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RELATIONSHIP BETWEEN FIRE RESISTANCE AND FIRE TOLERANCE

ANALYZED

by T. Z. Harmathy

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SOMMAIRE

Une grande partie de l'information recueillie au sujet de la résistance au feu des éléments séparateurs peut être utilisée dans la conception des bâtiments réalisée en fonction de la sécurité-incendie, si l'on convertit les valeurs de tolérance au feu en valeurs de résistance. Les méthodes de conversion suivantes sont étudiées de façon critique: la première est basée sur le concept des surfaces ayant les mêmes rapports tempérture-temps, la deuxième sur le concept des mêmes températures maximales et la troisième est basée sur la méthode de Law; des instruments de conversion pratiques sont présentés dans cet article. En ce qui concerne les éléments porteurs des bâtiments, il n'est pas recommandé de fonder la conception du bâtiment en fonction de la sécurité-incendie sur l'information disponible sur la résistance au feu.

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Relationship Between Fire Resistance and Fire Tolerance†

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Much of the information accumulated on the fire resistance of 'dividing' elements can be utilized in the design of buildings for fire safety, if the fire tolerance values are converted into fire resistances. Three methods of conversion, one based on the concept of equal temperature–time areas, the second on the concept of equal maximum temperatures, and the third, the Law method, are critically examined and handy conversion tools presented. In the case of the 'key' elements of buildings, basing the fire safety design on fire resistance information is not recommended.

INTRODUCTION

With the realization that fire resistant compartmentation may not be as effective a measure of protecting buildings against the spread of fire as previously thought, many fire scientists have become quite vocal about the need for a thorough revision of the existing fire protection practices. Yet progress toward new, knowledge-based fire protection practices seems to be very slow owing, possibly, to the fear that the multitude of data accumulated over the years on the fire resistance of building elements (as determined from standard fire tests) may be rendered useless by the new practices which, though logically coherent, will lack, for the time being, experimental foundations.

The purpose of this paper is to show that that fear is not justified; in fact, the bulk of the information available on the fire resistance of building elements may be well utilized in the newer concepts of providing fire safety in buildings.

FIRE SEVERITY PARAMETERS

The essence of these newer concepts is that the nature and extent of fire safety measures should be geared to the expected severity of potential fires.

It has long been usual to characterize the severity of compartment fires by their temperature histories for the postflashover period.[1–3] Unfortunately this method of characterization is not fully justifiable, because it suggests that compartments with well insulated boundaries are liable to suffer fires of increased severity.[4]

The author has suggested[4–7] that the severity of compartment fires be characterized by the following three parameters: 1. the duration of fully developed fire, τ; 2. the overall penetration flux, $\bar{q}$, i.e. the heat flux absorbed by the compartment boundaries, averaged spatially over the boundary surfaces and temporally over the period of full development; and 3. the average temperature of the compartment gases (average 'fire temperature'), $\bar{T}_g$, averaged spatially over the compartment volume and temporally over the period of full development.

These parameters can be evaluated from simple calculations.[4–7]

In examining the possible effect of fire on the various boundary elements of a compartment, the penetration fluxes for the individual boundaries are of interest. Noting that the over all penetration flux, $\bar{q}$, depends on the material property group $k/\kappa^{1/2}$, interpreted as

$$\frac{k}{\kappa^{1/2}}=\frac{1}{A_t}\sum_i A_i\frac{k_i}{\kappa_i^{1/2}} \tag{1}$$

where

$$A_t=\sum_i A_i \tag{2}$$

and that the rate of heat absorption by the various boundary surfaces is proportional to $k_i/\kappa_i^{1/2}$, the penetration flux for the individual boundaries can be expressed as

$$\bar{q}_i=\bar{q}\frac{k_i}{k}\sqrt{\frac{\kappa}{\kappa_i}} \tag{3}$$

In Eqns (1)–(3) the subscript i (=1, 2, 3, ...) refers to the various boundary elements and their lining materials.

After examining the import of the fire severity parameters, it can be seen that of the three two are sufficient for uniquely characterizing a fire: the duration of fully developed fire, τ, and the overall penetration flux, $\bar{q}$. In discussing the relation between the conventional and the newer concepts of providing fire safety, it may be necessary to select the average temperature of the compartment gases, $\bar{T}_g$, instead of $\bar{q}$, as the mate for τ. It must be emphasized, however, that the selection of $\bar{T}_g$ for characterizing the severity of a fire can be objected to on the same ground as the selection of the complete temperature history of the fire.

It is convenient to define the nominal duration of the entire postflashover period (period of full development plus decay period) by multiplying τ by an empirical factor,

$$\tau_{pf}=\lambda\tau \tag{4}$$

$\lambda \simeq 1.15$ is recommended for cellulosic materials and

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char-forming plastics. For non-charring plastics and liquid fuels $\lambda \simeq 1$.

In summary, the three possible ways of characterizing postflashover fires are illustrated in Fig. 1. Figure 1a shows the temperature-based characterization, either conventionally, by a fire-temperature versus time curve, or by the fire severity parameters $\bar{T}_g$ and τ. Figure 1b depicts the preferred method, characterization by $\bar{q}$ and τ, as well as by the individual penetration fluxes, $\bar{q}_1$, $\bar{q}_2$, $\bar{q}_3$, . . . applicable to the various elements of the compartment boundary. (It is seen that $\bar{q}$, $\bar{q}_1$, $\bar{q}_2$, $\bar{q}_3$, . . . represent time-averaged values of quantities that are subject to moderate variation.)

