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DIVISION OF MECHANICAL ENGINEERING
OTTAWA, CANADA
LABORATORY MEMORANDUM
SECTION ENGINE LABORATORY

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SUBJECT The Effect of Variable Mass on the
Critical Speed of a Shaft

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Visiting Professor
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ISSUED TO

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SUMMARY

During the writer's short stay at the Engine Laboratory of the National Research Council, some discussions were held with Mr. P. Y. Kim and Dr. I. R. G. Lowe regarding shaft vibration. The writer had carried out some experiments in respect to the effect of external effects on the critical speeds of shafts but these had not been developed for publication. It is hoped that this Memo will serve to generate a discussion on the subject and lead to further work. Specifically, the problem deals with the existence of shaft motions for engine systems that have not been reported elsewhere or have been presented incompletely.

LABORATORY MEMORANDUM

1.0 INTRODUCTION

This discussion deals with transverse vibrations in shafts. The general subject of critical speeds of whirling speeds of shafts is well established but a short review is given in order to establish a framework and indicate the areas of interest in respect to this note. (See any of the listed references.)

The basic system considered is a single disk on a weightless shaft. It is assumed that a small unbalance exists. Damping or frictional effects may be neglected in the typical analysis, although such effects may lead to other specific problems.

The system may be more complicated if more-disks are added. Good results are often obtained neglecting the shaft mass, but distributed mass system can be readily analyzed.

Refinements in the analysis include taking account of shear deformation and rotary inertia during shaft deflection, and considering axial loads on the shaft.

The foregoing comments on the critical speeds of rotating shafts in fact apply to the analysis of transverse natural vibrations of beams.

A rotating shaft, however, may have additional effects due to the gyroscopic action of a disk which may serve to stiffen the system.

2.0 CRITICAL SPEED

Consider a typical case:

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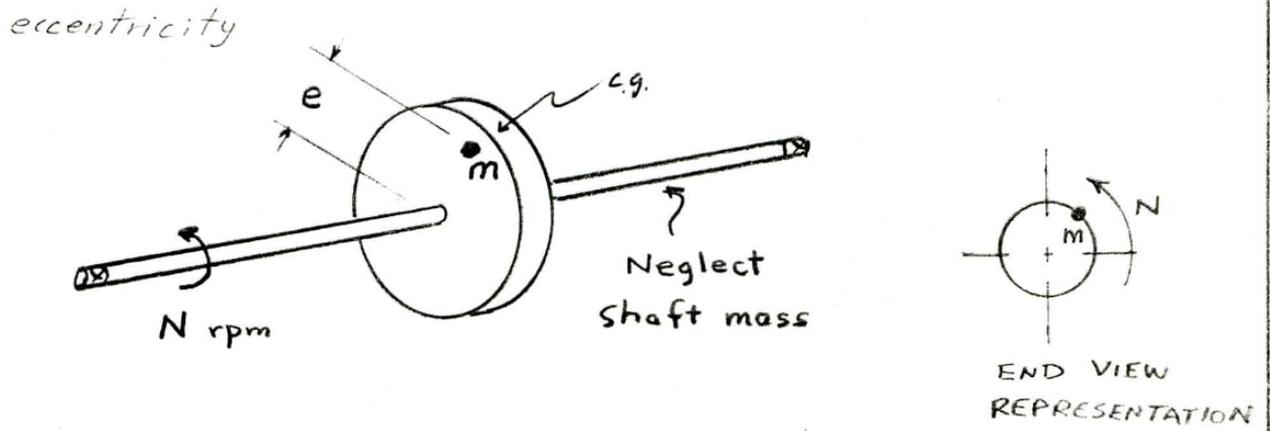


FIG 1 BASIC SYSTEM

The amplitude of the vibration may be shown in the response curve.

