha_{23} B*C + \alpha_{123} A*B*C$$

(12)Equation (12) includes all the terms (i.e. main effects, two-factor interaction effects and the three-factor interaction effect) in the model, but some terms may not have a significant effect on the response. In case of a two–level factorial design, the coefficients are calculated as:

$$\alpha_0 = \text{Overall average},\ \alpha_1 = [A]/2,\ \alpha_2 = [B]/2,\ \alpha_3 = [C]/2,$$
$$\alpha_{12} = [AB]/2,\ \alpha_{13} = [AC]/2,\ \alpha_{23} = [BC]/2,\ \alpha_{123} = [ABC]/2 \qquad (13)$$

In (13), [A] is the effect of factor A, [AC] is the interaction effect of factors A and C which represents the dependence of the effect of factor A on the level of factor C (or vice versa), and so on. Hence performing a $2^3$ factorial design (two-levels for three factors), may give an appropriate approximation of the objective function. With the available data we have no information about $\alpha_{23}$ (interaction of the factors V and β) and $\alpha_{123}$ (interaction of all three factors).

Now, an answer can be provided for the second question: How would we design an experiment in the future so as to conserve time and cost? To give an approximate quantity on the time and cost saving that could be made, noting the previous paragraph, with a $2^3$ factorial design, performing only eight runs, we might obtain a good approximation of the objective function g. Then, to check for the curvatures in the responses, the design could be augmented with axial runs to create a central composite design, which is a very effective design for fitting a second-order response surface model.

The full CCD for three factors is 14 runs plus the center-point runs. Center-point has coordinates (l/d, β) of (10,10) in Fig. 4. Note that if we have performed the $2^3$ design, only six axial runs plus the center-point runs should be augmented to it. Axial-runs have coordinates (8,10), (12,10), (10,0) and (10,20) in Fig. 4. It is usual to replicate the center-point runs. With e.g. three replications for the center-point the design totals to 17 runs. The present data for the resistance and static yaw tests totalled 16+55=71 runs! The difference between the figures, shows the time and cost saving.

## V. Conclusion

In this paper, two types of manoeuvering experiments, resistance and static yaw, were considered and the experimental data obtained for each of them were analyzed. From a statistical design of experiment (DOE) point of view, the effects of the main factors in each type of experiment were studied. Then an appropriate Response Surface Model (RSM) was fitted to the data. The RSM provided an acceptable prediction capability.

Finally, some discussion on the derivation of a unified response for both types of experiment and the possible saving of time and cost was presented. With a statistically designed experiment, we can derive the adequate regression equation, which gives the hydrodynamic force versus the main factor effects and interaction effects. On the other hand, as was illustrated for the present data, the number of runs for a statistically designed experiment is several times less than the regular one-factor-at-time experiment; 17 versus 71 run which means a great save in time and cost. In future, statistical DOE will be applied to the underwater vehicle experiments.


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