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# Floor Vibrations from Aerobics

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# Floor vibrations from aerobics[1]

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Recent vibration problems with floors used for aerobics have shown the need for better guidelines for structural design and evaluation. Such guidelines were introduced for the first time in the Supplement to the 1985 National Building Code (NBC), but more recent experience with one particular floor has shown the need for some improvements to these guidelines. The paper describes the investigation of the floor and the use of the floor to estimate the loading function for aerobics, and recommends changes to the NBC design criteria. The paper provides guidance on estimating parameters used in the criteria and discusses repair alternatives related to the floor problem.

*Key words*: floor vibration, aerobics, design criteria, repair.

De récents problèmes de vibration des planchers soumis à des activités de danse aérobique mettent en lumière la nécessité d'élaborer de meilleures lignes de conduite concernant l'évaluation et al conception de ces ouvrages. De telles lignes de conduite ont été introduites pour la première fois dans le Supplément du Code national du bâtiment du Canada (CNBC) 1985; cependant, des expériences récentes ont démontré la nécessité d'apporter des améliorations à celles-ci. Cet article décrit une étude d'un plancher et de l'utilisation de celui-ci en vue d'estimer le chargement relatif à la danse aérobique, et de recommander des modifications aux critères de conception du CNBC. Cet article propose un guide en vue d'évaluer les paramètres utilisés dans l'élaboration du critère et aborde les solutions permettant de corriger un problème de vibration des planchers.

*Mots clés* : vibration des planchers, danse aérobique, critère de conception, réparation.

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## Introduction

Increased spans combined with new human activities such as aerobics have resulted in an increasing number of floor vibration problems in recent years. A commentary to the 1985 National Building Code (NBC 1985) introduced new design criteria for controlling such vibration to levels acceptable for human reaction. The criteria are based on a periodic loading function for aerobics (Allen *et al.* 1985) which includes two sinusoidal loading components, one at the jumping frequency (first harmonic up to 3 Hz) and the other at twice the jumping frequency (second harmonic up to 6 Hz).

Recently, after a considerable effort by the designers to ensure a sufficiently high natural frequency to avoid resonance, a problem of excessive floor vibration occurred in a health club. This paper investigates the factors that contributed to the problem and makes recommendations for incorporation in the 1990 NBC Commentary.

## Investigation of health club floor

### *Floor structure*

The floor structure of the health club (Fig. 1) consists of a noncomposite 90 mm concrete on metal deck supported by steel joists 1.8 m deep (0.85 m spacing) spanning 20 m over a swimming pool. The joists are supported on a block wall on one side and on girder and columns on the other. Partitions separate the floor into three main activity areas, two gyms used for aerobics and a weight lifting area, shown in Fig. 2.

### *History of the problem*

The initial floor system was designed for a minimum natural frequency of 7 Hz, which satisfied the 1985 NBC criterion (NBC 1985) for jumping exercises, considering first and second harmonics only. During the construction the designers elected to reinforce the bottom chords of the joists in an effort to increase the natural frequency to greater than 9 Hz. This was in consideration of possible problems from a third harmonic.

A problem was first encountered, however, with annoying vibration at 4.5 Hz, which is twice the jumping frequency. This was attributed to interaction between the floor and the roof via the partitions, since 4.5 Hz corresponded to the natural frequency of the roof. The partitions were therefore separated from the roof.

The problem remained, however, but became associated with a natural frequency of 5 Hz. It was determined that a major contributing factor was the flexibility of the girder supports; shims were therefore inserted between the girders and a block wall, which increased the natural frequency of the floor to nearly 7 Hz.

The problem still remained, but now became associated with three times the jumping frequency, with vibration acceleration amplitudes of approximately 6.5% $g$. An attempt was made to minimize the vibration from the third harmonic by means of tuned dampers. This alteration of floor properties reduced the acceleration to 5% $g$, but this was not sufficient to make the floor acceptable. A major factor in this problem was that the floor supports a shared occupancy separated by a partition, with aerobics on one end of the span and weight lifting on the other. The acceleration criterion of 5% $g$ for jumping exercises recommended in the NBC Commentary (NBC 1985) is therefore too high for a shared occupancy such as this.

Further vibration tests were then carried out by the author

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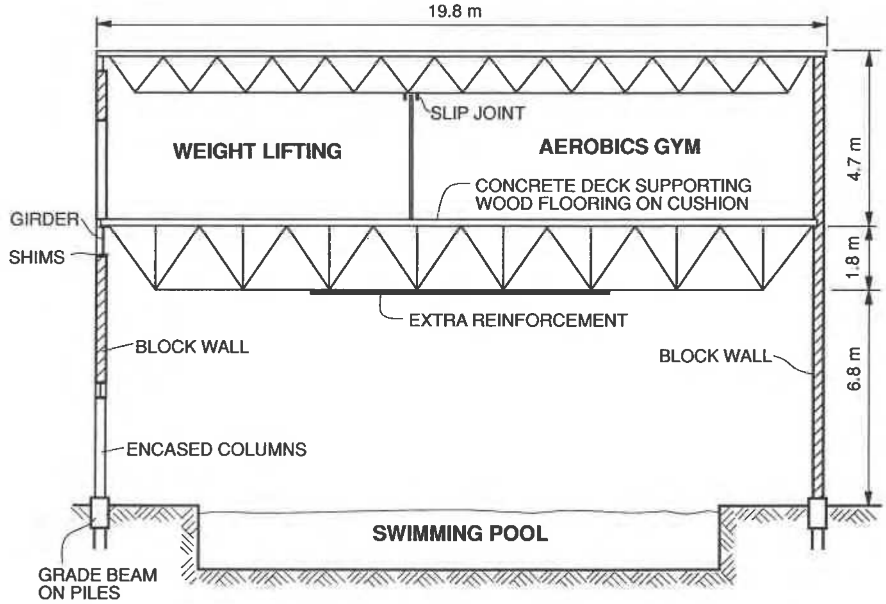


FIG. 1. Floor structure of health club (section B–B of Fig. 2).