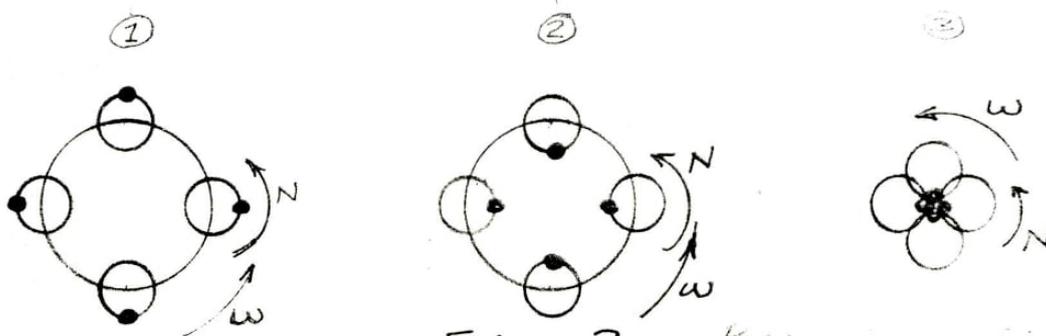
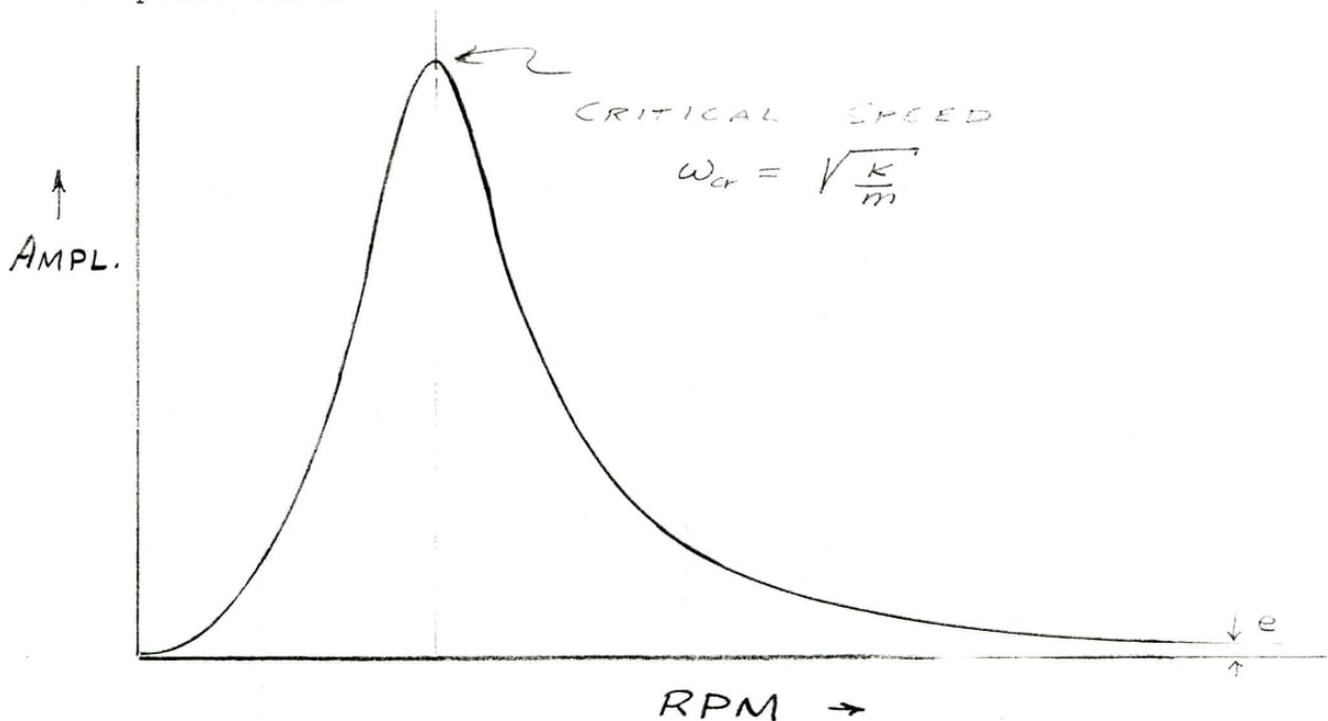


FIG 2

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Note that the unbalanced mass swings to the outside below the critical speed. At higher rotational speeds, the mass tends to remain fixed and the shaft whirls around it. Theoretically, at the critical speed, the phase angle is 90° . The phase angle between the unbalanced mass and the deflection varies as a function of damping or resistance to the motion as the shaft and disk whirl.

