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Galitzine, N.

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SUBJECT EFFECTS OF COMPRESSIBILITY IN WING-IMMERSED FANS FOR LIFTING VTOL AIRCRAFT

PREPARED BY N. Galitzine

ISSUED TO Internal

TABLE OF CONTENTS

NATIONAL RESEARCH COUNCIL

DIVISION OF MECHANICAL ENGINEERING

LABORATORY MEMORANDUM

EFFECTS OF COMPRESSIBILITY IN WING-IMMERSED LIFTING FANS FOR VTOL AIRCRAFT

1.0 INTRODUCTION

In reference 1 a rather simplified theory of axial-flow wing-immersed lifting fans for VTOL aircraft was presented for the purpose of preliminary design and size and weight estimation.

At the time of writing disc loadings of about 200-400 lb/ft^2 were considered, and it was felt that the effects of compressibility on the design of the fans could be neglected.

Reference 2 investigated the effects of fan disc loading on the weight and cost of VTOL powerplants with gross disc loading varying from 200 to 1000 lb/ft^2. The resulting curves of weight and cost were lowest but rather flat in the 500-1000 lb/ft^2 region, showing optima lying between 500 and 750 lb/ft^2, depending upon the criteria of either weight or cost.

In view of these conclusions, demonstrating the desirability of using the higher disc loadings, it was felt appropriate to introduce some questions of compressibility. The present memorandum is intended primarily as a record of some not very recent work on that subject.

Two main effects of compressibility are apparent, the one dealing with the geometry of the fan and its influence on disc loading limits, the other with the efficiency of the fan, as affecting the drive power requirements.

The memorandum here deals only with a particular type of fan, necessarily very shallow for wing-immersion, in which the prime object is to accelerate as efficiently as possible a large mass flow of air to high velocity in the shortest possible distance. This precludes the multi-staging of compression with a final expansion through a nozzle, such as in effect is done in turbojet engines.

2.0 GENERAL RELATIONSHIPS

Considering the fan as an air accelerating device, producing thrust as a result of its exit axial velocity

V ft/sec, after a change from 0 to V in the interior, the general form of the relationship between the thrust, power, and efficiency, remains substantially the same, whether compressibility is taken into account or not.

This is demonstrated below, for as in reference 1, neglecting any pressure terms on the outlet faces, the thrust X produced by the fan

$$X = \frac{WV}{g} = \frac{\rho A V^2}{g} \text{ lb.}$$

Where W = mass flow of air through the fan, lb/sec.

ρ = density of the air at outlet, lb/ft^3.

A = net cross-sectional area, or the active or blade-swept area of the fan at outlet, ft^2.

The kinetic energy of the fan discharge

$$= \frac{WV^2}{2g} = \frac{\rho A V^3}{2g} \text{ ft. lb./sec.}$$

and if the overall efficiency of the fan $= \eta$, then

$$\frac{\rho A V^3}{2g} = \eta Y$$

where Y = input power to drive the fan, ft. lb./sec.

By derivation from the above relations ~~and~~

$$\frac{X}{Y} = \frac{2\eta}{V} \quad \text{and} \quad V = \sqrt{\frac{gX}{\rho A}}$$

which when V is eliminated give

$$\frac{X}{A} = \frac{4\rho\eta^2}{g(X/Y)^2}$$

This is an expression for the net disc loading X/A in terms of the power loading X/Y, the efficiency η, and the air density ρ.

The only difference in this from reference 1 is that the density ρ and area A are specifically to be taken as being at the outlet of the fan.

If compressibility is neglected, ρ and A are constant through the fan, but if it is taken into account, the density varies, thus also affecting the area and the internal geometry of the fan.

As the disc loading rises so does the variation in density, and also the axial velocity V of the air increases until, in combination with the peripheral velocity of the fan blades, a relative velocity is reached which falls into the transonic region, with a consequent drag rise of the fan blades aerofoils, and loss in fan efficiency η.

It is these two effects of compressibility that are discussed in the following sections of the memorandum.