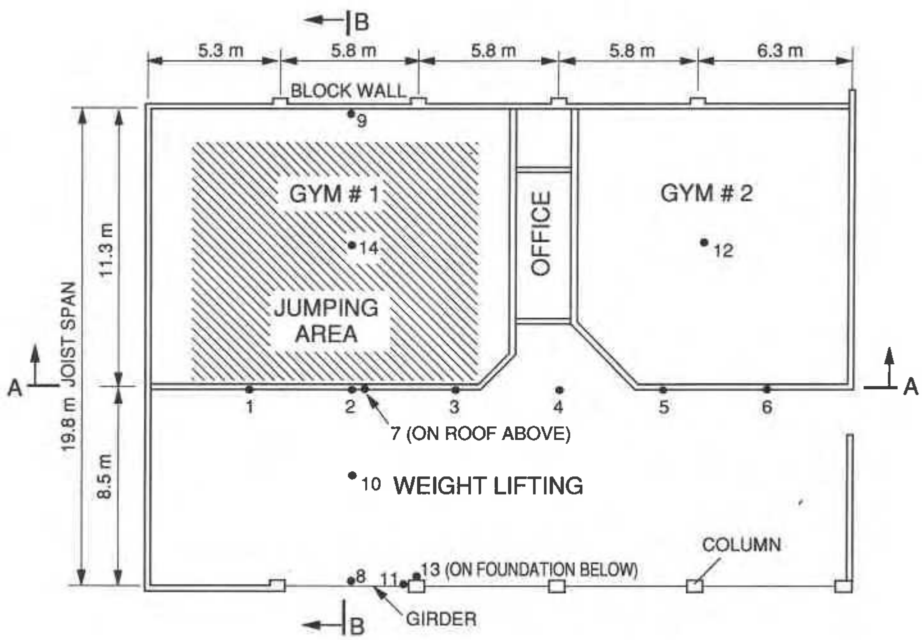


FIG. 2. Layout of health club floor showing acceleration locations.

to determine the contributing factors to the problem and to see if changes to the NBC Commentary were needed.

*Vibration tests*

Tests were carried out to determine the dynamic properties of the floor and the response of the floor to aerobic exercises. The instrumentation consisted of 14 accelerometers located as shown in Fig. 2 (two accelerometers, 12 and 14, were removed during aerobics). The data were collected on a Compaq portable 386 computer and, for backup, on tape. A computer program developed at the Institute for Research in Construction (IRC) was used for analysing the data (Fourier spectra for frequency content of response, cross spectra for mode shape).

*Dynamic floor properties*

A number of heel impact tests were carried out on the floor to determine natural vibration frequencies, mode shapes, and damping ratios. Figure 3 shows typical responses for two locations, one in the gym 1 area (accelerometer 2), the other in the gym 2 area (accelerometer 5). Figure 4 shows the mode shapes and frequencies determined from Fourier and cross spectra.

A strong feature of the results is the near independence or separation of floor response in the two gym areas, and the presence of one strong mode in each area, 6.7 Hz in gym 1 and 7.6 Hz in gym 2. The partitions, particularly those between the office and the two gym areas (see Fig. 2), appear to be instrumental in this separation effect. There are two weaker modes in gym 1 (7.7 and 8.4 Hz) and one in gym 2

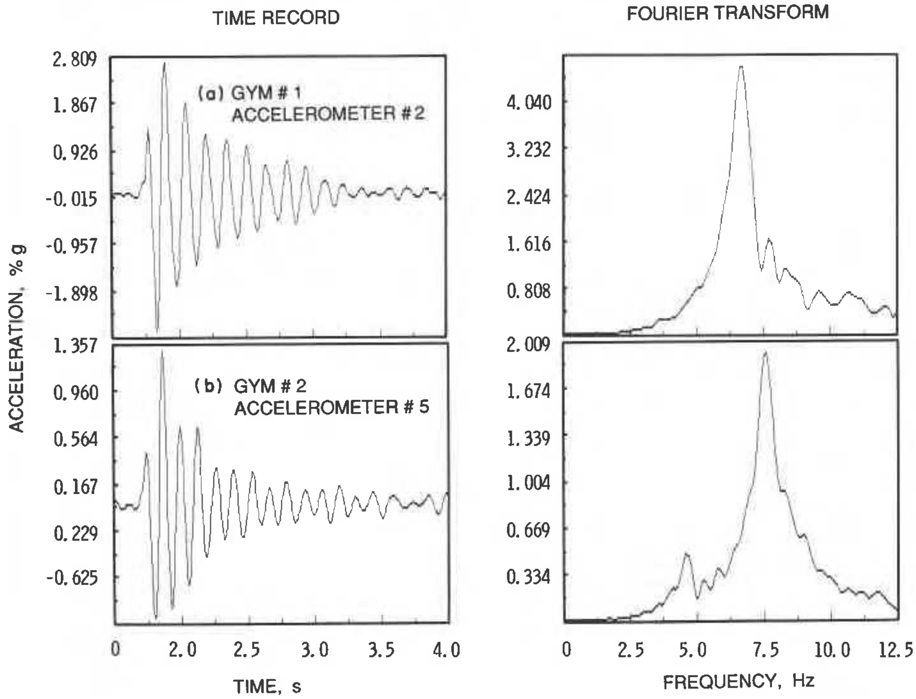


FIG. 3. Floor response to heel impact.