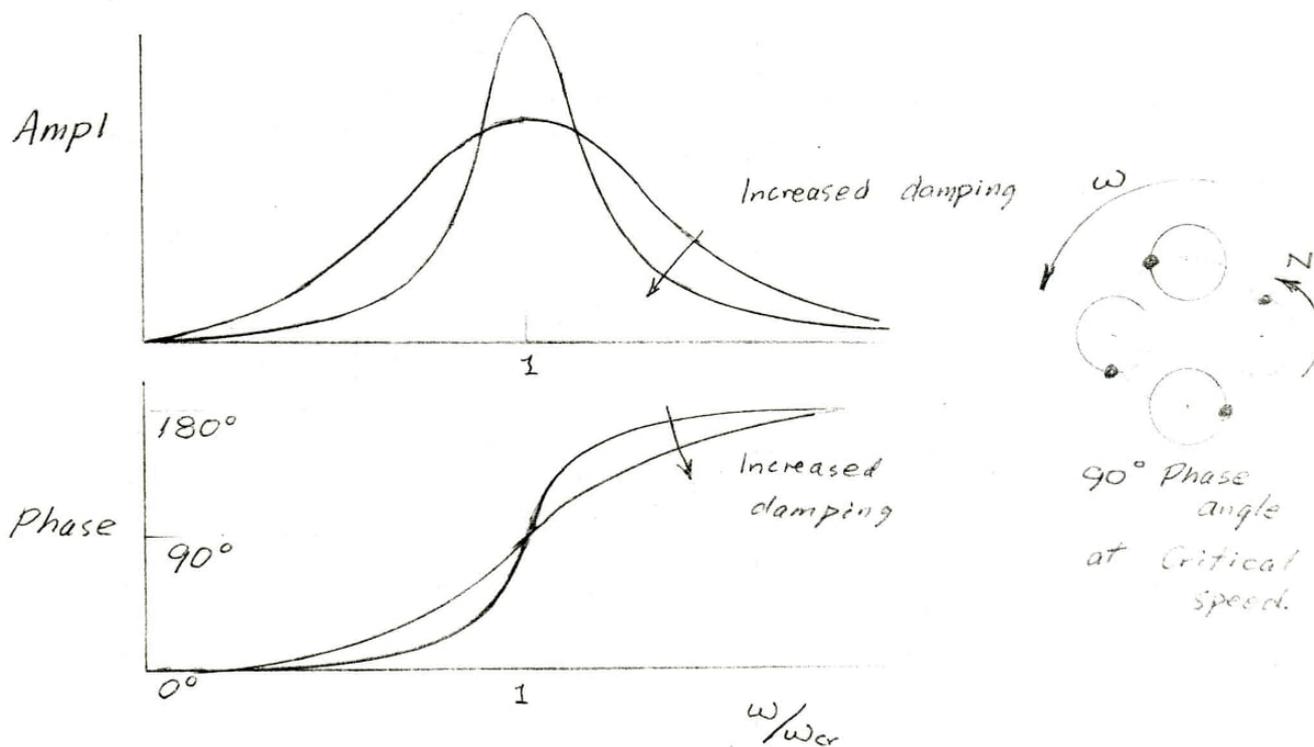


FIG 3 Phase Angle

The forces acting during vibration are indicated in the next sketch.

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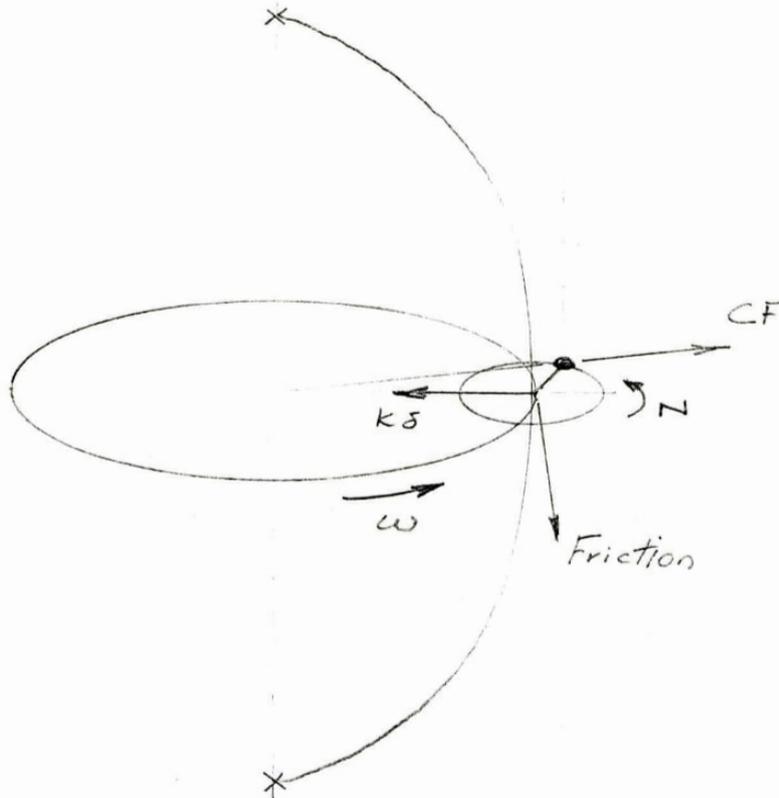


