Fundamentals of Safety for Structures
Knoll, F.

Publisher's version / Version de l'éditeur:
https://doi.org/10.4224/20331469
Technical Translation (National Research Council of Canada), 1969

NRC
TT - 1367

NRC
TT - 1367

NATIONAL RESEARCH COUNCIL OF CANADA

TECHNICAL TRANSLATION 1367

THE FUNDAMENTALS OF SAFETY FOR STRUCTURES

BY

FRANZ KNOLL

ETH, ZURICH. DOCTORAL THESIS
PROM. NR. 3701, 1965. 47 P.

TRANSLATED BY

D. A. SINCLAIR

THIS IS THE ONE HUNDRED AND EIGHTY - SECOND OF THE SERIES OF TRANSLATIONS
PREPARED FOR THE DIVISION OF BUILDING RESEARCH

OTTAWA

1969

PREFACE

To design a structure safely against failure the designer - or actually the code writer - needs to know not only the expected strength of the structure and the expected load, including their respective variations, but also he must decide on what safety factor will be required to make the structure sufficiently safe yet not uneconomical. A great deal of research has been done in the past on the strength of structures and some on the expected loads, but very little has been done about the safety factors and their method of implementation. The latter is now receiving serious attention.

This thesis, "Fundamentals of Safety for Structures", by Franz Knoll, studies the problem of structural safety in both its theoretical and practical aspects and discusses especially the influence of gross "human error". The author proposes a system of safety factors for reinforced concrete structures somewhat similar to the one proposed by the British Institute of Structural Engineers in 1955.

The Division is indebted to Mr. D. Sinclair for preparing this translation.

Ottawa
1969

R.F. Legget
Director

NATIONAL RESEARCH COUNCIL OF CANADA

Technical Translation 1367

Title:	The fundamentals of safety for structures. (Grundsätzliche zur Sicherheit der Tragwerke)
Author:	Franz Knoll.
Reference:	ETH, Zurich. Doctoral Thesis, Prom. Nr. 3701, 1965, 47 p.
Translator:	D. A. Sinclair, Translations Section, National Science Library.

FOREWORD

In using statistical methods in the field of structural safety it has been found that these methods are not sufficient to provide final solutions. Although statistical results can be formulated, they often do not agree with observations in actual practice.

The aim of the present thesis is to trace the results of statistical methods for safety problems through to their application in design practice. In addition, questions are analyzed which go beyond the range of exact scientific theory. The purpose is to determine, on the basis of the assumptions made, the limits of application of statistical laws and to discuss them with the help of some examples from the field of reinforced concrete.

Sincere thanks are expressed to Professor B. Thürlimann for his generous help. I am much obliged also to Professor F. Kobold for his valuable suggestions and comments which have been of great assistance. Appreciation is also expressed to my colleagues at the Institute of Structural Engineering and the Institute for Geodesy and Photogrammetry who assisted me in many fruitful discussions.

The present study has been made possible by the financial assistance of the Federal Fund for Economic Studies (Volkswirtschaftskredit). I am grateful to the members of the Board of this Fund.

ETH, Zurich, Spring 1965 Franz Knoll

TABLE OF CONTENTS

LIST OF SYMBOLS

(In some cases the chapters in which the symbols are used are shown in brackets)

A	Constant
A_g	Estimated value of A
A_t	Actual true value of A
B	Constant
E	Favourable event (3.1)
E'	Unfavourable event, damage (3.2, 3.3)
$F(x)$	Probability function of x (4.2)
Fe	Steel section
M	Bending moment
M_p	Plastic moment (4.2)
N	Normal force
P	Load (3.4)
P_N	Live load
P_{Pr}	Test load
S	Safety factor (2.1, 3.6)
$\bar{S}$	Required safety factor, median of the safety factor distribution (4.2, 4.3)
S'	Additional safety factor (5.4.5)
S^*	Auxiliary safety factor
T	Load-carrying capacity (3.3)
$W(E)$	Probability of the event E

X_i, Y_i	Parameters of safety (5.2 and following)
Z	Safety zone (3.6)
b	Width of cross-section
d, d'	Dimensions of cross-section
e	Eccentricity
f_i	Linear functions
f(x)	Distribution (probability density) of x (4.2)
g	Factor of the distribution function (4.3)
g	Dead weight
h	Effective depth
i, j	Indices
k	Reduction factor
ℓ	Length
ℓ_i	Observations, measured values
m	Integer
m_i	Material factors
n	Integer
p	Distributed load
q_i	Individual loads
v_x	Coefficient of variation of x (4.1)
w	Distributed wind load
z	Target function

Φ	Auxiliary function
α_i	Parameter of construction properties
β_D	Compressive strength of concrete
ϵ	Deviation
χ	Auxiliary function
λ_i	Load factors (4.5.2)
μ	Percentage of steel
μ_{Gr}	Critical percentage of steel
$\mu_{\log x}$	Median of the log normal distribution of x
ξ	Ratio
σ_F	Yield strength of steel
σ_b	Concrete stress
σ_e	Steel stress
σ_x	Standard deviation of x (4.1)
$\sigma_{\log x}$	Logarithmic standard deviation of x
φ ψ	Angles

SOME SYMBOLS FROM MATHEMATICS AND LOGIC ALGEBRA

$A \wedge B$	All numbers belonging to set A as well as set B
$A \vee B$	All numbers belonging either to set A or to set B or to both sets
$X > Y$	All numbers described by the inequality given
sign (K)	Sign function of K, sign of K
$\prod_{\ell}^{n} (x_i)$	Product of all x_i for $\ell \leq i \leq n$

THE FUNDAMENTALS OF SAFETY FOR STRUCTURES

1. INTRODUCTION

In modern civil engineering safety occupies an important place among the requirements of design. Owing to requirements of economy, dependability and durability, we are compelled to apply safety considerations in every project as a criterion for the estimated value of the planned structures. Increasing demands for economy lead to smaller dimensions of the structural elements, thus requiring better and clearer formulations of the safety concept and of the conclusions deriving therefrom. However, good formulations are only forthcoming when we trace the concept in question back to its fundamental principles. From these principles the tools of science - logic, mathematics and empirical methods - must be used to develop the relationship of the concept to the final structure. In our case, we are concerned with load-bearing structures and everything that affects them and their behaviour. As recent research has shown, the safety problem can be regarded as a problem in statistics and the theory of probability. However, we must always ask ourselves whether the solutions we obtain through these two tools are actually practicable, i.e. whether the results can be used to generate further conclusions, e.g. quantitative conclusions.

In many cases we find that this is indeed possible to the extent required for the case in question. This applies particularly to the case of mass fabrication (see 8.1.) and where phenomena of nature and their effects are involved. The latter can often be observed under the same, or very similar conditions, and accordingly reliable results can be attained by applying the laws of statistics. If data are then analysed by statistical methods, we can arrive at an estimate of their accuracy which can be used in turn for the determination of comparative weights. The weight of an observation is a measure of its validity and thus plays a very important part in all smoothing processes.

In the present study we shall discuss the application of these methods to the properties of structures, and the application of the results obtained therefrom to the safety problem. For common cases of structures and the conditions under which they are erected, it cannot be directly inferred that the laws and rules of statistics remain valid. The structure is neither a product of a natural process nor can it be considered as something that is produced many times under the same conditions, like a factory-produced article. On the contrary, in almost all structures it is necessary to take into account to a greater or lesser extent the influence of human action, which may involve the limited ability, and even of the arbitrariness of individual people. Such effects cannot be considered random, and hence the most important condition for applying the laws of statistics is absent. Moreover, most load-bearing structures, with few exceptions, constitute individual cases. We must therefore forego the "law of large numbers" which is another important condition for the effectiveness of statistical operations. These arguments, of course, must for the present be regarded as suppositions which, however, come to mind spontaneously even in a rather superficial approach. Later we shall look into them and discuss them in fuller detail.

If safety is formulated as the result of a probability consideration, it will have a form that is not directly applicable to design practice. We are therefore compelled to convert the concept of safety into comparative values which will fit more comfortably into the framework of structural calculations. At the same time different aspects which cannot be immediately reconciled must be taken into account. Here again we have a problem which can be solved satisfactorily only through an optimization process. In addition to purely quantitative arguments, the problem also contains, of course, formal and practical-procedural aspects, so that it cannot be solved with arithmetical or algebraic methods. In general terms, a tentative solution is to be worked out which will provide the safety coefficients (comparative values) for load-bearing structures in reinforced concrete. Most of the arguments which will be applied in the present study are applicable, insofar as they do not relate to specific reinforced concrete problems, but to other forms of construction as well.

For a clearer understanding, the suggested solution will then be applied to two typical examples for the dimensioning of reinforced concrete. In this way we will be able to test the procedural suitability of the safety coefficients for practical design purposes.

2. ASSUMPTIONS, STATEMENT OF THE PROBLEM

2.1 Historical Background (27)

In the course of time structures have evolved from a few basic forms to the present-day multiplicity. Besides protecting man from external dangers - storms, high and low temperatures, enemies - new purposes are continually being assigned to them: storage of goods, space for meetings and other occasions, the facilitating of transportation, space for industrial production and exhibitions, esthetic and monumental effects, etc.

This development is paralleled by another: buildings become consumers' goods, and are thus an important factor in the economy. As already remarked above, this finds its expression in the demand for more economical construction methods, but in turn entails a new type of danger. Although at first buildings were merely a means for the personal protection of people and because of their simplicity, constituted no danger, nowadays, they are becoming more and more significant sources of danger. This development can be traced back to the beginnings of history. It is punctuated by a number of spectacular collapses, for example, the tower of Babel.

Engineering has been confronted with this new risk ever since it began operating at the limits of its performance. The safety problem consists in estimating the dangers arising out of the structures themselves and finding ways of keeping the risk at an acceptable level.

When we look back over the history of safety in building we find that ever since building "became dangerous" certain general rules have been applied. Until the nineteenth century, of course, they could not be scientifically formulated, because no theoretical premises were available, and construction was carried on by intuition and personal insight. Thus, the rules formed part of the traditional art of building, but could not have been abstracted from it. Admittedly attempts were made again and again to express the problem in terms of numerical comparisons. However, this could not produce the required results before the principles of the theory of structures were known, and engineers had become aware of them. English engineers of the eighteenth century were the first to test the safety of individual supporting members under load and to draw conclusions from the results of this with regard to the safety of the completed structure. This applied to the compressive strength of masonry and cast iron rods being used at that time, and in which the direct transmission of vertical forces to the foundation was regarded as essential. For the first time the value of safety had been formulated, for comparison purposes in the form of the safety factor. It is defined as the ratio of the maximum test load (ultimate bearing capacity) to the working load of the structure:

$$S = \frac{P_{Pr}}{P_N} = \frac{T}{P} \tag{1}$$

After the introduction of structural statics and the theory of elasticity as the basis of design, a further development in the safety factor took place. Allowable stresses were formulated and structures were dimensioned in such a way that under the working load no point of the structure exceeded the limit imposed by the allowable stresses.

During the first half of the twentieth century new ways of looking at things have appeared. On the one hand, it became necessary to supplement the first order elastic theory by one of second order and by the theory of stability. Eventually the demand arose for a theory by which structures would no longer be studied merely with regard to their elastic behaviour, but beyond their elastic limit to their breaking strength. This would make it possible, to introduce a collapse load criterion as a design basis in lieu of the deformation criterion of the theory of elasticity used hitherto. The elastic theory of strength of materials was now supplemented by a plastic theory.

At the same time, designers began to realize that the data, and hence the results of structural calculations, were fraught with errors, sometimes very considerable ones. This is an extremely important fact where appraisals of safety are concerned, and now one tries to relate the safety concept to statistical analysis on errors and deviations of the data on structure.

This brings about a new sort of formulation in construction engineering. It is, indeed, much closer to the true meaning of the safety concept than the comparative values developed earlier, such as safety factors, allowable stresses, etc. In contrast to these values, however, it requires a more accurate and complete investigation of all the data and processes which have to do with safety in structures. It cannot seek support in mere or partial "common sense", as was always done previously in connection with safety considerations. However, a safety appraisal based on statistical information is only possible in specific, exceptional cases.

This situation will continue until we know a great deal more about the properties and actual behaviour of structures.

At the same time,as already mentioned above,there are other important objections to the adoption of safety concepts based on purely statistical considerations. It will be the main task of the present study to determine to what extent knowledge of the statistical properties of structures can presently be applied in construction engineering for the determination of safety.

2.2 Statement of Problem, Classification

Before we can deal explicitly with the matter of safety, we must first be able to describe it clearly. This involves especially conceptual, and as far as possible, mathematical formulation with all the terms relating thereto. Since this formulation deals only partially with physical objects that can be perceived by the senses, considerable space will have to be devoted to the development of concepts. This will be attempted in the third part of the investigation.

Once the various concepts have been assembled and their relationships established, we can then turn to the mathematical representation and the derivation of formulae. These must be discussed and analysed together with the methods of structural theory and statistics. More specifically, this will involve the study of the degree by which the data used in design depart, randomly and non-randomly, from the quantities actually represented. This will be taken up in Section 4: representation and calculation of safety.

As was indicated above in connection with the historical and introductory remarks, safety is one of the most important properties of structures. It is of both economic and social significance. Since structures are planned for the future, the required degree of safety must be an integral part of design. It involves the appraisal of all possible errors and deviations which may occur during the design stage; it must take into account all flaws which may be expected in the execution of the structure and estimate the consequence of these "sins". This of course does not mean that all these deviations must be accepted by the safety appraisal, since we do consider their frequency and take account of their consequences. On the contrary, the safety verification must also contain the provisions that have been taken or are foreseen for the avoidance of errors and their consequences; for it is primarily these provisions which constitute a counterweight to the deviations and uncertainties which cannot be avoided.

Safety, accordingly, becomes a very complex concept and a number of terms will have to be explained in (3.2).