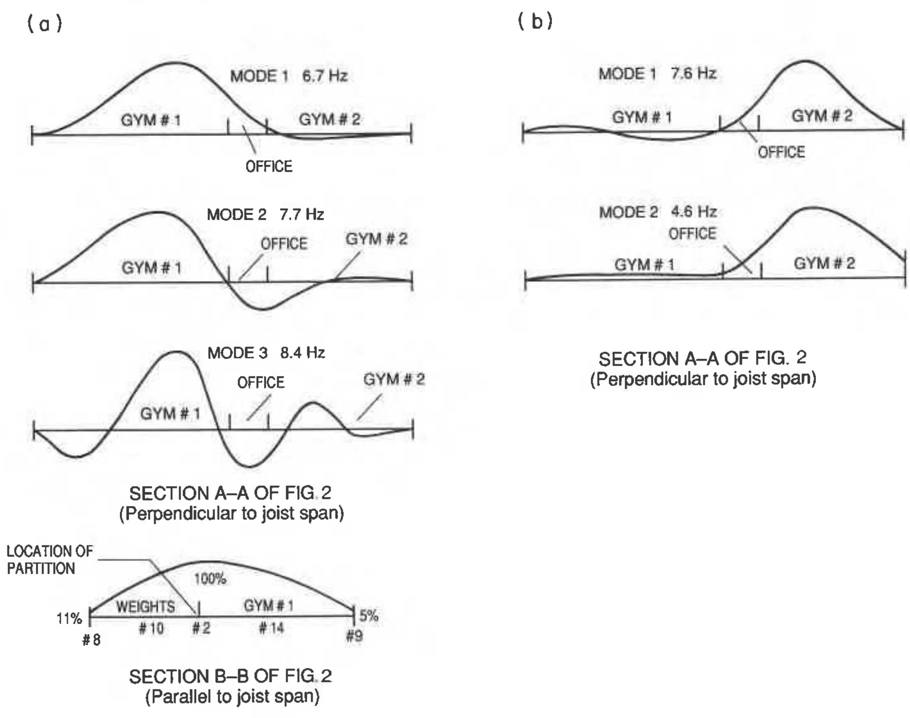


FIG. 4. Vibration modes derived from heel impact in (a) gym 1 and (b) gym 2 of health club floor.

(4.6 Hz). The 4.6 Hz mode is probably interaction with the roof structure (4.5 Hz natural frequency) due to lack of complete separation between the partitions in gym 2 and the roof. Modal damping ratios were estimated from band widths of Fourier spectra as well as from logarithmic decay to be approximately 0.04 for all modal responses.

The mode shape for the 6.7 Hz mode parallel to the joist span in Fig. 4*a* shows that the supports participate in the vibration. The measured support displacements relative to the maximum modal displacement were found to be 11% at the girder support (accelerometer 8) and 5% at the block wall (accelerometer 9). Of the 11% displacement at the girder support, 4% occurred at the column support (accelerometer 11) and 1% occurred at the foundation level (accelerometer 13). This information will be used later in the calculation of natural frequency.

### *Floor response to aerobics*

Vibration tests were carried out to determine the response of the floor structure to aerobics and, using the floor structure

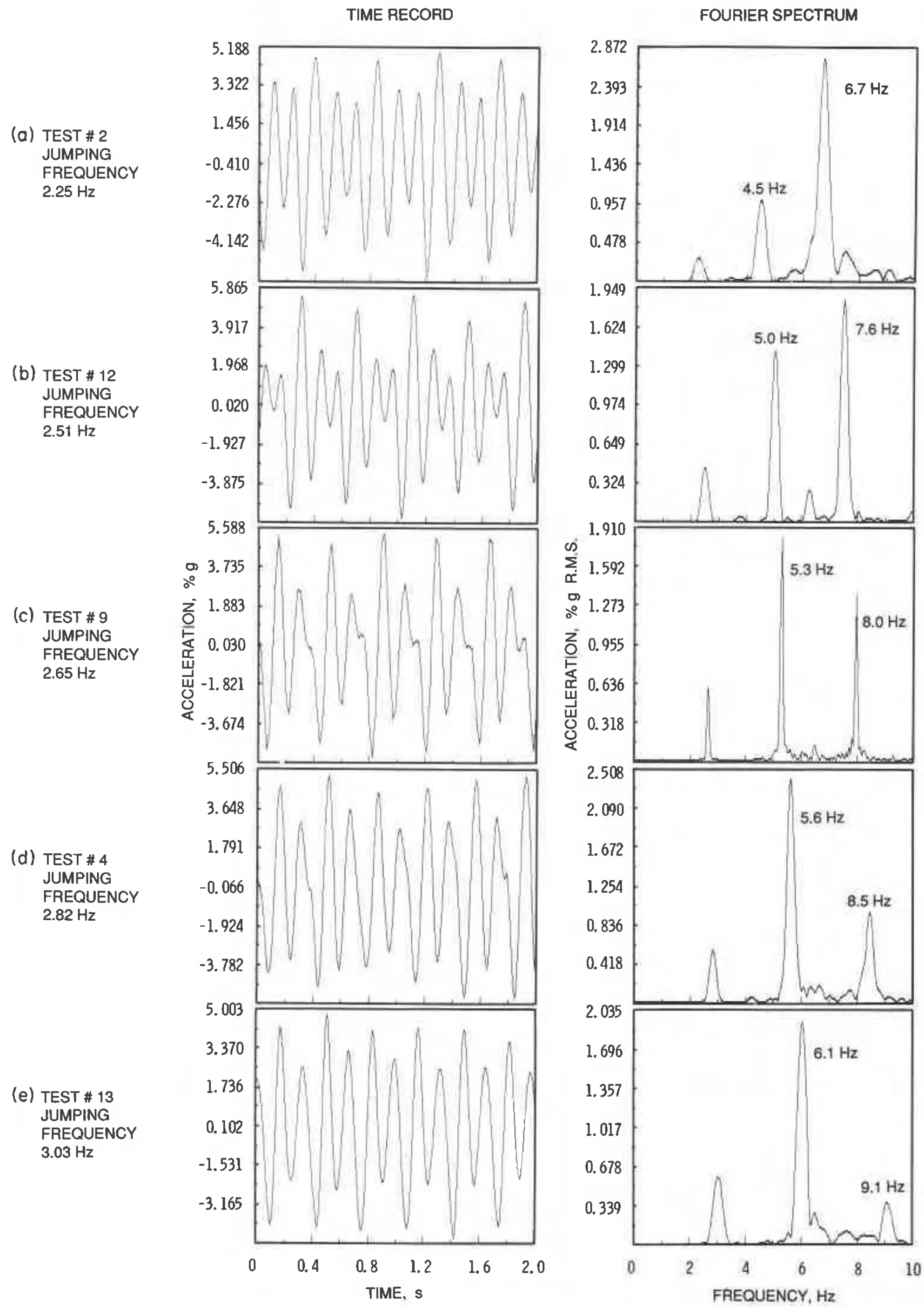


FIG. 5. Results of aerobics and jumping tests.