FIG 4 DAMPING EFFECTS

The friction force opposes the shaft whirl and is assumed to be acting on the centerline. The three forces are not concurrent and it can be seen that a torque must be applied to cause the shaft to increase speed beyond this value and reach the critical speed. It is possible to estimate the additional torque required for pulling through the critical speed, although little has been done in this respect.

3.0 SECONDARY CRITICAL SPEED

If a shaft is in a horizontal position and carries unbalanced disks, then gravity effect on the disks can produce a critical speed of the second order (den Hartog - p. 247). This vibration occurs at one half the regular natural frequency.

Another effect of this type is caused by variable flexibility of the shaft. This is of a similar nature but more pronounced ~~than~~ gravity effect.

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4.0 SELF-EXCITED VIBRATIONS

Several vibrations of this type may be possible in shafts. The first is a dry-friction motion, found 'on' shafts with guides rather than bearings. Oil whip motions may occur in the case of systems with journal bearings. Finally hysteresis induced motions, especially if lightly shrunk disks are used may occur.

5.0 COUPLED SYSTEMS - VARIABLE MASS

In the study of torsional vibration, the effect of coupling with transverse motion is well noted (den Hartog p. 172, Timoshenko p. 261, Thomson p. 248). Specifically a reciprocating mass on a crank appears to have variable mass in respect to torsional oscillation. Similarly, but not well documented, such a mass should have an effect on lateral vibrations or whirling speeds.

A simple case is that of a crankshaft with piston:

