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Analysis of thin rod flexural acoustic wave gravimetric sensors

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An analysis of thin rod flexural acoustic wave gravimetric sensors is presented. The diameter of the thin rod is much less than a wavelength. The lowest order flexural acoustic mode, F_{11}, is of interest. For small added masses, the calculated mass sensitivity is $-1/(2\rho a)$, where ρ is the density and a is the radius of the thin rod. Measurements of velocity dispersion of the F_{11} mode in 21-μm-diam gold wires are shown to agree well with the theoretical calculation. Devices based on thin rods are shown to potentially have the same advantages as plate-mode gravimetric sensors.

For the past two decades there has been growing interest in the development of integrated sensors involving acoustic waves. Sensors based on bulk (BAW),[1,2] surface (SAW),[3,4] and flexural plate acoustic waves[4,5] were reported. Most of these sensors operate in a gaseous medium, although a few are used with liquids or solids, to determine concentrations of biological and chemical substances, as well as mechanical quantities such as viscosity and acceleration, temperature, pressure, etc.

According to a recent theoretical analysis and experimental study by Wenzel and White,[6] flexural plate wave gravimetric sensors have higher mass sensitivity at low operating frequencies (a few MHz) than the BAW or SAW counterparts. The mass sensitivity S_m can be defined as[6]

$$S_m = \lim_{\Delta m_s \to 0} \frac{1}{V_0} \frac{\Delta V}{\Delta m_s}, \tag{1}$$

which can be rewritten as

$$\Delta V/V_0 = S_m \Delta m_s, \tag{2}$$

where Δm_s is the uniformly distributed mass per unit area added to the surface of the device; $\Delta V = (V - V_0)$, V_0 and V are the unloaded and loaded phase velocity, respectively. The sensitivity constant S_m was found to be approximately $-1/(2\rho h)$ for the lowest order antisymmetric plate mode (A_0), where ρ is the material density and h is the thickness of the plate. The higher achievable sensitivity for plate-wave sensors relies on the fact that h can be made very small.

In a thin isotropic rod there exist longitudinal, torsional, and flexural acoustic modes.[7,8] The criterion for being "thin" is that the rod diameter is much less than an acoustic wavelength. Our work originates from the fact that the lowest flexural acoustic mode, F_{11}, of a thin isotropic rod exhibits almost the same velocity characteristics as the A_0 mode of a thin isotropic plate.[7,8] For example, F_{11} and A_0 are the only modes with a lower phase velocity than the Rayleigh acoustic wave velocity of the bulk material of which the rod or plate are made. Furthermore, the velocity of the F_{11} or the A_0 mode is small if the product of the operating frequency f, and the rod radius a or the plate thickness h, is small. Therefore we expect that devices based on thin rods can potentially have the same advantages as plate-mode gravimetric sensors. Although many thin rod sensors, which employ longitudinal or torsional types of acoustic modes, have been developed by Lynnworth,[9] our work concentrates on the F_{11} mode in thin rods (e.g., $a = 10.5$ μm).

Let us consider an isotropic acoustic rod with radius a, Young's modulus E, and material density ρ. In the low-frequency limit the flexural mode F_{11} can be correctly described by the elementary theory of flexure, which provides the equation of motion:[10,11]

$$m \frac{\partial^2 U}{\partial t^2} + EI \frac{\partial^4 U}{\partial z^4} - T \frac{\partial^2 U}{\partial z^2} = 0, \tag{3}$$

where U is the transverse displacement of the F_{11} mode, z is the propagation distance along the rod, $m = \pi a^2 \rho$ is the mass per unit length, T is the tension exerted on the thin rod, and $I = \pi a^4/4$ is the moment of inertia of the cross section about the neutral plane.[10]