as an approximate measuring device, to estimate the loading function. This has previously been carried out for a gymnasium (Rainer and Swallow 1986) to determine the dynamic load factor for the first harmonic. So far the only data obtained on the loading function, including all harmonics, have been for 1−8 people jumping (Rainer *et al.* 1988; Pernica 1990). These may, however, not be representative for a typical large aerobics class, particularly for the higher harmonics. On this floor, normal aerobics exercises (most high impact and some low impact) and controlled jumping jacks at preset frequencies were employed. For high-impact aerobics and jumping jacks, both feet leave the floor, whereas for low-impact aerobics, one foot is always on the floor.

Figure 5 shows typical responses for different jumping frequencies in gym 1 (accelerometer 2). The Fourier spectra clearly show the presence of strong responses at three distinct frequencies, each corresponding to a harmonic of the jumping frequency. A fourth harmonic was also identified, but it was too small to be significant. Some spectra (Figs. 5*a*, *b*, and *e*) also show small off-harmonic amplitudes at a modal frequency

TABLE 1. Dynamic load factors estimated from aerobics exercises

| Test No. | Jumping frequency (Hz) | Dynamic load factors | | | Calculated participant loading† (kPa) | Number of people |
|---|---|---|---|---|---|---|
| | | $\alpha_1$* | $\alpha_2$ | $\alpha_3$ | | |
| *High impact* | | | | | | |
| 2, jumpng | 2.25 | 1.5 | 0.80 | 0.15 | 0.10 | 13–15 |
| 11, jumping | 2.42 | 1.5 | 0.50 | 0.08 | 0.14 | 18 |
| 12, jumping | 2.51 | 1.5 | 0.60 | 0.14 | 0.12 | 18 |
| 5, aerobics | 2.54 | 1.5 | 0.61 | 0.11 | 0.09 | 14 |
| 3, jumping | 2.57 | 1.5 | 0.63 | 0.15 | 0.10 | 13–15 |
| 9, aerobics | 2.65 | 1.5 | 0.50 | 0.08 | 0.14 | 21–25 |
| 10, aerobics | 2.72 | 1.5 | 0.64 | 0.13 | 0.15 | 21–25 |
| 4, jumping | 2.82 | 1.5 | 0.58 | 0.09 | 0.11 | 13–15 |
| 13, jumping | 3.03 | 1.5 | 0.30 | 0.06 | 0.10 | 10–15 |
| *Low impact* | | | | | | |
| 5, aerobics | 2.57 | 1.2 | 0.22 | 0.06 | 0.05 | 10–14 |
| 6, aerobics (gym 2) | 2.57 | 1.2 | 0.24 | 0.06 | 0.04 | ? |

*This value is assumed, based on direct force measurements (Pernica 1990).

†This includes a correction to $w_p$ calculated from [1] to take into account the actual loaded area in Fig. 1.

of the floor; other spectra (not shown) showed greater off-harmonic amplitudes at a modal frequency if there were changes in the aerobic action within the time span of the Fourier analysis. The off-harmonic amplitudes are free vibrations due to initial and nonrepetitive impulses.

Dynamic amplification or resonance effect is strongly evident in the Fourier spectra of Fig. 5:

- The third harmonic of the jumping frequency is in resonance with the fundamental mode of the floor (6.7 Hz) in Fig. 5*a*, with the second mode (7.7 Hz) in Fig. 5*b*, and with the third mode (8.4 Hz) in Fig. 5*d*. The third harmonic in Fig. 5*c* falls between two closely spaced modal frequencies (0.7 Hz apart) and therefore is considerably amplified. The third harmonic in Fig. 5*e* is beyond the major modal frequencies; the response is seen to be small.
- The floor response at the second harmonic of the jumping frequency increases dramatically as it approaches the fundamental natural frequency, 6.7 Hz, as shown in Figs. 5*a*–5*e*. Similarly, the floor response at the first harmonic increases with increasing frequency, but less dramatically than at the second harmonic.

Figure 5 demonstrates that aerobics performed by a large group of people can be represented by a periodic loading function consisting of three sinusoidal loading components (instead of two assumed in NBC 1985) at frequencies equal to the first three harmonics of the jumping frequency. The response to this loading function can be determined by classical dynamic analysis. When the frequency of the fundamental mode is equal to or greater than the third harmonic, Figs. 3 and 5*a* indicate that the response of the structure can be adequately represented by the response of a single-degree-of-freedom (SDOF) spring–mass system having a natural frequency equal to that of the fundamental mode, with negligible influence from the higher modes.

*Determination of loading function*

Results such as shown in Fig. 5 can be used to estimate the loading function, in particular the dynamic load factor for each harmonic. If only one mode of vibration occurs, then the dynamic load factors can be obtained from the steady-state solution (Allen *et al.* 1985):

$$[1] \quad \frac{a_i}{g} = \frac{1.3\alpha_i w_p / w_t}{\sqrt{\left[\left(\frac{f_0}{if}\right)^2 - 1\right]^2 + \left[2\beta\,\frac{f_0}{if}\right]^2}}$$

where $a_i$ is the peak acceleration due to the $i$th harmonic of the loading function, $\alpha_i$ the dynamic load factor for the $i$th harmonic, $w_p$ the equivalent uniformly distributed load (UDL) for the participants, $w_t$ the total floor weight (excluding jumpers), $f_0$ the natural frequency of the floor, $f$ the jumping frequency, and $\beta$ the damping ratio (0.04 for this floor). Jumpers are not included in $w_t$ because they act as an external repeated force.