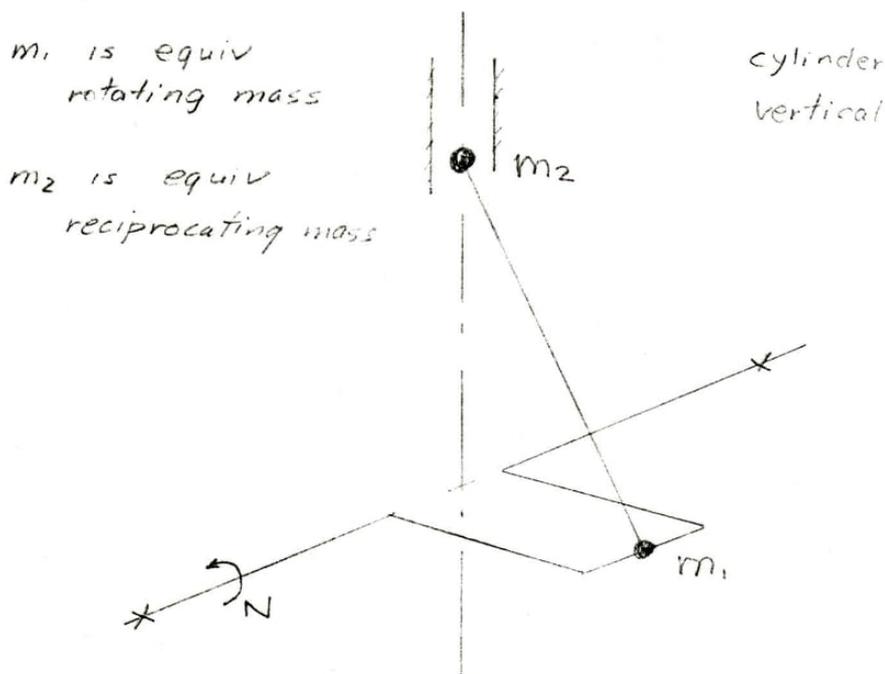


FIG 5

Slider Crank system

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For the case of torsional vibrations, the apparent mass depends on the crank angle (see references). For the case of transverse vibration, the apparent mass is different for horizontal or vertical transverse vibration. That is, the shaft sees the mass m_2 only for vertical motions. Note that the torque fluctuations due to inertia and pressure loadings (as well as torsional vibrations) may also be coupled to the transverse vibration.

The writer has carried out tests on an idealized version of a variable mass system (Cochkanoff, Ref. 4). The system consists of a rotating mass and a non-rotating mass fixed by a rigid bar to the shaft. Thus the additional mass is seen in one plane but not the other. This system does not have the added complication of a crank with its complex transverse deflection problems but retains the variable mass aspect.

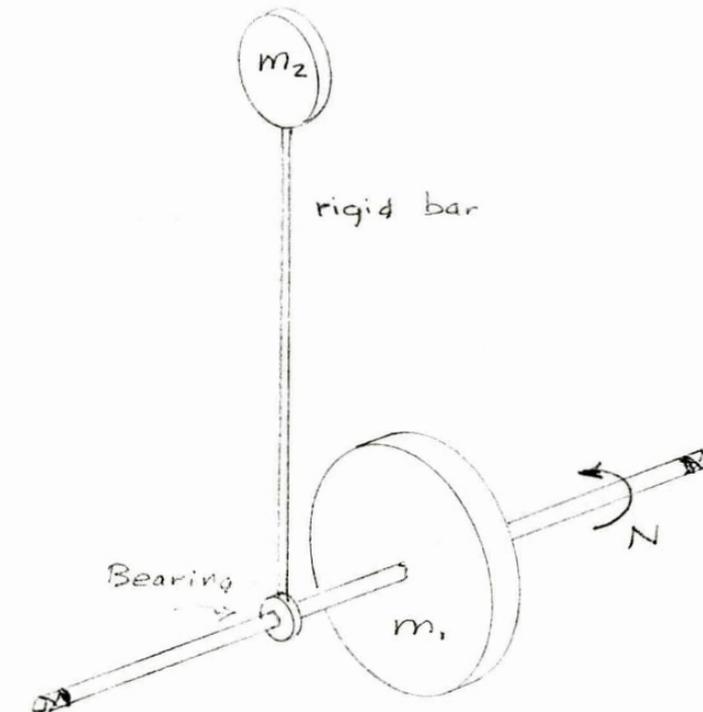


FIG 6 Variable Mass System

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Tests were carried out on this system with the non-rotating mass suspended vertically. Two distinct critical speeds were observed. The mass is greatest for vertical motion and thus the vertical frequency is lower. The shaft motion was picked up in both horizontal and vertical directions and was used to construct Lissajous figures of the motion.

The response curves and shaft motions are described below.

The system behaviour below and above the critical speeds is similar to the simple case. That is the unbalanced mass is either on the outside or on the inside.

At and between the critical speeds, the shaft behaviour was found to be somewhat different than reported elsewhere. Initially the whirl is in the same direction as the rotation. At the maximum amplitude of this vertical vibration, the phase angle changes and the whirl appears to "collapse" and then grow out again. The phase angle for vertical motion is now 180° whereas the horizontal motion is still below its critical speed and retains a zero phase angle.

For this motion to exist, the whirl must be in the opposite sense to the shaft rotation.

At the higher natural frequency, a similar transient effect must take place in order for the unbalanced mass to take up an inner position for horizontal motion.

The existence of these cusps in the response curves and the collapse and passing through of the motion of whirl has not been reported elsewhere. (It was first noted in the author's unpublished thesis.) It is considered that this observation can be useful in defining the exact value of the critical speed for such cases.

Further, it is hoped that this analysis may prove useful in establishing transverse natural frequencies for small engines where the variable mass problem may introduce a significant factor in critical speed calculations.

