**Effect of stress path geometry on soil brittleness**

Law, K. T.



**Publisher's version / Version de l'éditeur:**

*Géotechnique, 31, 2, pp. 279-287, 1981-06*

9010

National Research Council Canada
Conseil national de recherches Canada

# EFFECT OF STRESS PATH GEOMETRY ON SOIL BRITTLENESS

by K.T. Law



Reprinted from
Geotechnique
Vol. 31, No. 2, June 1981
p. 279 - 287



DBR Paper No. 995
Division of Building Research

Price $1.00 OTTAWA NRCC 19677

## SOMMAIRE

On a réalisé une étude de l'effet de la géométrie des trajectoires des contraintes sur la fragilité du sol telle que déterminée en fonction de mesures en laboratoire. Cette étude démontre qu'il existe une relation explicite entre la fragilité du sol et la géométrie des trajectoires des contraintes, pour des sols possédant des enveloppes de résistance données aux charges de pointe et ultérieures. Dans le cas particulier d'essais triaxiaux, on a considéré quatre directions de cisaillement sous l'effet de contraintes effectives. Les résultats indiquent que les essais triaxiaux drainés conventionnels donnent les valeurs de fragilité les plus élevées, lorsque les trajectoires des contraintes atteignent la valeur de rupture en présence de la même contrainte radiale, ou lorsqu'elles débutent à la même pression de consolidation avant le cisaillement. On commente les implications pratiques de cette étude.

LAW, K. T. (1981). *Géotechnique* **31**, No. 2, No. 2, 279–287

# Effect of stress path geometry on soil brittleness

K. T. LAW*

ANALYZED

A study has been carried out on the effect of stress path geometry on drained soil brittleness as measured in the laboratory. It shows that an explicit relationship exists between brittleness and stress path geometry for soils of given peak and post-peak strength envelopes. For the particular case of triaxial testing, four typical effective stress paths of shearing have been considered. The results indicate that the conventional drained triaxial test yields the highest brittleness, either when the stress paths are reaching failure at the same confining stress or when they start at the same consolidation pressure prior to shearing. The practical implications of the study are discussed.

Une étude a été réalisée sur l'effet de la géométrie du chemin des contraintes sur la fragilité de sols drainés, tel que mesuré en laboratoire. Cette étude montre qu'il existe un rapport explicite entre la fragilité et la géométrie du chemin des contraintes pour des sols à résistances de pic et residuelle données. En ce qui concerne le cas particulier d'essais triaxiaux, on a considéré quatre chemin de contrainte efficaces typiques de cisaillement. Les résultats montrent que c'est avec l'essai triaxial drainé classique qu'on obtient la plus haute fragilité, que les chemins de contrainte atteignent la rupture à la même tension de confinement ou qu'ils commencent à la même pression de consolidation avant cisaillement. Les implications pratiques sont analysées dans cette étude.

## NOTATION

| | |
|---|---|
| $c'$ | cohesion in terms of effective stress |
| $I_B$ | brittleness index |
| $K$ | $\sigma_{3o}'/\sigma_{1o}'$ |
| $p'$ | $(\sigma_1'+\sigma_3')/2$ |
| $q$ | $(\sigma_1'-\sigma_3')/2$ |
| $w_p$ | plastic limit of soil, per cent |
| $w_n$ | natural water content of soil, per cent |
| $w_l$ | liquid limit of soil, per cent |
| $\alpha$ | angle between stress path and $p'$ axis |
| $\sigma_1'$ | major principal effective stress |
| $\sigma_3'$ | minor principal effective stress |
| $\sigma_n'$ | effective normal stress |
| $\sigma_{oct}'$ | $(\sigma_1'+2\sigma_3')/3$ |
| $\sigma_{1o}'$ | major principal effective stress at end of consolidation |
| $\sigma_{3o}'$ | minor principal effective stress at end of consolidation |
| $\phi'$ | angle of internal friction in terms of effective stress |
| $\tau$ | shear stress |

*Subscripts*

| | |
|---|---|
| p | peak strength state |
| r | post-peak strength state |
| f | failure condition |
| o | end of consolidation |

## INTRODUCTION

A brittle soil is characterized by a stress–strain relation that displays a distinct peak strength $q_f$ followed by a decrease to post-peak strength $q_r$. When such a soil is encountered in building an earth structure, progressive failure may take place. This type of failure involves straining beyond the peak at some particular point in the earth mass, causing a decrease in strength at that point. This action will induce additional stresses on the neighbouring soil, and cause it also to strain beyond the peak. Thus a progressive failure within the earth mass is initiated.

The brittleness of soil has been the subject of many studies. Skempton (1964) presented evidence that soil sheared to the post-peak state acquires a certain reorientation of clay particles. Kenney (1977) showed that post-peak strength depends on mineral composition and chemical state. Haefeli (1965) stated that brittleness increases with the peak effective cohesion. For a very stiff sensitive clay, Conlon (1966) demonstrated that post-peak strength decrease is associated with interparticle bond breakage. Bishop (1971) introduced the brittleness index defined as

$$I_B = (q_f - q_r)/q_f \qquad (1)$$

He pointed out that this index varies with soil type, and that for a given soil it increases with decreasing effective normal pressure.

