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Publisher's version / Version de l'éditeur:

IAHS Publication, 162, pp. 605-612, 1987

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Appeared in
Avalanche Formation, Movement and Effects
(Proceedings of the Davos Symposium, September 1986)
IAHS Publication No. 162, 1987
p. 605-612
(IRC Paper No. 1543)

NRCC 29067

Canada

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An equation for calculating snow avalanche run-up against barriers

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ABSTRACT The well known Voellmy runout equation is routinely used to dimension protective barriers for snow avalanches. Takahashi and Yoshida (1979) derived an alternative runout equation for debris flows, which accounts for transfer of momentum and thrust between the main body of the debris wave and the frontal part which moves into the runout zone. This paper shows that their equation can be easily adapted to snow avalanches within the context of the Voellmy model. The adaptation is especially unambiguous when applied to the calculation of run-up; i.e. runout on a steep adverse slope. A numerical comparison indicates that the Voellmy equation underestimates run-up by at least 50% compared to the proposed method.

Une équation pour calculer la hauteur d'avancée des avalanches sur le front des barrages protectifs

RESUME L'équation de distance d'avancée d'après Voellmy est souvent utilise pour dimensionner le hauteur des barrages protectifs contre les avalanches. Takahashi et Yoshida (1979) ont dérivé une équation alternative pour les coulées de boue. Cette équation tient compte du transfert d'impulsion et la poussée éntre la partie principale de la masse mouvante des débris et la partie frontale qui monte la pente de la barrage. Cette équation a éte adaptée pour les problèmes des avalanches, dans le contexte du modèle dynamique de Voellmy. On peut démontrer que le hauteur evaluée par la theorie de Takahashi et Yoshida est 50% plus grande que celle calculée par l'équation de Voellmy.

INTRODUCTION

Barriers, usually in the form of earth embankments, are sometimes used to shorten the runout distance of snow avalanches (Fig 1). Their height should exceed the expected run-up of the design avalanche front, H. The standard method of estimating the run-up height (e.g. Mears, 1981) utilizes an equation derived from lumped-mass energy balance, due originally to Voellmy (1955):

$$H = \frac{v_o{}^2 \sin\theta}{2g(\mu\cos\theta - \sin\theta + v_o{}^2/2\xi h_o)} \qquad (1)$$

where: v_o is the mean approach velocity,
H is the run-up height,
h_o is the approach flow depth,
θ is the slope angle of the upstream face of the barrier,
μ is the friction coefficient and
ξ is the turbulent resistance coefficient of avalanche motion (Voellmy, 1955).

Equation (1) is based on the assumption that the kinetic energy of the avalanche mass, moving at velocity v_o, is converted into potential energy corresponding to the run-up height, H, after

FIG. 1 An avalanche barrier in Rogers Pass, Selkirk Mountains, British Columbia. (Photo courtesy P.A. Schaerer, National Research Council.)

subtracting work done against the frictional and turbulent resisting forces. It is implicitly assumed that the entire mass of the avalanche is being raised to the height H, as shown schematically in Fig. 2a. This is an obvious idealization. In reality, a large part of the avalanche mass remains behind the barrier and only the leading front is driven upwards, as shown in Fig. 2b. Momentum, as well as thrust force, are transferred from the main body of the avalanche into the leading front, increasing the run-up height. As shown in the following section, the mechanics of this process can be simulated by a relatively simple mathematical procedure.

THE RUNOUT EQUATION OF TAKAHASHI AND YOSHIDA

Takahashi and Yoshida (1979)* derived a momentum equilibrium equation which describes the run-up process illustrated in