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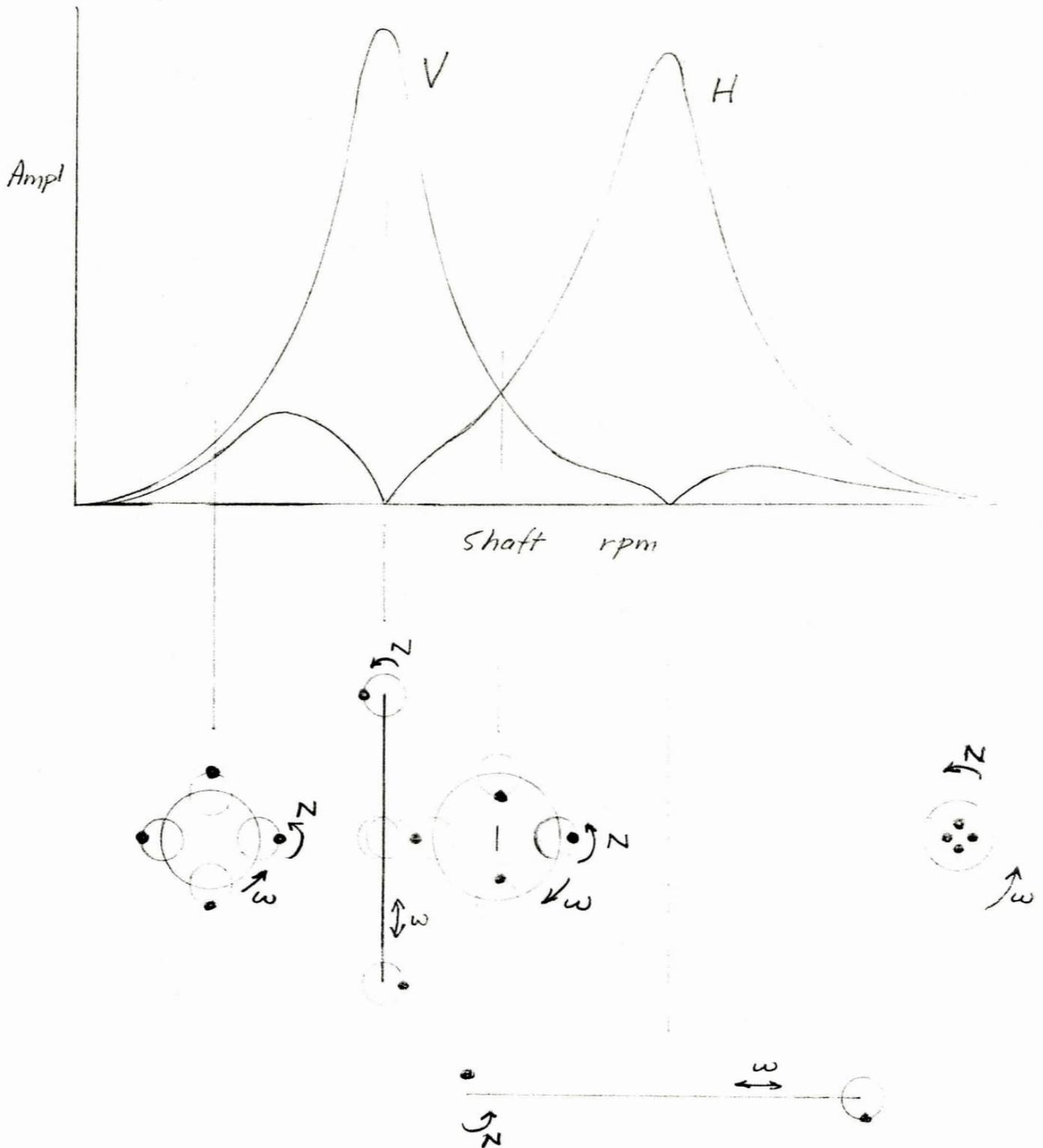


FIG 7 Critical Speeds

An interesting feature was observed in the tests. The motions at the peaks (and cusps) were not truly horizontal and vertical, but inclined at an angle. Thus, when the vertical motion was a maximum and the horizontal motion was zero, the resultant Lissajous figure tended to be a line inclined somewhat to the vertical.

An interesting extension of this work is noted in the next section.