Recent studies on Champlain Sea clay illustrate two different views on brittleness. Based on conventional constant strain rate triaxial tests, Lo & Morin (1972) and Lefebvre & La Rochelle (1974) reported clearly defined peak and post-peak strength envelopes for soils failing in the stress range commonly encountered in natural slope failure. On the other hand, Mitchell (1975) maintained that the soil behaves as a perfectly plastic material, based on incremental loading

**Discussion on this Paper closes 1 September, 1981. For further details see inside back cover.**

*Division of Building Research, National Research Council of Canada.



2785515

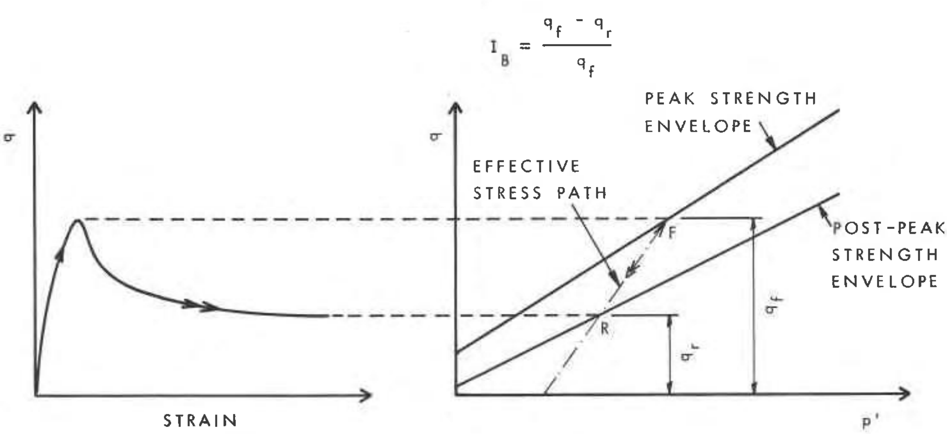


**Fig. 1. Evaluation of brittleness for a triaxial test on soil**

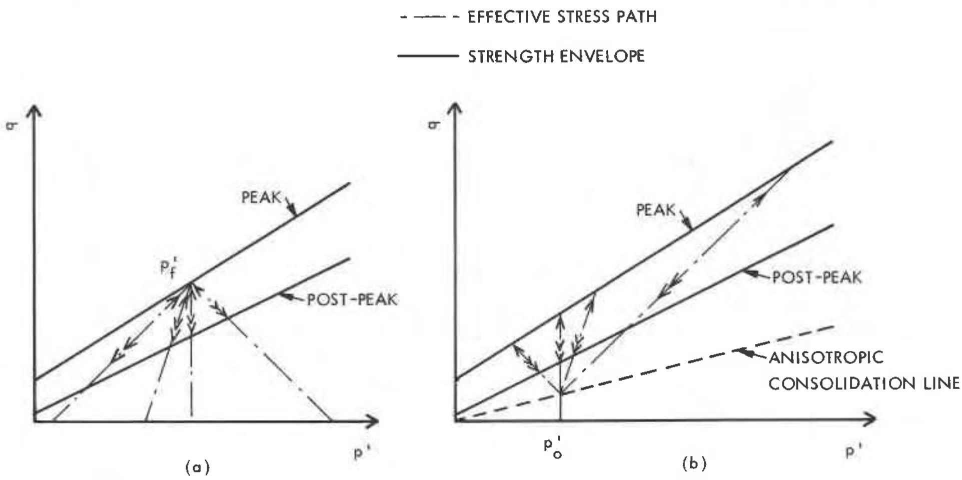


**Fig. 2. Cases of stress paths under study (a) reaching failure at same point; (b) starting from same point**

triaxial tests at constant mean effective normal pressure. These different views are derived from tests conducted along different stress paths, which are known to have an important influence on some aspects of soil behaviour (e.g. Lambe, 1967; Simons, 1971; Campanella & Vaid, 1973).

In view of the wide occurrence of brittle soils, it is worth reconsidering soil brittleness as measured in the laboratory. Some of the points, and controversy, raised by the studies mentioned above may be explained by examining the geometric effect of stress paths. This effect stems from the properties of the intersection between stress paths and strength envelopes, both quantities being represented by lines in a two-dimensional stress plot. From Fig. 1 it may be seen that brittleness is related to the geometry of the stress path for given peak and residual strength envelopes of a particular soil; this relationship will now be quantified.

## POST-PEAK STRENGTH DECREASE ALONG EFFECTIVE STRESS PATHS IN TRIAXIAL TESTS

Consider a soil with the peak and post-peak strength parameters denoted by $\phi_p', c_p'$ and $\phi_r', c_r'$, where $\phi'$ and $c'$ are the angle of internal friction and cohesion in terms of effective stresses, and subscripts p and r refer to peak and post-peak conditions respectively. The strength parameters are assumed to be independent of effective stress paths.