Equation [1] can be solved for $\alpha_i$ after measuring or estimating all other parameters. The dynamic load factor for the first harmonic, $\alpha_1$, however, can be obtained more reliably from other test data (Pernica 1990; Allen *et al.* 1985; Rainer and Swallow 1986) than the equivalent UDL for participants, $w_p$, can be estimated for this set of tests. Equation [1] was therefore used first to calculate $w_p$ from the response to the first harmonic by assuming $\alpha_1 = 1.5$ for high-impact aerobics and 1.2 for low-impact aerobics (Pernica 1990), and then to compute the dynamic load factors for the second and third harmonics. This approach was confirmed by also estimating $w_p$ directly from the number of participants.

The heel impact responses in gym 1 show the presence of one strong fundamental mode and a few weaker ones. Estimation of response at any harmonic when its forcing frequency ($if$) is 7 Hz or less is therefore based on a SDOF system corresponding to the fundamental mode. When the frequency of the third harmonic is greater than 7 Hz, however, the third haramonic can be in resonance with the second or third modes

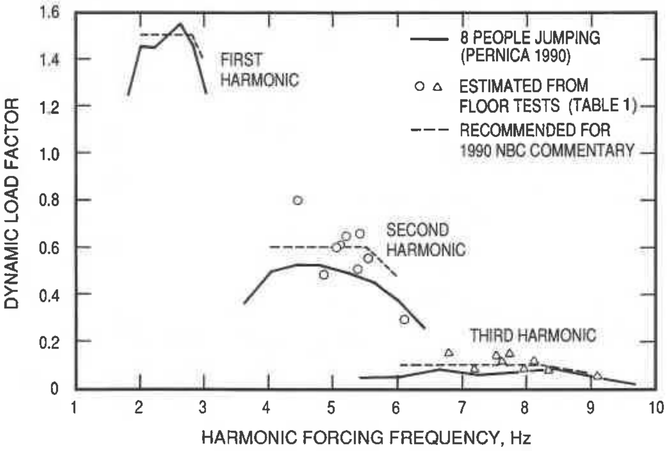


FIG. 6. Dynamic load factors for aerobics (high impact).

of the floor. The contribution of loading to each vibration mode is determined by the product of the load configuration and mode shape (Clough and Penzien 1975). Based on a comparison of the load configuration in Fig. 1 and the mode shapes for gym 1 in Fig. 4, it is estimated that 50% of the third harmonic loading excites the first mode, 35% the second mode, and 15% the third mode. Variations in this assumption do not affect the results appreciably.

Estimated dynamic load factors based on these assumptions are given in Table 1 for 11 aerobics and controlled jumping tests. The estimated dynamic load factors are compared in Fig. 6 with direct force measurements (Pernica 1990). On the basis of Fig. 6, expected dynamic load factors for high-impact aerobics are 1.5 for the first harmonic, 0.6 for the second harmonic, and 0.1 for the third harmonic. Expected dynamic load factors for low-impact aerobics from Table 1 are approximately 1.2 for the first harmonic, 0.3 for the second harmonic, and 0.05 for the third harmonic. Expected rather than extreme values are appropriate for the serviceability limit states.

Jumping frequency for the aerobics exercises carried out at the health club varied progressively from 2.25 to 2.6 Hz for low-impact and 2.5 to 2.75 Hz for high-impact aerobics. Controlled tests beyond 2.75 Hz were carried out (up to 3.1 Hz), but it became difficult to maintain coordination for a large group of people; consequently, the dynamic load factors for all harmonics decreased substantially as shown in Fig. 6. A maximum jumping frequency associated with the above dynamic load factors of 2.75 Hz is therefore recommended for design.

The weight of participants per square metre of floor area, $w_p$, is assumed to be 0.4 kPa in the 1985 NBC Commentary (NBC 1985). This corresponds to a fairly dense distribution of people (approximately 1.5 m$^2$ per person) which makes it difficult to exercise aerobics motions without hitting someone else or a wall. A more reasonable assumption of 0.2 kPa is therefore indicated.

*Calculation of natural frequency*

Resonance is the most important factor affecting aerobics vibration, hence natural frequency is the most important structural design parameter. The problem is to get the natural frequency away from the three harmonics. The designer of the health club floor tried to do this but, even after the girder supports were shimmed, the natural frequency turned out to be substantially less than predicted.

Calculations indicate that the natural frequency of the joists due to bending is 9.1 Hz assuming, as recommended for long-span joist floors (Canadian Standards Association 1984), composite action (for noncomposite design) and one-way simply supported behaviour. Figure 4*a* shows that, after shimming, the displacement of girder supports is 11% of the total maximum floor displacement or 12% of the maximum joist displacement. The flexibility of the floor system is therefore 1.12 times that of the joists alone and, since frequency is proportional to $\sqrt{\text{stiffness/mass}}$, this would indicate that the frequency of the floor system is $9.1/\sqrt{1.12} = 8.6$ Hz. The measured floor frequency, however, was 6.7 Hz.

The reason for this difference can be found in shear deformation of the joists. To ensure that the natural frequency was greater than 9 Hz, the joists were made very deep ($L/d$ = 11) and the bottom chord was stiffened by welding on an additional bar along the bottom flange (Fig. 1). Calculations show that the deflection due to deformation (elongation or shortening) in the web members alone is two-thirds of the bending deflection. The resulting natural frequency of the joists is therefore $9.1/\sqrt{1.67} = 7.04$ Hz. The flexibility of the supports further reduces the frequency to $7.04/\sqrt{1.12} = 6.65$ Hz, nearly equal to that measured.

The natural frequency of the floor system had been calculated using the moment of inertia, $I$, assuming composite action. Shear was accounted for by assuming that it corresponds to a loss in moment of inertia (Chien and Ritchie 1984), taken as 15% for noncomposite action or 6% for composite action. This corresponds to a frequency reduction of only 3% compared to nearly 30% determined above.