Assume the F_{11} mode traveling along the thin rod has the displacement U of the form $U = Ae^{j(\omega t - \beta z)}$, where A is the amplitude, $\omega = 2\pi f$, and f is the operating frequency. β is the propagation constant. Equation (3) can be rewritten as

$$-m\omega^2 + EI\beta^4 + T\beta^2 = 0. \tag{4}$$

The phase velocity $V = \omega/\beta$ and the group velocity $V_g = \partial\omega/\partial\beta$ of F_{11} mode can then be expressed as follows:

$$V = \left\{ \left[\left(\frac{T}{2m} \right)^2 + \frac{EI\omega^2}{m} \right]^{1/2} + \frac{T}{2m} \right\}^{1/2}, \tag{5}$$

$$V_g = \frac{T}{m} \frac{1}{V} + \frac{2EI\omega^2}{m} \frac{1}{V^3}. \tag{6}$$

In this letter three special cases described below are of interest. The corresponding phase velocity, V, from Eq. (5) is also given.

a. $T = 0$ (no tension):

$$V^0 = \left(\frac{EI\omega^2}{m} \right)^{1/4} = (\omega)^{1/2} a \left(\frac{\pi E}{4m} \right)^{1/4}. \tag{7}$$

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From Eqs. (6) and (7) we can also obtain

$$V_g^0 = 2V^0. \tag{8}$$

It is interesting to note that Eqs. (7) and (8) are very similar to the velocity expressions for the A_0 mode.[12]

b. $T^2/4m \ll EI\omega^2$ (small T):

$$V \sim \left(\frac{EI\omega^2}{m}\right)^{1/4}\left[1+\left(\frac{T^2}{16mEI\omega^2}\right)^{1/2}\right]$$

$$= V^0\left[1+\left(\frac{T^2}{16mEI\omega^2}\right)^{1/2}\right]. \tag{9}$$

c. $T^2/4m \gg EI\omega^2$ (large T):

$$V \sim (T/m)^{1/2}(1 + mEI\omega^2/2T^2). \tag{10}$$

If the second term of the rhs of Eq. (10) is much less than 1, V can be expressed as $(T/m)^{1/2}$ which is a constant.

Since the sensitivity S_m for the gravimetric sensor geometry is of interest, it is convenient to introduce the concept of mass per unit area, m_s, in a fashion similar to that adopted in Ref. 6:

$$m_s = m/2\pi a = \rho a/2. \tag{11}$$

Following the approach in Ref. 6 we assume that when the sensor is loaded, the mass per unit area, m_s, is changed by an amount of $\Delta m_s (= \Delta m/2\pi a)$ with negligible change in the rod diameter and Young's modulus, where Δm and Δm_s are the mass change per unit length and area, respectively. Equation (7) can then be written as

$$V^0 + \Delta V^0 = (\omega)^{1/2}\, a[\pi E/4(m+\Delta m)]^{1/4}. \tag{12}$$

Using Eqs. (1), (12), and (11) we can obtain

$$S_m = -1/2\rho a, \tag{13}$$

which is the mass sensitivity of our proposed thin rod gravimetric sensor with no tension exerted on the thin rod ($T = 0$).

As expected, Eq. (13) is very similar to the mass sensitivity for a plate-mode, A_0, gravimetric sensor [Eq. (18); $S_m = -1/(2\rho h)$] in Ref. 6. We can visualize that if the rod diameter is reduced, the mass sensitivity S_m increases. Similar to the analysis and claim for plate-mode devices reported in Ref. 6, the proposed thin rod flexural acoustic wave gravimetric sensors also have higher sensitivity at low operating frequencies (a few MHz) than the BAW and SAW counterparts. For BAW and SAW gravimetric sensors, S_m is directly proportional to acoustic frequency, f.[6] It is noted that operation at lower frequency means lower acoustic absorption losses in the device. In addition, acoustic experiments with an operating frequency at a few MHz are commonly convenient.

