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Splitting Tensile Force in Rectangular Plates Fixed Along One Side and Loaded at the Opposite Side

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PREFACE

The Division of Building Research has been interested in precast concrete construction in general for a number of years and more recently has been involved in an active research programme on the connections between precast structural members. One of the more fundamental problems in this area is the bearing capacity of concrete plates under a concentrated load, as for instance in the head of a column carrying one or more beams.

The results of the Swedish studies presented in this paper (one of a series by the same authors) contribute greatly to an understanding of the stress distribution and of the mechanism involved in the failure of concrete subjected to such loads. This translation is therefore provided to make this information more readily available to designers and others involved in the field of precast concrete.

The Division is indebted to one of its own members, W.R. Schriever, Head of the Building Structures Section for this translation.

Ottawa
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R.F. Legget
Director

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Authors: R. Hiltcher and G. Florin

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Research Council

THE SPLITTING TENSILE FORCE IN RECTANGULAR PLATES
FIXED ALONG ONE SIDE AND LOADED AT THE OPPOSITE SIDE

Abstract

The problem of the distribution of splitting tensile stresses in a plate under load, a problem which so far has been solved rigorously only for a load on an infinitely long strip, is studied by photo-elastic model methods for a plate of finite dimension which is fixed along one side and loaded at the other side by a relatively concentrated symmetrical load (as for instance a concrete wall on rock or a foundation with the load of a beam or heavy machines, on top, etc.).

The basis for an accurate design of the reinforcing against the splitting forces resulting from such a load is given in the form of a diagram for the resultant splitting tensile force and its location as a function of the plate height and the load concentration relative to the plate width.

1. Introduction

It has been known for a long time that under a load which acts on wall-like plate, tensile stresses are produced which lead to splitting cracks in the plate (Fig. 1). Morsch and Bleich derived the first calculation methods for this case. It is only with the advent of prestressed concrete, in which problems of reinforcing such danger zones under the anchor blocks arose, that the solution of this question became urgent. Approximate solutions were produced in rapid succession by Guyon⁽¹⁾, Sievers, Magnel and Ban. Iyengar⁽²⁾ finally derived an exact solution for the infinitely long half-strip loaded on its narrow side. If the type of problem arises in which the splitting force in a concrete wall under the influence of a heavy load concentrated on a small distance must be calculated, then the solution for the infinitely long half-strip is not sufficient. A wall usually is of rather limited height and is, in general, fixed on the opposite side in the foundation or on rock. This fixation usually results in a considerable reduction of the splitting effect. Since the problem just described does not lend itself easily to a rigorous solution, photostress model measurements were resorted to at an early stage.⁽³⁻⁵⁾ The development in that last few years of photostress methods using a lateral extensometer and polariscope led to a procedure of high accuracy⁽⁶⁾ which now offers, when used with a sufficient number of models for the tests, the basis for a general and exact experimental solution of the problem.

2. Test Planning

Figure 1 shows a basic sketch of the model investigated with the terms used. The elastic plate is fixed along its bottom side into an elastic semi-

infinite body of the same modulus as the plate, corresponding to the fixation of a concrete wall in rock or in a foundation block. In the model this fixation was achieved by gluing the 10 mm thick model plate made of Araldit in a sufficiently large block of the same material. The width of the load was the same for all tests, that is, $a = 12$ mm. This load width, a , represents the basic measure for the presentation of the stress distribution under the load. The lateral stress σ_x (Fig. 1) which is of interest here, is a compressive stress directly under the load, but changes, at a distance roughly equal to the width of the load, to a tensile stress as a result of the spreading of the concentrated load over the width of the plate. This tensile stress reaches a maximum and finally diminishes to 0 farther from the load, at a distance at which the stress distribution has reached its final form (parallel stress lines). The splitting effect which results from this stress distribution can be characterized by the resulting splitting tensile force H which has a distance from the loaded edge of y_H .

The tensile stresses σ_x depend mainly on two parameters:

(a) The relative load width a/b , or the inverse value b/a , which is called the "load concentration factor". For the load distributed uniformly over the plate width, i.e. for a load concentration factor $b/a = 1$, no tensile stress occurred under the load. For the other extreme case, however, $b/a = \infty$, the splitting effect reaches a maximum asymptotically.

