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RÉSUMÉ

Les crêtes de pression de la glace de mer de première année sont modélisées sous forme de prismes bidimensionnels se comportant selon la loi de Mohr-Coulomb. La répartition des contraintes passives est obtenue en utilisant une solution similaire et des polynomes approximatifs de la variable dépendante. Ces analyses permettent de calculer les forces horizontales dans les couvertures de glace liées à la formation de crêtes de pression flottantes et dans certains cas, aux amas de glace le long des côtes.

STRESSES IN FIRST-YEAR ICE PRESSURE RIDGES

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ABSTRACT

First-year sea ice pressure ridges are modelled as two-dimensional wedges of a Mohr-Coulomb material and the passive stress distribution is obtained using a similarity solution and approximate polynomial forms of the dependent variable. Analysis gives the horizontal forces in ice covers associated with formation of floating pressure ridges and some cases of rubble pile-up against shores.

INTRODUCTION

Ice floes driven by wind and current often fail on encountering other ice floes or fixed obstacles, by crushing, flexure or buckling, with resulting acummulation of ice blocks. These masses form a variety of features such as pressure ridges, pile-ups on shores, and rubble fields surrounding offshore structures. There have been extensive field, laboratory, and analytical investigations of the frequency of occurrence and the characteristics of such ridges and pile-ups, and of associated loads. Kovacs and Sodhi (1) presented a comprehensive review of the literature on ice rubble pile-ups; and a number of recent studies were included in the Proceedings of the Workshop on Sea Ice Ridging and Pile-up (2). Early investigations of ice rubble features and geometrical models of ridges were reported by Zubov (3); recent investigation of first-year pressure ridges in the Beaufort Sea have been presented by Tucker and Govoni (4).

An analytical model of the kinematics of pressure ridge formation was developed by Parmerter and Coon (5), who obtained lower bounds for the associated forces by equating the work done by the advancing ice sheet to the increase in the potential energy of the ridge. This approach has been adopted in all subsequent studies that estimate ridging or pile-up forces. Some authors have used semi-empirical formulas of soil mechanics to include frictional forces as well. Kry (6) discussed ice failure modes associated with rubble formation and the resulting forces on wide offshore structures. Mellor (7) treated brash ice as a Mohr-Coulomb material and considered some simple cases related to pressure ridging.

Laboratory experiments on model ice rubble performed by Keinonen and Nyman (8) and by Prodanovic (9) suggest that the bulk rubble obeys the Mohr-Coulomb yield criterion. In other experiments by Tatinclaux and Cheng (10) the rate effect on shear resistance was examined. The present study is concerned with the development of solutions for stress distributions in first-year pressure ridges and some cases of rubble pile-up.

GOVERNING EQUATIONS

Deformation of bulk rubble during ridge building or rubble piling is usually comprised of relative motion and rearrangement of the blocks, in addition to failure of individual blocks. This mode of deformation is similar to the behaviour of granular materials and soils. Both observation and experimental evidence suggest that in situations of present interest the rubble may obey the Mohr-Coulomb yield criterion. In the following analysis bulk rubble is considered to be a rigid-plastic continuum undergoing quasi-static, two-dimensional deformation; there are no experimental data or field observations of possible volume changes during deformation. A pressure-voids relation would be required if compressiblity were to be considered. The ice block concentration (or voids ratio) is therefore assumed to be constant. As rubble-piling events usually occur over short periods (1), temperature and its effect on the deformation process are ignored.

The equations of equilibrium are

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = \gamma \cos \theta \qquad (1)$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + 2\frac{\tau_{r\theta}}{r} = -\gamma \sin \theta \qquad (2)$$

where γ is the unit weight (or buoyancy of submerged rubble) and σ_r, σ_θ and $\tau_{r\theta}$ are normal and shear stress components, as shown in Fig. 1. Compressive stresses are considered positive throughout the present analysis.