The ultimate purpose of a consideration of the safety problem is to present the concept in a form in which it becomes a usable instrument for the dimensioning of structures. The complexity of structures and the conditions to which they are subjected mean that no generally applicable formulae and rules can be found for numerical safety values. From the formal, procedural and practical points of view, special requirements for comparative values arise in many cases which cannot all be satisfied simultaneously. As a consequence we are compelled to look for a system of safety margins which will satisfy the different requirements as far as possible. The procedure by which this is achieved is discussed in connection with some examples from reinforced concrete construction. In the limited space at our disposal it is not possible to take up special cases and deal with them exhaustively. Hence, the application of the safety coefficients is shown in two simple examples as far as their numerical application in the dimensioning of structures (6).

2.3 Assumptions, Limitations

(a) All the types of construction dealt with in this investigation are bodies at rest. Their structural behaviour is not influenced by any motion, and no dynamic forces occur.

(b) Similar restrictions have been made for the loads. Pulsating, or momentarily applied loads, or loads which can produce long-term effects (creep, shrinkage, fatigue) are not considered. In this connection the reader is referred to the works of Freudenthal (21, 22, 223).

(c) Only stress problems of first order are considered. In simple cases, especially in the case of statically determinate systems, the results of the investigation, i.e. the safety coefficients, can also be applied to stability cases. However, if it is necessary to study these in conjunction with statically indeterminate systems, the calculation will be correspondingly more complex on account of the non-linear relationship between the load and stress. It must then be conducted similarly to the calculation for the case of a plastic design by the mechanism method, as explained in Example 6.2.

(d) It is assumed that plane sections remain plane, wherever it simplifies the calculation. The elastic and plastic theories of the strength of materials are assumed to be such good approximations that their use entails no error that is not negligible compared with other deviations.

(e) Special structures such as dams and military engineering structures, cannot be taken into consideration, since entirely different considerations apply to them.

(f) If a load on a structure comprises several different forces simultaneously, it is assumed that the position of the point of application of the forces, as well as their direction and the ratio between them does not change with an increase of load. This assumption is dropped in Example 6.2. In the appendix (8.2) special problems deriving from this assumption and others related to it are taken up.

(g) It is assumed that the expressions and laws derived from statistics and the theory of probability are always applied to independent variables. This is not everywhere demonstrably the case. Thus the structural results for this reason are approximations. Since, however, we have no information on possible correlations in the statistical data of structures, then by taking into account the relationship between certain data we would only be introducing new unknowns - the correlation coefficients - into the investigation, without possessing the means for their determination.

(h) The data and numerical values used in the examples and in the discussion, are derived, as far as possible, from the literature on corresponding investigations. However, since we are far from having information available about all quantities, the gap must be bridged by estimates ("educated guesses"), which we base on practice and building materials common in Switzerland at the present time(41 - 45).

3. DEVELOPMENT OF CONCEPTS

3.1 The Concept of Safety *

The confidence level* is a term borrowed from the theory of probability. It was already appropriately recognized before this mathematical discipline made it possible to interpret it quantitatively. Safety signifies a high probability that a well-defined event (E) will occur (i.e. that the structure will not fail. Transl.). The confidence level is smaller than or equal to unity. If it is exactly one (100%), it is equivalent to absolute certainty. However, this is never the case in the field of structures; there is always the possibility of an alternative event (E') the probability of which does not wholly vanish.

We shall use the term safety, or confidence level, hereinafter to mean the probability of occurrence of the event E, which for the present we shall refer to simply as "favourable event". This is contrasted with the event E', which can be generally described by the word "failure". We shall have more to say below (3.2) concerning the specific importance of the two events.

The following logical relations exist between the two events:

$$E \wedge E' = 0$$
$$E \vee E' = 1 \qquad (2)$$

In plain language this can be expressed as follows: E and E' are alternatives, i.e. they are mutually exclusive and there are no possible events except E and E'. To prove that this assumption is correct in its application to the safety of structures can be a problem in logic. Presumably, this would only lead to a further assumption, which could not be confirmed, in turn, except with empirical considerations. However, since it is plausible that either "something" or "nothing" happens to a building, we may dispense with further discussion of this purely formal matter.

By analogy with logic, we may write in terms of probability calculus:

$$W(E) + W(E') = 1 \qquad (3)$$

* In German one employs the word "Sicherheit" both for safety and confidence level (Transl.)

where W (E) is the probability of E, i. e. the safety, and W (E') is the probability of damage E'.

For the concept of safety to represent something concrete, a description of one of the two events is necessary. In most discussions, this is omitted, since it is assumed to be obvious. Often, however, clarity concerning the event in question is only apparent and therefore, in what follows, the event will be defined wherever necessary.

Safety (confidence) is usually only slightly less than one. It is simpler, therefore, to consider the complementary probability W (E') of failure, which is a small quantity and can therefore be dealt with without loss of accuracy. Furthermore, we have a distinct equivalent between the probability and the frequency of failures. Moreover, the term failure can be positively and more simply described than would be possible for the event E of safety itself, whose definition would always have to include a list of all possible unfavourable events that must not occur.

Here it should be emphasized that safety (confidence level) can only be represented logically by an expression of theoretical probability. Dangerously false conclusions may result by designating some comparative value such as the safety factor, as "safety". Safety can have values only between zero and one:

$$O < W(E) < 1 \tag{4}$$

3.2 Definition of Safety, Concept of Damage

So far we have been speaking generally about failure, i. e. an unfavourable event that may happen to a structure with a probability W (E'), and should therefore be avoided. Since its occurrence can never be completely eliminated, we have to be satisfied with the reduction of its probability to a satisfactory level.

The content of the term "failure" requires explanation, because it may have widely differing interpretations.

From the history of construction engineering and from daily observations it is well known that failures can occur anywhere in many different forms and degrees. The essential criterion is usually the effect of the failure on the continued existence of the structure and its serviceability. Failure is said to have occurred whenever the structure is impaired in any one of its functions. Moreover, one sort of damage does not exclude others. In what follows we shall include under events E' only those which change the load-carrying capacity of the structure. All other effects such as aesthetic flaws or the reduction of insulating or impermeability properties, are not considered in this investigation. We shall have more to say concerning deformations in this connection. However, since no general statements can be made about it, this matter must be deferred for treatment with reference to specific examples.

Only the action of forces are to be regarded as causes of failure. Corrosion, chemical decomposition, wear, etc. are generally long-term influences, and thus do not fall within the scope of the present investigation. Several kinds of failure may occur to the same structure. These are to be distinguished by the subscript i, which can assume any number designating failure against which one wants to guard. E' then becomes the sum total of all events E'_i, or in the nomenclature of symbolic logic:

$$E'_i \subset E' \qquad i = 1, 2, \ldots \tag{5}$$

Besides the different types of failure, the location of the damage shall also be embraced by the subscript i.

Now, if we represent the safety with respect to a single case of failure E'_i:

$$W(E_i) = 1 - W(E'_i) \tag{5a}$$

this is only a significant statement if i describes the single failure which is of importance in this case. Often this is not so, and one would like to know the probability (confidence) that none of the several types of failure E'_i will occur, i.e. that "nothing" will happen to the structure. This is the logical meaning of "safety", and it is in this sense that we understand it, unless otherwise noted. It can be represented by the formula

$$W(E) = \Pi_i \left[1 - W(E'_i)\right] \tag{6}$$

which holds true, of course, only as long as the various E'_i are mutually exclusive, or, in statistical terms as long as there is no correlation between their probabilities. This condition, generally speaking, is not satisfied, and if one nevertheless proceeds as if it were, one arrives eventually at an underestimate of the safety, as may easily be verified. A better premise is to consider the sequence in time of the different instances of failure. That is to say, once the first failure has occurred, then, by definition, the structure is already impaired with respect to its bearing behaviour, and is thus changed. It must now be subject to a further investigation on the basis of its new characteristics. We consider load-bearing structures, therefore, only until the first failure of an essential nature, as far as the bearing behaviour is concerned, has occurred. In general, therefore, we may write:

$$W(E) = 1 - W(E') \tag{6a}$$

where E' is the first failure occurring.

For the discussion of the safety problem still another application of the term failure is necessary for the sake of the statistical treatment. It must enable one to describe the failure event E' in technical terms. For this purpose a number of additional concepts are needed, which we shall discuss briefly.

3.3 The Load-Bearing Capacity

As a measure of the strength of a load-bearing structure we take the magnitude of applied forces which will produce the first failure. This is generally called the bearing capacity and denoted by the symbol T, regardless of the form in which it is written.

The theory of elasticity uses, as a criterion for the bearing capacity attained, a state of deformation such that whenever this state is reached at any point, the investigation of the structure is terminated. Such a criterion does not usually correspond to significant damage and does not necessarily modify the structure in such a way as to alter its bearing behaviour. However, it does, as a rule, define the limits of validity of the theory of elasticity, beyond which its application leads to incorrect results. In this sense, therefore, the elastic limit must be used in all elastic calculations rather than a true instance of failure.

In the theory of plasticity on the other hand the criterion of bearing capacity is generally a so-called failure or collapse, the occurrence of which, as a rule, signifies the collapse of the load-bearing structure, i.e. a structure that has suffered such a failure is for all practical purposes unserviceable. Collapse occurs whenever the deformation at any point in the structure increases still further without any finite increase of load being applied. Since this can happen without exceeding the limit according to the theory of elasticity (stability problems, etc.) such a definition is not sufficiently accurate. A better description of the collapse event is found in an energy consideration: failure occurs when the total work performed by external forces is dissipated in the structure, i.e. is converted into non-mechanical energy (principle of virtual displacements).

The bearing capacity can be expressed in various forms: in terms of the external forces (supported load); in the form of cross-sectional forces (e.g. plastic moment); as a stress or even a strain, as is customary in the theory of elasticity. If statically indeterminate systems are investigated for their collapse load, then properly speaking the form of supported load must be used, because cross-sectional forces and stresses can no longer be definitely verified. If we did use cross-sectional forces, this always means that the calculation is only an approximation of the theory of plasticity.

The description of the specific collapse process (mechanism) is exceedingly important for the determination of the bearing capacity. The consequence of this will be evident in a single simple case from example 6.2. That is, as soon as the load-bearing structure data are regarded as static quantities it cannot be verified in what way the limit of the strength is reached, i.e. various forms of damage may occur before collapse.

3.4. Loading

The load on a structure generally means a group of forces which are imposed on it externally. As a rule, stresses and strains are produced which in some cases bear a simple relationship to the size of the load. In that case, it is logical to represent the load and the bearing capacity in the same form. In an elastic calculation this usually takes the form of stresses, whereas in statically determinate systems the form of the cross-sectional forces may be used to calculate the collapse load, or, for all cases, as a proportion of the ultimate load. In the latter case, however, the loading configuration (location and relative magnitude of individual forces) must be known, so that in going from zero up to the ultimate load only a single value, the absolute value of the load, need be changed. If the individual load components are not proportional, or if their locations and directions change, then instead of the system of safety coefficients dealt with in this investigation, a somewhat more general method must be applied, which, however, can easily be derived from what is shown here. An example will be found in (226).

In the design stage, which, of course, must include a safety consideration, the load cannot be predicted. Nevertheless values must be assumed. Under these circumstances estimates are employed derived by analogy from loads on already existing structures, or which have been codified in standards for typical cases.

These values are merely for calculation purposes. However, their relationships to actual loads is of decisive importance for the safety problem. We shall return to this question again (5.2.13). For the present when we speak of loading or design load we are referring to such a calculation value, which is generally denoted by the symbol P.

3.5 The Structure. Types and Properties

Structures are solid bodies which transmit forces applied to them to a foundation. One of these forces (loads) is always the weight of the structure itself. Both the structure and its foundation may be movable, but according to 2.3, no additional forces are assumed to arise as a result of such motions. Similarly, the relative speeds of any two points of an individual element are assumed to be negligible (no shocks, vibrations, etc.).

For purposes of structural calculation a structure may be divided into separate parts, each of which is thereafter treated as an individual element. This is not always an advantage, as we shall see; however, the concept of structure should contain this possibility.

Structures possess an important structural property, which we may call "circuitry" by analogy with electrical systems. In both cases there are resistances;for both cases there are two possible kinds of elementary circuit, i.e. connection in series and connection in parallel. In the theory of structures this determines how the forces are transmitted, i.e. the "play of forces" in the structure; in the electrical circuit the kind of connection determines the distribution of the currents.

Two small examples may illustrate the meaning of the circuit concept as applied to load-bearing structures: connection in series corresponds to a chain, while connection in parallel corresponds to a cable (bundle of wires). One cannot, of course, derive quantitative physical results directly from the electrical analogy. Therefore we shall refrain from any further statement about the similarity, and the consequences of the type of structural connection itself will be considered.

The difference between the two kinds of connection is very important with reference to the safety problem. It can be very easily understood from the two examples already mentioned.

A chain is as strong as its weakest link (Fig. 1). A rope yields only when the last fibre yields (Fig. 2). In the second case, of course, an additional condition must be satisfied, namely that the material from which the rope is made possesses an adequate plastic range.

All load-bearing structures can be represented as circuit diagrams, where, on detailed consideration, a combination of the two possibilities is always involved: each load-bearing part can be divided lengthwise (main bearing direction) into elements which are connected to each other in series. At the same time each cross-section can be divided into individual fibres providing connection in parallel. However, this possibility will be of no further interest here. Instead we shall consider the load-bearing structure as a whole, i. e. as a circuit made up of various elements (rods, plates, slabs, etc.). We then find that there are both series and parallel circuits, as well as combinations of the two. A number of simple examples are represented in Figures 1 to 5. For the case of connection in parallel the load-bearing structure can be divided into individual paths (lines of force), where the same bars are participating in the transmission of the force but in different ways.

In terms of structural theory connection in series means a statically determinate system, i. e. one where there is only one distinct path over which forces can be transmitted. Structures connected in parallel are statically indeterminate, and the number of (statically determinate) paths is greater by one than the degree of static indeterminacy. Connection in parallel applies, of course, only if the different paths are not reunited (Example 5).