The evaluation of the brittleness index $I_B$ and its dependence on stress path for a typical triaxial test are illustrated in Fig. 1, in which $q$ is plotted against $p'$ where $q = (\sigma_1' - \sigma_3')/2$ and $p' = (\sigma_1' + \sigma_3')/2$ and $\sigma_1'$ and $\sigma_3'$ are the effective major and minor principal stresses. The term post-peak strength used here refers to the strength at moderate strain where the stress–strain curves stabilize beyond the

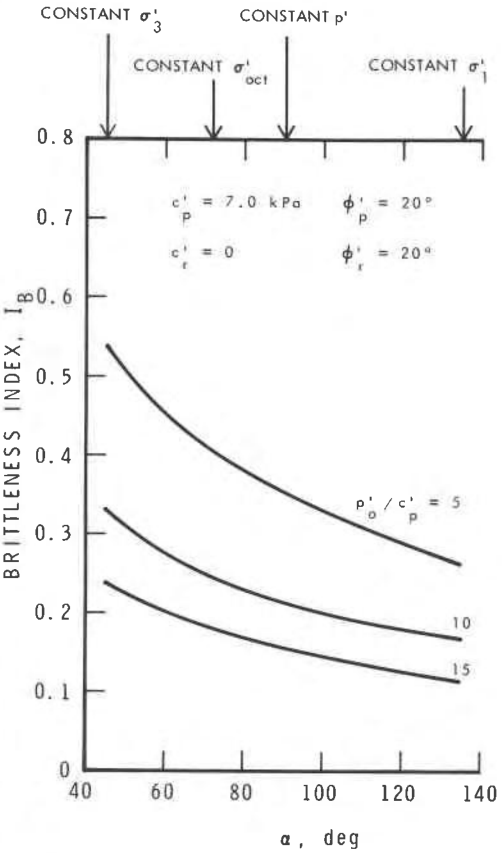


Fig. 3. Variation of brittleness with stress paths reaching failure at the same $p'$ value for soil A

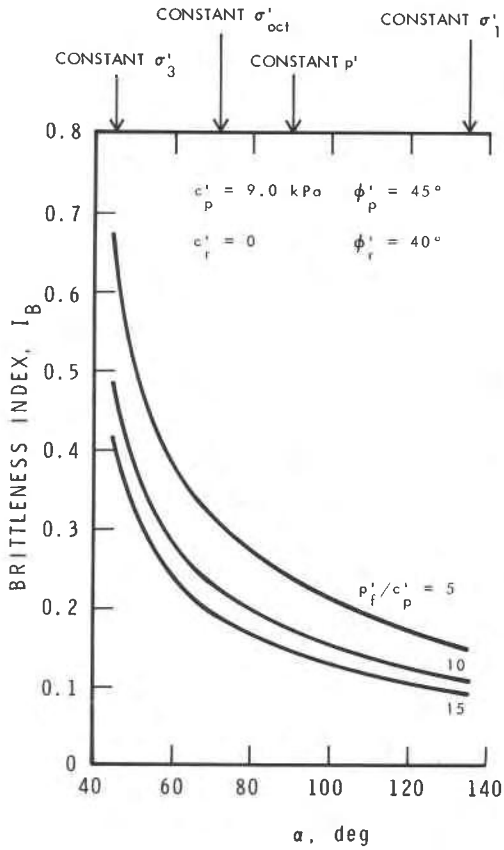


Fig. 4. Variation of brittleness with stress paths reaching failure at the same $p'$ value for soil B

peak. This strength can be readily determined by conducting triaxial tests for bonded clays (La Rochelle, 1967). The evaluation of the brittleness index of soils whose post-peak strength cannot be determined by the triaxial test will be given later.

For simplicity only a linear effective stress path is considered, expressed as

$$q = a + p' \tan\alpha \tag{2}$$

where $a$ is a constant related to consolidation pressure and $\tan\alpha$ is a constant related to test type.

The equations for the peak and post-peak strength envelopes can be written as

$$q = c_p' \cos\phi_p' + p' \sin\phi_p' \tag{3}$$

and

$$q = c_r' \cos\phi_r' + p' \sin\phi_r' \tag{4}$$

The brittleness index $I_B$ can be evaluated by solving, simultaneously, equations (2) and (3) and equations (2) and (4). Solutions of these equations are obtained for two cases (Fig. 2) in order to illustrate how $I_B$ is affected by effective stress paths. In the first case, failure at the same $p'$ value is reached from different paths; in the second case, all paths start at the same consolidation pressure.

*Failure at the same $p'$ value*

This failure condition permits comparison of the brittleness of a given soil sheared to failure at the same effective confining stress, but along different stress paths. This will clearly reveal the influence of stress path because variations in behaviour arising from different effective stress ranges will be eliminated.

Let $p_f'$ be the $p'$ value at failure for the peak condition. The expression for $I_B$ is

$$I_B = \frac{p_f'(\sin\phi_p' - \sin\phi_r') + (c_p' \cos\phi_p' - c_r' \cos\phi_r')}{(1 - \sin\phi_r'/\tan\alpha)(c_p' \cos\phi_p' + p_f' \sin\phi_p')} \tag{5}$$

with the restriction that $\sin\phi_p' < \tan\alpha$.

The variation of $I_B$ with respect to $\alpha$ and $p_f'/c_p'$ was studied for two soils of typical strength

**Table 1. Strength characteristics of soils studied**

| Soil | Typical properties | | | Strength parameters | | | |
|---|---|---|---|---|---|---|---|
| | | | | Peak | | Post-peak | |
| | $w_p$ | $w_n$ | $w_l$ | $c_p'$: kPa | $\phi_p'$: degrees | $c_r'$: kPa | $\phi_p'$: degrees |
| A | 30 | 31 | 82 | 7 | 20 | 0 | 20 |
| B | 25 | 70 | 60 | 9 | 45 | 0 | 40 |

$w_p$, $w_n$ and $w_l$ are plastic limit, natural water content and liquid limit in per cent respectively.

