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View Planning as a Set Covering Problem

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Abstract

A new theoretical framework for the view planning problem is presented. In this framework, view planning is defined as an instance of the well-known set covering problem from the field of combinatorial optimization. We include an image-based registration constraint and express the result in the form of an integer programming problem.

Keywords: view planning, range sensors, object reconstruction, inspection, geometric modeling

1 Introduction

The view planning problem (VPP) for 3D object reconstruction and inspection by active range cameras remains an open problem. The imaging environment comprises a range sensor [3], the object, associated fixtures and a positioning system. While inspection benefits from a detailed object model, object reconstruction commences with little a priori knowledge and must undertake both scene exploration and precision measurement. Most current techniques combine these activities in iterative exploration/measurement steps exploiting current knowledge for cues to the next-best-view (NBV). The non-model-based view planning literature is reviewed at [14], [10].

An alternative approach is model-based view planning [10] which partitions scene exploration from precision measurement. As its starting point, the present work assumes that a separate, pre-programmed and rapid scene exploration phase has been completed, resulting in a polygonal mesh *rough model* capturing an approximation of object geometry. The rough model, or CAD model in the inspection case, is used to compute one or more measurability matrices which become the knowledge base for planning precision measurement. The issue of how rough the rough model can be and still reliably guide precise 3D geometric sensing has been addressed at [11].

Theoretical treatment of view planning has been limited. Tarbox and Gottschlich [15] introduced the measurability matrix concept in a model-based approach to inspection. They showed that the VPP was isomorphic to the set covering problem (SCP) which is known to be NP-complete [6]. Scott et al [10] extended that work to object reconstruction with a set theoretical framework and introduced the concept of performance-oriented view planning - that is, planning with respect to specific technical criteria in a model specification. Whaite and Ferrie [16] presented an autonomous exploration theory based on minimization of model uncertainty. Yuan [17] used mass vector chains in a global approach to view planning. Most other work in the field relies on a variety of heuristic techniques without a theoretical framework.

2 View Planning as an Integer Programming Problem

2.1 Theoretical Framework

We have previously developed [10] a set theory based formulation of the VPP in terms of a measurability mapping between viewpoint space V and object surface space S. The size of V and S are v and s, respectively. Recognizing the VPP as an instance of the SCP admits its expression as a classical 1/0 integer programming problem (IP), a sub-class of linear programming problems (LP). The view planning problem can then be expressed as the problem of covering the rows of a binary s-by-v measurability matrix $\mathbf{M} = [m_{ij}]$ by a minimal sub-set of the columns. In this notation, rows correspond to surface points (vertices in the rough model) and columns to candidate viewpoints. Candidate viewpoints are generated from the geometric scene knowledge embedded in the rough model.

It is instructive to partition $\mathbf{M}$ into column vectors $\mathbf{M}_{S,j}$ and row vectors $\mathbf{M}_{i,V}$. The set S_j of surface elements measurable by a single viewpoint $\mathbf{v}_j$ is defined by the corresponding column vector $\mathbf{M}_{S,j}$ of the measurability matrix. Similarly, the region V_i of viewpoint space from which a given surface element $\mathbf{s}_i$ is measurable is defined by the corresponding row vector $\mathbf{M}_{i,V}$ of the measurability matrix.

Computing the measurability matrix has complexity $\mathcal{O}(vs^2)$. For a typical dense model reconstruction, $s \approx 10^5$. Without simplification, viewpoint space discretization levels can be very high ($\approx 10^{11}$). Complexity reduction techniques [10] include sparse sampling in surface and viewpoint space as well as segmenting the object surface into patches. In this case, separate measurability matrices are computed for each segmented patch and v and s refer to subsets associated with individual patches. The theoretical framework presented here applies regardless of the representation.