This shows that the procedure of determining joist frequency on the basis of moment of inertia can lead to substantial error, particularly for deep joists or trusses necessary for this type of application. It is therefore recommended that the following procedure, based on deflection rather than moment of inertia, be used for determining the natural frequency of a joist floor structure (Clough and Penzien 1975):

$$[2a] \quad f_0 = \frac{1}{2\pi}\sqrt{\frac{\text{stiffness}}{\text{mass}}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}}$$

where $\Delta$ is the mid-span deflection of an equivalent SDOF spring–mass system due to its mass weight. For a joist floor structure, this can be approximated by

$$[2b] \quad \Delta = \frac{(\Delta_B + \Delta_G)}{1.3} + \Delta_S$$

where $\Delta_B$ is the deflection of the joist due to bending (chord strain) and shear (web strain), $\Delta_G$ is the deflection of the girder due to bending and shear, and $\Delta_S$ is the shortening of the column or wall support (axial strain); and where all member deflections result from the total mass weight supported by the member (excluding the jumpers). Both supports are considered and the most flexible one is used in the calculation. The factor 1.3 in [2*b*], as in [1], is determined by dynamic principles, to be described later. Generally, $\Delta_S$ is small except for multistorey buildings (Allen 1990).

For joist floors on rigid supports ($\Delta_G$ and $\Delta_S$ equal to zero), [2] assumes one-way action. In fact, as shown in Fig. 4, the action is two-way. For strongly orthotropic simply supported plates such as joist floors, the lower natural frequencies are determined from (Ohlsson 1988)

$$[3] \quad f_{0n} = f_0\sqrt{1 + 2\left(\frac{nL}{B}\right)^2\frac{D_{xy}}{D_x} + \left(\frac{nL}{B}\right)^4\frac{D_y}{D_x}}$$

where $f_0$ is the one-way fundamental frequency in the joist direction, $L$ the joist span, $B$ the floor width perpendicular to joist span, $n$ the mode, $D_x$ the unit stiffness in the joist direction, $D_y$ the unit stiffness in the deck direction, and $D_{xy}$ the unit torsional stiffness in the deck. Only modes close to the fundamental ($n = 1$) will be excited by the aerobics loading configuration. Also the ratio of $D_y$ or $D_{xy}$ to $D_x$ is of order 0.01 or less (approximately 0.002 for this floor). According to [3], the fundamental frequency, $f_{01}$, is very close to the one-way frequency except for floors of narrow width. A higher mode (gym 1) or a narrow floor width (gym 2) results in an increased frequency according to [3]. Except for narrow floor widths, natural frequency should therefore be estimated by [2], assuming one-way action.

As a check on [2], the following mass weight deflections were estimated for the health club floor: $\Delta_B = 3.81$ (bending) + 2.54 (shear) = 6.35 mm, $\Delta_S = 0.3$ mm (axial strain). The deflection of the shimmed girder, $\Delta_G$, cannot be calculated, but, from the modal configuration, is estimated to be 0.5 mm. The resulting natural frequency of 6.7 Hz from [2] is equal to the measured frequency. With the shims removed, the estimated girder deflection is 4.4 mm for bending only and 4.7 mm including shear. Without the shims, the column carries a greater load over its height and its axial shortening is estimated to be 0.8 mm. The resulting natural frequency of 5.2 Hz calculated from [2] is close to the measured frequency of 5 Hz.

Natural frequency can also be obtained by finite element analysis, provided all deformations, including shear and column deformations as well as flexural deformations, are taken into account.

### Repair alternatives for health club

After some efforts at increasing the natural frequency and reducing resonance vibration with tuned dampers, the problem with the health club floor still remained.

There are a number of reasons why tuned dampers did not work well for this problem. First, it is necessary to tune the dampers to all significant modes that may be in resonance with a harmonic of the jumping frequency, in this case the 6.7 and 7.7 Hz modes. Second, the jumping frequency varies and therefore perfect resonance rarely occurs. Application of [1] with damping ratio, $\beta$, in the range 0.02–0.05 shows that damping is effective in reducing vibration only for forcing frequencies within approximately 5% of the natural frequency. For example, second harmonic vibration in Fig. 5*e* (6.1 Hz), although considerably amplified, is not significantly reduced by dampers tuned to 6.7 Hz. Finally, because damping ratios for the partitioned floor are already fairly high (0.04), the tuned dampers must incorporate considerable mass and damping to make a significant difference to the floor response. Tuned dampers are more effective when the damping ratio of the floor structure is small.

Altering stiffness, usually by increasing it, is another remedial measure. It some cases this is relatively easy, e.g., providing a column at mid-span. In other cases, such as an aerobics floor over a swimming pool, this is difficult. A 10 Hz floor is needed to get satisfactory performance for the health club. A 10 Hz frequency corresponds to a total mid-span dead load deflection, $\Delta$ in [2], of 2.5 mm compared with 5.7 mm in the present floor; to achieve this with the existing support conditions would require a joist deflection of 2.3 mm. Various schemes can be used to achieve this stiffness, such as stiffening the floor by post-tensioned cables and struts (inverted hangar system), but the cost is prohibitive. A less-expensive method is to incorporate very stiff intersecting structural partitions anchored into the floor and roof structure, but this would require a fixed layout and is still expensive.

TABLE 2. Recommended acceleration limits for vibrations due to rhythmic activities

| Occupancies affected by the vibration | Acceleration limit (% $g$) |
|---|---|
| Office and residential | 0.4–0.7 |
| Dining and weightlifting | 1.5–2.5 |
| Rhythmic activity only | 4–7 |

Change of the activity to allow only low-impact aerobics would reduce the vibration by approximately half, but this restricts the activity. Another possibility to reduce the higher harmonics would be to round out the sudden force change as the feet touch and leave the floor by means of a soft cushion below the floor deck. Aerobics floors in fact do incorporate a cushion below the wood deck to prevent injury to the legs. Significant reduction of the second and third harmonics, however, would require a much softer cushion than is presently used; as a result the floor would respond somewhat like a trampoline.