3.0 FAN GEOMETRY.

For the purpose of the present analysis the fan may be split up into two distinct parts, as shown in figure 1.

a) The Bellmouth in which the air is accelerated from a static condition to an axial velocity V at the expense of a pressure drop ΔP.

b) The fan rotor in which the pressure drop ΔP is regained at the expense of the drive power Y.

Also necessary for any fan with a single rotor are stator guide vanes to correct for swirl at the outlet. These vanes may be placed either at the inlet or the outlet of the rotor, and with two counter-moving rotors there is in fact no need for them because the two rotors may be used to correct each other. In any case conditions of flow through the vanes are such that no large differences of pressure occur, thus permitting omission of them from any further discussion in this memorandum.

3.1 Constant Area Rotor.

The more obvious type of fan rotor is located in a parallel duct following the Bellmouth, and is therefore called a constant area rotor. Referring to figure 1, with subscripts 0, 1 and 2 denoting the ambient static, rotor inlet, and rotor outlet conditions, the following ideal (reversible adiabatic) relations are established if compressibility is taken into account:-

$$\text{Thrust } X = \frac{\rho_2 A_2 V_2^2}{g} \text{ and } A_2 = A_1 = A \text{ constant.}$$

Net disc loading based on the active or blade-swept area A

$$\frac{X}{A} = \frac{\rho_2 V_2^2}{g}$$

$$\text{By continuity } \rho_1 V_1 = \rho_2 V_2 \therefore V_2 = \frac{\rho_1}{\rho_2} V_1$$

$$\text{and } \rho_2 = \rho_o \qquad \rho_1 = \frac{P_1}{RT_1} = \rho_o \frac{P_1}{P_o} \left(\frac{P_o}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Therefore

$$\frac{X}{A} = 2P_o \frac{\gamma}{\gamma - 1} \left(\frac{P_1}{P_o}\right)^2 \left(\frac{P_o}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \left[\left(\frac{P_o}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

and

$$\frac{V_1}{V_2} = \frac{P_o}{P_1} \left(\frac{P_1}{P_o}\right)^{\frac{\gamma-1}{\gamma}}$$

There is a limiting net disc loading when the Bellmouth throat chokes and P_1/P_o reaches the critical value of 0.53.

With P_o = standard atmosphere, 14.7 lb./in.^{2}

$\gamma = 1.4$ and $\gamma-1/\gamma = 0.286$

The limiting disc loading

$$\frac{X}{A} = 2\text{x}14.7\text{x}144\text{x } \frac{1}{0.286} \text{ x } (0.53)^2 \left(\frac{1}{0.53}\right)^{0.286} \left[\left(\frac{1}{0.53}\right)^{0.286} - 1\right] = 995 \text{ lb./ft.}^2$$

and $\frac{V_1}{V_2} = 1.57$

In an actual case where the adiabatic compression efficiency of the rotor may be say 85%, the corresponding figures are not too different, with

$X/A = 1000$ lb./ft.^{2}, $V_1/V_2 = 1.56$

Conditions at the limit are illustrated in figure 1

Thus it is seen that in a constant area rotor, besides the limiting net disc loading of about 1000 lb./ft.^{2}, there is an undesirable excess of axial air acceleration before the rotor with a deceleration in the rotor itself, a condition quite probably prejudicial to efficiency. Also, in an actual case, with a hub occupying part of the gross disc area, and inlet guide vanes perhaps preceding the rotor stage, the gross disc loading limit would be below the above net figure, and therefore inadequate to fulfill the required top limit of reference 2 (1000 lb./ft.^{2}).

3.2 Constant Velocity Rotor.

Following up the conclusions of the previous section, it is further obvious that a rotor with at least constant axial velocity would be preferable.