(b) The relative plate height h/b . This determines the reduction of the splitting effect for smaller plate heights due to the effect of fixation along the lower edge, and varies between $h/b = 0$ and $h/b = \infty$. The latter value is the limiting case for the infinitely high half-strip according to Iyengar.

In order to produce a full solution to the problem the parameters b/a and h/b must be varied in the model tests between the limits indicated above, with the "infinite" values being reproduced by sufficiently large values, because of the asymptotic condition of the splitting effect. The test series were arranged in such a way that first the basic models with relative heights of $h/a = 5$, $h/a = 10$, and $h/a = 23$ were made. The load concentration factor b/a was produced by stepwise reduction of the plate width to values $b/a = 29$, 16, 12, 8 and 4. An additional test with a still lower model with a relative plate height of $h/b = 0.2$ was also made, so that the full investigation covered 16 different models.

In practice, accurate stress distribution under the load is usually not well known. In planning tests it is therefore necessary to decide with which load distribution the investigation shall be made. In Fig. 2 some typical cases are shown. A series of preliminary tests showed that the resulting splitting tensile force in the case of a rather rigid loading body (Fig. 2a)

is approximately 10 percent smaller, and in the case of a cylindrical loading body (Fig. 2c), resulting in an elliptical stress distribution because of a higher load concentration, approximately 10 percent greater than for the relatively rare case of a uniformly distributed load (Fig. 2b). The fact that the differences are so small can be explained by stating that the characteristic pressure distribution under the load spreads out rapidly according to the principle of St. Venant. Since tensile stresses occur only at a certain distance from the edge of the plate (Fig. 1), the differences in the load distribution at the surface is of small significance for them. The model tests were therefore carried out only for a uniformly distributed load. A corresponding correction can be made for possible other known load distributions.

The fact that splitting tensile stresses occur only at a certain distance below the loaded edge is very favourable for measurement with a lateral extensometer. Since the lateral strain in the immediate vicinity of the applied load is restrained, a three-dimensional state of stress occurs there which makes measurements with a lateral extensometer questionable. Measurements concerning this⁽⁶⁾ have, however, shown that even under a loading body which is glued on (in the present case the load was only set on), the effect on the lateral extensometer measurement at a distance of $2 - 3a$ from the upper edge was only 1 - 2 percent.

Concerning the application of the model results to the full size prototype, it must be stated first that the stress distribution in the present two dimensional case is independent of the lateral contraction coefficient and naturally also of the elastic modulus of the material. The results of the model tests, presented in dimensionless form, are therefore directly applicable to any other elastic material. In order to preserve geometric similarity between the model (M) and the prototype (H) in the loaded condition, the strains must be the same in both cases:

$$\epsilon_M = \epsilon_H \quad \text{or} \quad \sigma_M/E_M = \sigma_H/E_H.$$

The same relationship is also valid for the mean load intensity $Q = P/a \cdot t$ (where t is the thickness of the plate) which also has the unit of a stress,

$$q_M/E_M = q_H/E_H,$$

whereby q/E is called the relative load intensity. For concrete with $q = \sigma_b = 100 \text{ kg/sq cm}$ and $E_b = 300,000 \text{ kg/sq cm}$ we obtain $q/E = 0.33 \cdot 10^{-3}$, whereas one would prefer for photostress methods to have a load intensity as high as possible in order to produced stresses and strains which are easy to measure. In order to determine to what extent one may deviate in the present case from the strict similarity without getting into significant errors, a

study was made with four different loads on the same model. The results are shown on Fig. 3. It can be seen from this that a relative load intensity of $2 \cdot 10^{-3}$, i.e. a six-fold increase is still just acceptable. Figure 3 incidentally gives an idea of the reliability of the photostress measurements by means of the polariscope and lateral extensometer.