**Table 2. Relation of α values and triaxial test types**

| α: degrees | Triaxial test type |
|---|---|
| 45 | Conventional drained test |
| 71·6 | Constant $\sigma_{oct}'$ test |
| 90 | Constant $p'$ test |
| 135 | Test with constant $\sigma_1'$ and decreasing $\sigma_3'$ |

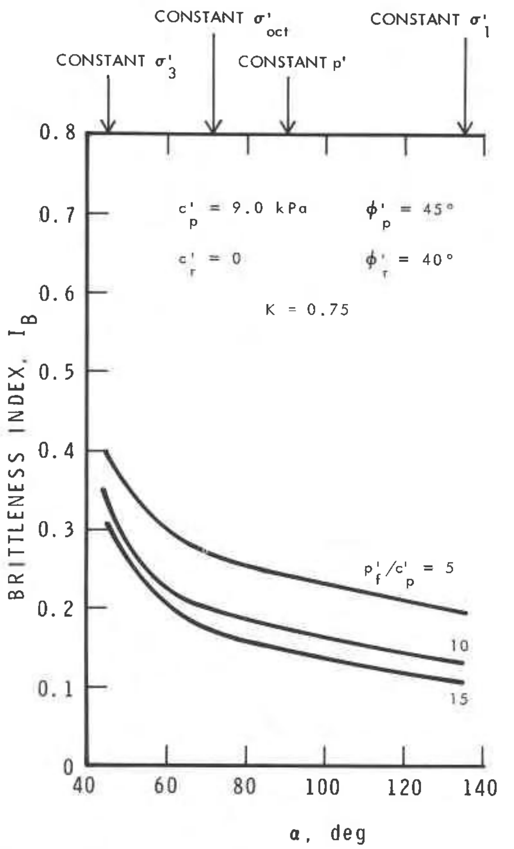


**Fig. 5. Variation of brittleness with stress paths starting from the same consolidation pressure for soil B**

parameter (Table 1). Soil A corresponds to the brown London clay (Skempton, 1977) and soil B to the Champlain Sea clay of Eastern Canada (Eden & Mitchell, 1973). The results of the study and indications of four triaxial test types are shown in Figs 3 and 4. The four triaxial test types are associated with specific α values (Table 2). Based on the results the following observations can be made.

(a) Higher $p'$ value at failure yields lower brittleness, as was stated by Bishop (1967). Furthermore, the present study shows that this is also true for all the admissible linear effective stress paths.
(b) For a given path, brittleness increases with $c_p'$ (Haefeli, 1965).
(c) Brittleness is significantly affected by effective stress path. Among the four triaxial test types the conventional drained test yields the highest brittle behaviour, while the constant $\sigma_1'$ test ($\alpha = 135°$) yields the lowest. The constant $\sigma_{oct}'$ and $p'$ tests measure a brittleness appreciably lower than that of the conventional drained test; for example, the $I_B$ drops from 0·68 in the conventional test to 0·15 in the constant $\sigma_1'$ test for soil B failing at $p' = 45$ kPa.

Understanding of the relationship between brittleness and stress path makes it quite clear why there has been a difference of opinion regarding brittle behaviour. Lo & Morin (1972) and Lefebvre & La Rochelle (1974) all used the conventional drained tests, and observed substantial brittleness. Mitchell (1975), on the other hand, used the constant $\sigma_{oct}'$ test and measured far less brittleness. The total absence of brittle behaviour that he reported was due to the use of incremental loading procedures, which were incapable of giving any meaningful post-peak behaviour. His earlier constant $\sigma_{oct}'$ tests (Eden & Mitchell, 1970) at constant strain rate did indicate some small, yet definite, strength decrease beyond the peak.

*Stress paths starting from the same consolidation pressure*

Many laboratory tests on undisturbed soil samples reported in the literature have been carried