Now, if we again consider the rules governing chains and ropes, it becomes generally clear how important the kind of connection is in relation to the safety: in the case of connection in series only, the failure of any one member results in the collapse of the entire system (here, of course, care is required in the demarcation of the system). There are cases such as the cantilever bridge, where not all members are necessarily affected by a failure. However, the section to which the force is applied, at least, will move.

In the case of connection in parallel, on the other hand, all paths must be exhausted at at least one place before the structure yields (Example 6.2.). A general theory of structure circuitry cannot be given here, since owing to the many simplifications of structural systems it would be rather confusing. As a rule, however, these simplifications mean a reduction in the labour involved in the structural calculation, but then the true characteristics of the general case are no longer recognizable, and only from such a general case can generally valid theorems be simply formulated. We shall therefore restrict ourselves to the basic properties of the two kinds of circuitry.

The importance of the circuitry properties of the structure as a whole only becomes meaningful when we use as a criterion of the bearing capacity, a kinematic mechanism. In the theory of elasticity this is not the case. Here the criterion, more simply, is as follows: the load-bearing structure has failed whenever the yield limit has been surpassed at any point, i. e. if Hooke's law ceases to operate in any fibre. On the other hand, this states nothing about the "real" bearing capacity, i e. the maximum load that can be transmitted to the foundation.

For the sake of the subsequent discussion it is also necessary to establish the concept of the "structural properties". For this, we consider the structure as the sum of all those conditions or circumstances which affect its bearing behaviour. Besides the intrinsic properties such as circuit paths, geometry, strength, etc. this includes also certain external circumstances, such as load, behaviour of the foundation and indirectly the end use of the structure. This does not conform to ordinary linguistic usage, but is equivalent to an expansion of the concept "property". However, it will help to avoid many complications.

3.6 Comparison between Bearing Capacity and Load

Safety verification

If we know the internal properties of the structure and the load applied, the inequality of the confidence level can be set up. It will contain the resistance (T) present in the structure and the load (P) to be represented in the same units, so as to examine its bearing safety. Generally speaking, there are three possible inequalities:

$$T > P \tag{9}$$

This means that the strength exceeds the load and the structure will therefore support it.

$$T = P \tag{9a}$$

If the bearing capacity is exactly equal to the load, then the limit of safety has been reached. This will be explained more fully below.

$$T < P \tag{10}$$

If the bearing capacity is less than the load, then under the given circumstances the structure will fail, i. e. it will become damaged. The inequality of the confidence level is a physical formulation of the events E and E', as discussed in 3.1 and 3.2. The probability expressions derived therefrom will thus be as follows:

$$W(E) + W(E') = W(T > P) + W(T < P) = 1 \tag{11}$$

and

$$1 - W(T > P) - W(T < P) = W(T = P) = 0 \tag{12}$$

This is easy to understand if we assign to the indefinite state $T = P$ an infinitesimal change of one side

$$P \rightarrow P + dP \tag{13}$$

to the unfavourable event E', as is customary in statistics.

Nevertheless, it is helpful to use the limiting event $T = P$, because in a single expression it points to the two possibilities E and E'. It describes exactly the safety criterion, even though its occurrence possesses a probability of zero.

If in place of the bare inequality, which of course contains only qualitative information, we use an arithmetical expression, the numerical results of this will be a comparative value for the safety, e.g. the safety factor

$$S = \frac{T}{P} \tag{14}$$

or the safety zone

$$Z = T - P \tag{15}$$

These quantities make quantitative assertions about safety, but must not be confused with the true safety concept (3.1.). It is indeed true for a specific, well-defined structure in general, that the safety factor increases proportionally with the safety:

$$\mathrm{sign}\left(\frac{dS}{dT}\right) = \mathrm{sign}\left(\frac{dW(E)}{dT}\right) \tag{16}$$

and

$$\mathrm{sign}\left(\frac{dS}{dP}\right) = \mathrm{sign}\left(\frac{dW(E)}{dP}\right) \tag{17}$$

For two different structures, however, a larger safety factor does not necessarily mean greater safety. Furthermore, for the above relations to hold certain conditions must be satisfied:

$$\frac{dT}{d\alpha} \geq \frac{dP}{d\alpha} \tag{18}$$

i.e. for a change of bearing capacity due to a change in any one parameter α of the properties of the structure the bearing capacity must be increased by a greater amount than the dead load, or conversely, for a reduction of bearing capacity with α the dead load must be decreased to a greater extent. This is especially important in the case of long spans, where a strengthening of the main structural elements entails a considerable increase of dead weight. Similarly, this point must be taken into account for the cases of partial stresses of different sign (8.2.).

4. REPRESENTATION AND CALCULATION OF SAFETY

4.1 Deviations, Errors, Uncertainties

Classical statics applied to structural engineering disregards the fact that the actual properties of constructions projected for the future are not known at the time of the safety verification. For the calculation estimates (assumptions) are employed which are as similar as possible to the values expected in practice.

Once these assumptions are made, then the calculation no longer takes into account deviations of the "actual" from the design values, but relegates them to the category of "errors", which must be compensated by suitable safety margins. This summary treatment of the deviations results from two facts, namely that deviations or "errors" can never be wholly avoided, and that no information is available concerning their specific size. Otherwise, the assumptions could have been improved.

The relationship between errors and safety margins means that the deviations and their properties have to be used as a basis for the determination of safety measures. In order to discuss the procedure for this, we shall first review the errors and their statistical laws as well as the kinds of information which we possess for this purpose.

In the classical theory of errors, one studies errors of observation made during the measurement of geometric quantities. These errors are first divided into three classes: coarse, systematic and random.

Thereafter the theory of errors is concerned only with the properties of the random errors, on the assumption that the other two classes have already been eliminated by the application of suitable measures.

Such a classification is favourable under the assumptions that apply in surveying: the errors are small compared with the size of the quantities measured, so that their frequency distribution is independent of the size of the quantity being measured. Moreover, the size and distribution of the errors can be determined directly, since large series of measurements of similar quantities made with the same instrument are available. At the same time the quantities measured are always simple geometrical values which can be reduced to clear, physical concepts.

Nevertheless, this classification is a purely practical one, that is its foundations are wholly empirical. Modern statistics, in which the theory of the small errors of observation constitutes a special case, assumes no such distinction a priori. It strives, instead, to determine not only the frequency and size of the errors, but also their causes.

For further discussion it is better to introduce the concept of deviations in place of errors. As in the theory of errors, deviations are defined as the difference between design and observed (measured) value of a given magnitude:

$$\varepsilon = A_t - A_g$$

All we can learn concerning the unknown deviations with A_t are facts of a statistical nature: measurements of actual properties can be carried out on existing or ruined buildings, or on specimen structures. These are usually much more accurate than any later assumptions (statistical estimates) will be, but they are not referred to the same object, i. e. to the structure projected for the future. The results of such measurements are incorporated in frequency diagrams (histograms), which can be approximated by suitable analytical functions (distribution functions).

If statistical methods are applied to the construction as a whole, there are three reasons why it is impossible to obtain satisfactory, and in many cases even usable results.

1. There are properties of a construction which cannot be observed at all before the construction exists (behaviour of the foundation, effects of neighbouring buildings, etc.). Nevertheless these, too, have to be represented by estimates in any calculation of the eventual bearing behaviour. The deviations thus introduced further impair the accuracy and reliability of the results of the structural calculation. However, they do not always arise, and for the present will be disregarded.

2. The usual construction is a function of a great many properties of different kinds (parameters). Almost always there is only a single, or at best a few similar structures available for observation. However, if we want to get results with statistical methods which are reliable enough to permit drawing of conclusions for future constructions on the basis of analogy, one must have recourse to the "law of large numbers", i.e. the number of observations must be as large a multiple as possible of the number of parameters to be determined.

Individual properties can be observed singly. In such cases a measurement usually leads to a useful result. This holds true for certain strength properties of the building materials, and for climatic quantities.

Other structural properties, however, are so closely intertwined that the expenditure necessary for the many observations is practically no longer manageable. Furthermore, various quantities, for example the behaviour of the structure in the failure region which is so important from a safety point of view, can only be observed in conjunction with the destruction of completed buildings. Buildings are always being demolished, of course, but generally speaking these have been erected by earlier building techniques, and thus, in a time of rapid change in the technology of construction, do not yield a great deal of information. Attempts have been made to overcome this difficulty - i.e. the lack of items for statistical observations, by the application of test procedures to specimens. However, since the latter are produced and tested under conditions very different from the real ones occurring in the structure, the results of such tests cannot be accepted immediately as reliable (e.g. strength of concrete specimens).

3. Among the deviations occurring in the construction properties, there are almost always some which are caused not by "standard" influences, but which occur just at one specific place in one specific structure. Generally speaking such deviations must be ascribed to the direct human influence. As stated in the introduction, statistical methods fail wherever the human will comes directly into play, and this holds true for almost all properties of structures. The exceptions are few and structures in which everything can be regarded as ramdom are still fewer. As long as we do not possess as complete information on the principles of human behaviour as we do about certain fabrication processes or natural occurrences, this

objection to the application of statistics to safety problems for structures must stand. We shall return to this extremely important point in subsequent sections (4.4; 5.2.15).

In the classical theory of errors, errors are distinguished according to size (coarse errors) and according to sign (systematic errors) for the practical requirements of calculating errors and corrections. This must be supplemented by another problem, namely the question of the cause of the errors, which to a certain extent, of course, is implicit in the theoretical classification. Where safety is concerned, however, this aspect takes on great importance, so that closer attention must be given to it.

We distinguish between:

1. Deviations not affected directly by human beings. These are to some extent unavoidable, and we shall refer to them hereinafter as random.

2. Deviations due directly to human failure - in the form of negligence, ignorance, whim. These can largely be eliminated by suitable measures, which, of course, entail a corresponding expense. These will all be lumped together as coarse errors.

We shall not undertake any subdivision of systematic errors here. This can be justified qualitatively as follows. Systematic deviations affect each item in a series of observations in the same manner. Hence, they do not properly belong among the deviations at all but are actually properties of the items in question. They generally stem from the method of measurement or the means of measurement employed, and since they do play a quantitative role, they must be eliminated by the improvement of both methods and means. An example would be the testing of specimens of concrete. The means of measurement are probably good enough; the strength of the specimen can be measured very accurately. Systematic errors arise in the application of the data obtained to the actual structure. Basically, they can be attributed to a defective analogy, and the result is that the testing of the specimens for their strength is often of little value.

The classification of errors with respect to their causes can probably not be carried out more sharply than is done with the assumptions of the theory of error. Nevertheless it will be more suitable for further discussion.

We shall also try to determine the mathematical properties of the two classes of errors (end of this Section). For the present we shall consider the cause of a coarse or avoidable error as being simply a departure from a rule, or carelessness in the erection and use of the structure.

For random deviations distribution functions can as a rule be determined. For coarse errors we must assume that they can acquire arbitrary values within wide limits.

This is very evident in all construction accidents. In almost every case the effect of human unreliability has ultimately been found to be at the root of a coarse error, which then leads to the failure of the structure. The investigations arising out of this are carried out on the initiative of the judicial authorities who determine the responsibility, and only in the rarest of cases do they find the cause of a disaster to be anything other than human action and negligence.

However, we are not concerned here with the legal aspects of such occurrences, but rather how to avoid them. The judicial interpretation of an accident can only take place after it happens. It is a matter of hindsight. The purpose of the safety verification, however, is to formulate a prejudgment for any structure that will provide a picture of its future behaviour and give grounds for the prediction that it will carry its load. For this purpose we must devise methods that provide information on the occurrence of deviations and prevent these from becoming so large as to cause accidents.

The question of the nature of such methods can be answered quite clearly and simply for the two classes of deviations separately:

- Against unavoidable random deviations, safety margins offer protection.
- Against the consequences of human incapacity or coarse errors, inspection measures are used.

Since no exact criterion is known by which the two classes of deviations could be separated, this brief recipe is correspondingly vague. Nevertheless, it can be shown to be more or less logical to follow this plan.

As a measure of the random deviations we use the "mean square deviation" (mean error) which can be estimated from a series of observations by the formula:

$$\sigma_x = \sqrt{\frac{\Sigma(\bar{\chi} - \ell_i)^2}{n-1}} \tag{19}$$

where $\bar{\chi}$ is the median of the distribution function, ℓ_i the individual observation, n the number of observations. Where a forecast for the future is required we put in place of it an expectation value which is extrapolated from observations analogous to the future construction. This method of error estimation has proved very effective in the calculation of adjustments by the method of least squares (Gauss) and in statistics, because it leads to simple calculations and has certain additional advantages over other methods. The median of a distribution is given by the rule that half of all measured values must be smaller, and the other half larger. For symmetrical distributions it coincides with the mean value, and for distributions which appear similar to the Gaussian standard distribution, it coincides with the mode(maximum frequency).

In many cases one relates the mean error to the measured value (median), and obtains the coefficient of variation.

$$v_x = \frac{\sigma_x}{\bar{\chi}} \tag{20}$$

This is suitable for certain operations, and for quantitative discussions it is better than the mean error itself.

Most structures have many different properties which appear in turn as parameters of the structural calculation or dimensioning. In order to avoid getting lost in the consideration of these parameters and their deviations, it is necessary to arrange the parameters into categories according to certain principles. For the following four groups of parameters we shall assume that their distributions are mutually independent, i. e. there is no correlation between any two groups. The four basic parameters are:

1. Strength properties of the structural materials;
2. Geometry of the structure, structural system and dimensions of the cross-sections;
3. Loads;
4. Behaviour of the foundation.

These influence the bearing behaviour of the structure directly, and large enough deviations can occur in each of the four groups to affect the safety.

From recent investigations of individual parameters (especially strength properties and loads) certain conclusions with respect to their statistical properties can be reached(41-45).