Relocation is generally the best course of action. Vibration discomfort of the weight lifters in Fig. 2 would be reduced by approximately 90% if gym 2 were moved alongside gym 1 at the end of the building and combined into one gym, the weight lifting taking place at the other end. This is because the two ends of the floor are nearly independent in dynamic response. The partitions are an important factor in this and therefore their relocation would have to be done with care.

### Recommended changes to NBC commentary A

In addition to the above recommendations concerning the aerobics loading function and the calculation of natural frequency, the following changes are recommended for the NBC Commentary.

*Design criterion*

Design calculations can be based with reasonable accuracy on a periodic loading function containing harmonic loading components and an SDOF steady-state response. Each sinusoidal harmonic loading component produces a steady-state sinusoidal vibration, whose maximum acceleration, $a_i$, is given by [1]. The three sinusoidal vibrations occur simultaneously, and add together in phase when the periodic jumping forces are at a maximum. The effective maximum acceleration, $a_m$, for human reaction to this motion can be approximated by (see Apendix 2)

$$[4] \quad a_m = (a_1^{1.5} + a_2^{1.5} + a_3^{1.5})^{\frac{1}{1.5}}$$

The 1985 NBC Commentary recommends a criterion of 5% $g$ for jumping exercises and 2% $g$ for dining and dancing, the latter being a mixed occupancy more sensitive to vibration. The health club floor supported a mixed occupancy of weight lifting and aerobics; a criterion of 2% $g$ is therefore recommended for such occupancies. Office occupancies are very sensitive to vibration, with a recommended criterion of 0.5% $g$

(Canadian Standards Association 1984). Because human reaction depends on other factors besides occupancy (such as remoteness of vibration source), a range for each occupancy is recommended rather than a single value. These ranges are given in Table 2.

A useful design formula for minimum satisfactory natural frequency under sinusoidal loading is obtained from (Allen *et al.* 1985)

$$[5] \quad f_0 \geq f\sqrt{1 + \frac{1.3}{a_0/g}\frac{\alpha w_p}{w_t}}$$

where $f$ is the forcing frequency, $\alpha$ is the dynamic load factor, and $a_0/g$ is the acceleration limit. Aerobics, however, involves three harmonics. If the three harmonics occurred sequentially, then the minimum natural frequency for satisfactory performance could be determined by applying [5] to all harmonic loadings, $\alpha_i w_p$, at forcing frequencies, $if$, and choosing the maximum. All three harmonics, however, occur simultaneously in accordance with [4]. To account for this, the factor 1.3 in [5] should be increased by approximately 50% to account for the fact that the effective peak acceleration according to [4] is approximately 50% greater than the peak acceleration of the critical harmonic. The following design formula is therefore recommended for aerobics:

$$[6] \quad f_0 \geq if\sqrt{1 + \frac{2}{a_0/g}\frac{\alpha_i w_p}{w_t}}$$

Effective maximum acceleration for any assumed natural frequency can, however, be more accurately determined from [1] and [4]. This is useful when damping times mass is great enough to reduce third harmonic resonance to an acceptable level.

Application of [6] to the health club floor using an acceleration criterion of 2% $g$, with $w_p$ = 0.12 kPa (0.2 kPa over the gym area) and $w_t$ = 3.5 kPa, gives a minimum natural frequency of 9.6 Hz for both the second and the third harmonics. Application of [1] for a 10 Hz floor, with jumping at 2.75 Hz, results in harmonic peak accelerations of 0.55, 1.16, and 0.93% $g$ respectively, and the effective maximum acceleration from [4] is 1.87% $g$.

Application of [1] for a 6.7 Hz floor with jumping at 2.23 Hz, the critical jumping frequency for maximum response, results in harmonic peak accelerations of 0.84, 2.13, and 5.57% $g$ respectively, and the effective maximum acceleration from [4] is 6.62% $g$. The dynamic displacements corresponding to this motion are small, 1 mm approximately, as is the dynamic stress range, 10 MPa approximately.

*Determination of distributed load and floor weight*

Equations [1] and [6] apply to a simply supported beam with uniform load, $w_p$, and mass weight, $w_t$. For nonstandard cases, such as dynamic load applied to only part of the span or an extra mass weight supported by the beam, the values $w_p$ and $w_t$ are adjusted using the following principles (Clough and Penzien 1975):

- equivalent loading varies as the modal displacement ratio,
- equivalent mass varies as the square of the modal displacement ratio, where the modal displacement ratio is the ratio of modal displacement at any location to maximum modal displacement (usually at mid-span).

A vibrating floor structure can be represented as a simple spring–mass system with dynamic load and mass weight concentrated at the location of maximum floor displacement. For a simply supported beam under distributed dynamic load, $\alpha w_p s$ ($s$ is the beam spacing), vibrating in a sine wave configuration, the equivalent dynamic force concentrated at mid-span is equal to

$$\alpha w_p s \int_0^L \sin\frac{\pi x}{L}\,dx = 2\alpha w_p sL/\pi$$

and the equivalent mass weight concentrated at mid-span is equal to

$$w_t s \int_0^L \left(\sin\frac{\pi x}{L}\right)^2 dx = w_t sL/2$$

The ratio of these two quantities is approximately $1.3\alpha w_p/w_t$, which accounts for the factor 1.3 in [1] and [2*b*].

In the case where dynamic load is applied to part of the span, the equivalent value of $w_p$ is decreased according to the integral of the loaded area under the sine wave. In the case where a beam (or a column support) carries an extra mass weight, $W_c$, the equivalent mass weight concentrated at mid-span is equal to $W_c y^2$, where $y$ is the modal displacement ratio at the location supporting the extra mass weight. This concentrated weight can be expressed as an increase in distributed floor weight by equating $W_c y^2$ to $wsL/2$, where $w$ is the equivalent floor weight. The floor weight, $w_t$, in [1] and [6] is therefore increased by $2W_c y^2/sL$.