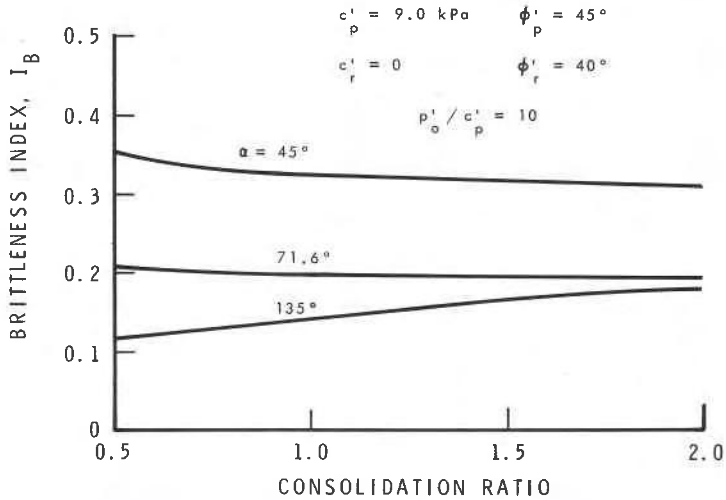


**Fig. 6. Variation of brittleness with consolidation ratio $K$ for soil B**

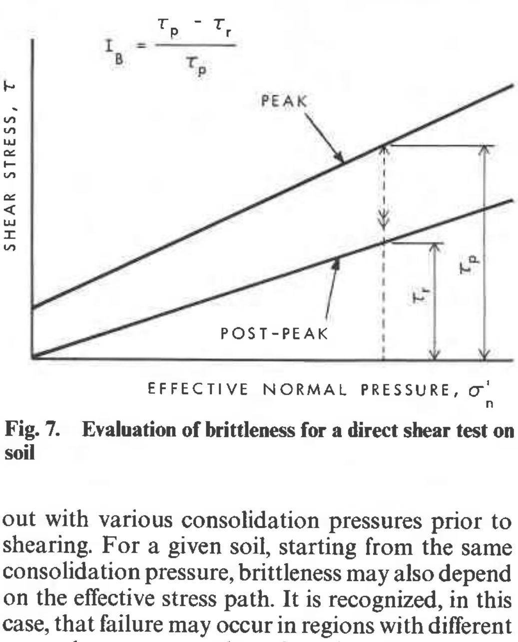


**Fig. 7. Evaluation of brittleness for a direct shear test on soil**

out with various consolidation pressures prior to shearing. For a given soil, starting from the same consolidation pressure, brittleness may also depend on the effective stress path. It is recognized, in this case, that failure may occur in regions with different strength parameters, thus obscuring the geometric effect of stress path on brittleness. Nevertheless, if the peak and post-peak strength parameters remain unchanged, the following expression for $I_B$ is obtained.

$$I_B = 1 - \frac{c_r' \cos \phi_r' + p_o' \sin \phi_r'(1-\eta/\tan \alpha)}{c_p' \cos \phi_p' + p_o' \sin \phi_p'(1-\eta/\tan \alpha)} \times \frac{\tan \alpha - \sin \phi_p'}{\tan \alpha - \sin \phi_r'} \tag{6}$$

where $\eta = (1-K)/(1+K)$ and $K = \sigma_{3o}'/\sigma_{1o}'$ while $\sigma_{3o}'$ and $\sigma_{1o}'$ are the effective minor and major principal stresses at the end of consolidation.

Based on equation (6), $I_B$ is independent of the effective stress path if $\phi_p' = \phi_r'$ and $K = 1{\cdot}0$ (isotropic consolidation). Apart from this condition, $I_B$ is, in general, significantly affected by $\alpha$, as is shown in Fig. 5 for soil B at various consolidation pressures. The variation of $I_B$ from $\alpha = 45°$ to $\alpha = 135°$ is less drastic than the previous case of failure at the same $p'$ value. Figure 5 also shows that brittleness increases with decreasing $p'$ and increasing $c_p'$, as in the previous case.

The consolidation ratio $K$ has only a small effect on brittleness (Fig. 6). When $\alpha = 90°$, $I_B$ is independent of $K$; when $\alpha < 90°$, $I_B$ decreases slightly with $K$; and when $\alpha > 90°$, $I_B$ increases slightly with $K$.

## POST-PEAK STRENGTH DECREASE ALONG DIFFERENT STRESS PATHS IN DIRECT SHEAR TESTS

The foregoing treatment assumes that the post-peak strength can be determined in the triaxial tests. There are other soils whose post-peak stress–strain relationships only stabilize at a very large displacement. For such soils the post-peak strength, which is often referred to as residual strength, may be determined using a direct shear (Skempton, 1964) or a ring shear apparatus (Bishop *et al.*, 1971). In this case, only the normal effective stress $\sigma_n'$ and the shear stress $\tau$ are known, while $p'$ and $q$ cannot be clearly defined. With $\sigma_n'$ and $\tau$ replacing $p'$ and $q$ respectively, a similar stress plot can be drawn, as shown in Fig. 7. The stress paths for tests at different $\sigma_n'$ will be represented by vertical lines on the plot and the corresponding expression for $I_B$ is

$$I_B = \frac{(c_p' - c_r') + \sigma_n'(\tan \phi_p' - \tan \phi_r')}{c_p' + \sigma_n' \tan \phi_p'} \tag{7}$$