Since the deviations which have a bearing on the safety are always of considerable size (coefficient of variation of 5% and more), the assumption of symmetry of the distribution functions which is applicable in the theory of small observation errors cannot be accepted here. This is to be expected, since for most construction properties the value zero signifies a boundary on one side, whereas symmetrical distributions are generally unbounded in either direction. As a rule, then, skewed distributions are introduced, the third moment of which

$$m_3 = \frac{\Sigma(\bar{\chi} - \ell_i)^3}{n} \tag{21}$$

does not vanish. The fact that certain quantities in construction (strength of materials, loads, etc.) must have a skewed distribution is also confirmed by corresponding histograms. An example of an asymmetrical distribution function is the logarithmic-normal distribution of the form:

$$f(\chi) = \frac{1}{\sigma \log \chi \cdot \chi \cdot \sqrt{2\pi}} \cdot \ell\chi p\left(-\frac{1}{2}\left(\frac{\log \chi - M \log \chi}{\sigma \log \chi}\right)^2\right) \tag{22}$$

which is transformed into a Gaussian normal distribution by taking the logarithm of the χ- axis.

Before we come to the discussion of safety on the basis of error distribution, we must recapitulate what little is available about those deviations which are caused by human carelessness, etc. Since these "coarse" errors occur everywhere and in many forms, no one has yet succeeded in collecting sufficient data to permit the drawing of conclusions that would go beyond purely quantitative, common-sense explanations.

If it be assumed that the coarse errors have a distribution, then at any rate certain statements may be made.

As in the case of the random deviations, large coarse deviations must be less frequent than small ones. This is obvious. The greater an oversight, the sooner it is perceived and corrected.

The distribution of coarse errors must also converge towards zero on both sides. This fact, of course, is really implied in what was stated previously, but it can be more specifically supported. Generally speaking, it can be shown that the parameters of the structure, as physical phenomena, are limited in their quantitative value, because it is technically impossible to go beyond certain limits. For example, where the zero point is a reasonable boundary, this is obviously true also for the coarse error. All strength values are in this category. Similar boundaries can also be formulated for loads: a bridge cannot be crossed by vehicles heavier than those already found on the roads of the country; one cannot put anything more into a full container. Similar statements can also be made directly, for example, about the bearing capacity of a structure with a loading test : the structure which withstands the test will not break under a smaller load, disregarding long-term effects, of course.

Further information about the coarse errors could be obtained from random samples, which can be observed with and without the influence of coarse errors. The difference between the two distributions is then the effect of the coarse errors.

This procedure has already been applied, e.g. to the distribution of strength of concrete cube specimens (42) which were made with carefully inspected and poorly inspected concrete. From the two histograms very basic differences appear, especially in the width of scatter of the specimen strength. This indicates that the following conclusion is correct: the majority of large (coarse) errors are avoidable and stem from lack of care on the part of human beings. We shall come to the same conclusions when we consider structures as a whole (4.4).

4.2 Deviations and Safety

In what follows we shall determine the relationship between random i.e. statistically represented errors and safety. As a working hypothesis the coarse deviations will be regarded as non-existent up to and including Section 4.3.

The content of the safety inequality

$$T > P \tag{23}$$

can be written in terms of a single quantity, for example the safety factor:

$$S > 1 \tag{24}$$

The damage E' or

$$T < P \tag{25}$$

is then present, if

$$S < 1 \tag{26}$$

Thus, from the value of the safety factor conclusions may be drawn concerning the safety of the structure. If the actual safety factor is known, then it will be evident whether the structure to which it applies will suffer damage or not. However, the safety factor with

$$S = \frac{T}{P} \tag{27}$$

is also a function of the properties of the structure, and the question now arises whether we can draw conclusions concerning the distribution of the safety factor from the distribution of the properties. For the present this will be assumed, so that, before proving it, we can first determine its consequences.

That is, if the distribution f (S) of the safety factor is known, then by a simple integration:

$$F(S*) = \int_{-\infty}^{S*} f(S)dS \tag{28}$$

the probability function, can be determined which for every value of S* gives the probability that S will be less than the value of S*:

$$F(S*) = W(S \leq S*) \tag{29}$$

Putting the critical value one for S*, we obtain the safety from:

$$1 - F(1) = 1 - W(S \leq 1) = W(S > 1) = W(E) \tag{30}$$

It can therefore be derived from the distribution function of the safety factor (Fig. 6).

Now, writing the safety factor as a function of the parameters of the properties of the structure:

$$S = S(\alpha_1, \ldots \alpha_i, \ldots \alpha_n) \tag{30a}$$

the distributions $f(\alpha_i)$ which are assumed to be known, the probability function of the safety factor can then be calculated by the formula:

$$F(S*) = \left\{ \int \ldots \int \left[\Pi_1 (f(\alpha_i)d\alpha_i) \right] \right\}_G \tag{31}$$

where the region G in the n-dimensional space of α_i is bounded by the inequality:

$$S(\alpha_1, \ldots .\alpha_n) \leq S^* \tag{32}$$

Differentiating with respect to S^*, we get from this the distribution function of the safety factor

$$f(S^*) = \frac{d}{dS^*} (F(S^*)) \tag{33}$$

Thus, for each structure it is possible to calculate the degree of safety at which the distributions of the properties of the structure and their relationships to the safety factor are given (Formula 30a). A simple example may be used to illustrate the basic mathematical procedure:

Example: A statically-determinate reinforced concrete beam is subjected to a bending moment from a uniformly distributed load. At mid-span it amounts to

$$\overline{M} = \frac{p \cdot \ell^2}{8} \tag{34}$$

The strength of a cross-section at mid-span is given by the plastic moment (collapse criterion):

$$\overline{M}_p = \sigma_F \cdot Fe \cdot h \left[1 - 0.6 \frac{\sigma_F \cdot Fe}{b \cdot h \cdot \beta_D} \right] \tag{35}$$

From this the safety factor can be calculated:

$$\overline{S} = \frac{\overline{M}_p}{\overline{M}} = \frac{\sigma_F \cdot Fe \cdot h \left[1 - 0.6 \frac{\sigma_F \cdot Fe}{b \cdot h \cdot \beta_D} \right]}{p \cdot \ell^2 \cdot \frac{1}{8}} \tag{36}$$

Let its numerical value be

$$\overline{S} = 2.0$$

Let all quantities be introduced by their medians (design value). Thus the design safety factor is equal to the median of the distribution. It will be referred to hereinafter as "nominal value". The elements of S with the most scatter are:

- the effective depth h
- the yield stress of the steel σ_F
- the assumed load p

Deviations of the other values are either unimportant (Fe, ℓ) or for formal reasons have little effect (b, β_D). They are neglected. Similarly, the scatter of the second term inside the bracket is disregarded, because it is small compared with unity and therefore has a small effect on the errors.

The variable elements can be summarized as follows:

$$\varphi = \frac{\sigma_F \cdot h}{p} \tag{37}$$

and similarly the invariable part:

$$A = \frac{8 \cdot Fe}{\ell^2} \cdot \left(1 - 0.6 \cdot \frac{Fe \cdot \sigma_F}{b \cdot h \cdot \beta_D} \right) \tag{38}$$

In order to avoid confusion, the variable quantities are renamed:

$$\sigma_F = \alpha_1$$
$$h = \alpha_2$$
$$p = \alpha_3$$

and we can now write the safety factor:

$$S = \frac{\alpha_1 \cdot \alpha_2}{\alpha_3} \cdot A \tag{39}$$

Let the distributions of the three variables be:

$$f(\alpha_1) = \frac{1}{\alpha_1 \cdot \sqrt{2\pi} \cdot \sigma_{\log \alpha_1}} \cdot \exp\left(-\frac{1}{2} \left(\frac{\log\alpha_1 - \mu_{\log \alpha_1}}{\sigma_{\log \alpha_1}} \right)^2 \right) \tag{40}$$

$$f(\alpha_2) = \frac{1}{4\sqrt{3}} \qquad \alpha_{2\,min} \leq \alpha_2 \leq \alpha_{2\,max} \tag{41}$$

$$f(\alpha_3) = \frac{1}{\alpha_3 \cdot \sqrt{2\pi} \cdot \sigma_{\log \alpha_3}} \cdot \exp\left(-\frac{1}{2} \left(\frac{\log \alpha_3 - \mu_{\log \alpha_3}}{\alpha_{\log \alpha_3}} \right)^2 \right) \tag{42}$$

with parameter values:

$$\begin{array}{ll} \mu_{\log \alpha_1} = \log 3.6 & \sigma_{\log \alpha_1} = \log \left(\frac{20}{19}\right) \\ \alpha_{2min} = 30 - 2\sqrt{3} & \alpha_{2max} = 30 + 2\sqrt{3} \\ \mu_{\log \alpha_3} = \log 0.1 & \sigma_{\log \alpha_3} = \log \left(\frac{12}{11}\right) \end{array} \tag{43}$$

The distributions of α_1 and α_3 are logarithmic-normal. From histograms of this quantity this appears reasonable for the yield stress of the steel. For the load, the logarithmic-normal distribution can be proposed as approximating the distribution of the extreme value in the absence of more accurate information. The effective depth is distributed rectangularly, corresponding more or less to the case of a tolerance check with limits of ± 2 cm. The nominal values of the three variable quantities, with

$$\bar{\alpha}_1 = \bar{\sigma}_F = e^{\log 3.6} = 3.6\,[t/cm^2] \tag{44}$$

$$\bar{\alpha}_2 = \bar{h} = 30\text{ cm}$$

$$\bar{\alpha}_3 = \bar{p} = e^{\log 0.1} = 0.1\,[t/m^1]$$

correspond more or less to the usual conditions, as do the coefficients of variation

$$\begin{aligned} v_1 &= e^{\sigma \log \alpha_1} \cong \pm 5\% \\ v_2 &= \pm 6.7\% \\ v_3 &= e^{\sigma \log \alpha_3} \cong \pm 8\% \end{aligned} \tag{45}$$

Here $\mu \log \alpha_1$ is the logarithm of the median of α_1, $\sigma_{\log \alpha_1}$ the mean quadratic deviation of the logarithm from the median.

We now get the probability function of the safety factor from the formula

$$F(S^*) = \int_{-\infty}^{S^*}\int\int f(A \cdot \alpha_1) \cdot f(\alpha_2) \cdot f(\alpha_3) \cdot d\alpha_1 \cdot d\alpha_2 \cdot d\alpha_3 \tag{46}$$

Since this calculation cannot be carried out by analytical methods, it was solved by a numerical procedure on a computer. The result is represented in Figure 7. For comparison, the normal distribution and the Chebyshev inequality for the same parameters (median, coefficient of variation) are also shown. A logarithmic scale is chosen for F(S) so that the value of the function remains clear even for small values of S.

The two comparison curves bear different relationships to the "actual" distribution of S:

The Chebyshev inequality constitutes an absolute upper limit for all distributions. However, it is unusable for this discussion, because it is still very high even for small values of S.

The Gaussian normal distribution is a better approximation. However, as will become evident in the example of Section 4.3., it also leads to impossible conclusions concerning the necessary margin of safety. Nevertheless, as Fig. 7 shows, it provides an upper bound for small values of S. Moreover, since it results in simple calculations we shall employ it as far as possible in further discussions(223).

The advantage of an approximation is that estimates of the safety can be calculated from statistical data on the parameters, without as much effort as in the example shown which applies to the empirical distributions directly. The symmetrical normal distribution can in general be regarded as an upper limit for the cases involved in structural engineering, because the data of structures are usually distributed asymmetrically with a limit at the zero point. This means that if the normal distribution is employed, the safety is underestimated, and such an approximation is therefore on the safe side.

A much closer approximation, of course, is the logarithmic-normal distribution, which will be taken up again in what follows (4.3).

4.3. Safety as a Design Parameter

Until now we have been dealing with the actual safety verification, in which the safety of a structure is determined after its data (nominal values, scatters) have been established.

Since in this sense safety verification is a judgment a posteriori, which can only be applied to a complete structural calculation, an iteration process is necessary for final dimensioning, so that the first results of the safety verification must be followed by a new structural calculation with different data, until the safety requirement e.g. in the form of a safety factor, is satisfied. Even cases where the required safety factor is exceeded have to be recalculated for economic reasons. Where the calculations are extended, however, such an iteration process is very laborious and time consuming. As a consequence the question has long been asked whether safety can be so formulated as to be incorporated in a direct dimensioning process, not requiring a repetition of the work of calculation. More clearly speaking, this means that the safety might be introduced as an explicit dimensioning parameter along with the other data of the projected structure.

The basic flow charts generated by these questions are shown in Figures 8 and 9. It is easy to see from these diagrams that direct dimensioning involves a great deal less work.

If direct dimensioning is applied the problem of the structural calculation is reversed: also, although in the trial and error method (safety verification) the dimensions of the structure are selected as initial values of the iteration procedure in the direct dimensioning process they are left open to variation. Instead the safety, or a value derived therefrom, is prescribed and must then be satisfied by varying the dimensioning values(effective depth, percentage of reinforcement, etc.).

Since the safety factor is the simplest form of safety coefficient, it will be used as a basis for the rest of the investigation. It can easily be converted into other comparative coefficients.

The safety of a structure is determined even before the data are chosen. However, this does not hold for safety coefficients (margins of safety) which depend directly on the static properties of the parameters according to 4.1. These, too, must first be assumed, before the actual process of dimensioning begins. The safety coefficient can then also be determined assuming of course, that the relative deviations coefficient of variation of the data, varied for dimensioning purposes, are independent of its nominal value. This assumption is not correct in particular for the cross-sectional dimensions (5.2.12). For this influence, therefore, a special gradation of the safety margins is required .

The attempt to derive safety coefficients from the statistical properties of structures is based on relations which were derived in the foregoing section. For the sake of simplicity we at first employ the Gaussian normal distribution. Where this is no longer a good approximation, we shall replace it by the logarithmic-normal distribution.

The true value of the safety factor can be regarded as a sum of the nominal value and deviation:

$$S = \bar{S} + \Delta S \tag{47}$$

The deviation ΔS is purely a random quantity in accordance with the working hypothesis of 4.2. Its distribution can be calculated from the distributions of the parameters of the safety factor (46). We now wish to determine the nominal value $\bar{S}$ which with the deviation permits the true value S to become greater than unity (24) for a sufficient confidence.