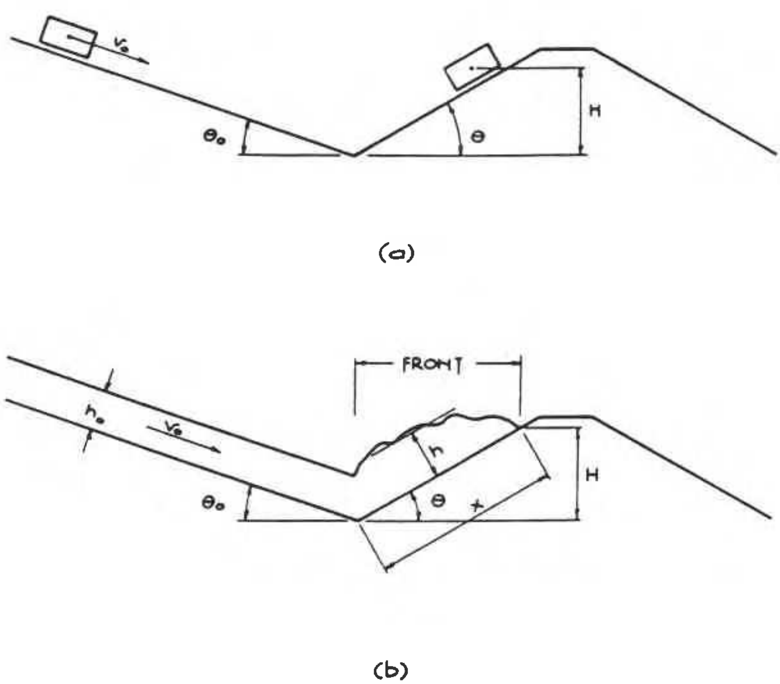

FIG. 2 (a) Lumped-mass representation of run-up.
(b) Leading front model analysed by Takahashi and Yoshida (1979).

* In Japanese. To the authors' knowledge, the theory has not yet appeared in English language publications.

Fig. 2b. Their derivation was intended for a special kind of coarse-grained debris flow and was carried out in terms of effective stress. It is therefore not directly applicable to snow avalanches. However, a parallel derivation based on the Voellmy model is given in the following. The expression equates the time rate of change of the leading front momentum with the sum of the applied forces:

$$\frac{d(h \rho vx)}{dt} = T1 + T2 + T3 - T4 - T5 \qquad (2)$$

where: h is the mean depth of the front,
x is its length at the time t,
v is the mean velocity and
ρ is the flow density, assumed constant and equal in the avalanche body and the front.

The force terms T1 to T5 are defined as follows:

- Gravity driving force (or resisting force, where θ is negative):

$$T1 = \rho ghx \sin\theta \qquad (2a)$$

- Momentum flux between the main body of the avalanche and the front, calculated as mass flux times velocity. It is assumed that the presence of the barrier does not influence flow conditions upstream, as would be the case in supercritical flow. The significance of this assumption upon the later stages of run-up should be investigated experimentally:

$$T2 = \rho h_o v_o{}^2 \cos(\theta_o - \theta) \qquad (2b)$$

where θ_o is the approach slope angle (all angles are positive downwards from the horizontal)

- Fluid thrust force between the main avalanche body and the front, assumed hydrostatic in distribution and magnitude:

$$T3 = 1/2 \rho g h_o{}^2 \cos\theta_o \cos(\theta_o - \theta) \qquad (2c)$$

- Frictional resisting force on the base of the leading front:

$$T4 = \rho ghx \cos\theta \mu \qquad (2d)$$

- Turbulent resisting (drag) force:

$$T5 = \frac{\rho gh\ x\ v^2}{\xi h} \qquad (2e)$$

The growth of the front must also satisfy the continuity equation:

$$hx = h_o v_o t \quad (3)$$

where t is the time elapsed since the first contact of the leading edge with the toe of the barrier. The equation assumes uniform width throughout the runout, a conservative assumption in case of channelled avalanches.

Substituting from (3) to (2) and rearranging, we obtain:

$$\frac{d\,(vt)}{dt} = V - tG - t\,v^2 D_o \quad (4)$$

where:

$$V = v_o \cos(\vartheta_o - \vartheta)\left(1 + \frac{g h_o \cos\vartheta_o}{2 v_o{}^2}\right) \quad (4a)$$

$$G = g\,(\cos\vartheta\,\mu - \sin\vartheta) \quad (4b)$$

$$D_o = g/(h\xi) \quad (4c)$$

The derivation of the term D_o implies that h is assumed constant both spatially and in terms of time - an assumption with little influence on the numerical results in the case of run-up, but one which could be the source of significant errors if the equation was used for estimating runout. In the notation of Perla et al (1980) D_o is termed D/M with D as the dynamic drag and M as the avalanche mass.

Equation (4) (a Riccati type differential equation) has been solved using the Runge-Kutta numerical solution, implemented on a microcomputer. Some simplification is possible, however. If the turbulent resistance term T5 is neglected ($D_o = 0$), the equation becomes separable and has a particular solution derived originally by Takahashi and Yoshida (1979):

$$H = \frac{V^2 \sin\vartheta}{G} \quad (5)$$

If terms T2, T3 and T5 are neglected, one recovers the Voellmy runout equation with zero turbulent resistance:

$$H = \frac{v_o{}^2 \sin\vartheta}{2G} \quad (6)$$

PARAMETRIC STUDY

To illustrate the results of the above formulas, a parametric study has been carried out using the Runge-Kutta solution for the following range of parameters:

θ = −33.6° (i.e. an embankment face slope, angled at 1.5 horizontal in 1 vertical)
μ = 0.15 to 0.45
ξ = 400 to 1000 m/sec^2
θ_o = 0 to 20°
v_o = 0 to 25 m/sec
and h_o = 1 to 5 m.