PENETRATION FLUX IN FIRE TESTS

If, as recommended, the penetration fluxes $\bar{q}_i(\bar{q}_1, \bar{q}_2, \bar{q}_3, \ldots)$ are selected (in addition to τ) for characterizing the severity of fire exposure of the various boundary surfaces of a compartment in real-world fires, the question necessarily arises, how these $\bar{q}_i$ compare with the (time-averaged) heat fluxes that penetrate these surfaces in standard fire resistance tests, $(\bar{q}_r)_i((\bar{q}_r)_1, (\bar{q}_r)_2, (\bar{q}_r)_3, \ldots)$. Some data reported by Seigel[8] and the author[9] showed that, as expected, the $(q_r)_i$ depend largely on the thermal properties of the construction material and on the duration of the test. Unfortunately they are known to depend also, in not readily quantifiable manners, on a number of other factors such as the moisture content of the test specimen, the type of furnace construction (especially the thermal properties of the furnace lining materials), and the combustion and radiation characteristics of the burning furnace fuel.

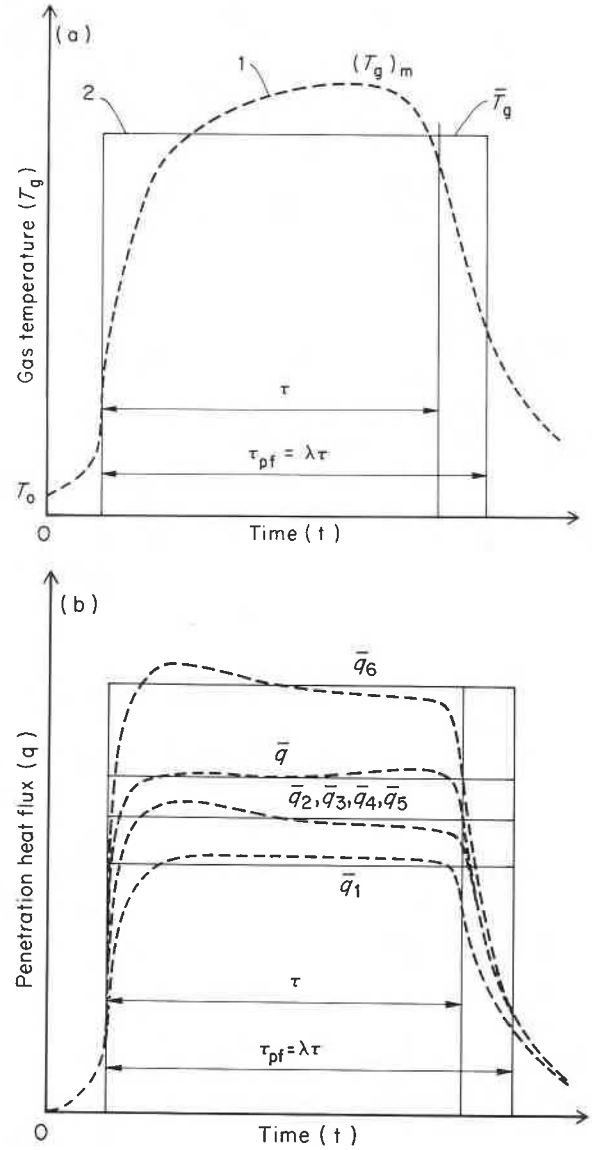

Figure 1. Means of characterizing fire: (a) temperature-based characterization: curve 1—by complete temperature versus time curve; curve 2—by parameters $\bar{T}_g$ and τ. (b) Characterization by parameters $\bar{q}$ and τ, and by the individual penetration fluxes $\bar{q}_1$, $\bar{q}_2$, $\bar{q}_3$, . . . (the dashed curves represent the assumed temporal variations of q, q_1, q_2, q_3, . . .).

There have been a few attempts to evaluate experimentally the rate of heat absorption by various constructions during standard fire resistance tests. Unfortunately, the experimental difficulties have proved substantial. It seems, therefore, easier and perhaps more accurate to estimate $(q_r)_i$ from theoretical studies, based on an acceptable model of heat transmission between the furnace and the test specimen.

In a multitude of computer simulations of fire resistance tests, the author[10] and Lie and Allen[11] modeled the 'standard' fire exposure as equivalent to the transmission of radiant heat to the specimen surface (whose emissivity was assumed to be about 0.9) from a black body whose temperature varied according to the temperature–time curve specified by ASTM Method E119. Although with this model the heat transfer by convection was neglected, it is believed that the error committed thereby was insignificant for the following reasons: (a) the convective contribution to the heat transfer is usually only a small fraction of the radiant heat transfer in most industrial furnaces;[12] (b) the neglect of convective heat transfer is compensated for by modeling the radiating medium (gases and furnace walls) as perfect radiator; and (c) if the coefficient of heat transfer is high, higher than, say, $80\ \mathrm{W\,m^{-2}\,K^{-1}}$, the rate of heat absorption by a solid is controlled largely by the group $k_i/\kappa_i^{1/2}$ and the magnitude of the coefficient of heat transfer is of secondary importance.[10] Since, in the case of well-designed test furnaces, the coefficient of heat transfer by radiation alone is sufficiently high, the contribution of convection to the rate of heat absorption by the test specimen is usually negligible. It is not surprising, therefore, that the described modeling of the furnace-to-specimen heat transmission in the computer studies always yielded results comparable with those derivable from standard fire tests.

Some results taken from those computer studies have been utilized in plotting in Fig. 2 the variation of the instantaneous penetration flux, $(q_r)_i$ (curves 1a to 4a) as well as the average (over the period 0 to t) penetration flux, $(\bar{q}_r)_i$ (curves 1b to 4b), during standard fire resistance tests, for four types of concretes; two normal weight (numbers 1 and 2) and two lightweight (numbers 3 and 4). The four concretes are identified by the values of $k_i/\kappa_i^{1/2}$ (applicable at room temperature) in the figure caption. These values reveal that concretes 1 and 2 can be regarded, as far as performance in fire is concerned, as limiting cases for normal weight concretes, and concretes 3 and 4 as limiting cases for lightweight concretes.[10] Consequently, for