3. Tests and Evaluation

In the main tests the stresses σ_x along the axis of symmetry and, to check the equilibrium conditions, the stresses σ_y in any chosen horizontal section were determined for 16 model variations as indicated above by measuring the principal stress sums and differences⁽⁶⁻⁸⁾. Figures 4-6 show, for fifteen of the investigated cases, the distribution of the splitting tensile stress σ_x in dimensionless form with the reference stress $q = P/at$. From these curves the resulting splitting tensile force H/P and its relative distance from the loaded edge y_H/a was determined by table integration. These two values, which in each case essentially describe the splitting effect, are plotted in Fig. 7 as functions of the load concentration factor b/a and the relative plate height h/b .

4. Results

From the two diagrams in Fig. 7 it follows that:

1. For the load uniformly distributed over the length of the plate, with a load concentration factor of $b/a = 1$, the splitting tensile force is 0.
2. With increasing load concentration b/a the splitting tensile force increases and reaches a practical limiting value for $b/a > 30$. From the tensile stress distribution obtained by Iyengar⁽²⁾ for the limiting case of $b/a = \infty$ and $h/b = \infty$ it can be calculated in an approximate way that for $b/a = 30$ approximately 90% of the limiting value has been reached, and that $H/P = 0.3$ (approximately) represents an upper limiting value of the splitting tensile force.
3. Fixation along the lower plate edge results in a reduction of the splitting tensile force. With relative plate height increasing from $h/b = 0$ to $h/b = \infty$ (case according to Iyengar) the splitting tensile force increases from 0 to a maximum value. The maximum is already practically reached for plates with a relative height of $h/b = 2$ to 3. For plates with a greater height than $h/b = 2$ (approximately) the influence of the fixation along the lower edge is no longer of any practical significance.
4. The resultant of the splitting tensile force occurs in general at a distance of $2-4a$ below the loaded edge. With increasing load concentration

b/a and with increasing relative slab height h/b the depth of the resultant increases also.

5. Considerations for the Design of the Reinforcing

The resultant splitting tensile force H/P and its location below the loaded edge y_H/a can be read from the diagrams in Fig. 7 for each case. A steel reinforcement area calculated in this way, and related to the line of force given in the diagram would balance the splitting tensile force. This, however, does not yet guarantee that no tensile stresses will occur and that there is absolute assurance against cracking. If all tensile stresses are to be avoided, then the reinforcement has to be distributed according to the accurate stress distribution in the diagrams of Fig. 4 - 6, as long as this appears at all worthwhile considering the small tensile stresses.

It should be noted here that it is not necessary to carry out model tests with the reinforcement according to the method given in reference 9. In the present case the reinforcement, designed properly for value and position, is positioned such that it will have its maximum efficiency. In contrast to this in reference 9 the problem of the tear tensile force of a loaded corner has been studied, whereby a saving up to 60% can be achieved by moving the reinforcement from the centre towards the edge of the plate. The solution of this statically indeterminate problem has so far only been achieved in model tests with reinforced models.

If one wants to apply a correction for the deviation of load distribution from the case of a uniformly distributed load, for which the tests were made, one should, as already mentioned in Section 2, first determine clearly what the actual load distribution is. The case of the rigid body and that of a uniformly distributed load will be rather rare in practice, as will the special case of a cylindrical body with an elliptical load pressure distribution (wheel on a rail). Load distributions in the form of a bell-shaped curve under only moderately thick plates, such as shown for instance in Fig. 2d, will, however, be quite frequent. The concentration of the load intensity towards the line of symmetry is in this case even higher than for the case of the elliptical pressure distribution. Since, however, the accurate load distribution is known only very rarely, and since, furthermore, it is rarely possible to develop a splitting tensile stress diagram for each load distribution, the estimated load distribution is replaced by a corresponding elliptical distribution with the reduced width a' , approximately as shown in Fig. 2d (45° distribution), for which distribution a correction of +10% compared to the case of the uniformly distributed load is known. From the diagram for the splitting tensile force one can see furthermore whether or not the present case is sensitive to a not quite accurate determination of the load width a' .

On the basis of similar considerations it should be possible, by means of the splitting tensile diagram for a plate, to obtain at least an estimate of the splitting tensile effect for three-dimensional problems such as for the case of a real load on a crane runway on a concrete wall.