We represent the deviation as a multiple of the standard deviation.

$$\Delta S = g \cdot \sigma_S \tag{48}$$

and finally solve for g, which is related to the distribution of S.

For example if we decide, as a safety rule that every millionth construction, on the average, may be allowed to fail from random causes, then from a table for the error integral (normal distribution(145):

$$g \sim 4.5$$

and we obtain the necessary margin of safety from the relation:

$$1 \leq \bar{S} - g \cdot \sigma_S = \bar{S} - 4.5\, v_S \cdot \bar{S} \tag{49}$$

to

$$\bar{S} \geq \frac{1}{1 - g \cdot v_S} = \frac{1}{1 - 4.5 \cdot v_S} \tag{50}$$

For this value, according to the table

$$F(1) = W(S \leq 1) = 10^{-6} \tag{51}$$

as decided. Thus the problem, fundamentally, is solved. We demanded a certain confidence level (safety), and gave the distribution function of the safety factor. From this we found the necessary nominal value of the safety factor. This can now be substituted in the dimensioning like any other parameter, for example, the assumed load, and the bearing capacity is brought by suitable variations of the cross-sectional values or material properties to a value which satisfies the dimensioning conditions

$$\bar{P} \cdot \bar{S} \leq \bar{\bar{T}} \tag{52}$$

From the expression for the safety factor (50), however, it now becomes apparent that the value of $\bar{S}$ increases very rapidly for higher variation coefficients v_S. If the variation coefficient is precisely

$$v_S = \frac{1}{g} \tag{53}$$

an infinitely large safety margin would have to be used, in order to attain the desired safety. This obviously does not correspond to actual conditions, and consequently it is clear that the symmetrical normal distribution is no longer applicable as an approximation. We therefore replace it by the logarithmic-normal distribution and obtain, in accordance with (50) the equation for determining the nominal value of the safety factor:

$$\exp\,(\mu_{\log} S - g \cdot \sigma_{\log} S) = 1 \tag{54}$$

with

$$\mu_{\log S} = \log \bar{S} \tag{55}$$

This leads to

$$1 = \exp\,(0) = \exp(\log \bar{S} - g \cdot \sigma_{\log S}) \tag{56}$$

and the necessary safety factor is obtained as

$$\bar{S} = \exp\,(g \cdot \sigma_{\log S}) \tag{57}$$

For comparison the relation to the safety factor for the normal and logarithmic-normal distribution is represented in Fig. 10 for two different coefficients of variation.

The choice of the logarithmic-normal distribution as a better approximation of the "actual" distribution derived from the different distributions of the individual parameters, needs a still better basis than has hitherto been given. The central limit theorem states: if A be the sum of n quantities which may have any distributions at all, the distribution of A comes closer and closer to a normal distribution with increasing n. Similarly, the following may be derived: if B be the product of m factors, which may also have any distributions, then with increasing m the distribution of B comes closer and closer to a logarithmic-normal distribution. The central limit theorem of course, holds true only under certain, but rather broad conditions especially on the skewness of the distributions involved.

The question now is whether these results can also be applied to the safety factor. And in fact, the safety factor generally can be written as the product of its parameters. This is shown in example 4.2. for the case of pure bending. If we assume that factor A of the non-scattering elements nevertheless possesses a distribution, the safety factor is a product of four terms

$$S = A \cdot \sigma_F \cdot h \cdot \frac{1}{p} \tag{58}$$

Departures from the asymptotic logarithmic-normal distribution may also be due to the fact that m = 4 is too small a number. However, since some of the individual distributions of the parameters already resembled logarithmic-normal ones, this limitation is compensated for.

The strongest objection that can be made against the approximation using the logarithmic-normal distribution has to do with such stresses as pure bending, which act on inhomogeneous cross-sections (reinforced concrete). In this case two different materials are added together to produce the bearing capacity of a cross-section, and the safety factor contains a sum. In most cases, however, one building material will outweigh the other in the sum, and thus its distribution will determine that of S, in which case the product function is again approximated (see for example for axial pressure in 4.5.1).

Finally, let us now give the two rules for the transfer of the errors of the parameters to that of the safety factors:

For standard distribution, we may write, according to the method of least squares:

$$\sigma_S = \left(\sum_1^n \sigma_i^2 \right)^{\frac{1}{2}} \tag{59}$$

For the logarithmic-normal distribution, accordingly,

$$\sigma_{\log S} = \left(\sum \sigma^2_{\log \alpha_i} \right)^{\frac{1}{2}} \tag{60}$$

The two equations will be found in the literature under the name "law of error propagation" or " theorem of the addition of variants" (σ^2). In this simple form they hold true only for non-correlated quantities α_i.

With the determination of the necessary safety factor from the statistical data of the structure, a feasible path has been found of determining the safety margins on the basis of statistical principles. Of course, two important conditions must be satisfied, so that this path will lead to correct results.

1. The statistical data must be given with sufficient accuracy. At the present time, however, this can rarely be expected. Primarily, we would need much better information on loads and tolerances, and on certain properties of the foundation. For the present, therefore, the applicability of the statistical method is restricted to a very small and special category of structures: systems which are highly indeterminate (connected in parallel); simple cases of prefabricated parts under a load that has been studied statistically (wind, snow, hydraulic discharges, etc.).

2. The method can only be used when the applied distribution functions also hold for the extreme deviations. This has not yet been proved by observations even for purely random parameters. Moreover, it is precisely the large, rare deviations in which the generally very strong influence of the coarse (human) errors must be expected. In order to investigate this objection in somewhat greater detail, the next section deals specifically with the extreme deviations.

4.4 Extreme Deviations, Empirical Values

For certain structural parameters statistical results are available. Others can be secured at a reasonable cost. However, they relate to comparatively small samplings - a few hundred to a few thousand. For the distributions obtained, this means that good information may be available concerning the most common values close to the median or mean value, but not the extreme ones, one tail of which is of decisive importance for the problem of safety. Such information could only be gained if much larger samplings could be taken. Their necessary size is approximately inversely proportional to the relative frequency of those deviations about which one would like to have information and whose probability is of the same order to magnitude as the frequency of the cases of damage W (E') which one is willing to accept. If the safety is to be of the order of $1\text{-}10^{-5}$ or $1\text{-}10^{-6}$, then samplings would have to be available embracing at least 10^5 to 10^6 measured values in order to have a good idea of the frequency of such rare occurrences. This is possible, however, only at an extremely high technical cost, and therefore will probably have to be foregone for the present in the field of structures.

Hypotheses can of course be constructed on the frequency of rare events in the theory of extreme value statistics(12, 13). This can even be done on the basis of distributions which have been determined from smaller samplings, by considering only the behaviour of the extreme values of the distribution. Such hypothetical approximations, however, are really useful only when they can be supported by a very great deal of observation material, for example, for long-term measurements of climatic quantities, river discharges, etc. A suggestion was made by Weibull for the distribution of the extreme deviations in the strength of building materials, derived from elementary strength considerations. It has the following form (12, 13):

$$F(\alpha) = 1 - \exp\left(-\left(\frac{\alpha}{a}\right)^k\right) \quad k > 0 \tag{61}$$

where α is the variable (strength quantity), a the expected extreme value.

For most structures, however, such expressions are inapplicable for two reasons:

1. Besides the well-known parameters there are others forwhich no equivalent information is available, but for which deviations of the same order of magnitude must be expected. There is then not much sense in introducing an influence with higher accuracy when other quantities are represented only with rough estimates.

2. In all structures the influences which generate large deviations outside the "natural" scatter, play a decisive part. These are again the "coarse" errors, about which we have already spoken. According to their effects, probably individual parameters can be assigned to them (overloading, design errors, defects of material, errors in the design calculations, etc.). Their cause must be sought outside the structure in the personnel executing the construction (designers, contractors, masons, etc.), and finally in the user.

For these two reasons there is no point to a further discussion of the particular extreme values for which information is available; let us turn rather to the consequences resulting from the fact that the statistical results and general considerations do not suffice to determine the margin of safety for the general case of the usual type of construction.

There are two problems to be considered:

1. Is it possible to get information on safety which will already incorporate the influence of the coarse errors?

2. What is the best way of dealing with coarse errors? i.e. how can their frequency and magnitude be reduced, and what are the likely consequences with respect to the choice of a margin of safety?

In what follows we shall discuss the first of these questions. The second one will be dealt with in more detail below. (5.2.15. ff).

Construction as a whole can also be regarded as a population in the statistical sense, which can be observed. Without referring to specific events, we find in all structures which have hitherto stood or which are still standing, a qualification which is closely related to safety, namely the relative frequency of failures among structures. This is a direct measurement value with respect to safety on a very large sample.

It is not possible, of course, to formulate exact numerical values in this, because all failures do not have equal significance, and not all are accessible to investigation. However, the "general experience" can be very well summarized in the form of estimates.

On the other hand, we know by what methods and with what margins of safety construction has been practised at any given time in the past. For example, since 1951 a collapse safety factor of 1.8 for bending elements reinforced with high grade steel has been used. For buildings put up since that time, therefore, it may be assumed that they have been dimensioned with this requirement in mind. These will form the basis of discussion of further examples.

The value 1.8 is designated in the Standard (53) as the "minimum collapse safety". In this connection it may further be remarked that the term "safe" is not admissible for the safety factor or for any other comparative safety value. If structures put up by elastic methods are recalculated using the ultimate load procedure, on the average a somewhat higher safety factor, namely $\bar{S} = 1.9$, is obtained. This value may be considered a median of the distribution of the "actual safety factors" of bending elements.

From this information a second value is also accessible, namely the relative frequency of structures which have failed for any reason at all. It is between 10^{-3} and 10^{-5}. This includes all cases of negligence, unreliability and other coarse errors. Accurate numerical values are very difficult to obtain, since for understandable reasons not all cases of damage are made public. Furthermore, before such figures can be formulated, it would be necessary to carry out an evaluation of the individual events (it is not the same if a major bridge or merely a secondary element of a high building fails). For spectacular, far-reaching accidents one can always deduce a value which lies between certain limits. Generally speaking serious accidents of this kind do not become public knowledge. *

From the statistical data on the properties of structures we can, for example with the aid of a logarithmic-normal distribution as an approximation, calculate the distribution curve of the purely random deviations for the safety factor. It is only a hypothesis, of course, if we employ this particular distribution for the rare random deviations as well. However, this hypothesis has some support from the results of the two preceding sections.

Using formula (35) for calculation of the bending resistance for the plastic moment, and putting the following estimated mean values for the coefficients of variation

$$v_1 = \pm 8\%$$

$$v_2 = \pm 5\%$$

$$v_3 = \pm 5\%$$

---

* Experience in North America tends to indicate the opposite. (Trans.)

we get for the "random" fracture probability with

$$g = 5.8$$

the value

$$W(E') = W(S \leq 1) = 1.3 \cdot 10^{-8} \tag{62}$$

Because of the vagueness of the assumptions made, this result, of course, is a rather crude estimate. The choice of numerical values employed must therefore be reviewed briefly. For the coefficient of variation of strength a mean value is taken from a series of American tests on the yield limit of reinforcing steel. This value is generally responsible for the resistance of beams subject to bending stresses. For accuracy of size (geometry of a cross-section, v_2), which depends strongly on the dimensions of the members, a scatter of 5% is surely too high for large cross-sections. For small members, which constitute the majority, however, such an uncertainty must be employed, especially for the effective depth (location of steel). The estimation of the coefficient of variation of loading (v_3) should actually be preceded by an extreme value calculation. Very different standards are employed for different kinds of loads. For example, for the loading of residential houses a value is prescribed that is generally much too high and which in reality is never attained. On the other hand, estimates of loads due to natural forces are usually based in part on meteorological observations spread out over a very long time. In other cases, for example warehouses, the posted load also represents the assumed load, without taking specifically into account the fact that excess loading may very well occur. The coefficient of variation of 5% is therefore simply a convenient substitute for the extremely variable types and magnitudes of deviations of the assumed loads.

In contrast to the example of Section 4.2 in this case the logarithmic-normal function was used for all distributions in order to simplify the calculation.

Since from here on the discussion is in terms of magnitude, however, these assumptions, some of which are only rough approximations, will not too greatly affect the conclusions which are to be drawn from them.

For example, if we compare the two values for the probability of collapse as derived from general experience ($W[E'] \sim 10^{-3}$ to 10^{-5}), and as determined from random deviations (10^{-8}), the difference becomes very clear in a quantitative way.

Thus, many more cases of failure occur than would be expected from random deviations of the structural properties. This result confirms the supposition of Section 4.1., that almost all failures are the direct result of human shortcomings.

Consequently, we can base another conclusion on what corresponds essentially to the brief formulation about the effect of inspections (4.1.):

Margins of safety which would give better protection against coarse errors would have to be made disproportionately high. This would lead, in turn, to unjustifiable costs. Instead of increasing the safety factors, therefore, means must be sought to eliminate these errors as far as possible. The best such means, of course, are inspections.

Coarse errors, with respect to their kind and manner, size, sign and place of occurrence, are almost unlimited. It is therefore reasonable to say that they cannot be eliminated by simple, stereotyped measures. Inspection procedures must be varied depending on the kinds of error that may be expected. Errors in the structural calculation can be detected only by means of mathematical checks, which, however, must not be restricted to mere arithmetical checks. There must be genuine checks, i.e. procedures that are as far as possible independent of the original calculation. Rough errors in the dimensioning of supporting members, on the other hand, can be eliminated by simple visual inspection, or by checking the measurements with the simplest methods, etc.

The following principle applies: the checks applied must be as varied as the error possibilities. Admittedly, for technical reasons, this principle is never wholly satisfied, but must remain as a guide line. In the present paper we shall have to forego considering inspection and checking measures in detail, because this would involve a recapitulation of the whole technology of construction, and the most important feature of inspection, namely the care with which it is carried out, must always be left to the individuals involved. Therefore, there is not much point to issue regulations on inspection except where generally essential routine verifications are concerned.

As a rule the checks are not difficult, but in many cases involve additional cost and loss of time which are worthwhile only when we consider all structures together.