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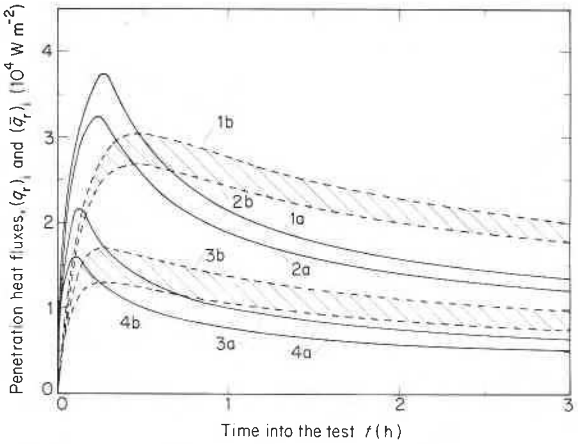

Figure 2. Penetration heat fluxes during standard fire tests: ——— instantaneous penetration fluxes; – – – – time-averaged penetration fluxes; curves 1a and 1b: normal weight concrete, $k_i/\kappa_i^{1/2}=2130$; curves 2a and 2b: normal weight concrete $k_i/\kappa_i^{1/2}=1500$; curves 3a and 3b: lightweight concrete, $k_i/\kappa_i^{1/2}=780$; curves 4a and 4b: lightweight concrete, $k_i/\kappa_i^{1/2}=500$.

ordinary normal weight and lightweight constructions, realistic values of $(\bar{q}_r)_i$ are expected to lie between curves 1b and 2b, and 3b and 4b, respectively (lined areas in Fig. 2).

In tests of up to 1 h duration $(\bar{q}_r)_i$ may be chosen roughly as 27 500 W m^{-2} for normal weight, and as 15 000 W m^{-2} for light concrete constructions. For constructions made of other materials, interpolation is permissible based on the value of the property group $k_i/\kappa_i^{1/2}$. It may be added here that the thickness of the test specimen, unless unusually small, has no noticeable influence on the value of $(\bar{q}_r)_i$.

FIRE RESISTANCE VERSUS FIRE TOLERANCE

There are two problems intrinsic to the use of fire resistant compartmentation as a technique of fire defense: how to decide on what constitutes the minimum period of satisfactory performance of the compartment boundaries, and how to judge the acceptability of the boundaries for the specified performances.

With respect to the first problem, decisions on the minimum period of satisfactory performance are usually made authoritatively by the writers of building codes. As to the latter, the common practice at present is to judge the acceptability of a compartment boundary for a specified minimum performance on the basis of its *fire resistance*, as determined, in general, by subjecting a representative specimen of the construction to a standard fire test, e.g. in North America the ASTM E119 test.

The practice of complying with authoritative decisions in choosing the minimum periods of satisfactory performance, t^*, and identifying the periods of satisfactory performance with the fire resistance values, r, developed for the various constructions either by fire tests or by reasonings or calculations based on the fire test philosophy, will be referred to here as *fire resistance allotment*.

Clearly, the condition of acceptability of a construction is

$$r \geq t^* \;(\equiv r_{\min}) \tag{5}$$

There are many advocates of the idea that competent engineers be given a larger degree of freedom in working out the most appropriate measures for the containment of possible fires. This greater freedom would include examination of the performance of the planned compartment boundaries under the relevant conditions, and the making of decisions on their acceptability.

In contradistinction to fire resistance allotment, the practice involving decisions by qualified engineers on the necessary and adequate measures of fire containment will be referred to as *fire tolerance design.* The term *fire tolerance*, to be denoted by θ, will be used in a specific sense, to mean period of satisfactory performance of a building element under the applicable fire severity conditions (as characterized by $\bar{q}_i$, or, less desirably, by $\bar{T}_g$), determined by either engineering studies or by non-standard tests; the limit of satisfactory performance being defined (for reasons to be explained in the next section) either (a) by heat transmission and structural failure criteria, similar to those of the standard test methods, in the case of 'dividing elements' or (b) in the case of 'key elements', by the same criteria for fire exposure on one side or the other, but by structural failure criteria only for fire exposure on both sides.

The condition of acceptability is

$$\theta \geq \theta_{\min} \;(\equiv \tau_{\mathrm{pf}}) \tag{6}$$

Because of some basic faults behind the philosophy of fire resistance allotment,[6] the movement for the recognition of the fire tolerance design is steadily gaining momentum.

DIVIDING ELEMENTS AND KEY ELEMENTS

In defining the meaning of fire tolerance, differentiation has been made between dividing and key elements from the point of view of the applicable failure criteria. The logical background for this differentiation is as follows.

The term dividing element is to be applied to those components of a building that are not essential parts of the principal structural network. Clearly, once the fire has reached, via any path, the reverse side of a dividing element, its structural failure (collapse) can in no way influence either the further course of the fire or the performance of the building as a whole. Consequently, the performance of a dividing element in fire can be evaluated on the assumption that the element is exposed to fire on one side only.

In contrast, the structural failure or major deformation of the essential load-bearing elements, the so-called key elements, of a building may result in extensive damage and loss of life and, therefore, must not be allowed to happen even after an eventual spread of the fire to their reverse sides. Consequently, the performance of every key element of a building must be judged on the assumption of a two-sided fire exposure, provided that such exposure is conceivable and, if it is, that it represents an adverse situation.

It has been shown[6] that the exposure of a construction to fire on both sides (not necessarily simultaneously) indeed creates a situation which is more adverse than that arising from a one-sided exposure. Since in a

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standard fire test a construction is exposed to the test fire on one side only (except columns), there are fundamental obstacles to the utilization of fire resistance data (derived from either fire tests or reasonings or calculations based on the fire test philosophy) in the design of key elements for fire safety. Obviously, such obstacles do not exist in the case of the dividing elements and, therefore, the utilization of fire resistance data in the design by dividing elements for fire safety is mainly a problem of convertibility between fire tolerance and fire resistance.