Assuming therefore that $V_1 = V_2 = V$ constant, the ideal relations come out as below:-

Thrust $X = \frac{\rho_2 A_2{}^2 V^2}{g}$

Net disc loading based on largest active rotor area $= \frac{X}{A_1}$

By continuity $\rho_1 A_1 = \rho_2 A_2 \qquad \therefore A_2 = \frac{\rho_1}{\rho_2} A_1$

Also $V = \sqrt{2\, gRTO \frac{\gamma}{\gamma-1} \left[1 - \left(\frac{P_1}{P_o}\right)^{\frac{\gamma-1}{\gamma}}\right]}$

$\rho_2 = \rho_o$ and $\rho_1 = \frac{P_1}{RT_1} = \rho_o \frac{P_1}{P_o} \left(\frac{P_o}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \rho_o \left(\frac{P_1}{P_o}\right)^{\frac{1}{\gamma}}$

Therefore

$$\frac{X}{A_1} = 2\, P_o \frac{\gamma}{\gamma - 1} \left(\frac{P_1}{P_o}\right) \left[\left(\frac{P_o}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

and

$$\frac{A_2}{A_1} = \frac{P_1}{P_o} \left(\frac{P_o}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_1}{P_o}\right)^{\frac{1}{\gamma}}$$

The limiting disc loading in a standard atmosphere, when the bellmouth throat chokes and $P_1/P_o = 0.53$,

$$\frac{X}{A_1} = 2 \times 14.7 \times 144 \times \frac{1}{0.286} \times 0.53 \left[\left(\frac{1}{0.53}\right)^{0.286} - 1 \right] = 1560 \text{ lb/ft}^2$$

and $\frac{A_2}{A_1} = 0.636$

In an actual case, with rotor adiabatic compression efficiency = 85%, the limiting disc loading is the same 1560 lb./ft.^{2}, but the area ratio is slightly different, with $A_2/A_1 = 0.655$.

At the above limiting disc loading the rotor becomes all supersonic, since the axial velocity is exactly sonic, and the relative velocity therefore supersonic.

Under and approaching such circumstances operation is likely to be very unstable, with efficient performance only at the design point (Reference 3).

However, as pointed out later in the memorandum, operation is likely to be satisfactory at the gross disc loading of 1000 lb./ft.^{2}, which is the horizon of the present study. Conditions at this loading are illustrated in Figure 1.

The constant velocity rotor therefore probably covers any gross disc loading that may at present be foreseen as practicable. The necessary area ratio A_2/A_1 is plotted, against net disc loading, in Figure 2, using an assumed compression efficiency of 85%.

4.0 FAN EFFICIENCY.

The fan overall efficiency in the general relationship

$$\frac{X}{A} = \frac{4\rho\eta^2}{g(x/y)^2}$$

is probably best obtainable from experimental tests, since it covers all the losses in the bellmouth and stator guide vanes, etc., as well as the losses in the rotor itself.

However, rough comparative values may perhaps be usefully derived from the rotor efficiency, which is itself traceable to the basic rotor blade aerofoil characteristics of lift and drag, as shown in Reference 1.

The rotor efficiency η_r, though related to, is not of course to be confused with the adiabatic compression efficiency of the previous sections.

From reference 1:-

$$\eta_r = \frac{1 - \frac{D}{L}\, 2\phi \frac{1}{1 + \theta}}{1 + \frac{D}{L}\, \frac{2}{3\phi}\, \frac{1 + \theta + (\theta)^2}{1 + \theta}}$$

Where L/D = aerofoil lift/drag ratio

ϕ = axial/tip velocity ratio

θ = hub/tip radius (or diameter) ratio

Reference 1 and subsequent analysis also established that approximately $\theta = K\phi$, where K = 0.68 for single rotor fans, and 0.38 for counter-moving fans. These limits of the hub/tip ratio θ are based on the fact that the hub is the most critical part of the fan blade design. Here the relative air velocity is at its minimum along the blade length, due to the reduced peripheral speed, so that the lift coefficient C_L is strained to the limit in order to meet the required pressure rise through the fan rotor. The values of 0.68ϕ and 0.38ϕ for θ have been calculated on the basis of $C_L = 1.9$ as the limit for an isolated aerofoil, which is then considered in a row or cascade of other aerofoils, and corrected accordingly.

There is a large amount of literature on aerofoil characteristics. References 4 and 5 were particularly chosen as applicable to the present study. Their lift/drag ratios versus Mach number for 6% two dimensional aerofoils are combined in Figure 3.

Taking as example the constant velocity rotor, it is first necessary to find an expression connecting the axial air velocity and the fan tip speed with the disc loading, in order to relate disc loading to the rotor efficiency η_R .