4.5 Formulation of Safety Margins

In order to facilitate the practical application of safety margins, and at the same time to make them adaptable in as many ways as possible to the individual conditions, the basic form, i.e. the simple safety factor, was broken down into components. These were to be assigned variously to the individual design values (structural properties) in order to eliminate their uncertainties. A larger safety margin is assigned to an uncertain value than to one in which the magnitude is known very exactly(70). Other information is also incorporated in it, for example, the relative importance of the structure, expressed as the degree of seriousness of the consequences of any collapse. Thus, greater safety margins are demanded for steel railroad bridges than for buildings, etc. When we study the variation of safety margins from these points of view, we are compelled, on account of the variety of individual arguments, to replace the simple safety factor by a system of partial safety margins which can be combined in various ways to produce the desired variation.

Obviously, load and strength are quantities which are largely independent of each other. Nor are the strength properties of different structural materials influenced by each other, so that from this standpoint a separate partial safety factor should be introduced for each such group of parameters.

In what follows we shall briefly review the main possibilities for analysis of the safety factor, in particular outlining the formal arguments. It would lead us too far afield to discuss all the possible applications in detail.

Since the simple safety factor $\bar{S}$ is to be dealt with in greater detail, it will not be considered in the comparative considerations of this chapter. It is a special form of safety margin and need not be introduced for comparisons of the different systems of partial factors.

4.5.1 The material factors

Construction materials have various statistical properties, as may be expected in view of the different materials and different manufacturing methods. Since the strength of a material is one of the most important safety parameters, special factors have been introduced in order to compensate for the expected deviations right at the beginning. These factors reduce the mathematically assumed strength value by an amount that corresponds roughly to the expected extreme deviation. These are the material factors m_i .

Their value was determined primarily from general experience, and especially from earlier safety regulations (U.S.A., U.S.S.R. 117,121). When it is stated that the material factors have been determined solely from statistical considerations, this must be considered at the very least doubtful, because, as we shall see below, it is not possible to get agreement between the laws of error propagation and a system of material factors.

In addition to the actual material properties and their statistical significance, other conditions are usually taken into account by the material factors as well, namely the importance of the structure in question, geometric uncertainties, load assumptions, etc. Depending on how many of the material factors have been assigned, they either embrace the entire safety margin or only a partial one which then must be supplemented by other coefficients, for example, load factors.

This raises a first difficulty. The material factors have to take into account influences which do not correspond to their function. Their determination, and probably also their classification thus becomes very difficult. Of course, where we have the case of a structure constructed from a single material, the use of material factors will become, from the formal point of view, once more equivalent to the single safety factor, and the disadvantage is not incurred. In the case of reinforced concrete, where two fundamentally different structural materials are involved, this is not so simple. Therefore, a brief formal investigation of this case will be carried out.

For the sake of simplicity we assume that the yield stress (σ_F) of the steel, and the compressive strength (β_D) of the concrete are the only parameters which may have substantial deviations. The assumed load shall be taken to be so accurate that no errors resulting therefrom are of any consequence.

For the formal investigation we employ an example: we shall determine the strength of a cross-section axially loaded under a compressive stress. The rated value is:

$$T = T_{concrete} + T_{steel} = F_B \cdot \bar{\beta} + F_S \cdot \bar{\sigma}_F \qquad (63)$$

The coefficient of variation of the ultimate load, which is also that of the safety factor, is calculated by the "law of error propagation":

$$\begin{aligned} v_S &= \frac{1}{\bar{T}} \cdot \sqrt{\left(\frac{\delta T}{d\beta} \cdot v\beta \cdot \beta\right)^2 + \left(\frac{\delta T}{d\sigma_F} \cdot v\sigma \cdot \delta_F\right)^2} \\ &= \sqrt{\left(\frac{T_{concrete}}{\bar{T}} \cdot v\beta\right)^2 + \left(\frac{\bar{T}_{steel}}{\bar{T}} \cdot v\sigma\right)^2} \\ &= \sqrt{(\varkappa_B v_B)^2 + (\varkappa_S v_S)^2} \end{aligned} \qquad (64)$$

If the safety W (S > 1) with characteristic factor g is prescribed, we can then determine the necessary safety factor as

$$\bar{S} = \frac{1}{1 - g \cdot v_S} \qquad (65)$$

(Here we use the simple normal distribution as a basis. For the logarithmic-normal distribution the same formal conclusion is obtained, but in this case it cannot be represented as clearly.)

On the other hand:

$$\bar{S} = \frac{\bar{T}}{m_B \cdot \bar{T}_{concrete} + m_S \cdot \bar{T}_{steel}} \tag{66}$$

and, by substitution of the first equation in the second we get the following formula:

$$1 = \frac{1 - g \cdot \sqrt{(\varkappa_S v_S)^2 + (\varkappa_B v_B)^2}}{m_B \varkappa_B + m_S \varkappa_S} \tag{67}$$

This is a functional equation for the material factors m_B and m_S, provided we observe the following subordinate conditions, which must be satisfied for plausible reasons:

The material factors must not depend on the cross-section values ($\varkappa_i$):

$$\frac{dm_B}{d\varkappa_B} = \frac{dm_S}{d\varkappa_B} = \frac{dm_B}{d\varkappa_S} = \frac{dm_S}{d\varkappa_S} = 0 \tag{68}$$

Nor may they depend on each other:

$$\frac{dm_B}{\partial m_S} = 0 \ ; \quad \frac{dm_S}{\partial m_B} = 0 \tag{69}$$

Thus, no solution of the functional equation (67) is possible, as can easily be shown by a transformation. This means that no system of material factors can be found which will do justice to the uncertainties in the strength of the two materials. There is no simple relationship between the material factors and the quantities linked additively to the scattering values; material factors, therefore, are not formally correct in this case.

Practical experience with various building standards have shown, however, that the approximative use of material factors for most cases results either in excessive risks or excessive expenditures(223). This is because the functional equation, whose form in any event depends on the choice of the individual distributions, can be adequately expressed by a stepped approximation of the safety coefficients (in this case the material factors). Therefore, a system of material factors should not be rejected arbitrarily. However, it must be emphasized that with such an approximation the safety coefficients can no longer be derived from the statistical weights of the structural materials, but are the product of a practical compensating process; in other words, a system of material factors (or other partial factors) must be based in turn on general experience, and hence to some extent, therefore, on the discretion of the people involved.

This also affects the possibility of a reconstruction of the true safety from the safety coefficients. Such a reconstruction is not directly possible and a special safety verification must therefore be carried out in each case.

4.5.2 Load factors

Basically, it does not matter whether the safety margins in the design stage are applied to the bearing capacity (material factor) or to the assumed loads (load factors). If only material factors are used, it may be objected that the type of load assumption and its uncertainties cannot be taken into account. The reverse holds true for the load factors.

In general it may be said that

$$m_i \leq 1 \tag{70}$$

and for the load factors

$$\lambda_i \geq 1 \tag{71}$$

Beyond this there is nothing which distinguishes the load factors fundamentally from the material factors. Formally one could deduce also for the load factors (e.g. for the case where the load is a sum of various components of various accuracy), that they, like the material factors, cannot be made to conform to the law of error propagation. Generally speaking, therefore, the true safety of a structure cannot be determined from the load factors, either. However, this is not a matter of great importance, since for lack of information we are at present unable in any case to reconstruct the quantitative relationship between safety and safety coefficients.

Material factors and load factors can also be used in combination(54, 55). In this way the statistical weights of the two parts of the safety inequality (T and P) can be taken into account separately, in which case the same formal objections apply as for the material and load factors.

The load factors will be taken up again below in connection with a special problem (Appendix 8.2).

4.5.3. Allowable stresses

Design by the allowable stress method is based on the theory of elasticity to the extent that load, stress and strain are proportional. The allowable stresses correspond more or less to a system of material factors. For the same reasons, therefore, they are not completely satisfactory. Furthermore, they cannot be used for investigations beyond the scope of the theory of elasticity and must be replaced by other systems of safety coefficients for such a case. If allowable stresses are used in the design, this means designing in accordance with a criterion of strain, which in many cases is illogical. The time when a structure shows certain (very small) deformations is unimportant; on the other hand, it is essential to know what load it will withstand (collapse criterion, 3.3.).

There is another argument against the use of allowable stresses: the measured strength values cannot be used in the calculation; certain standardized classes of construction materials, which perhaps are out of date, have to be observed. For example in the SIA standards two kinds of reinforcement steel are distinguished by the allowable stresses; only the respective yield points are taken into account. Whether or not these steels have uniform strength, which would conform to the safety concept, is disregarded. The adaptation of allowable stresses to this matter would be extremely complicated.

Nevertheless, we cannot yet dispense with the allowable stresses as safety coefficients, since they are the traditional form of the safety margin which has been used in recent decades for most designs. The industry has become accustomed to their use. Moreover, the elastic theory will have to be applied to many problems of the theory of structures and problems of dimensioning for some time to come, because other, more general methods have not yet been researched fully enough in order to permit designing for the collapse load in all cases. It should also be borne in mind that the changeover from allowable stresses to other forms of safety margin will constitute a considerable readjustment to which all builders will have to adjust before new methods can be generally introduced.

Consequently, the method of allowable stresses must continue to be applied as a useful instrument for some time to come, side by side with more up-to-date procedures, which can develop as time goes on.

4.5.4. Conclusions

From the discussion of various possibilities for breaking down the safety factor into components, it is evident that this always entails disadvantages, at least of a formal nature.

Hence, there is no other system of safety coefficients apart from the simple safety factor by which the safety margin can be determined correctly with respect to the error distribution in the given case. (No general proof of this contention has been given, but this could be done for any division of the safety factors into multiplicative or additive components, as have been carried out in the case of the material factors. This proposition does not hold for the safety zone Z (15), which can be derived from the safety factor without breaking it down.)

The formal criterion, of course, is only of theoretical value as long as safety margins cannot be based solely on statistical values. That is to say, as soon as we drop the working hypothesis of 4.2. it is no longer possible to form a "correct" system of safety coefficients, but only one that is as close as possible.

There are other considerations, as well, especially procedural ones. As we shall show in Section 8.2, there are problems which can be solved more readily with one of the systems (load factors) than with others. However, the solution of the simple safety factor has the advantage of greater clarity and easier manipulation, especially for plastic methods of design. Therefore, the problem will be treated with the aid of the simple safety factor, right up to the numerical application of a suggested system to two typical design examples (6.1; 6.2).

5. THE VARIATION OF THE SAFETY FACTORS

5.1 Conditions

Before drawing up a usable system of safety factors, it is first necessary to examine a number of procedural aspects and practical conditions so as to determine how such a system can be developed.

The first and most important condition is the need for simplicity. In other words, the introduction of the margin of safety into the design should not complicate it substantially.

A second, more fundamentally theoretical rule is as follows: the application of margins of safety must not entail any additional errors which would introduce appreciable deviations in the results. In other words the true nature of the structural problem must not be distorted by the safety margin, as may easily happen through the clumsy application of material factors or allowable stresses.

The third and most important principle concerning safety coefficients is that these must do justice to all influences through a proper variation which affects the available or required safety. An example of an influence which helps to determine the available safety is the strength of the material, while a parameter of the required safety would be, for example, the number of people or the value of the goods which might be endangered by the possible collapse of a structure.

A system of safety margins must satisfy these three main conditions. They are to some extent contradictory, so that no one solution is "correct," but at best can be an optimum, a situation that already had to be accepted in connection with the formal aspects of the problem.

5.2 Compilation of Basic Variables

The following suggestion for the choice of variables, which are important for the numerical determination of the safety factor, is not the only possibility. Not all special problems can be encompassed in it in a simple manner. Therefore, this compilation should be regarded as a simplified prescription of "how it might be done."

Since the prevention of collapse is generally more important than protection against other damage (deformations, cracking, etc.), it is tacitly assumed that the compilation is everywhere based on this criterion. In accordance with the observations under 5.1, we distinguish between two groups of variables:

- 5.2.1 Sources of errors and deviations as well as controls that are applied and which determine the actual safety (of the finished structure).

-5.2.2 Ways of achieving the required safety.

5.2.1 Variables of actual safety

5.2.1.1 Uncertainty in material properties

In reinforced concrete two materials are always involved which have very different properties both statistically and structurally. In the case of steel the yield stress is generally employed, while for concrete it is the expected compressive strength which is used in the calculation as a representative quantity. The two materials do not always contribute to the bearing capacity according to their proportions. For example, in pure bending of weakly reinforced cross-sections ($\mu < \mu_{Gr}$) virtually only the strength of the reinforcement under tensile strength is employed, because this is what determines the moment that the cross-section is capable of transmitting (33). In the case of heavily reinforced cross-sections ($\mu > \mu_{Gr}$), on the other hand, it is the strength of the concrete compression zones which is critical. As an illustration see examples in 6.2 and 6.1. The latter example, of course, deals with a section in compression, which behaves from this point of view, however, like the compression zone in an over-reinforced beam.

Different safety margins must be used, depending on which material mainly determines the bearing capacity. For poured concrete, for example, the margins are much greater than for reinforcement steel which is obtained from a manufacturing process. In concrete, moreover, a greater frequency of coarse errors must be expected, since it is produced and finished with less strictly trained personnel and less well developed methods. This also has a great bearing on the safety margins.

5.2.1.2 Inaccuracy of dimensions

The geometry of the cross-sections, especially the effective depth h and the external dimension, also have a direct influence on the bearing capacity, according to the formulae for the cross-sectional strength. Different designs and different methods of production mean that the cross-sections are not all susceptible to such errors of measurement to the same extent.

In the case of poured concrete structures, greater deviations occur than in prefabricated elements (8.1). When the dimensions are small formwork errors and errors in positioning the reinforcement are relatively much more important than in ver massive structures. In poured concrete errors of measurement of a few centimetres must be expected. In the variation coefficients, however, these are related to the dimensions themselves, so that for slender parts a deviation of 1 to 2 cm is sufficient to result in a considerable decrease in the actual bearing capacity, whereas the same error in very deep girders has no appreciable effect.