CONCEPT OF EQUAL TEMPERATURE–TIME AREAS (Method 1)

Kawagoe[1, 13] was the first to consider the problem of utilization of fire resistance data in a rational fire tolerance design. Asserting that the severities of compartment fires can be characterized by their temperature versus time curves, he suggested that all fires, real-world fires as well as standard test fires, be compared on the basis of the area under their temperature history curves above a base level, T_b, 673 K (400 °C) for normal weight concrete and similar constructions, and 773 K (500 °C) for lightweight concrete and similar constructions. This concept, as applied to the comparison of a standard test fire and a real-world fire, is illustrated in Fig. 3.

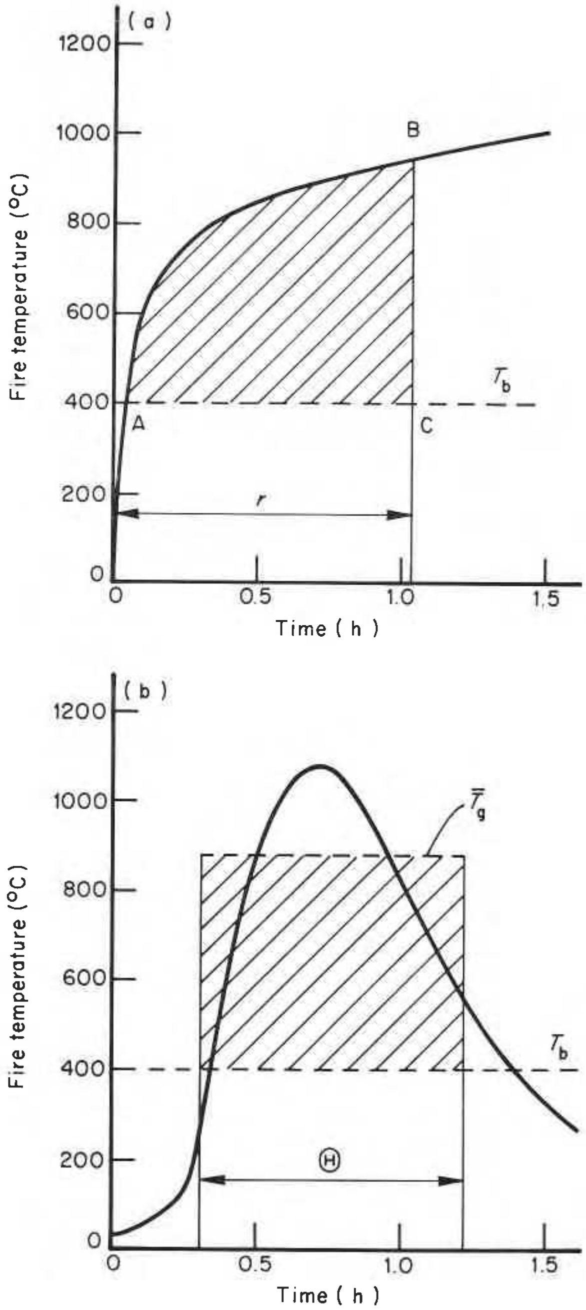

Figure 3. Concept of equal temperature–time areas: (a) standard fire test; (b) a real-world fire.

The following equation applies:

$$\text{area (ABC)} = (\bar{T}_g - T_b)\theta \tag{7}$$

To bring this equation into tractable forms, it is necessary to express area (ABC) in terms of fire resistance, *r*, in other words, in terms of the time of successfully endured exposure to a test fire. To develop such expressions, two area (ABC) versus *r* plots have been prepared (corresponding to the two values of T_b; using the standard ASTM E119 furnace-temperature versus time curve) in Fig. 4. These curves seem to be amenable to approximation by straight lines and the following equations:

$$\text{for } T_b = 673\text{ K} \quad \text{area (ABC)} = 555\,(r - 1000) \tag{8}$$

$$\text{for } T_b = 773\text{ K} \quad \text{area (ABC)} = 444\,(r - 1000) \tag{9}$$

By combining Eqns (8) and (9) with Eqn (7), two correlations are obtained between fire tolerance and fire resistance. They are, for normal weight concrete constructions and the like:

$$r = 1000 + \frac{\theta}{555}(\bar{T}_g - 673) \tag{10}$$

for lightweight concrete constructions and the like:

$$r = 1000 + \frac{\theta}{444}(\bar{T}_g - 773) \tag{11}$$

These equations suggest that, from the point of view of fire resistance requirement, short and hot fires are equivalent to longer and relatively cool fires.

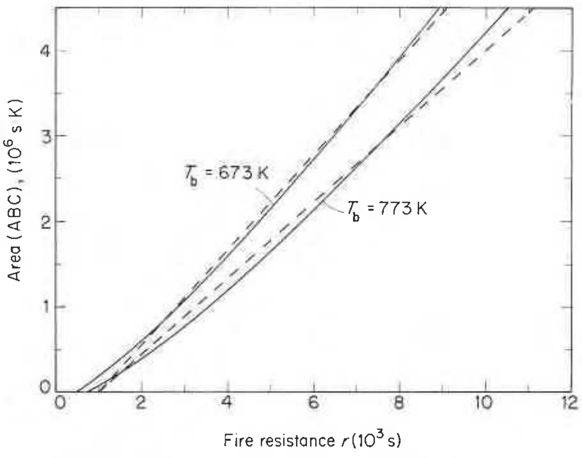

Figure 4. Temperature–time area versus fire resistance plots for standard ASTM E119 fire exposure, at two values of the base temperature. Dashed lines represent straight line approximations for the curves.

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CONCEPT OF EQUAL MAXIMUM TEMPERATURES (Method 2)

For protected steel and reinforced or prestressed concrete constructions the concept of equal maximum temperatures is a more realistic one. This concept states that the corresponding values of fire tolerance and fire resistance are those that pertain to identical maximum temperatures reached by a steel component of deciding importance in real-world fires and standard test fires.[14–18] The concept is illustrated in Fig. 5.