Gross disc loading, based on the whole fan area, as in reference 2, is of prime interest. Denoting this as Z,

$$Z = \frac{X}{\pi\tau^2}$$

where τ = fan tip radius at the rotor inlet.
Assuming the rotor inlet as the point where the hub relation $\Theta = K\phi$ applies, the blade swept area A_1 at the inlet

$$= \pi\left[\tau^2 - (\Theta\tau)^2\right] = \pi\tau^2\left[1 - \Theta^2\right]$$

Therefore

$$\frac{X}{A_1} = \frac{Z}{1 - (K\phi)^2}$$

With reference[2] to the section on the constant velocity rotor

$$X = \frac{\rho_2 A_2 V_2^2}{g} = \frac{\rho_1 A_1 V^2}{g} \quad \text{and} \quad \therefore \frac{X}{A_1} = \frac{\rho_1 V^2}{g}$$

$$\text{Therefore } Z = \frac{\rho_1 V^2}{g}\left[1 - (K\phi)^2\right]$$

$$\text{Now } V = \sqrt{2gRT_o \frac{\gamma}{\gamma-1}\left[1 - \left(\frac{P_1}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\text{and } \rho_1 = \rho_o \frac{P_1}{P_o}\left(\frac{P_o}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{from both of which}$$

$$Z = \frac{\rho_o V^2}{g} \left[1 - (K\phi)^2\right]\left[1 - \frac{V^2}{2gRT_o \frac{\gamma}{\gamma-1}}\right]^{\frac{1}{\gamma-1}}$$

Introducing the fan most suitable for high disc loading, with counter-rotation, when $K = 0.38$, gives

$$Z = \frac{\rho_o V^2}{g} \left[1 - 0.145 \left(\frac{V}{U}\right)^2\right]\left[1 - \frac{V^2}{2gRT_o \frac{\gamma}{\gamma-1}}\right]^{\frac{1}{\gamma-1}}$$

where U = fan tip speed.

It follows that rotor efficiency η_R may be obtained in terms of the disc loading Z and tip speed U, since Z gives V from the above expression, and V and U in turn give ϕ and hence Q.

Values of L/D in the expression for η_R are obtainable from Figure 3, since the Mach numbers of the relative air velocities from hub to tip may be derived from U and V. Relative velocity at the tip

$$= \sqrt{U^2 + V^2}$$

and at the hub $= \sqrt{Q^2 U^2 + V^2}$

Figure 4 has been derived in this manner, assuming that the Mach numbers of the relative air velocities are referred to the temperature T_1.

The use of maximum, rather than actual working, L/D values is justified on the grounds that approximate relative, not absolute, efficiencies have been sought. Unlike in aircraft wings, an aspect ratio of infinity may be assumed for the fan blades, since end losses must be negligible, thus resulting in some of the rather high L/D values which are shown.

Figure 4 shows the variation of relative rotor efficiency η_R with gross disc loading Z, at three different tip speeds. The range of gross disc loading covered is from near zero to the limit imposed by the choking of the fan bellmouth throat. At a tip speed of 1000 ft./sec. this limit is 1326 lb./ft.^{2}, equivalent to the net disc loading of 1560 lb./ft.^{2} previously found.

If it is assumed that the overall fan efficiency η is directly related to η_R, then values of η may be derived, and are also shown in Figure 4, where for simplicity they are related by a constant increment to η_R (rather than perhaps the more correct constant multiplier). The assumption is that η = 80% at a gross disc loading of 500 lb./ft.^{2} and tip speed = 800 ft./sec.

The fan efficiency η then obtainable at a gross disc loading of 1000 lb./ft.^{2} and tip speed 800 ft./sec., for example, is 70%. At these conditions, the corresponding axial velocity is also (by coincidence) 800 ft./sec. and rotor tip inlet relative = 1130 ft./sec. or Mach 1.07. Reference 6 has demonstrated that stable efficient operation is possible over a wide range of speeds, at reliable inlet Mach numbers of at least 1.1 and under.