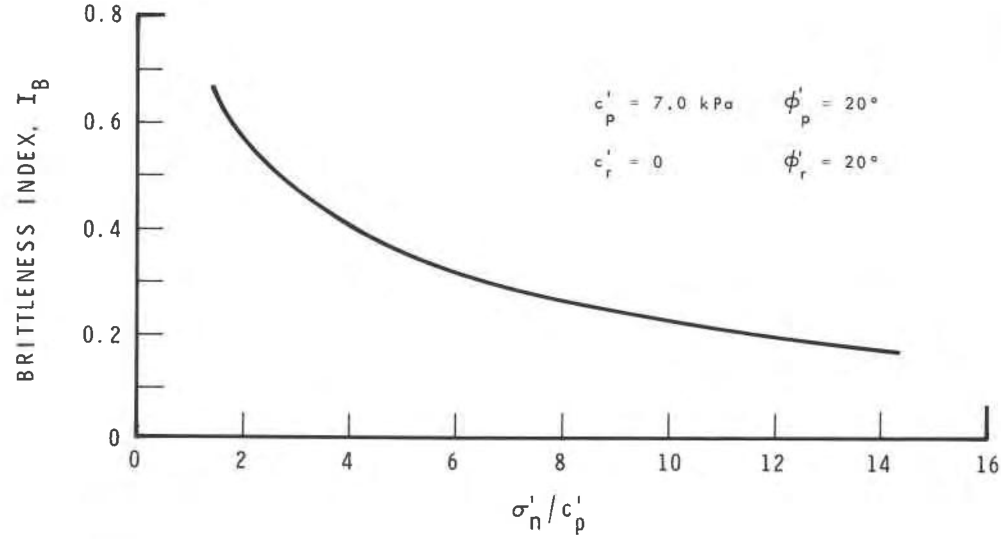


**Fig. 8. Variation of brittleness with effective normal pressure for a direct shear test on soil A**

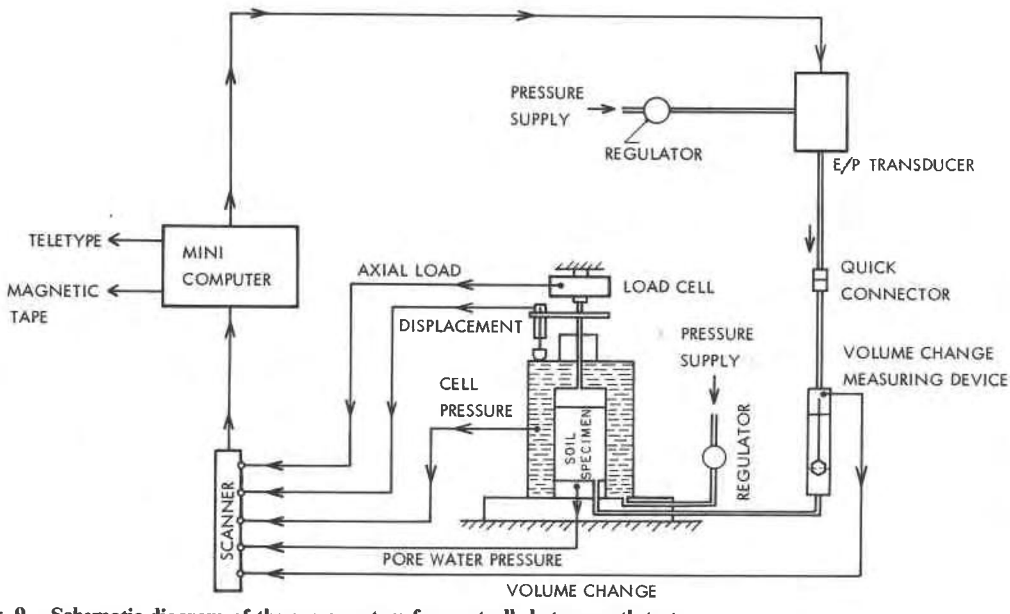


**Fig. 9. Schematic diagram of the servo-system for controlled stress path tests**

It can be shown that in general $I_B$ decreases with increasing effective normal stress; Fig. 8 shows such a relationship for soil A.

## EXPERIMENTAL STUDY

To illustrate the case of triaxial tests, experiments were conducted on undisturbed soil samples from Rockcliffe, Ottawa, Ontario. The soil is a stiff fissured Champlain Sea clay with natural moisture content ranging from 65 to 70%. Its characteristics have been described in detail by Mitchell (1970). The tests were carried out along various effective stress paths and under a constant strain rate so that the entire stress–strain curve could be determined. All the stress paths listed in Table 2 have been tried.

Use was made of a servo-system which is shown schematically in Fig. 9. It consists of two main components: a mini-computer and an electric-pneumatic (E/P) transducer. The mini-computer collects and analyses data during a triaxial test and

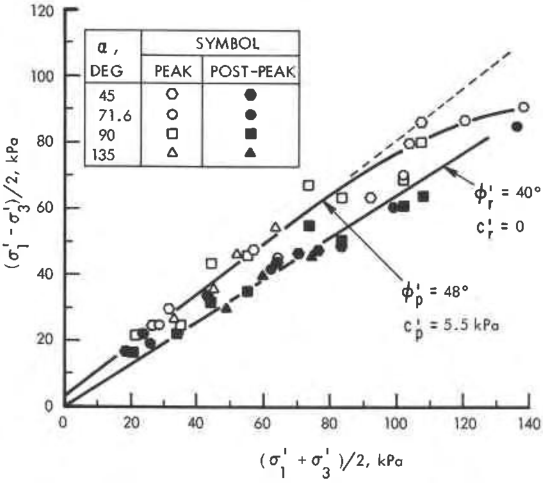


**Fig. 10. Summary of test results**

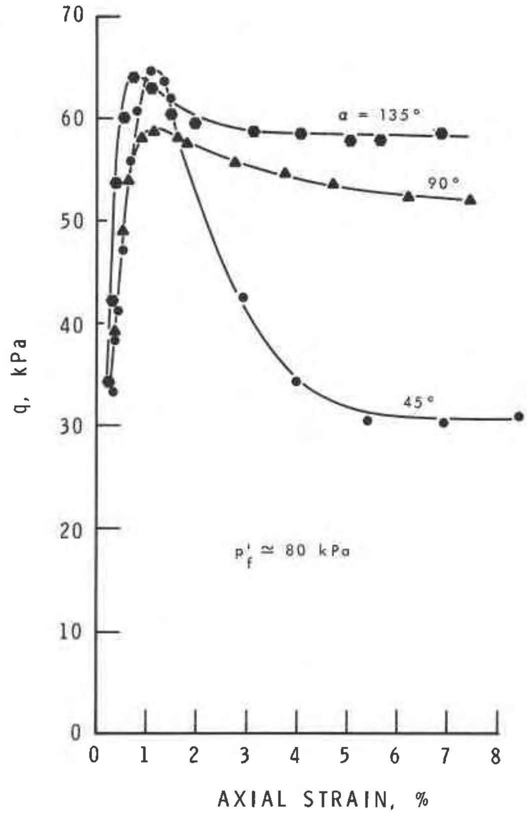


**Fig. 11. Stress–strain curves for tests reaching failure at the same $p'$ value along different effective stress paths**

transmits an appropriate voltage to the E/P transducer. The E/P transducer regulates a pressure proportional to the transmitted voltage. The regulated pressure is the back pressure applied on the soil specimen sheared at a constant strain rate in a triaxial cell. Thus, the back pressure is varied automatically so that the given effective stress is followed whether during the loading (pre-peak) or unloading (post-peak) stage. Other details of the system are described by Law (1980).