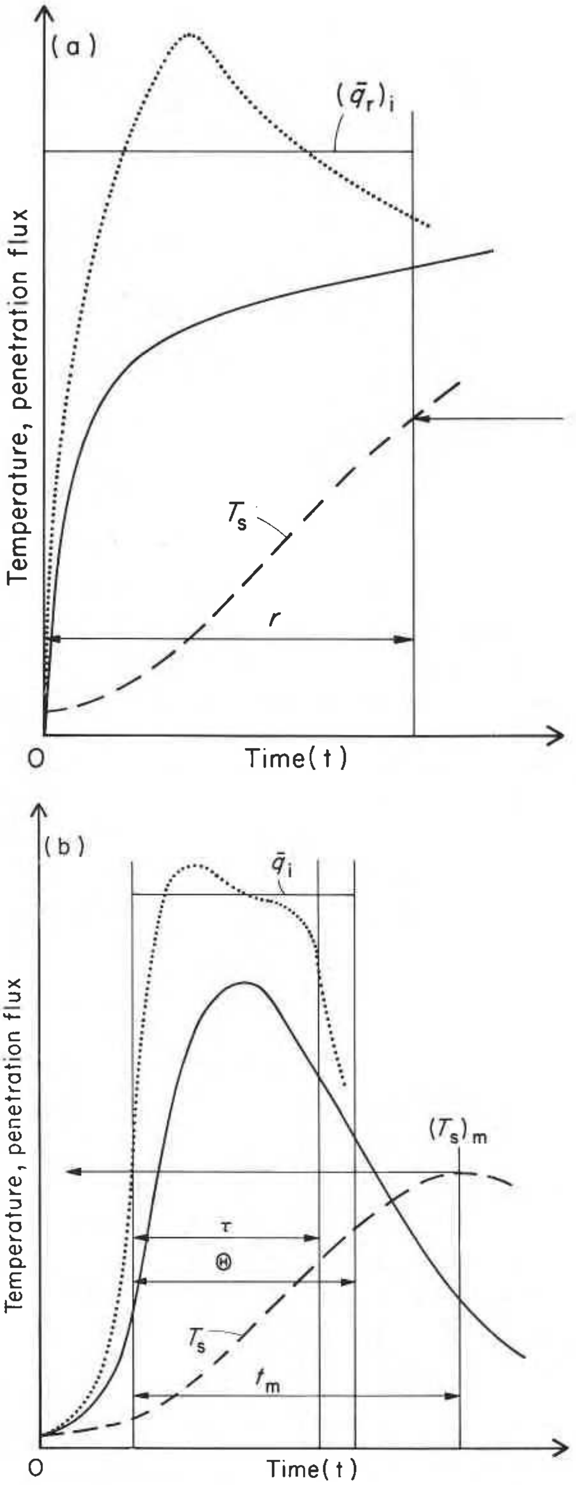

Figure 5. Concept of equal maximum temperatures: (a) standard fire test; (b) a real-world fire. ——— Fire temperature. – – – – Temperature of a steel component of deciding importance. . . . . . . Penetration heat flux.

To develop a relationship between fire tolerance and fire resistance, first the temperature of the crucial steel component, T_s (at a distance a below the surface of the concrete or other protective material) is to be expressed for the real-world fire with the air of the penetration flux, $\bar{q}_i$. Because the maximum temperature of that steel component is reached inevitably following the end of the nominal fire exposure† (Fig. 5b), only the $t > \theta$ region of its temperature history is of interest. For this region,[19]

$$\frac{k_i(T_s - T_0)}{\bar{q}_i a} = 2\left(\frac{\kappa_i t}{a^2}\right)^{1/2}\left\{ierfc\,\frac{1}{2}\left(\frac{a^2}{\kappa_i t}\right)^{1/2} - \left(1-\frac{\theta}{t}\right)^{1/2} ierfc\,\frac{1}{2}\left[\left(\frac{a^2}{\kappa_i t}\right)^{1/2}\left(1-\frac{\theta}{t}\right)^{-1/2}\right]\right\} \qquad (12)$$

After differentiating with respect to t, and making dT_s/dt equal to zero, an expression is obtained for the time, t_m, at which the crucial steel component reaches its maximum temperature, $(T_s)_m$[20]:

$$\frac{\kappa_i t_m}{a^2} = -\frac{1}{2}\,\frac{\dfrac{\theta}{t_m}}{\left(1-\dfrac{\theta}{t_m}\right)\ln\left(1-\dfrac{\theta}{t_m}\right)} \qquad (13)$$

By substituting into Eqn (12) sets of corresponding values of θ/t_m and $\kappa_i t_m/a^2$ obtained from Eqn (13), a relationship results between $k_i[(T_s)_m - T_0]/\bar{q}_i a$ and $\kappa_i t_m/a^2$. This, again utilizing corresponding values of θ/t_m and $\kappa_i t_m/a^2$, can be converted into a relationship between

$$\frac{k_i[(T_s)_m - T_0]}{\bar{q}_i a} \text{ and } \frac{\kappa_i \theta}{a^2}$$

which is plotted as curve 1 in Fig. 6.

With test fires, as with real-world fires, the severity of fire exposure is more aptly characterized by the (average) penetration heat flux, $(\bar{q}_r)_i$, than by the standard temperature–time curve. Using $(\bar{q}_r)_i$ as datum information, the temperature history of the same crucial steel

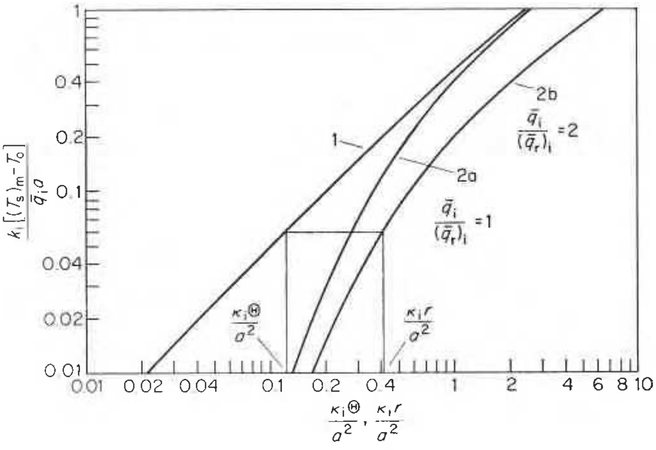

Figure 6. Information used in developing the relationship between fire resistance and fire tolerance.