Informally, the goal of view planning is to find the smallest set of views satisfying measurability constraints. More formally, the problem can be written as follows. Here, we define $\mathbf{X}$ as a vector of binary viewpoint variables x_j such that $x_j = 1$ if column j is in the solution and $x_j = 0$ if it is not.

$$\text{Minimize } Z = \sum_{j=1}^{v} x_j \tag{1}$$

$$\text{subject to } \sum_{j=1}^{v} m_{ij} x_j \geq 1\,;\; i = 1, \ldots, s\,;\; i \in S \tag{2}$$

$$x_j \in \{0, 1\}\,;\; j = 1, \ldots, v\,;\; j \in V \tag{3}$$

$\mathbf{X}$ spans viewpoint space as sampled by the viewpoint generation stage. The optimal $\mathbf{X}$ is the minimal set of viewpoints covering S - that is, the next-best-view set N. The constraints defined by equation 2 ensure that each row of the measurability matrix is covered by at least one viewpoint. Equation 3 applies an integer constraint on the viewpoint variable. Occasionally, there may be no feasible solution, in which case we wish to have a solution for the minimal number of non-compliant constraints, that is - maximum surface covering.

2.2 Positioning System Movement Constraint

The preceding formulation is known as the unicost set covering problem. Modifying equation 1 by assigning costs c_j to each viewpoint restates the problem as a non-unicost SCP. This would be appropriate for an imaging environment with non-uniform and significant movement "costs" associated with viewpoints. In general, this will not be the case and the unicost formulation will suffice for our purposes. The non-unicost IP is a somewhat more difficult programming problem.

$$\text{Minimize } Z = \sum_{j=1}^{v} c_j x_j \tag{4}$$

A separate issue relates to physical limitations on the range of motion and degrees of freedom of the positioning system which effect viewpoint feasibility. There is no need to include such constraints in the above IP formulation. It is more efficient to avoid generating infeasible viewpoints at the viewpoint generation stage.

2.3 Image-Based Registration Constraint

A common imaging problem arises when positioning system precision is inferior to that of the sensor. It is then necessary to employ image-based registration to bring images into a common reference frame with a precision comparable to that of surface measurements. The iterative closest point (ICP) algorithm [4] and its more recent enhancements is widely used for image-based registration. It works by minimizing errors in the geometric fit between overlapping range image segments. In this section we show how an image-based registration constraint can be expressed within the IP formulation of the VPP.

Image-based registration requires sufficient overlap between range images[1]. A degree of image overlap is also necessary for image integration. As a first approximation, we specify a point overlap constraint which is a necessary but not sufficient condition for image-based registration. In general, we need to add a geometric complexity requirement to the overlap region to fully constrain registration in all directions and rotations. It will be apparent from the simpler point overlap constraint how, in principal, we can formulate a more stringent constraint for overlap with geometric complexity. This is being pursued.

There is a direct correspondence between viewpoints and range images. Image overlap can therefore be determined from the degree of viewpoint correlation. For the purposes of view planning, we define the cross-correlation σ_{kj} of two viewpoints v_k and v_j as the dot product of the respective column vectors $\mathbf{M}_{S,k}$ and $\mathbf{M}_{S,j}$ of the measurability matrix, normalized[2] by the maximum viewpoint coverage of any viewpoint in the candidate viewpoint set, i.e. $m_S = max|\mathbf{M}_{S,k}|\ \forall k \in V$.

$$\sigma_{kj} = \frac{\mathbf{M}_{S,k} \cdot \mathbf{M}_{S,j}}{m_S} \tag{5}$$

To register image (viewpoint) v_k with image (viewpoint) v_j, we require that their cross-correlation exceed some image-based registration threshold, typically around 20%.

$$\sigma_{kj} \geq t_r \tag{6}$$

Let the binary variable $X_{kj} = 1$ if $\sigma_{kj} \geq t_r$ and $X_{kj} = 0$ otherwise. We can then compute the symmetric v-by-v cross-correlation matrix $\Sigma = [X_{kj}]$. Normally, $X_{kk} = 1$. In the rare case where $X_{kk} = 0$, meaning the image is so sparse it could not even register with itself, we drop the associated viewpoint from the candidate viewpoint list and reformulate the IP set covering problem accordingly.