5.0 CONCLUSIONS.

1. In an axial flow fan, of which the rotor has a constant cross-sectional flow area, compression choking in the throat of the bellmouth inlet limits the net disc loading, based on the net flow area, to about 1000 lb./ft.^{2}. The limit of gross disc loading, based on the gross area, is correspondingly lower, depending upon the area occupied by the hub. There is also a deceleration of the axial velocity in the rotor which is probably prejudicial to efficiency at high disc loadings.

2. In a fan, of which the rotor has a constant axial flow-velocity, and correspondingly decreasing flow area, the limit of net disc loading is 1560 lb./ft.^{2}. Gross disc loading is correspondingly lower than this figure, depending upon the hub size but adequate to cover the horizon of the present memorandum, which is 1000 lb./ft.^{2}. The required flow area reduction at the loading limit is about 35%.

3. Due to compressibility effects as a result of high Mach number, particularly at the tip, the efficiency of a fan decreases with disc loading, at disc loadings over about 500 lb./ft.^{2}. Thus in a fan with finite hub size, (which itself depends on the disc loading), and where the efficiency might be say 80% at a gross disc loading of 500 lb./ft.^{2}, tip speed 800 ft./sec., the corresponding efficiency at 1000 lb./ft.^{2} is estimated to be roughly 70%, and at the

limits of disc loading even lower (50 - 55%).

4. The loss in efficiency at high disc loading implies higher power to drive the fan, and hence greater size and weight of the power generator. Multi-staging of compression in the fan, and final expansion through a nozzle may be good for efficiency, but is very likely not possible in wing-immersion, and probably merely shifts the burden of size and weight increase from the power generator to the fan.

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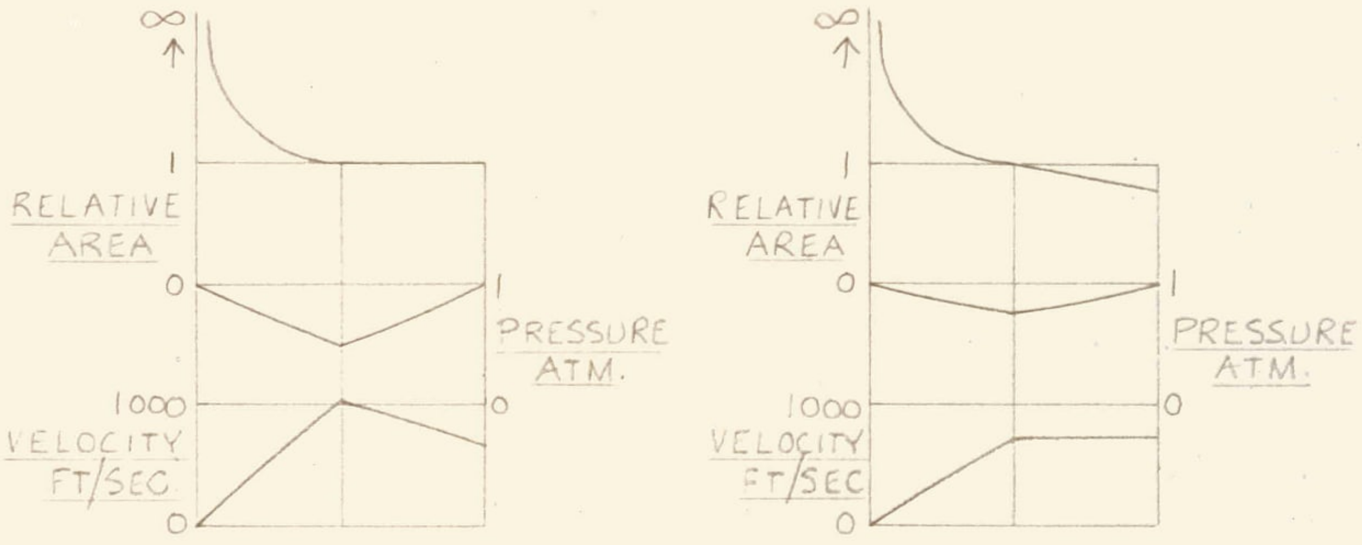
FIG. 1.