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† It is assumed that spalling or disintegration of the protective material is not liable to occur.

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component in a test fire is described by the following equation:

$$\frac{k_i(T_s-T_0)}{(\bar{q}_r)_i a}=2\left(\frac{\kappa_i t}{a^2}\right)^{1/2} ierfc\,\frac{1}{2}\left(\frac{a^2}{\kappa_i t}\right)^{1/2} \qquad (14)$$

But according to the concept of equal maximum temperatures, the standard fire resistance is interpreted as the time at which T_s attains the value $(T_s)_m$ in the corresponding real-world fire, i.e., $t=r$ when $T_s=(T_s)_m$. Thus, the concept of equal maximum temperatures is expressed by the equation

$$\frac{k_i[(T_s)_m-T_0]}{\bar{q}_i a}=2\,\frac{(\bar{q}_r)_i}{\bar{q}_i}\left(\frac{\kappa_i r}{a^2}\right)^{1/2} ierfc\,\frac{1}{2}\left(\frac{a^2}{\kappa_i r}\right)^{1/2} \qquad (15)$$

This equation defines a family of curves

$$\frac{k_i[(T_s)_m-T_0]}{\bar{q}_i a} \text{ versus } \frac{\kappa_i r}{a^2}$$

with $\bar{q}_i/(\bar{q}_r)_i$ as a parameter. Two members of the family are also plotted in Fig. 6; one (curve 2a) for $\bar{q}_i/(\bar{q}_r)_i=1$ and the other (curve 2b) for $\bar{q}_i/(\bar{q}_r)_i=2$.

If complemented by further curves of the family, representing a spectrum of realistic values for $\bar{q}_i/(\bar{q}_r)_i$, Fig. 6 will offer a convenient way of graphically developing a relationship between the fire resistance and fire tolerance in the following dimensionless form:

$$\frac{r}{\theta}=f\left(\frac{\kappa_i\theta}{a^2}, \frac{\bar{q}_i}{(\bar{q}_r)_i}\right) \qquad (16)$$

The procedure is illustrated in Fig. 6.

To find sets of corresponding values of $\kappa_i\theta/a^2$ and r/θ for, say, $\bar{q}_i/(\bar{q}_r)_i=2$, read $\kappa_i\theta/a^2$ and $\kappa_i r/a^2$ from curves 1 and 2b respectively, at various levels of the ordinate, $k_i[(T_s)_m-T_0]/\bar{q}_i a$, then obtain r/θ by dividing $\kappa_i r/a^2$ by $\kappa_i\theta/a^2$.

Figure 7 is a graphical presentation of the relationship expressed by Eqn (16) for those ranges of the three variables which are important in the solution of practical problems.

LAW METHOD (Method 3)

It is also possible to calculate the minimum fire resistance requirements directly from the characteristics of the fire compartment, without resorting to the use of the fire severity parameters. The method was worked out by Law and her colleagues in Britain.[21–23] The following empirical relationship was recommended:

$$r_{\min}=78\,\frac{G}{\sqrt{A_W(A_t-A_F)}} \qquad (17)$$

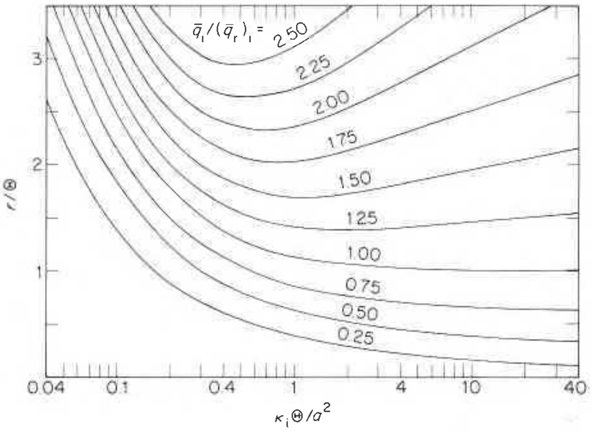

Figure 7. Dimensionless presentation of the relationship between fire resistance and fire tolerance using the concept of equal maximum temperatures.

This equation was later slightly modified[24] to include an additional variable, namely the height of window.

The Law method suggests that for a given compartment (i.e. for $G=$const and $(A_t-A_F)\simeq$const) the fire resistance requirement is inversely proportional to $A_W{}^{1/2}$, in other words, to the square root of rate of entry of air into the compartment. If one assumes that the minimum fire resistance requirement for a specific compartment boundary is roughly proportional to the product $\bar{q}\tau$ (referred to as the fire severity product by this author[5]), such a relationship indeed seems to be justifiable (see Fig. 9 in ref. 5). Unfortunately, combining $\bar{q}$ and τ into a single fire severity parameter is not strictly correct.

NUMERICAL EXAMPLES

The application of the preceding three methods will now be illustrated on two numerical examples. Both relate to a compartment similar to that used in the experiments of Butcher *et al.*[25, 26]

Example 1

Information on compartment. Dimensional details:†

$$A_F=28.6\text{ m}^2,\ A_t=130.2\text{ m}^2,\ A_W=5.6\text{ m}^2,$$

$$H_C=2.9\text{ m},\ h=1.83\text{ m}$$

Fire load (cellulosic): $G=436$ kg, specific surface of fuel $\phi=0.13\text{ m}^2\text{ kg}^{-1}$. Average thermal properties of compartment lining materials: $k=0.616\text{ W m}^{-1}\text{ K}^{-1}$, $\kappa=0.326\times10^{-6}\text{ m}^2\text{ s}^{-1}$.