We can now observe that Σ specifies viewpoint adjacency in registration terms. We therefore define a *viewpoint registration-adjacency matrix* $A = [a_{kj}]$ such that $a_{kj} = X_{kj}$, $i \neq j$, and $a_{kk} = 0$. The registration-adjacency matrix A has an associated *viewpoint registration graph* G_r

[1] Few view planning techniques in the literature incorporate a registration constraint. Pito [8] includes an explicit overlap requirement and mentions the need for shape complexity in the overlap area but does not implement it. Whaite and Ferrie [16] achieve image overlap by a conservative search strategy.

[2] There are several potential choices for a normalizing value. m_S has the advantage of guaranteeing σ_{kj} values in the range [0,1], is reciprocal and is independent of object or segmentation patch size, rough model sampling density and sensor characteristics.

encoding viewpoint connectivity in terms of inter-image registration potential. Consequently, we can express the image-based registration requirement by stating that

The viewpoint registration graph must be at least simply connected.

Connectivity information embedded in the registration-adjacency matrix can be used to determine the connectivity of the registration graph. The $(k,j)^{th}$ entry of the matrix product A^r defines the number (≥ 0) of different paths (including backtracking) between nodes k and j of length r in the graph. The maximum length of a non-repeating path in the registration graph is one less than the size v of the viewpoint set V. Consequently, the *cumulative registration-adjacency matrix* $C = [c_{kj}]$ shown in equation 7 defines the number of paths of all lengths from minimum to maximum connectivity between viewpoints. In principal, C can be computed directly from Σ as a pre-processing step prior to tackling the IP problem.

$$C = A^1 + A^2 + \cdots + A^{v-1} \tag{7}$$

For the registration graph to be connected, rows and columns of C corresponding to viewpoints in the next-best-view set N must be non-zero. Therefore, a necessary and sufficient condition for successful image-based view registration is

$$c_{kj} \geq 1\,;\; k,j \in N\,; k \neq j \tag{8}$$

As C is symmetric with all elements ≥ 0, and diagonal elements are irrelevant, the equation can be simplified as follows, where $n = |N|$ and, generally, $n \ll v$.

$$c_{kj} \geq 1\,;\; k = 1, \cdots, (n-1)\,;\; j = k+1, \cdots, n\,;\; k,j \in N \tag{9}$$

Furthermore, C defines the registration connectivity of all viewpoints in V whereas we wish to apply the global registration constraint only on $N \subset V$. Then, if $x_l = 0$, ignore c_{kj} for all $k, j = l$. We can achieve the desired effect over the full problem domain by changing the registration constraint to

$$c_{kj} \geq x_k x_j\,;\; k = 1, \cdots, (v-1)\,;\; j = k+1, \cdots, v\,;\; k,j \in V \tag{10}$$

2.4 Registration Constraint Example

A simple example illustrates the above concept. Consider the set of four candidate viewpoints shown with their global registration graph in Figure 1. The base registration adjacency matrix A can be written[3] as

[3]As we are interested only in simple connectivity, we take the liberty of using boolean algebra in matrix manipulation and simplification.

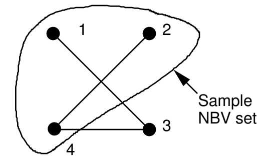

Figure 1: Example Registration Graph and NBV Cut

$$\begin{bmatrix} 0 & 0 & x_1x_3 & 0 \\ 0 & 0 & 0 & x_2x_4 \\ x_1x_3 & 0 & 0 & x_3x_4 \\ 0 & x_2x_4 & x_3x_4 & 0 \end{bmatrix}$$

from which we obtain the cumulative registration adjacency matrix $C = A^1 + A^2 + A^3$

$$\begin{bmatrix} x_1x_3 & x_1x_2x_3x_4 & x_1x_3 & x_1x_3x_4 \\ x_1x_2x_3x_4 & x_2x_4 & x_2x_3x_4 & x_2x_4 \\ x_1x_3 & x_2x_3x_4 & x_3(x_1+x_4) & x_3x_4 \\ x_1x_3x_4 & x_2x_4 & x_3x_4 & x_4(x_2+x_3) \end{bmatrix}$$

or simply as the following where "-" signifies "don't care".