6.0 FLEXIBLE BEARINGS

In this case the shaft is on bearings which are not rigid but exhibit different flexibilities in the two orthogonal directions. It is easy to see that the apparent stiffness of the shaft can be used to determine the critical speed and that two distinct values will be obtained. den Hartog (p. 229) discusses this case. His response curves are not quantitative and the curves do not represent the correct detail of the motion as **observed** by this writer. In fact the curves also have cusps and thus the amplitude of vertical motion is zero at the horizontal critical speed and vice versa.

This behaviour, in fact is **entirely** of the type discussed under the case of variable mass or coupled vibration type. In relatively highly damped systems, the amplitude peak can be easily defined because of the sharp zero noted in the orthogonal motion.

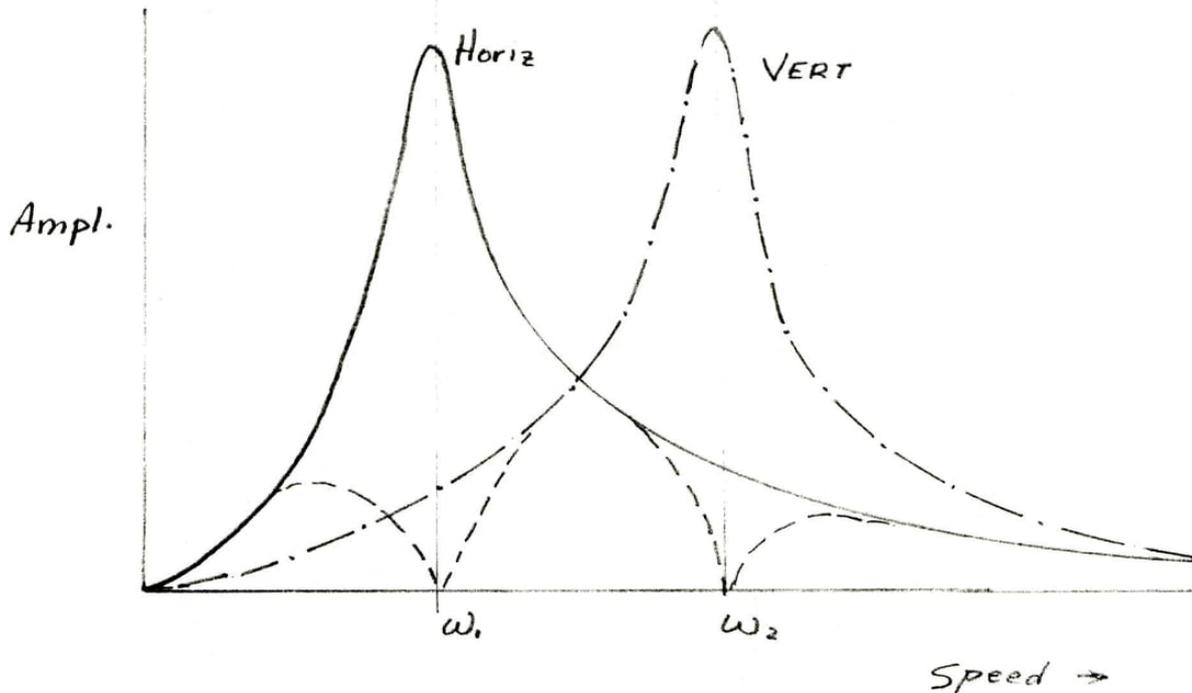
It is assumed that the shaft system is more flexible in the horizontal plane. Thus the horizontal natural frequency is lower than the vertical natural frequency. Again the whirling motion "collapses" and reforms a circle.

Again it can be deduced that the whirl direction between the natural frequency must be opposite in direction to the shaft rotation. Thompson (p. 85) notes this but does not refer to the reasons nor does he mention the transient condition of zero amplitude necessary for this to be achieved. His equations do not reflect this fact either.

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Basic Curves as shown in den Hartog.
Cusps shown as observed by writer

FIG. 8 VIBRATION OF SHAFT WITH FLEX BRGS.

7.0 RECOMMENDATIONS

It would be most worthwhile to consider the more detailed modelling of a shaft with variable mass. This would involve a more thorough analysis of the dynamics of a small engine.

The validity of using a model of the type described should be reviewed.

The tests undertaken by the author could be repeated to verify the conclusions.

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