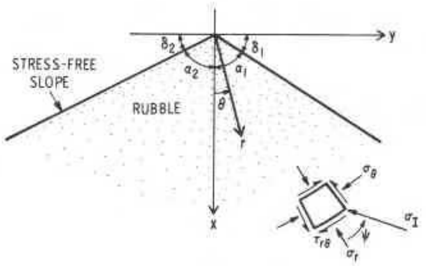

Figure 1 Sketch of two-dimensional wedge

The Mohr-Coulomb yield criterion is satisfied by expressing the stress components as follows

$$\begin{aligned} \sigma_r &= \sigma + q \cos 2\psi \\ \sigma_\theta &= \sigma - q \cos 2\psi \\ \tau_{r\theta} &= q \sin 2\psi \end{aligned} \tag{3}$$

and
$$q = \sigma \sin\phi + C \cos\phi \tag{4}$$

where ϕ is the angle of internal friction, C is the cohesion, ψ is the angle between the major principal stress and the r-direction, and σ is the average normal stress.

$$\sigma = \frac{\sigma_\theta + \sigma_r}{2} = \frac{\sigma_I + \sigma_{II}}{2} \tag{5}$$

σ_I and σ_{II} are the major and minor principal stresses, respectively.

This set of equations is sufficient for determining the components of the stress tensor (or the variables, σ and ψ) and can be reduced to two first-order, quasi-linear, hyperbolic partial differential equations. The same equations are also used in studying the critical equilibrium of granular materials. The procedure of numerical solution and its application to several cases was given by Sokolovski (11).

Experimental results provide very little information about the nature of the cohesion term, C. Here it is assumed that C depends on the average normal stress. The use of a pressure-dependent cohesion makes it possible to satisfy the boundary conditions while maintaining a critical state everywhere within the boundaries and, in addition, may be physically justified. Contact area and bond strength between individual blocks are expected to be proportional to normal contact forces and, in turn, to the over-all average normal stresses. A similar assumption was used for granular materials in bins and hoppers by Jenike and Johanson (12). The following simple form for cohesion is employed

$$C = k\,\sigma \tag{6}$$

The governing equations can thus be written in a form similar to that for cohesionless materials by using an equivalent angle of internal friction

$$\phi' = \sin^{-1}(\sin\phi + k \cos\phi) \tag{7}$$

SOLUTION

The critical equilibrium of a two-dimensional wedge (Fig. 1) is assumed to simulate an actively forming ridge or rubble pile. This problem admits the "similarity" or "radial stress" solution of Sokolovski (11). Average normal stress becomes proportional to distance, r, from the apex of the wedge. The solution has the form

$$\sigma = \gamma\, r\, S(\theta) \tag{8}$$

$$\psi = \psi(\theta) \tag{9}$$

Substituting Eq. (8) and (9) in (3) and using (4), (6) and (7), Eq. (1) and (2) may be reduced to two ordinary differential equations for the unknown functions S and ψ.

$$\frac{d\psi}{d\theta} = \frac{\cos\theta - \sin\phi' \cos(2\psi + \theta) - S\cos^2\phi'}{2S\sin\phi'(\cos 2\psi - \sin\phi')} - 1 \tag{10}$$

$$\frac{dS}{d\theta} = \frac{-\sin(2\psi + \theta) + S \sin 2\psi}{\cos 2\psi - \sin\phi'} \tag{11}$$

The stress characteristics form angles $\pm\left(\frac{\pi}{4} - \frac{\phi'}{2}\right)$ with the principal stress direction. Notice that a singularity exists when $\cos 2\psi = \sin\phi'$. This corresponds to a radial stress characteristic.

Both Sokolovski (11) and Marais (13) used direct integration of Eq. (10) and (11) to obtain solutions for the active case. Nadai (14) also developed an approximate solution for a special case of the active state. A review of the available active state solutions was given by Marais (13). The present problem, however, corresponds to the passive state (the horizontal normal stresses are larger than the vertical normal stresses); direct integration of Eq. (10) and (11) does not yield an appropriate solution that satisfies all boundary conditions. Marais (13) also reported that a passive state solution could not be obtained in a manner similar to that of the active state. It is possible that these difficulties are caused by the occurrence of stress discontinuities. Savage and Yong (15) presented an analysis of similar discontinuities that exist in cohesionless granular materials. Numerical integration would require repeated trials to locate such discontinuities. A simpler approach is used in the present study to obtain an approximate solution. The exact distributions of S and ψ, which may be discontinuous, are approximated by continuous functions expressed in powers of θ.