Characteristics of a potential fire. Fire severity parameters from calculations[4–7]: $\tau=1162$ s, $\bar{q}=14\,060\text{ W m}^{-2}$, $\bar{T}_g=820$ K. Required minimum fire tolerance period (for cellulosic fuel $\lambda=1.15$): $\theta_{\min}=1.15\times1162=1336$ s.

Information on the structure to be protected. Reinforced lightweight concrete ceiling slab. Thermal properties: $k_i=0.62\text{ W m}^{-1}\text{ K}^{-1}$, $\kappa_i=0.28\times10^{-6}\text{ m}^2\text{s}^{-1}$. Distance of reinforcing bars from the fire exposed surface: $a=0.03$ m.

Miscellaneous information. Pre-fire temperature: $T_0=293$ K, estimated penetration heat flux in standard fire test (for lightweight concrete): $(\bar{q}_r)_i=15\,000\text{ W m}^{-2}$.

Solutions. Method 1—based on the concept of equal temperature–time areas. Use Eqn (11)

$$r_{\min}=1000+\frac{1336}{444}(820-773)=1141\text{ s }(0.32\text{ h})$$

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† Some of the variables listed here do not appear in any of the formulas presented in this paper; their values are needed, however, in the calculation of the fire severity parameters.[4–7]

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Method 2—based on the concept of equal maximum temperatures. Introductory calculations:

$$\frac{\kappa_i \theta_{\min}}{a^2}=\frac{0.28\times10^{-6}\times1336}{0.03^2}=0.416$$

Use Eqn (3)

$$\bar{q}_i=14\,060\,\frac{0.62}{0.616}\sqrt{\frac{0.326\times10^{-6}}{0.28\times10^{-6}}}=15\,270\ \mathrm{W\ m^{-2}}$$

$$\frac{\bar{q}_i}{(\bar{q}_r)_i}=\frac{15\,270}{15\,000}=1.02$$

From Fig. 7, $r_{\min}/\theta_{\min}=1.40$, hence $r_{\min}=1.40\times1336=1870$ s (0.52 h).

Method 3—Law method. Use Eqn (17)

$$r_{\min}=78\,\frac{436}{\sqrt{5.6\times(130.2-28.6)}}=1430\ \mathrm{s}\ (0.40\ \mathrm{h})$$

Example 2

Information on compartment. Same as in example 1, except that the fire load, G, is 1744 kg.

Characteristics of potential fire. Fire severity parameters from calculations:[4–7] $\tau=2250$ s, $\bar{q}=19\,820$ W m^{-2}, $\bar{T}_g=1076$ K. Minimum fire tolerance period required: $\theta_{\min}=1.15\times2250=2590$ s.

Information on structure to be protected, and miscellaneous information. Same as in example 1.

Solutions. Method 1. Use Eqn (11)

$$r_{\min}=1000+\frac{2590}{444}(1076-773)=2770\ \mathrm{s}\ (0.77\ \mathrm{h})$$

Method 2

$$\frac{\kappa_i \theta_{\min}}{a^2}=\frac{0.28\times10^{-6}\times2590}{0.03^2}=0.806$$

Use Eqn (3)

$$\bar{q}_i=19\,820\,\frac{0.62}{0.616}\sqrt{\frac{0.326\times10^{-6}}{0.28\times10^{-6}}}=21\,530\ \mathrm{W\ m^{-2}}$$

$$\frac{\bar{q}_i}{(\bar{q}_r)_i}=\frac{21\,530}{15\,000}=1.44$$

From Fig. 7, $r_{\min}/\theta_{\min}=1.64$, hence $r_{\min}=1.64\times2590=4250$ s (1.18 h).

Method 3. Use Eqn (17)

$$r_{\min}=78\,\frac{1744}{\sqrt{5.6\times(130.2-28.6)}}=5700\ \mathrm{s}\ (1.58\ \mathrm{h})$$

Discrepancies similar to, or even larger than, those occurring in the selected examples can be expected as normal.

DISCUSSION

The relative merits of the described three methods of conversion between fire tolerance and fire resistance can perhaps be best discussed in connection with the results of some additional calculations presented in

Table 1. Calculated fire resistance requirements for three reinforced concrete beams exposed to real-world compartment fires at three fire loads (cellulosic)[a]

(A) Fire severity parameters and related information

Specific fire load G/A_F (kg m^{-2})	Insulated compartment						Non-insulated compartment					
	τ (s)	$\bar{q}$ (W m^{-2})	T_g (K)	θ (s)	$\bar{q}_i$ (W m^{-2})	$\frac{q_i}{(\bar{q}_r)_i}$	τ (s)	q (W m^{-2})	T_g (K)	θ (s)	$\bar{q}_i$ (W m^{-2})	$\frac{q_i}{(q_r)_i}$
15	1162	10 168	782	1336	28 470	1.04	1162	11 442	754	1336	21 477	0.79
30	1162	22 649	1159	1336	63 417	2.31	1162	27 389	1065	1336	51 409	1.87
60	1910	23 532	1386	2197	65 870	2.40	1910	29 979	1268	2197	56 271	2.05

(B) Miscellaneous input information and results for insulated compartment (IC) and non-insulated compartment (NIC)

Specific fire load G/A_F (kg m^{-2})	Beam			Fire resistance requirement consistent with fire tolerance design (h)						
				Method 1		Method 2		Method 3		Ref. 27[b]
	Size (m)	a (m)	$\frac{\kappa_i\theta}{a^2}$	IC	NIC	IC	NIC	IC	NIC	IC
15	0.6×0.3	0.0508	0.304	0.351	0.332	0.575	0.483	0.342	0.342	0.496
	0.4×0.2	0.0465	0.363			0.538	0.456			0.578
	0.35×0.15	0.0575	0.237			0.631	0.546			0.597
30	0.6×0.3	0.0508	0.304	0.603	0.540	1.043	0.883	0.684	0.684	0.889
	0.4×0.2	0.0465	0.363			1.021	0.854			1.025
	0.35×0.15	0.0575	0.237			1.095	0.939			1.037
60	0.6×0.3	0.0508	0.501	1.062	0.932	1.727	1.459	1.369	1.369	1.371
	0.4×0.2	0.0465	0.596			1.733	1.446			1.467
	0.35×0.15	0.0575	0.390			1.738	1.483			1.547

[a] Information on compartment: $A_F=12.43$ m^2, $A_t=66.48$ m^2, $A_W=2.58$ m^2, $h=2.18$ m, $k/\kappa^{1/2}=783$ J s$^{-1/2}$ m^{-2} K^{-1} (insulated compartment), $k/\kappa^{1/2}=1168$ J s$^{-1/2}$ m^{-2} K^{-1} (non-insulated compartment).