The resistance parameter range was chosen following the results of Buser and Frutiger (1980). The mean flow depth of the front, h was assumed as twice the approach flow depth, i.e. $h = 2h_o$. If the two depths were assumed equal, the resulting difference in run-up height would be less than 15% for the range of variables studied (the latter assumption produces the lesser H). The significance of this assumption would need to be much more cautiously evaluated if the equations were to be used for the prediction of runout instead of run-up on steep adverse slopes.

Further conclusions from the parametric study indicate that the influence of the resistance parameters, μ and ξ, amounts to a

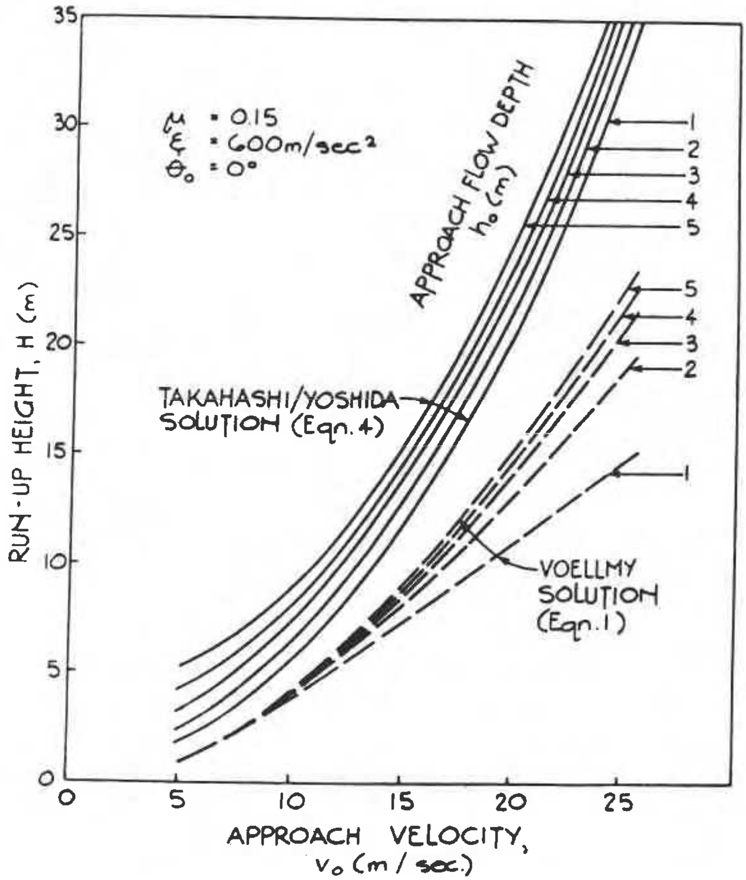

FIG. 3 Run-up height predictions for a barrier with a face slope of 1.5 horizontal in/vertical.

maximum of plus/minus 15% over the range considered. The approach slope angle, θ_o, has no significant influence, except at velocities less than 10 m/sec combined with approach flow depths of the order of 1 m or less. In the latter case, the run-up decreases with increasing approach angle.

Figure 3 shows the dependence of run-up height on approach velocity and flow depth for a typical set of resistance coefficients (μ = 0.15 and ξ = 600 m/sec^2 given by deQuervain, 1977). The diagram can be used as an approximate guide for design, within the precision limitations indicated earlier. The required input - approach velocity and flow depth - can be estimated using standard techniques.

Figure 3 includes contours of the Voellmy solution (Eqn 1) as dashed lines, for comparison. In general, the Takahashi-Yoshida method estimates a run-up height at least 50% greater than predicted by the Voellmy method. This is considered to be the result of momentum flux and thrust force transfer neglected in Eqn (1).

A more rigorous recent version of the Voellmy equation (Salm, 1979) produces run-up estimates within approximately 5% of Eqn (1), i.e. also substantially less than the values shown in Figure 3.

The parametric study also showed that the simplified solution, Eqn. (5), overestimates run-up by a margin of 10 to 36% in comparison to the complete numerical solution of Eqn (4). This is an acceptable error margin on the conservative side. The closed-form solution could therefore be used in practical calculations.

CONCLUSION

The Takahashi-Yoshida runout equation has been adapted for estimating snow avalanche run-up against barriers. The equation should be verified against field measurements. However, its physical basis is unquestionably more realistic than that of the earlier Voellmy equation. Initial results indicate that the improved theory predicts significantly greater run-up height.

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