Boundary Conditions

Boundary conditions at the side slopes of the wedge are assumed to correspond to those of an infinite slope. The stress function, S, simply vanishes, but the value of the angle, ψ, has to be derived from the stress distribution near the stress-free surface. It can be shown (see for example, Marais (13)) that a region in the vicinity of the side slope must exist where stresses are determined by conditions of an infinite slope. The present approach, however, requires only the values of ψ at the boundaries. The angle between the major principal stress and the vertical direction for an infinite slope is constant:

$$\tilde{\psi} = \psi + \theta = \text{constant} \tag{12}$$

As a result of this hypothesis, $\frac{d\psi}{d\theta} = -1$. Consequently, Eq. (10) gives

$$S = \frac{\cos\theta - \sin\phi' \cos(2\tilde{\psi} - \theta)}{\cos^2\phi'} \tag{13}$$

At the side slope $S = 0$ and $\theta = \frac{\pi}{2} - \delta$. Equation (13) gives two values of $\tilde{\psi}$ corresponding to the active and passive states. For the passive state

$$2\,\tilde{\psi} = \pi - \delta - \sin^{-1}\left(\frac{\sin\delta}{\sin\phi'}\right) \qquad (14)$$

Approximate Solution

Analysis is simplified by considering the stress angle, ψ, to be a linear function of θ. This assumption was used by Nadai (14) for the active state, and is in reasonable agreement with the numerical solutions of Sokolovski (11) and Marais (13). The passive state may be somewhat different, but the linear distribution still agrees with the exact solution for the limiting case of horizontal sides (apex angle = π) or an infinite slope of any inclination. Thus, the stress angle is given by

$$\psi = \psi_2^* + (\psi_1^* - \psi_2^*)\left(\frac{\theta + \alpha_2}{\alpha_1 + \alpha_2}\right) \qquad (15)$$

where ψ_1^* and ψ_2^* are the values of ψ at the side slopes. These could be determined from Eq. (12) and (14), and α_1 and α_2 are the angles between the side slopes and the vertical direction. For symmetrical wedges, only half the wedge need be studied and the value of ψ at the centreline will be $\frac{\pi}{2}$. Equation (15) then reduces to

$$\psi = \frac{\pi}{2} + \left(\psi_1 - \frac{\pi}{2}\right)\left(\frac{\theta}{\alpha_1}\right) \qquad (16)$$

The stress function, S, is approximated by a polynomial form

$$S = S_o + S_1\,\theta + S_2\,\theta^2 + S_3\,\theta^3 + \ldots \qquad (17)$$

The coefficients S_o, S_1, . . . are determined by satisfying the boundary conditions and the equilibrium of forces on sectors of the wedge. Terms of order greater than θ^2 are neglected. As $S = 0$ at the boundaries, Eq. (17) may be written as

$$S = S_o\left[\,1 + \frac{(\alpha_1 - \alpha_2)}{\alpha_1\,\alpha_2}\,\theta - \frac{\theta^2}{\alpha_1\,\alpha_2}\,\right] \qquad (18)$$

For symmetrical wedges $(\alpha_1 = \alpha_2)$, S is an even function of θ and Eq. (18) reduces to

$$S = S_o\left[\,1 - \left(\frac{\theta}{\alpha_1}\right)^2\,\right] \qquad (19)$$

The unknown coefficient, S_o, may be obtained from the balance of vertical forces acting on a sector of the wedge of radius r. This condition is

$$\frac{1}{2}\gamma\, r^2\,(\alpha_1 + \alpha_2) = \int_{-\alpha_2}^{\alpha_1} (\sigma_r \cos\theta - \tau_{r\theta}\sin\theta)\, r\, d\theta \qquad (20)$$

Substituting Eq. (15) and (18) in (8) and (3), the condition given by Eq. (20), gives S_o. The integration in Eq. (20) can be obtained in a closed form, albeit a very lengthy one. It is simpler and more practical to evaluate it numerically.

The additional term $S_3\theta^3$ was included in an extension of Eq. (18) to test the convergence of the present solution for asymmetrical wedges. The additional condition was the equilibrium of horizontal forces on a sector of the wedge. The results, in this case, were almost identical to those obtained by neglecting $S_3\theta^3$.

The present solution approximates the exact solution by power series expansion. It implies that the yield criterion and similarity condition are satisfied everywhere. The equilibrium of forces, however, is only satisfied in an average sense over the wedge. It could be satisfied over any arbitrary number of smaller sectors of the wedge by including more terms in the power series of Eq. (17).

RESULTS

Values of S_o are presented in Fig. 2 for symmetrical wedges and in Fig. 3 for asymmetrical wedges with one horizontal side, $\delta_2 = 0$, Stress distributions in wedges of any side slope can be obtained, as well, from the present analysis. As expected, stresses increase with increasing equivalent angles of internal friction, ϕ', and with decreasing side slope, δ. The calculated values of S_o are less, by 3%, than the exact values for a horizontal surface $\left([1 - \sin\phi']^{-1}\right)$ owing to approximation of the exact solution cosine function by a parabola. The error is expected to decrease for smaller apex angles (larger δ). When the analysis was extended to the active state to examine its accuracy, the results were in good agreement with Marais' numerical values (13). The maximum error was 3% for horizontal surfaces and decreased for larger values of δ.