In conclusion the authors wish to thank Tor Hafstad for the conscientious execution of the tests.

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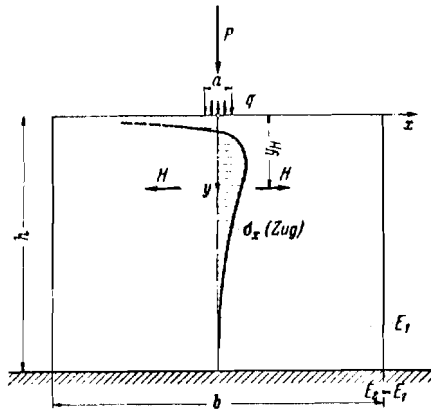


Fig. 1

Diagrammatic sketch and nomenclature

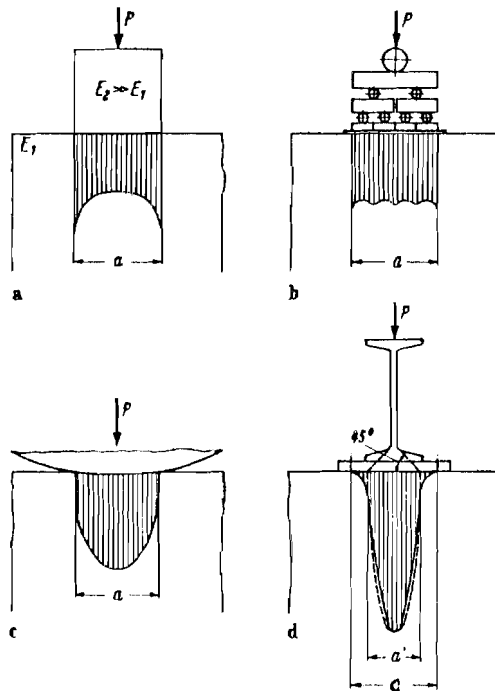


Fig. 2

Different load distributions

(a) Pressure of a rigid body on a soft semi-infinite body; (b) Arrangement for the production of a nearly truly uniformly distributed load; (c) Elliptical load distribution under a cylindrical body; (d) Load distribution under a base plate of a structural section. This distribution can be replaced in practice by the elliptical distribution indicated with a reduced load width a'

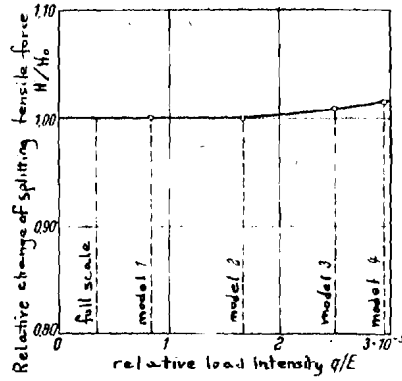


Fig. 3

Dependence of the splitting tensile force on the relative load intensity in the model test

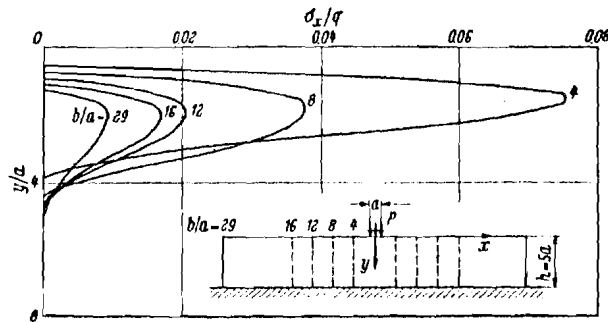


Fig. 4

Splitting tensile stresses σ_x/q as a function of the distance y/a from the loaded edge for various load concentrations b/a for a constant model height $h/b = 5$