For most specimens tested, shearing beyond the peak was accompanied by the formation of a shear plane. To evaluate the shear stress for this stage, two corrections were applied and they are similar to those described by La Rochelle (1967). First, an area correction was incorporated to account for the reduction in contact area due to the relative movement between the upper and the lower blocks separated by the shear plane. Second, the resistance of the membrane and side drains arising from the relative movement was determined experimentally. A series of correction curves were obtained for various effective pressures and strains and were applied in deducing the shear stress.

The summary of test results is shown in Fig. 10. The peak and post-peak strength envelopes are independent of stress path for this soil and can be represented by $c_p' = 5{\cdot}5$ kPa, $\phi_p' = 48°$ and $c_r' = 0$, $\phi_r' = 40°$, respectively. The upper end of the peak strength envelope is curved and may be represented by another set of parameters. For the present discussion the linear portion will be considered.

The stress–strain curves from samples tested at $\alpha = 45$, 90 and 135° are compared in Fig. 11. All reach about the same peak strength, but drop to different post-peak strengths depending on stress paths. Figure 12 shows the stress–strain curves of two other specimens (tested at $\alpha = 45$ and $71{\cdot}6°$) consolidated to the same pressure. They failed at different $p'$ values, but largely in regions with the same strength characteristics. The relative brittleness derived from these tests is, within experimental scatter, in line with that from the theory (Fig. 13).

## DISCUSSION OF RESULTS

The brittleness index $I_B$ of a soil, expressed in terms of its peak and post-peak strengths, is only a quantity describing a certain soil behaviour under a given set of conditions. The geometry of the stress path along which shear takes place is one of these conditions and has been studied in detail. Without specifying the conditions, $I_B$ is not very useful for engineering applications. A better characterization of brittle behaviour would be to specify the peak and post-peak strength envelopes. This would yield the proper strengths associated with the stress paths that might be encountered in the field.

Only linear stress paths and linear strength envelopes have been examined so far. If one or both are non-linear, as on the upper end of the strength envelope (Fig. 10), reasoning similar to the linear case will lead to the same conclusion that brittle-

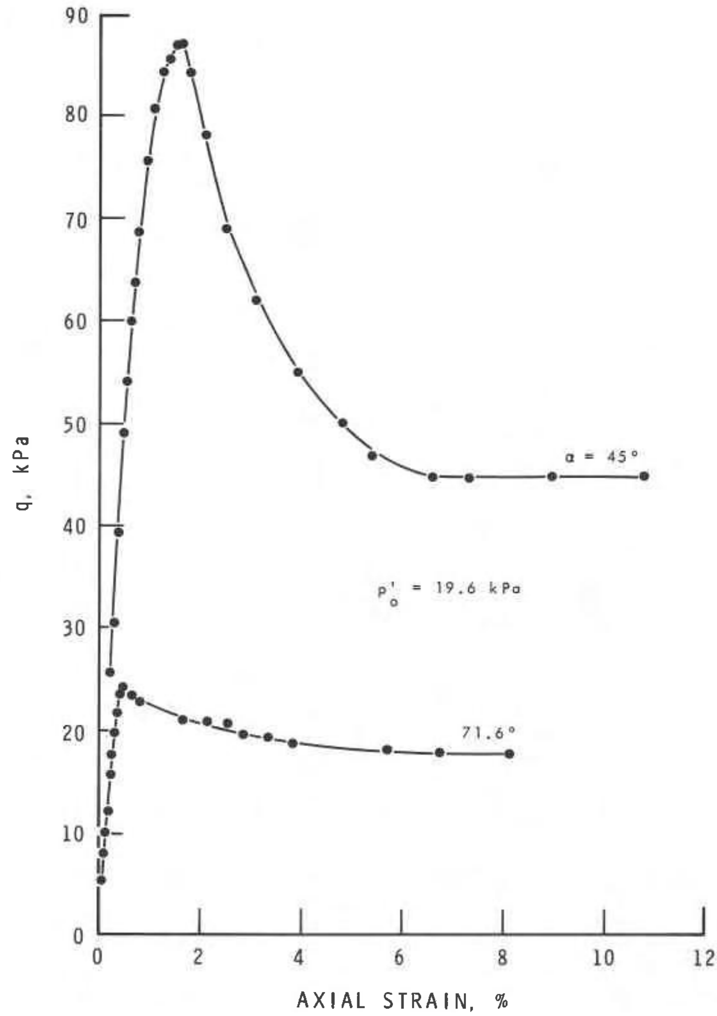


**Fig. 12. Stress–strain curves for tests consolidated to the same pressure but failed along different effective stress paths**

ness is affected by the geometry of stress paths. Also, for extension load such as that occurring beyond the toe of an embankment, the same conclusion is still valid provided that the peak and post-peak strength envelopes are different in that stress space.

## SUMMARY AND CONCLUSIONS

When a soil displays strength characteristics that can be described by peak and post-peak strength envelopes, its brittleness can be related to the geometry of stress path of shearing. A mathematical examination of this relationship has been carried out to support published experience and to explain the discrepancy noted by researchers with regard to brittle behaviour of Champlain Sea clay. Triaxial tests were conducted on a stiff Champlain Sea clay along various stress paths to demonstrate the predicted dependence. The results of this study can be summarized as follows.