The horizontal force in the ice sheet adjacent to a symmetrical rubble wedge (sail or keel) may be determined by integrating the normal stress along the vertical axis ($\theta = 0$),

$$F = \int_0^H \sigma_\theta\, dr \qquad (21)$$

From Eq. (3 to 8, 19)

$$F = \frac{1}{2}\,\gamma\, H^2\, S_o\,(1 - \sin\phi' \cos 2\psi_o) \qquad (22)$$

where H is height of wedge (from apex to base, see Figure 4a) and the angle, ψ_o, is $\frac{\pi}{2}$. The order of these forces may be illustrated by considering an example of typical values for a first-year pressure ridge of a symmetrical triangular sail ($\phi' = 60°$,* $\delta = 25°$ and $\gamma = 6200$ N/m^3). The force associated with keel formation may be taken as approximately equal to that associated with the sail (since weight of the sail equals buoyancy of the keel). Thus, for a sail height of 3 m the force in the ice sheet during ridge building would be 2.34×10^5 N/m.

Asymmetrical rubble features, as shown in Fig. 4b, may have sail and keel apexes at different vertical

*This is the value of ϕ' for an internal friction angle, $\phi = 50°$ and a cohesion coefficient, $k = 0.15$.

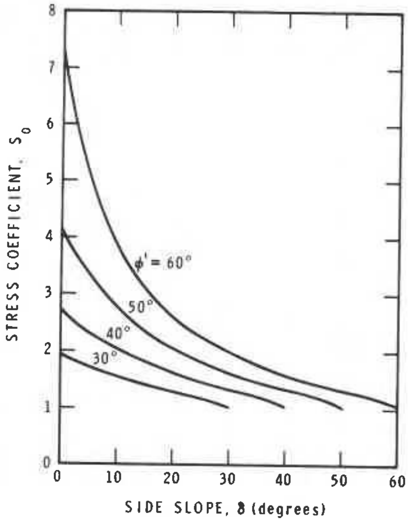

Figure 2 Stress coefficient, S_o, for symmetrical wedges

planes. The horizontal force in the ice sheet should be estimated by including the shear stresses on part of the horizontal base of the wedges in addition to the normal stresses on the vertical planes through the apexes. Note that ψ_o can be determined from the boundary values and linear distribution of ψ given by Eq. (14) and (15). Normal and shear stresses on the horizontal base of the wedge are given by

$$\sigma_{\substack{x \\ y}} = \frac{\gamma SH}{\cos\theta} \left[1 \pm \sin\phi' \cos 2(\psi + \theta)\right] \tag{23}$$

$$\tau_{xy} = \frac{\gamma SH}{\cos\theta} \sin\phi' \sin 2(\psi + \theta) \tag{24}$$

The stresses on the base of the sail of a ridge would not be in equilibrium with those on the base of the keel if both sail and keel had a common plane boundary at the water level. Instead, at water level a layer of rubble or an ice sheet must exist that has stresses from the sail and keel acting on different sides. Integration of the shear stresses along this layer gives the same total horizontal force in the ice cover as that predicted by Eq. (22).

The force obtained from the potential energy approach for a symmetrical wedge is (Kovacs and Sodhi (1)).

$$F_p = \frac{1}{2} \gamma_i Ht \tag{25}$$

where γ_i and t are the unit weight and thickness of ice blocks, respectively. The ratio of the forces predicted from the present analysis to those from the potential energy approach is

$$\frac{F}{F_p} = \frac{H}{t} \frac{\gamma}{\gamma_i} S_o (1 + \sin\phi') \tag{26}$$

Typical values of the variables in Eq. (26) give ratios of the order of 20.

The preceding analysis applies to newly formed, floating rubble with a wedge-shaped sail or keel and two plane, stress-free sides. Grounded rubble, on the other hand, would have different boundary conditions.