Information concerning the beams: $\kappa_i=0.587\times10^{-6}$ m^2 s^{-1}, $k_i/\kappa_i^{\frac{1}{2}}=2192$ J s$^{-1/2}$ m^{-2} K^{-1}.

Penetration heat flux in standard fire test: $(\bar{q}_r)_i=27\,500$ W m^{-2}.

[b] Obtained by computer simulations of the fire processes.

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Table 1. These calculations relate to the fire resistance requirements for three reinforced concrete beams under exposure to real-world compartment fires similar to those produced experimentally by Arnault *et al.*[18] In their experiments they used three 'specific' fire loads ($G/A_F = 15$, 30 and 60 kg m^{-2}). While, in the series to be considered here, the compartment dimensions and ventilation were kept constant, the compartment lining materials were selected to represent two kinds of boundary conditions, namely—in the authors' words—'insulated' and 'non-insulated' compartment boundaries.

All essential information concerning the compartment and the concrete beams is given in Table 1. (The material properties have been estimated by the author on the basis of his own measurements performed on materials similar to those described in the Arnault *et al.*[18] report.)

The fire severity parameters (calculations[4–7]) and some related information are given in section A of Table 1; the results are summarized in section B.

The following observations can be made: (1) Method 1, by utilizing the variables θ and $\bar{T}_g$, is responsive to all those factors on which the severity of fire depends, such as fire load, ventilation, compartment dimensions, and compartment lining. It is responsive, in a crude way (by the selection of the appropriate base temperature level, T_b) also to the nature of the material of the construction considered. It is not responsive to the thickness of protective cover on the crucial load-bearing component of the construction.

(2) Method 2, by utilizing the variables θ and $\bar{q}_i$, is responsive to all important characteristics of the compartment fire (as determined by fire load, ventilation, compartment dimensions, and compartment lining). By utilizing κ_i and a, it is also responsive to the nature of the construction considered, namely to its material and to the thickness of the protective cover on its crucial load-bearing component.

(3) Method 3, by depending on G, A_W and A_t, is responsive to fire load, ventilation, and overall compartment dimensions. It is not responsive either to the quality of compartment insulation, as determined by the thermal properties of the lining materials, or to the nature of the construction considered, as determined by its material and by the thickness of cover on its crucial load-bearing component.

These findings are summarized in Table 2. As to Method 3, the significance of not taking account of the nature of the construction considered (material and protective cover on the crucial load-bearing component) has not yet been investigated systematically. It is well known, however, from the experiments of Arnault *et al.*,[18] that the error resulting from neglecting the insulating quality of the compartment boundaries may be quite substantial.

Because it relies on the most realistic input information and on the use of a graph (Fig. 7) prepared without resort to empiricism, Method 2 should, as a rule, be preferred over the other two methods.

The results of several computer studies performed by Schneider and Haksever[27]—for the same beams and fire conditions (for 'insulated' compartment boundaries only)—are also included in Table 1. It can be seen that, except for the largest fire load, Method 2 gives the best approximations.

Table 2. Characterization of three methods of conversion between fire tolerance and fire resistance

	Responsive to					
	Compartment characteristics				Construction characteristics	
Method	Fire load	Ventilation	Compartment dimensions	Compartment lining	Material	Thickness of protective cover
1	Yes	Yes	Yes	Yes	?	No
2	Yes	Yes	Yes	Yes	Yes	Yes
3	Yes	Yes	Yes	No	No	No

CONCLUSIONS

As far as dividing elements are concerned, the available massive information on the fire resistance of building elements can be well utilized in the design for fire safety. Unfortunately, because standard fire tests do not yield information on performance of building elements under two-sided fire exposure, fire resistance information cannot be utilized in the design of such elements which are relied upon for the integrity of the building even under two-sided exposure. The design of these key elements must be based on special studies similar to those described in a previous paper.[6]

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APPENDIX I: NOMENCLATURE

a	distance of steel from fire-exposed surface, (m)
A	area (m^2)
G	total fire load (kg)
h	height of window (m)
H	height (m)
i	$=1, 2, 3, \ldots$
ierfc	integral of the error function of
k	thermal conductivity; without subscript: average thermal conductivity (W m^{-1} K^{-1})
q	penetration heat flux (W m^{-2})
$\bar{q}$	temporal average penetration heat flux either during the fully developed period of real-world fires or during standard test fires; without subscript: that averaged spacially for all compartment boundaries; with subscript: that for an individual compartment boundary (W m^{-2})
r	fire resistance (s)
t	time (s)
t^*	authoritatively specified minimum period of satisfactory performance (s)
T	temperature (K)
$\bar{T}$	average temperature during the fully developed period of real-world fires (K)
θ	fire tolerance (s)
κ	thermal diffusivity; without subscript: average thermal diffusivity (m^2 s^{-1})
λ	factor (Eqn (4)), $\simeq 1$, 1.15, dimensionless
τ	fire duration; without subscript: that of fully developed fire; with subscript: nominal fire duration (s)
ϕ	specific surface of combustibles (m^2 kg^{-1})

Subscripts

b	of base level
C	of compartment
F	of floor
g	of compartment gases
i	for the *i*th boundary element and its lining material
m	maximum
min	minimum
0	under pre-fire conditions
pf	for the postflashover period
r	under standard fire resistance test conditions
s	of steel
t	total for the compartment
W	of window