(a) Soil brittleness increases with decreasing effective confining stress or with increasing effective cohesion.
(b) For a given soil failing at the same effective confining pressure, the conventional drained test yields the highest brittleness, followed by constant $\sigma_{oct}'$, constant $p'$ and lastly by constant $\sigma_1'$ tests.
(c) The same order as is listed in (b) was observed for soil consolidated to the same confining stress prior to shear under a constant strain rate.

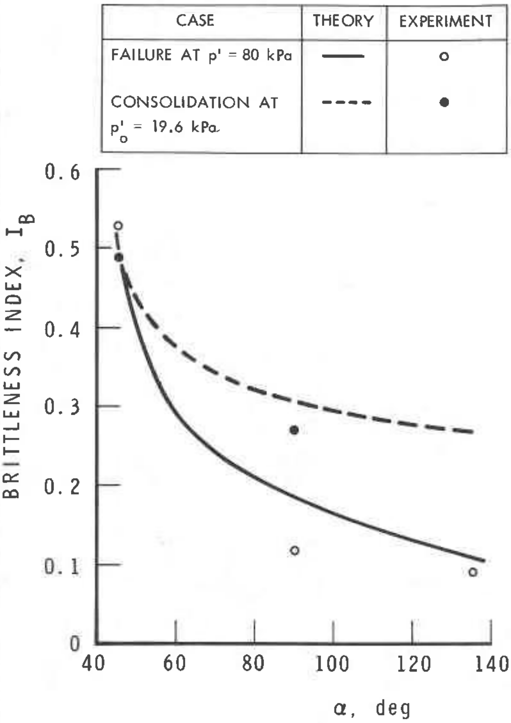


**Fig. 13. Theoretical and experimental variation of brittleness with stress paths**

## ACKNOWLEDGEMENT

This Paper is a contribution from the Division of Building Research, National Research Council of Canada, and is published with the approval of the Director of the Division.

## REFERENCES

Bishop, A. W. (1971). The influence of progressive failure on the choice of the method of stability analysis. *Géotechnique* **21**, No. 2, 168–172.

Bishop, A. W., Green, G. E., Ganga, V. K., Andresen, A. & Brown, J. D. (1971). A new ring shear apparatus and its application to the measurement of residual strength. *Géotechnique* **21**, No. 4, 273–328.

Campanella, R. G. & Vaid, Y. P. (1973). Influence of stress path on the plane strain behaviour of a sensitive clay. *Proc. 8th Int. Conf. Soil Mech., Moscow* **1**, 1, 85–92.

Conlon, R. J. (1966). Landslide on the Toulnustouc River, Quebec. *Can. Geotech. J.* **3**, 113–144.

Eden, W. J. & Mitchell, R. J. (1970). The mechanics of landslides in Leda clay. *Can. Geotech. J.* **7**, 285–296.

Eden, W. J. & Mitchell, R. J. (1973). Landslides in sensitive marine clay in Eastern Canada. *Highw. Res. Rec.* No. 463, 18–27.

Haefeli, R. (1965). Creep and progressive failure in snow, soil, rock and ice. *Proc. 6th Int. Conf. Soil Mech., Montreal* **3**, 134–148.

Kenney, T. C. (1977). Residual strengths of mineral mixtures. *Proc. 9th Int. Conf. Soil Mech., Tokyo* **1**, 155–160.

Lambe, T. W. (1967). Stress path method. *J. Soil Mech. Fdns Div., Am. Soc. Civ. Engrs* **93**, SM6, 309–331.

La Rochelle, P. (1967). Membrane, drains and area correction in triaxial test on soil samples failing along a single shear plane. *3rd Pan Am. Conf. Soil Mech.* **1**, 273–291.

Law, K. T. (1980). A servo system for controlled stress path tests. *Proceedings of symposium on shear strength of soils, Chicago.* Philadelphia: American Society for Testing and Materials.

Lefebvre, G. & La Rochelle, P. (1974). The analysis of two slope failures in cemented Champlain clays. *Can. Geotech. J.* **11**, 89–108.

Lo, K. Y. & Morin, J. P. (1972). Strength anisotropy and time effects of two sensitive clays. *Can. Geotech. J.* **9**, 261–277.

Mitchell, R. J. (1970). On the yielding and mechanical strength of Leda clays. *Can. Geotech. J.* **7**, 297–312.

Mitchell, R. J. (1975). Strength parameters for permanent slopes in Champlain Sea clays. *Can. Geotech. J.* **12**, 447–455.

Simons, N. E. (1971). The stress path method for settlement analysis applied to London clay. *Proceedings of the Roscoe Memorial Symposium.* Cambridge: Foulis, 241–252.

Skempton, A. W. (1964). Long-term stability of clay slopes. *Géotechnique* **14**, No. 2, 77–101.

Skempton, A. W. (1977). Slope stability of cuttings in brown London clay. *Proc. 9th Int. Conf. Soil Mech., Tokyo* **3**, 261–270.

Publications of the Division may be obtained by mailing the appropriate remittance (a Bank, Express, or Post Office Money Order, or a cheque, made payable to the Receiver General of Canada, credit NRC) to the National Research Council of Canada, Ottawa. K1A 0R6. Stamps are not acceptable.

A list of all publications of the Division is available and may be obtained from the Publications Section, Division of Building Research, National Research Council of Canada, Ottawa. KIA 0R6.