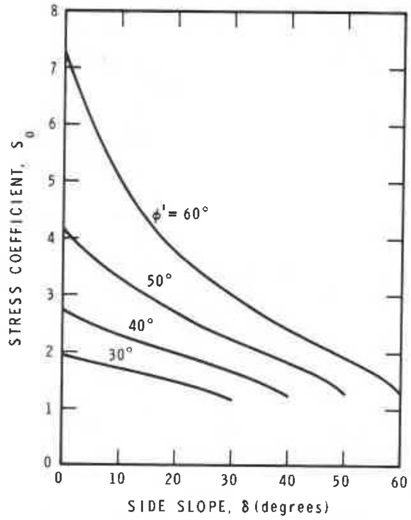

Figure 3 Stress coefficient, S_o, for asymetrical wedges with one horizontal side ($\delta_2 = 0$)

The similarity solution could be extended to treat such cases only if the grounding stress on the keel side were proportional to the distance from the apex.

Shore pile-ups occurring above water level may correspond to the above analysis if the rubble is pressed against an obstacle in such a way that a passive stress state exists. An ice sheet or a layer of rubble advancing on the shore would be subjected to resistance due to rubble on its top surface (or the base of rubble wedge) and to the resistance of grounding on its bottom surface. Rubble forces can be estimated as for floating ice. Ground resistance may be assumed to be equal to the normal force from the rubble wedge multiplied by a friction coefficient between the soil and ice. Note that the shear stresses from the ground and the rubble wedge are not in equilibrium and act in a direction opposite to the movement of the advancing ice sheet. Other pile-up situations (not considered here) may occur where there are no vertical obstacles exerting horizontal force on

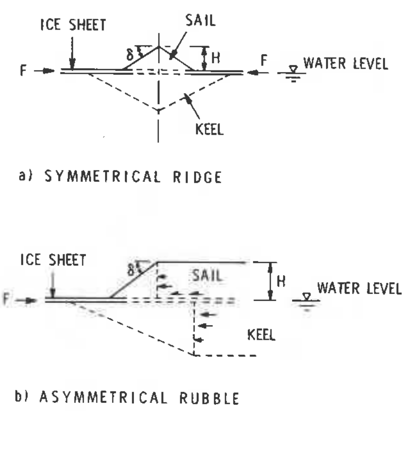

Figure 4 Ice rubble features

the wedge opposite the advancing ice sheet. A passive state might develop in parts of the wedge, while an active state might develop in others.

Ice sheet thickness does not influence stress distribution, according to the present analysis, except through its effect on bulk rubble properties. It could be related to rubble height by equating the predicted forces to ice sheet strength. This would require assumptions regarding the failure modes of the ice sheet.

CONCLUSION

An analytical model of ice rubble pile-ups and first-year ridges has been developed, in which the Mohr-Coulomb yield criterion and equilibrium equations are employed to describe the stress field. Deformation was assumed to be two-dimensional and quasi-static. Solutions were obtained using the similarity method and expressing the unknown stress function in a polynomial form. The resulting horizontal forces in the ice sheet seem to be reasonable and substantially exceed the lower limits obtained from an energy balance neglecting frictional dissipation. Stress distribution and total forces are related to bulk rubble properties and the geometry of the wedges.

This analysis deals only with the stress field at the critical state of equilibrium. Kinematics of the problem have not been considered. The continuity equation and a flow rule should be included in the governing equations in order to determine the strains and displacements of the rubble. At present there is no available information regarding the appropriate flow rule. Further experimental work will be needed to clarify this problem.

NOMENCLATURE

C	cohesion of bulk rubble
F	horizontal force in the ice sheet
F_p	value of F estimated using potential energy method
g	gravitational acceleration
H	height of wedge
k	parameter used in Eq. (6)
q	stress given by Eq. (4)
S	stress function
$S_o, S_1, \ldots$	coefficients used in Eq. (17)
t	thickness of ice blocks
α_1, α_2	angles between vertical direction and sides of the wedge
γ	unit weight of bulk rubble
γ_i	unit weight of ice blocks
δ	angle between horizontal and the stress-free surface
δ_1, δ_2	angle between the horizontal and sides 1 and 2 of wedge
σ	average normal stress
σ_I	major principal stress
σ_{II}	minor principal stress
σ_r	normal stress in r direction
σ_θ	normal stress in θ direction
σ_x	normal stress in x direction
σ_y	normal stress in y direction
$\tau_{r\theta}$	shear stress in r-θ direction
τ_{xy}	shear stress in x-y direction
ϕ'	angle of internal friction
ϕ	equivalent angle of internal friction
ψ	angle between σ_I and r direction
$\tilde{\psi}$	angle between σ_I and x direction
ψ_1^*, ψ_2^*	angle ψ at sides 1 and 2 of the wedge

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