This is taken into account, for example, in SIA Standard 162, where lower allowable stresses are prescribed for thin parts than for heavy ones. (Actually the difference is conceived with reference to deflections and the width of cracks, but would be similar for taking into account errors of measurement.)

Inaccuracies of dimensions have hitherto generally been disregarded in statistical investigations, presumably on the grounds that they should be lumped together with the coarse errors. Certainly there are some departures which must be ascribed to incorrect construction of the false work or support work, etc. Scatters of a few cm can never be wholly avoided. Consider only the upper reinforcement, which during assembly is forced out of position by the pouring and distributing of the concrete. Added to this there are inaccuracies of plans, the settling of form work, openings boxed out in the form work, etc.

Of great fundamental importance, moreover, is the fact that the position of reinforcement, in particular, cannot be corrected after the concrete has been poured, and that it can then only be observed and checked with difficulty. Hence we have no statistical information on these quantities and have only to rely on estimates for their consideration.

5.2.1.3 Uncertainty in load estimates

It is generally customary to apply maximum values of the load estimates and to introduce these into the calculation, since in most cases the maximum load determines the fate of the structure. Departures from the mathematical estimates, therefore, obey an extreme value distribution, and where statistical considerations are at all applicable, can be treated in the same way as the properties of the material.

In the case of loads which are applied directly by human beings, various possibilities of overloading must be taken into account. The loads are applied during occupancy, and checks of overloading must therefore take place during this time in order to be effective.

In those cases for which the maximum load will not be the sum for all kinds of all loads of the same sign, will be further discussed below (8.2).

5.2.1.4 Methods of design and calculation

Basically, all methods applied in the theory of structures are approximations. In the usual cases, however, the errors committed through simplifying assumptions concerning the mechanical behaviour (e.g. "elastic - ideally plastic") are small enough that they can be disregarded in comparison with the variance of the other parameters.

However, since the methods of mechanics and statics applied to structures do not always suffice to solve the problems of the theory of structures in this frame of reference, we often use still rougher approximations and simplifications (for example in calculating multi-dimensional bearing elements such as slabs, shells, etc.). This is often done even where an exact method might be known to exist, but could only be carried out with an inordinate amount of calculation.

Wherever possible, an attempt is then made to determine whether the approximate results are "on the safe side," i.e. whether the safety expected from the results is not overestimated. In many instances, however, this is not possible. One must then assume the existence of errors of unknown size and sign.

Such errors can sometimes be dealt with by studying special cases of a simple kind, or test items. This means that only cases of entirely new building materials, structures, and methods, are inaccessible to reliable estimation.

In one sense model tests which may give results on the basis of analogies that cannot be checked, and which are characterized by great inaccuracy and large systematic deviations, fall within this category. It is especially dangerous to attempt to draw quantitative conclusions from model tests, because often there is no way of verifying them.

Among cases which must be calculated with coarse approximations are certain instances of combined stress in the region of failure (shear and torsion with bending), and which have not yet been investigated thoroughly enough. In such cases empirical formulae are used which have been adapted to the results of tests. However, the farther the conditions of the specific instance of application differ from that of the test the less reliable does its application become.

Errors which stem from the application of approximation methods cannot be regarded as random. Rather, this constitutes a kind of "coarse" error, but one which cannot be avoided. We know too little about such errors to be able to base any variation of safety margins on them. The only rule that can be applied in order to avoid dangerous effects is a purely practical one, namely whenever unchecked approximations must be used one should try to verify the results by means of different approximations.

5.2.1.5 Reliability of checks

The effectiveness that one might derive from the reliability of inspection and control on the level of the safety margins is of an entirely different kind from the four preceding ones. It cannot be described by the error tolerance. Formally speaking all that can be said is that the checks reduce the frequency of large deviations.

Every check consists of an inspection of an object for coarse errors. If such errors are discovered in the course of inspection, they can generally be examined for their seriousness, and if necessary corrected. Thus every control that is applied reduces the possibility of coarse errors.

When it is a question of employing the effect of controls as another parameter for the variation of the safety margins, a number of questions arise, especially concerning how the checks are to be applied so that maximum effectiveness in the elimination of large deviations can be attained.

Schematically the many possibilities of large deviations and errors can be represented by a series of partial probabilities, the sum of which is approximately known. (The "Oder probability," of course holds only under the condition that the various individual probabilities belong to independent events, which, however, may be reasonably assumed in this case). That is to say:

$$\sum_{1}^{n} W_i \cong W(E') \tag{72}$$

where $W(E')$ is the frequency of failure relative to all structures, which according to (4.4) is in the order of magnitude of 10^{-3} to 10^{-5}.

Through checks and their effect, i.e. improvements, as large a number as possible of individual probabilities shall now be eliminated. The total error of probability is now reduced by the component

$$\sum_{1}^{m} W_i = \Delta W(E') = k \cdot W(E') \tag{73}$$

If all the W_i terms are equal, this can be simplified to

$$k\% = \frac{n-m}{n} \cdot 100 \tag{74}$$

where k gives the reduction. Now, if k is to be as large as possible, then for the general case we find that as far as possible the most dangerous - largest - individual probabilities are to be pursued, and indeed as many as possible. This appears obvious, but there are certain consequences which are not self evident: thus, it does not appear reasonable to examine at great expense and to correct a single parameter, for example the strength of the concrete or the accuracy of the formwork, if at the same time other sources of error such as load, behaviour of the foundation soil, etc. are neglected. The best results are obtained from the most logical possible distribution of the checking costs.

Another basic question may be formulated as follows: should the safety margins be changed at all according to the effectiveness of the inspections and controls? This would assume that the control would be subject to a reproduceable set of regulations, which in view of what has been said above meets with great difficulties.

Nevertheless this practice has already become customary in some cases (e.g. Danish Reinforced Concrete Standards 1939). This is possible where governmental agencies are responsible to carry out these inspections uniformly. How this might be organized in our own country, for example, cannot be discussed here.

A strong argument against the rewarding of controls in this way is that this itself is very difficult to check and consequently the responsibility would necessarily become obscure. Moreover, it would mean the legalization of an intolerable state of affairs, namely that it would be left to the individual to erect poorly or well verified structures: coarse errors can have all sorts of effects. If they are not eliminated as far as possible, then even bigger margins of safety will not help.

Another school of thought advocates variation of the margins of safety depending on the reliability of the controls: through the smaller consumption of materials that would be necessary as a result of the lower safety margins of well controlled structures, controls would become visibly and directly profitable. This would have an educational effect. Since controls have a favourable effect also on the random events (random according to our definition above), something would be gained in this respect as well.

The discussion and weighing of the pros and cons with respect to this point belong in the fields of economics and social science, and can therefore not be discussed further here. For the variation of the safety factor in the present example the placing of a premium on controls is not considered.

5.2.2 Variables of the required safety

5.2.2.1 The kind of stress

The failure of a cross-section occurs differently, depending on its structure and on the stress to which it is subjected. We distinguish between the ideal cases of "brittle" and "plastic" failure and, correspondingly, brittle and plastic cross-sections. This is incorrect inasmuch as one and the same cross-section may react differently under different forces. In every case, therefore, the kind of stress must be specified.

In the brittle failure, the transmission of force is suddently interrupted, without any visible deformations occurring beforehand, i.e. at a given place the supporting structure ceases to have continuity. The degree of static indeterminacy is usually thereby reduced by six (three-dimensional problem) or by three (two-dimensional problem), which must often lead to a local collapse. Plastic failure can be described more or less as follows: after the stress (load) has risen to a certain value - e.g. the plastic moment - the deformations begin to increase rapidly without any further increase of load. Only after the deformations have also increased to a certain, and generally visible extent, does the actual collapse take place, and is then usually brittle in nature. The degree of static indeterminacy is reduced only by one (3.5) as a result of a location (e.g. "plastic hinge"). Only in statically determinate systems does this signify exhaustion of the structure.

Plastic failure occurs in all members which under pure bending, eccentric tension and compression do not act brittlely for structural reasons (excessive reinforcement or no reinforcement).

The deciding factor here again is which material "yields" first. If it is the steel, the collapse is usually a plastic one; if it is the concrete, a brittle, sudden failure occurs.

The consequences of this difference is obvious. There are cases where collapse occurs suddenly without appreciable deformations, which might have been visible beforehand and give warning. On the other hand there are systems which indicate approaching exhaustion by large deflections.

From this one may draw conclusions concerning the course which any potential accident may follow, and it is reasonable to say that its consequences are attended by different degrees of danger, depending on which kind of failure is involved. This can be taken into account by varying the safety margin.

5.2.2.2 Circuitry of structural members

As for the cross-section and the individual member, so we may also discuss the course of a possible accident for an entire structure. For this the "circuitry" of the structure is the determining factor. In the case of the statically determinate system, corresponding to connection in a series, the elimination of any single force transmission is sufficient to exhaust the structure. Any slight increase of load then results in collapse. In the case of statically indeterminate structures, i.e. connected in parallel, there are always several possible ways in which the system might fail under a given load: there are several collapse mechanisms. For an illustration of this argument see example 6.2.

We can distinguish essentially between local, i.e. locally restricted failures, and the entire collapse. Collapse mechanisms involving large spaces must be prevented by correspondingly higher safety margins, and the case may even be imagined where a local collapse, possibly a harmless one, is deliberately invited by lower safety margins. It might be desirable for the structure to begin to "fail" at a point which is known and can be specifically observed. This is analogous to the fusing of electrical circuits and would be the means of preventing a very serious and extensive collapse.

Such a safety measure does not necessarily mean the complete "collapse" of the safety member, but would indicate such changes as cracks, or slight sagging. This is also the meaning of the principle of elastic dimensioning; one guards against large deformations and thus also against what these would entail, namely damage and collapse. The elastic protection indication only tells how and at what increase of load the collapse might begin, and therefore one never knows "how far one is from collapse," when the critical deformation is reached.

The "internal sequences" of a potential failure, i.e. the consequences for the structure itself, are what results from the circuit properties of the structure. For variation purposes, therefore, the question is against what possibilities (mechanisms) of collapse is it necessary to take special precautions through higher safety margins?

5.2.3 The structure and its surroundings

Every structure constitutes a danger for its contents and its surroundings. The latter, therefore, must also be included in a safety consideration.

There is a fundamental difference between purely economic values, which may be represented, say, by the costs of reconstruction, and human life and limb.

The risk taken by storing goods in a warehouse can be weighed against, say, the extra expense that would necessarily be involved in increasing the margins of safety. Hence it is possible to arrive at an "optimum" value of the safety margin from an economic standpoint, which, however, would in general be insupportable from the human point of view. Thus, material values alone do not determine the levels of the safety margins.

Human life and the danger to which it is exposed, however, cannot be simply expressed in material value scales. (Although this is done in a certain sense in life insurance, we shall disregard this here, because the establishment of such a scale cannot be based on technical considerations.) Nonetheless, it is reasonable to pay more attention to the safety of buildings which are constantly used by large numbers of people, than buildings which are only employed for the storage of commodities. This is especially true since people circulating in an auditorium, crossing a bridge, etc., often provide the determining load. As a scale or a relative graduation of safety margins one might use, for example, the number of people who might find themselves in the vicinity of the structure at the time of a presumed accident and would thus be endangered. This, however, would only give some idea of the relative size of the safety margin.

In order to establish it quantitatively, on the other hand, we have to draw on experience: "the safety coefficients used so far have been satisfactory (or not satisfactory) and thus there is not (is) a reason for increasing or lowering them."

Again we shall dispense with further discussion of this point, because it leads us into the fields of ethics and social sciences. Fundamentally, of course, one must have a clear conception of the problem of relative values as between human life and costs, which, of course, cannot be resolved with the tools at the disposal of the engineering sciences.

5.3 Formal Considerations in Connection with the Variation of Safety Margins

5.3.1 Variables

We shall represent the 8 variables discussed briefly in section 5.2, some of which cannot be represented mathematically, by the quantities $X_1 \dots X_8$. To X_i we assign values: $X_i = 1, 2, 3$, so that wherever the safety margin receives a low, intermediate or high value on account of the variable X_i, the respective numerical values 1, 2, 3 stand for the variable X_i. We divide these into not more than 3, and often only 2 classes, since any more detailed graduation would lead, at least for this fundamental investigation, to an excessively complex system.

The variables stand for the following arguments:

X_1 uncertainty concerning the properties of the materials

X_2 inaccuracy in dimensions

X_3 errors in estimated load

X_4 inaccuracy of calculation and design

X_5 reliability of controls

X_6 kind of stress

X_7 circuitry of structure, "internal consequences of failure"

X_8 contents and surroundings of structure, "external consequences of failure."

5.3.2 Reduction of the number of variables

With 8 variables each divided into 3 steps we would get a total of

$$3^8 = 6561$$

different safety factors. Such a system is too complex for ease of handling. Hence certain individual variables must be dropped, and others left undivided. The reasons for the procedure selected hereinafter are given in section 5.2. Only their essential features will be recalled here.

X_4 inaccuracy of calculation and design

X_5 reliability of controls.

Practically speaking no variation in respect to these two arguments can be carried out, because there are no adequate scales for the departures which result or are reduced by controls.

X_1 uncertainty of material properties

X_6 kind of stress.

The consequences of these two variables cannot be separated. Where the concrete determines the bearing capacity, a brittle behaviour must be accepted. Conversely, plastic strains usually occur wherever steel reinforcement determines the strength of a member. We combine these two variables into a single one, namely Y_1.

X_7 circuitry of the bearing capacity, "internal consequences of failure"

This variable must be dealt with separately, because its value will differ, depending on the method of design employed, in order to insure a reasonable behaviour with respect to failure. It is intended for application specifically in the plastic methods of design, and its use is described briefly in example 6.2.

5.3.3 Choice of numerical values

According to section 4 the fundamental quantitative theorem of the safety factor cannot be given a scientific basis. No reliable conclusions can be drawn either from statistical data or from considerations in the theory of probability, with regard to what values of safety margin should generally be chosen. We are therefore relying on the presently accepted safety margins as control values of our proposal, but leaving the way open, of course, to modified and increased variation with respect to new requirements in some cases.