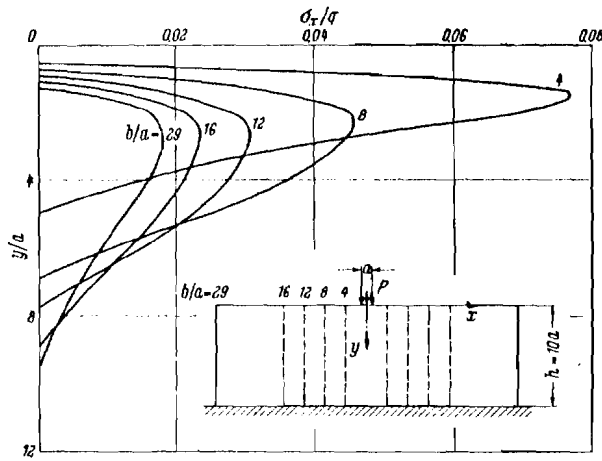


Fig. 5

Splitting tensile stresses σ_x/q as a function of the distance y/a from the loaded edge for various load concentrations b/a for a constant model height of $h/a = 10$

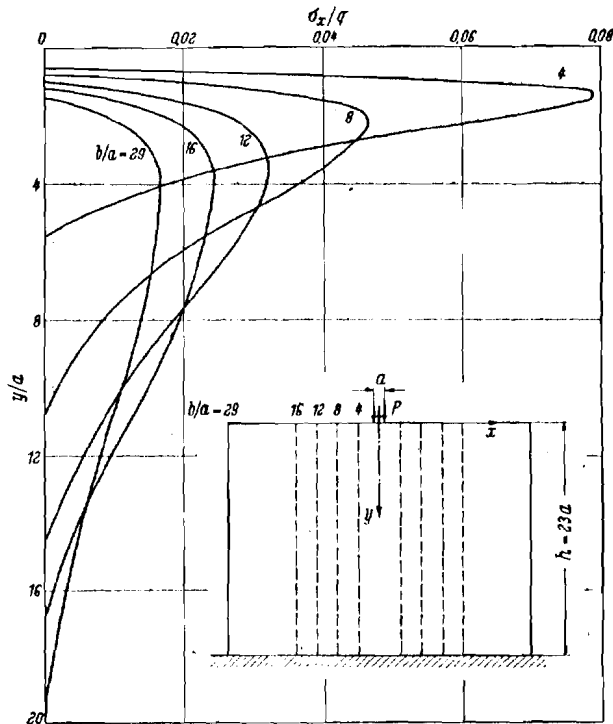


Fig. 6

Splitting tensile stresses σ_x/q as a function of the distance y/a from the loaded edge for various load concentrations b/a for a constant model height of $h/a = 23$

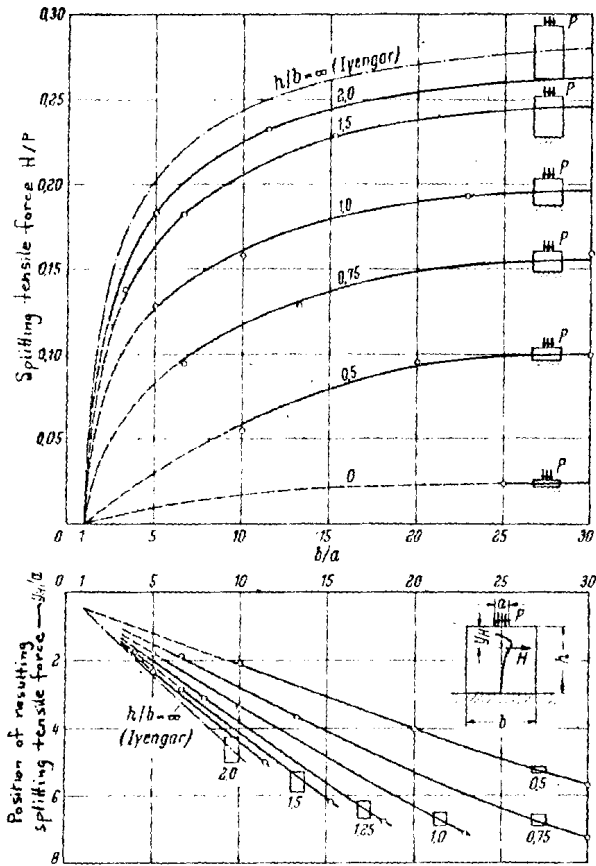


Fig. 7

The relative splitting tensile force H/P and its position y_H/a as functions of the load concentration b/a and the relative plate height h/b