## Conclusions

On the basis of the results derived from this investigation and from the data of the references cited, the following recommendations are made for the design of floors for vibrations due to aerobics:

*Acceleration criterion*. An acceleration criterion of 2% $g$ is recommended for a mixed occupancy of weight lifting and aerobics, similar to dining and dancing.

*Loading function*. The loading function for jumping exercises can be represented by a periodic function containing three sinusoidal harmonic components, $\sum_{i=1}^{3} \alpha_i w_p \sin 2\pi ift$, where (a) dynamic load factors, $\alpha_1 = 1.5$, $\alpha_2 = 0.6$, and $\alpha_3 = 0.1$; (b) maximum jumping frequency, $f$ = 2.75 Hz; and (c) maximum weight of participants over loaded area, $w_p$ = 0.2 kPa.

*Response*. The response of floor structures to aerobics can, for design purposes, be represented for each harmonic as a SDOF steady-state vibration, [1]. Equation [4], based on human reaction, is recommended for determining maximum effective acceleration from harmonic peak accelerations. Equation [6] is recommended as an approximate design criterion, but may be conservative when third harmonic response is within acceptable limits. A procedure is given in this paper for estimating distributed loading, $w_p$, and floor weight, $w_t$ for use in [1] and [6] for nonstandard cases.

*Estimation of natural frequency.* The natural frequency of long-span joist or truss floors can be substantially reduced by shear deformation in web members as well as flexibility of supports. Equation [2] provides a means for estimating natural frequency of such floors, taking all flexibilities into account.

*Repair alternatives*. Experience with this health club and others indicates that it is sometimes very difficult to repair an existing floor to alleviate vibration problems. Tuned dampers are often not effective, and increasing the stiffness to avoid resonance can be expensive. Usually the best alternative is to

TABLE A1. Acceleration ratios for steady-state jumping vibration

| Peak ($\Sigma a_i = 1.0$) | | | | Effective maximum, $a_m$ | |
|---|---|---|---|---|---|
| First harmonic $a_1$ | Second harmonic $a_2$ | Third harmonic $a_3$ | $a_{rmq}$ ([A3]) | $a_{rmq}/0.78254$ | Eq. [A5] |
| 0.1 | 0.3 | 0.6 | 0.583 | 0.745 | 0.759 |
| 0.1 | 0.4 | 0.5 | 0.578 | 0.738 | 0.741 |
| 0.1 | 0.5 | 0.4 | 0.582 | 0.743 | 0.741 |
| 0.1 | 0.8 | 0.1 | 0.658 | 0.841 | 0.846 |
| 0.8 | 0.2 | 0.0 | 0.662 | 0.847 | 0.865 |
| 1.0 | 0.0 | 0.0 | 0.78254 | 1.000 | 1.000 |

relocate. In doing so, one can sometimes take advantage of partition layout in isolating one end of the floor from vibrations generated at the other end.

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## Appendix 1. List of symbols

$a_i$ peak acceleration due to $i$th loading harmonic
$f$ jumping frequency
$f_0$ natural frequency of the floor structure
$g$ acceleration due to gravity
$i$ harmonic of jumping frequency ($i$ = 1, 2, 3)
$L$ span of floor joists or beams
$w_p$ equivalent uniformly distributed load of participants
$w_t$ equivalent uniformly distributed floor weight
$\alpha_i$ dynamic load factor for $i$th harmonic of the loading function
$\beta$ critical damping ratio
$\Delta$ deflection of an SDOF oscillator due to its mass weight
$\Delta_B$ deflection of joist due to the mass weight supported
$\Delta_G$ deflection of girder support due to the mass weight supported
$\Delta_S$ shortening of vertical support due to the mass weight supported

## Appendix 2. Effective maximum acceleration for steady-state jumping vibration

For any given steady-state vibration acceleration, $a(t)$, ISO (1989) recommends that human reaction be based on the following acceleration parameter (called the r.m.q. acceleration):

$$\text{[A1]} \quad a_{rmq} = \left[\frac{1}{T}\int_0^T a^4(t)\,dt\right]^{1/4}$$

where $T$ is the duration.

The steady-state response incorporating the first three harmonics of the jumping frequency, $f$, is given by

$$\text{[A2]} \quad a(t) = a_1 \cos 2\pi ft + a_2 \cos 4\pi ft + a_3 \cos 6\pi ft$$

where $a_i$ is the peak acceleration for the $i$th harmonic.

The three sinusoidal vibration components of [A2] add together in phase when the periodic jumping forces are at a maximum, i.e., at $t$ = 0, $1/f$, $2/f$, etc.

The acceleration parameter, $a_{rmq}$, is determined by substituting [A2] into [A1] and making the time window $T$ equal to the period $1/f$. The result of the integration is given by

$$\text{[A3]} \quad a_{rmq}^4 = \frac{3}{8}(a_1^4 + a_2^4 + a_3^4) + \frac{3}{2}(a_1^2a_2^2 + a_1^2a_3^2 + a_2^2a_3^2 + a_1a_2^2a_3) + \frac{1}{2}a_1^3a_3$$

Table A1 contains values of $a_{rmq}$ for some practical ratios of $a_i$ to their sum. The acceleration criterion contained in 1985 NBC Commentary is, however, related to the peak acceleration for steady-state sinusoidal vibration. It is simpler to retain this definition for practice and to make an adjustment for nonsinusoidal vibration. This is done in Table A1 by dividing $a_{rmq}$ by its value for simple sinusoidal vibration, 0.78254, and defining the ratio as the effective maximum acceleration, $a_m$.

A close approximation to [A3] for determining $a_m$ is an equation of the form:

$$\text{[A4]} \quad a_m = \sqrt[k]{\sum_{i=1}^{3} a_i^k}$$

By trial and error, a close correspondence to [A3] was obtained by setting $k$ = 1.5. Comparative values are given in the last two columns of Table A1. A vibration criterion for human reaction for three harmonic components is therefore given by

$$\text{[A5]} \quad a_m = (a_1^{1.5} + a_2^{1.5} + a_3^{1.5})^{\frac{1}{1.5}}$$

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