Pure gold wires of 10.5 μm radius were used as thin rod samples. They are chosen because of the uniform wire diameter. Figure 1 shows the calculated velocity dispersion relation for the lowest order flexural mode, F_{11} of the thin gold wire; the lower solid and dotted lines for F_{11} mode are calculated phase velocities based on exact[13] and approximated [Eq. (7)] dispersion formulas, respectively. The upper solid line represents the exact group velocity. It is clear

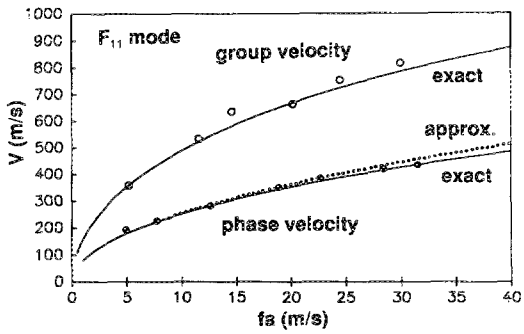

FIG. 1. Phase (solid line: exact; dashed line: approximation; solid circles: measured) and group velocity dispersion curves (solid line: exact; open circles: measured) of the F_{11} mode in a thin rod.

that Eq. (7) is very accurate for small fa values. In addition, F_{11} exhibits dispersion characteristics very similar to the zeroth antisymmetric plate mode.[5]

The basic arrangement for exciting and receiving thin rod flexural acoustic waves is shown in Fig. 2 . A 10.5-μm-radius gold wire is fixed at two posts by adhesive tapes. A piezoelectric longitudinal ultrasonic transducer is bonded at the end of a silica glass horn. The small displacements of the ultrasonic transducer are transformed into large displacements at the tip of the horn. The tip of the horn is adhesively bonded to the gold wire. When the longitudinal waves in the glass rod reach the thin tip, they excite the flexural acoustic F_{11} mode in the wire. The same type of transducer is also used at the receiving end. A similar excitation geometry and mechanism has been reported in Ref. 13.

We have adopted the standard continuous-wave (cw) technique to measure the phase velocity of the F_{11} mode. The measured phase difference of the received cw acoustic signal due to the change in propagation distance of the F_{11} mode is used to calculate the phase velocity. The measured phase velocities indicated as closed circles in Fig. 1 agree very well with the theoretical prediction which assumes zero tension. Radio-frequency (rf) pulses were used to measure the group velocity dispersion. The measured group velocities indicated as open circles in Fig. 1 also agree with theoretical calculations. For velocity measurements the operating frequency ranges from 0.4 to 3.0 MHz.

The F_{11} mode has been generated along an alumel wire coated with a piezoelectric thin film[14] and along a glass fiber by a noncontact laser technique.[15] The A_0 mode generated by an electrostriction method on a silicon nitride

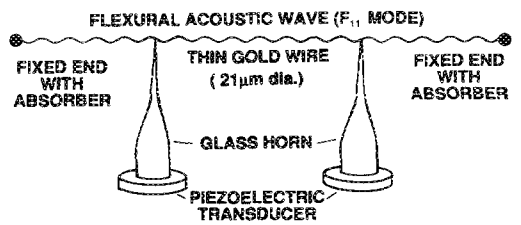

FIG. 2. Basic experimental arrangement for the velocity measurement of the F_{11} mode in a thin rod.

film has also been reported.[16] Since rod materials can be made of metals, glasses, polymers, etc., with relative ease, it is possible to find a suitable excitation and receiving geometry for a thin rod flexural acoustic wave gravimetric sensor made of a specific thin rod.

An analysis of the dispersion and sensitivity characteristics of thin rod gravimetric sensors based on the F_{11} flexural wave was presented. The diameter of the thin rod is much less than acoustic wavelength. The measured phase and group velocity dispersion curves agree well with theoretical calculations. For small added masses, the calculated mass sensitivity for thin rod gravimetric sensors is $-1/(2\rho a)$ provided that there is negligible change in a and E, where ρ, a, and E are the density, radius, and Young's modulus of the wire, respectively. The proposed devices potentially have the same advantages of high mass sensitivity which plate-mode sensors can offer over their BAW and SAW counterparts. Since thin rods may be produced directly from the absorptive coating materials normally used for SAW and plate-wave vapor sensors,[3–5] very high mass sensitivity could be obtained due to the potentially large ratio of $\Delta m/m$. In addition, the thin rod also has very low heat capacity and hence a rapid thermal response.

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