$$\begin{bmatrix} - & x_1x_2x_3x_4 & x_1x_3 & x_1x_3x_4 \\ - & - & x_2x_3x_4 & x_2x_4 \\ - & - & - & x_3x_4 \\ - & - & - & - \end{bmatrix}$$

Using equation 10 and removing redundancies, the foregoing simplifies to the following global registration constraints which can be validated by inspection of G_r. Testing the registration graph in Figure 1 against the constraints at equation set 11, we observe that the example candidate NBV set fails the registration test.

$$\begin{aligned} x_1x_2x_3x_4 &\geq x_1x_2 \\ x_1x_3x_4 &\geq x_1x_4 \\ x_2x_3x_4 &\geq x_2x_3 \end{aligned} \tag{11}$$

2.5 Observations on the Registration Constraint

A number of observations on the registration constraint are appropriate. Firstly, we note the constraints are non-linear, making the IP non-linear. However, we can solve for the set covering constraints alone and then enforce registration compliance as part of the fitness function. If required, a minimal number of views can be added to satisfy the registration constraint and render the solution feasible overall.

A second concern is the computational cost of formulating the registration constraints as calculation of C involves symbolic matrix operations on large matrices. The computational complexity of C is approximately $\mathcal{O}(v^3)$ operations. However, the matrix need be computed only once. Additionally, we again have the option to delay computation of C to only candidate solutions, in which case the dimensionality of the required computations is greatly reduced and can be numerical rather than symbolic. In the latter case, the computational complexity is approximately $\mathcal{O}(cn^3)$ operations, where c is the number of candidate solutions and $n \ll v$.

3 Summary and Conclusion

Summarizing, given one or more measurability matrices computed from a rough exploratory model, we can express the view planning problem as the following integer programming problem, where the objective function and constraints are as previously defined.

$$\text{Minimize } Z = \sum_{j=1}^{v} c_j x_j \tag{12}$$

subject to

$$\sum_{j=1}^{v} m_{ij} x_j \geq 1\,; i = 1, \ldots, s\,;\ i \in S \tag{13}$$

$$c_{kj} \geq x_k x_j\ ;\ k = 1, \cdots, (v-1)\,;\ j = k+1, \cdots, v\,;\ k, j \in V \tag{14}$$

$$x_j \in \{0, 1\};\ j = 1, \ldots, v\,;\ j \in V \tag{15}$$

Expressing the view planning task as an IP provides a compact mathematical formulation of the problem, opening up the rich research base in discrete combinatorial optimization. However, IP solution time can be highly unpredictable, depending on the problem formulation, data characteristics and problem size. Optimal solution methods such as branch-and-bound and cutting-plane techniques typically use an intelligent tree search of feasible solutions and are found in a variety of commercial LP/IP solvers. While guaranteeing optimal results, such exact methods can be computationally prohibitive even for modestly sized IPs. For most medium-to-large IPs, this leaves a choice of approximate and heuristic algorithms [9], including greedy search (GS) [5], simulated annealing (SN) [13], genetic algorithms (GA) [2], Lagrangian relaxation [1] and neural network [7] methods. Most published performance results [7], [1] deal with random, low density data sets. The VPP falls into the category of a medium-to-large IP with non-random data and moderate density [12].

Finally, we believe an even more important area awaits better theoretical underpinnings - notably, issues relating to the problem formulation, rather than its solution. Specifically, the field requires a sound theoretical basis for determining a suitable set of viewpoint variables and surface constraints - that is, optimal sampling of viewpoint space and the exploratory object model.

In conclusion, we have expressed a theoretical framework for the view planning problem as an integer programming problem including a registration constraint. The formulation is amenable to a variety of exact or approximate solution methods, depending on application requirements.

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