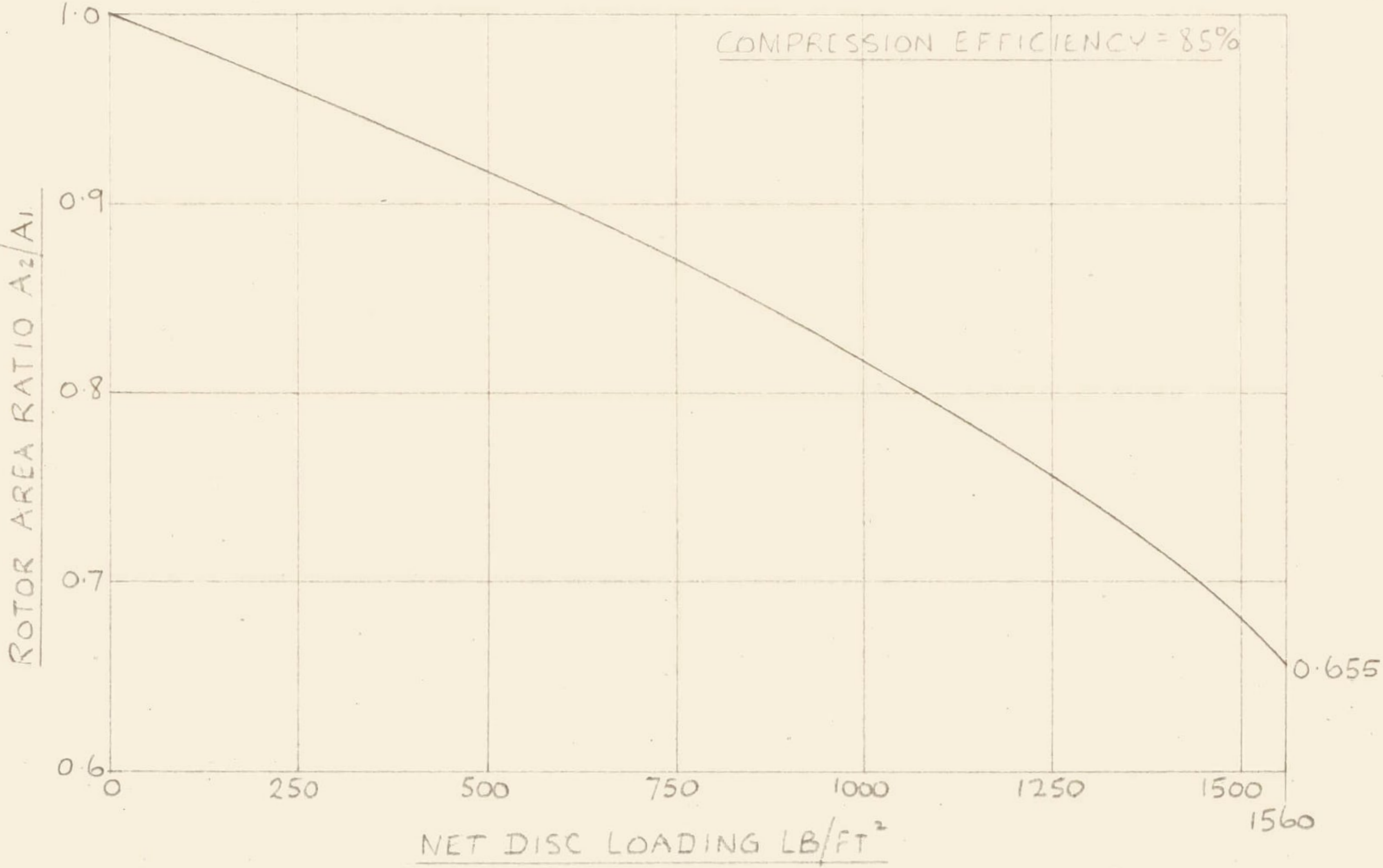


CONDITIONS FOR 1000 LB/FT² NET DISC LOADING

CONSTANT AREA ROTOR

CONSTANT VELOCITY ROTOR

DIAGRAMS OF FAN GEOMETRY



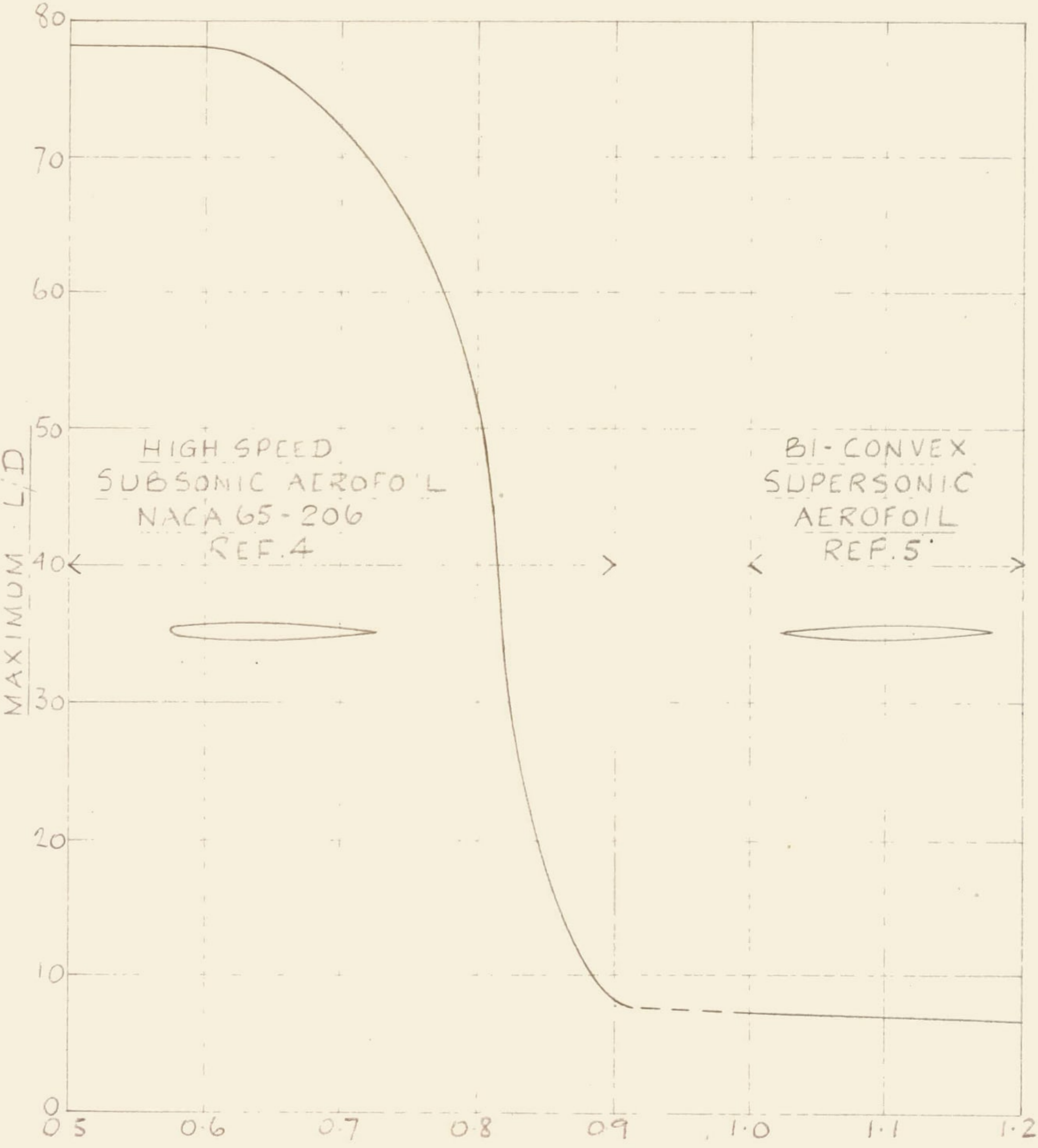
AREA REQUIREMENTS OF CONSTANT VELOCITY ROTOR

FIG. 2.

FIG. 3

MAXIMUM LIFT/DRAG RATIOS OF 6% AEROFOILS

(ASPECT RATIO = ∞)



MACH NUMBER

FIG.4

FAN RELATIVE EFFICIENCIES

(WITH COUNTER-ROTATION AND CONSTANT AXIAL VELOCITY)