The system of safety factors is represented in such a way that for each design a simple safety factor can be read from a table, once the table parameters (X_i) have been determined from a classification of design cases. The supplementary factor S' will be discussed separately in section 5.4.5, since for formal reasons it cannot be fitted into the table.

The classification of structures according to the table parameters is given in section 5.4. For the sake of simplicity, the nomenclature of the variables remaining from 5.3.2 after reduction can be summed up as follows:

Y_1 material properties and nature of stress

Y_2 dimensions

Y_3 load assumption

Y_4 content and surroundings of structure, risk and consequences

5.4 Classification of Designs (Suggestion in Short Form)

5.4.1 Material properties and stress*

$Y_1 = 1$ Cross-section with $\mu \leq \mu_{Gr} \cdot 0.8$
pure bending, bending and axially force

$Y_1 = 2$ All cross-sections with $\mu > \mu_{Gr} \cdot 0.8$
bending, bending with axially force,
axial compression

$Y_1 = 3$ All cases of shear and torsion (since shear and torsion never, or almost never occur alone, a rule must be added here, wherever it is necessary to design for shear or torsion. See 5.4.6)

5.4.2 Dimensions

$Y_2 = 1$ Cross-section for which effective depth is at least 18 cm and maximum reinforcement diameter is not more than $\frac{h}{8}$.

$Y_2 = 2$ All cross-sections for which the effective depth is less than 18 cm and/or the maximum reinforcement diameter exceeds $\frac{h}{8}$.

* Y_1 should probably be divided into more than 3 classes so that no very sharp and definitive boundary will occur at μ_{Gr} which, of course, does not conform to actual conditions.

5.4.3 Load assumption

$Y_3 = 1$ Structures for which the total dead weight exceeds half the maximum total load, and structures where the load is restricted by the geometric factors to the calculated value (containers, etc.).

$Y_3 = 2$ Structures for which the dead weight is less than half the maximum total load.

$Y_3 = 3$ Structures which are no longer being used for their original purposes; warehouses for mixed commodities, special cases where the true load is not accurately known.

5.4.4 Contents and surroundings of the structure

$Y_4 = 1$ Simple warehouses, sheds, machine shops; all structures in which numbers of people never remain for any length of time (for a proposed standard, of course, this classification would have to be formulated more precisely, which is not appropriate here).

$Y_4 = 2$ Apartment buildings and office buildings, vehicular bridges, pedestrian bridges, industrial buildings, hotels.

$Y_4 = 3$ Churches, auditoriums, department stores, theatres, moving picture houses, railroad bridges.

5.4.5 Function of the structural member

All supports, such as frame legs, columns, and walls contain a supplementary factor $S' = 1.4$ unless for some other reason they have not already been assigned to the class $Y_1 = 2$. In designing by the mechanism method, the supplementary factor holds for all collapse mechanisms which are not restricted to horizontal bearing members.

5.4.6 Schematic arrangement (Table 1)

In Table 1 the values of the safety factor $\bar{S}$, obtained in accordance with the variation of the class parameters are given: $\bar{S}(Y_1, Y_2, Y_3, Y_4)$.

The values of $\bar{S}$ can at the same time be considered medians of the estimated distribution of the actual safety factor. Accordingly, in the structural calculation, the bearing capacity and load, and hence also the other data should be represented by the medians of their distribution. Other than the safety factor $\bar{S}$, no supplementary reduction or other coefficients are used, since the entire safety margin required is combined in the factor $\bar{S}$.

TABLE 1

Y_3	Y_4 \ Y_1	1		2		3	
	Y_2	1	2	1	2	1	2
1	1	1111 1,5	1211 1,6	2111 2,4	2211 2,6	3111 2,4	3211 2,6
	2	1112 1,6	1212 1,7	2112 2,5	2212 2,7	3112 2,8	3212 3,0
	3	1113 1,8	1213 1,9	2113 3,0	2213 3,2	3113 3,2	3213 3,5
2	1	1121 1,5	1221 1,6	2121 2,5	2221 2,7	3121 2,5	3221 2,7
	2	1122 1,6	1222 1,8	2122 2,6	2222 2,8	3122 2,9	3222 3,1
	3	1123 1,8	1223 2,0	2123 3,2	2223 3,4	3123 3,5	3223 3,8
3	1	1131 1,6	1231 1,8	2131 2,6	2231 2,8	3131 2,6	3231 2,8
	2	1132 1,7	1232 1,9	2132 2,8	2232 3,0	3132 3,5	3232 3,8
	3	1133 2,0	1233 2,2	2133 3,4	2233 3,7	3133 4,0	3233 4,2

For illustration, let us review once more the conditions leading to this proposal:

1. All magnitudes employed in a structural calculation or design are estimates. Consequently the results of these calculations are also classed as estimates (T and P, S).

2. Since no unbroken relationship can be established between the safety factor and the statistical properties of the design parameters, a number of guide values for the safety margin must depend on traditional design practice (standards). This has been done for the values $\bar{S}$ (1112), $\bar{S}$ (1122), $\bar{S}$ (1222) and for the basic value $\bar{S}$ (2...) of all factors under $Y_1 = 2$. Reference for this are the collapse safety regulations in SIA-Standard 162(53).

In designing for shear ($Y_1 = 3$) no such reference can be cited, since this can be carried out only with elastic methods. The numbers in this column, therefore, must be used together with the best available collapse formula as soon as one becomes available. It has hitherto been customary to exclude the possibility of collapse of a girder subject to high shearing stress by the use of comparatively high safety margins. This is reasonable, since shear failure usually occurs very suddenly (brittlely), and because it is usually possible to avoid this kind of failure at small expense.

3. The supplementary factor S' which represents the importance of individual supporting members, should be used wherever a design detail falls into the category of "plastic hinge" according to the other design rules, where the function of the bearing member however demands a greater safety because its failure would have serious consequences (6.2).

4. Approximation of safety requirements by a graduated variation of the safety factor does not correspond to the actual nature of the problem. Especially in the case of parameters Y_2 and Y_3 the graduation may appear rather crude. However, it was employed in order to retain clarity. It would be entirely possible to do more justice to the conditions through a finer subdivision.

5. As the basic form of safety margin, the simple safety factor has a number of defects, in addition to many disadvantages. From the procedural point of view, and from formal considerations, it must be preferred to other possibilities. This is countered by certain practical arguments, especially the fact that it has hitherto not normally been used in the practice of reinforced concrete design, and thus would require a certain amount of rethinking. However, it is not the task of a technical investigation to determine what should be emphasized in the discussion of such points of view, because this involves the weighing of many different requirements, some of them of an entirely practical nature, against each other. The question of the best system of safety coefficients must be left open here. Another example for the choice of numerical values is found in the ICBR papers (225).

6. EXAMPLES FOR THE USE OF THE SAFETY FACTOR

6.1 Reinforced Concrete Section under Eccentric Load

(Example of safety verification according to 4.2; 5.)

The safety factor on a doubly reinforced concrete cross-section of rectangular-shape is to be verified.

The data are as follows:

$d = 24$ cm	$P = 40,000$ kg
$d' = 2$ cm	$e = 2$ cm
$b = 18$ cm	$Fe = 7.7\ cm^2$ (5 x 14)
$\beta = 250\ kg/cm^2$	$\mu = 1.78\ \%$
$\sigma_F = 3500\ kg/cm^2$	Fe, μ for one-sided reinforcement

The cross-section is assumed to be that of a short column (neither buckling nor plastic flow are considered here). The slight eccentricity results in $Y_1 = 2$. The effective depth is greater than 18 cm, thus $Y_2 = 1$. Suppose the column is being used in an auditorium: then $Y_4 = 3$. Let the assumed load fall into class $Y_3 = 2$. The dimensioning case therefore falls into the class 2123, and hence the safety factor according to Table 1 is $\bar{S} = 3.2$.

Assuming a parabolic stress distribution in the concrete at ultimate load ($\varepsilon_u = 3\ ‰$) and ideal elastic-plastic behaviour of the reinforcing steel, a mean bearing capacity is obtained of

$$\bar{T} = 128,000 \text{ kg}$$

The dimensioning expression is:

$$\bar{S} = 3.2 \leq \frac{\bar{T}}{\bar{P}} = \frac{128,000}{40,000}$$

and is thus satisfied.

Detailed calculation of the cross-sectional resistance T from the above data was omitted here, because the details of the process are very complex. The value of T is taken from a table. (34).

6.2 Dimensioning of a Steel Reinforced Concrete Framework

(Example of direct dimensioning)

A two-column framework is to be dimensioned so as to have a safety margin as given in 5.4. It is completely fixed on both sides and rectangular. Its cross-section is constant over the entire length (beam and columns) (Fig. 12).

Let this frame be used in a building with coefficients $Y_4 = 3$, $Y_3 = 2$. Its size means that $Y_2 = 1$, as can be estimated initially.

Logical dimensioning requires that a frame be protected against any possible type of collapse with whatever safety factor is necessary in the given case. Y_1 therefore varies, depending on the specific dimensioning detail in question. In order to prevent the example from becoming too lengthy, we will omit dimensioning for shear on the assumption that this will be undertaken later.

The data of the frame are as follows:

$$\ell = 5 \text{ m} \qquad p = 2000 \text{ kg/m}$$

$$W = 5000 \text{ kg} \qquad h = \ell = 5 \text{ m}$$

Initial dimensioning gives a beam depth d = 65 cm and cross-section width b = 30 cm. The resulting dead weight, g = 500 kg/m.

The calculation is then conducted in 2 different ways:

1. Determination of the force distribution by elastic methods and dimensioning of cross-sections with reference to the collapse criterion.

2. Dimensioning of the frame as a whole by the mechanism method (force distribution and dimensioning for the collapse state).

For the sake of comparison the results of a dimensioning according to the purely elastic method under the regulations of SIA (Standard 162 are also given.

6.2.1 Determination of the force distribution: elastic. Dimensioning according to collapse criterion.

The elastic calculation of the force distribution is of no interest here and therefore only the results are shown (limiting value curves) (Fig. 13).

From the limiting value curves of the nominal loads we can get the necessary cross-sectional resistances; at every point investigated the proper safety factors are assigned by which M and N must be multiplied in order to make them exactly equal to the non cross-sectional resistance (dimensioning condition).

From the table of safety factors, we get $\bar{S} = 1.8$ for the tie beam.

For the columns the necessary safety factor is also $\bar{S} = 1.8$, but this must be additionally multiplied by $S' = 1.4$ in accordance with (5.4.5), so that the ultimate safety factor of the column is given by

$$S' = 1.8 \cdot 1.4 = 2.5$$

For each cross-section it must additionally be shown that it will not behave in a brittle manner ($\mu \leq 0.8 \cdot \mu_{Gr}$), since otherwise it would fall within the class $Y_1 = 2$ and its safety factor would have to be increased accordingly.

The structure is investigated in four sections (1, 2a, 2b, 3). The necessary plastic moments are derived from the dimensioning condition:

$$M_p = \bar{S} \cdot M_{max}$$

The quantities assigned by sections are as follows:

Section	1	2a	2b	3
M_p^- in cmt	-1550	-5350	-3900	$\pm$ 0
M_p^+	+4000	+ 400	+ 300	+2700

Dimensioning may now be carried out with these quantities, substituting the nominal material properties. In doing so, of course, the influence of the normal force must also be taken into account and evidence must be given that all cross-sections will also behave plastically under the influence of the normal force. However, since this calculation is not involved in the manipulation of the safety factor it shall not be presented here. Since the steel percentage must not in any event exceed 1.25 %, the condition is immediately satisfied everywhere.

6.2.2 Dimensioning by the mechanism method

For calculation and dimensioning (the two here are more or less carried out as a single process) by the mechanism method we assume that other kinds of failure (fracture of cross-section due to shear, etc.) are not possible, in the same sense as in 6.2.1. There are then three kinds of collapse ("independent mechanisms" or "independent kinematic chains", Figures 14-16). They are distinguished by the position of the hinges, and a special distinction must also be made between whether the hinges in the corners of the frame develop in the region of a column or in the region of the tie beams. However, by suitable designing it can easily be assured that they will lie with high probability within the tie beam; hence no further consideration will be given of this point.

According to 5.4.5 the beam mechanism of Fig. 14 requires a safety factor $\bar{S}_1 = 1.8$.

The other two mechanisms, according to 5.4.5., must be secured with a total factor of $\bar{S}_2 = 1.8 \cdot 1.4 = 2.5$, similar to the calculation in 6.2.1., where again it is assumed that no brittle places will be found in the hinged region.

The difference between mechanism 1, on the one hand, and 2 and 3 on the other, may not appear very important in this simple, two-column frame. Mechanism 1 is an example of a purely local collapse, while the other two involve the entire structure. Failure of the tie beam alone endangers only the space directly below the beam; a collapse at one side, however, affects the actual region outside the structure. This essential difference is much more obvious in multi-column frames several storeys high, where a lateral displacement mechanism can result in collapse of an entire building, whereas a beam mechanism is always locally restricted.

Mechanism 3 shows one peculiarity. Theoretically the hinge in the beam does not occur in the middle of a frame. Its position depends on the resistance of the other hinges ($\chi = \xi \cdot \ell$).

For dimensioning purposes it is convenient to apply the safety factor to the load, which is being investigated at the time (g + p, g + W, etc.).

Now, we formulate the three equations between internal and external work according to the principle of virtual displacement. For the three mechanisms we get the following relations:

$$(p + g) \cdot \ell^2 \cdot \bar{S}_1 \cdot \varphi = (M^-_2 + 2 \cdot M^+_3 + M^-_4) \cdot \varphi \tag{77}$$

$$W \cdot \ell \cdot \bar{S}'_2 \cdot \varphi = (M^-_1 + M^+_2 + M^-_4 + M^+_5) \cdot \varphi \tag{78}$$

$$\left(W \cdot \ell + (g + p) \cdot \ell^2 \cdot \frac{1}{2-\xi}\right) \cdot \bar{S}'_2 \cdot \varphi$$

$$= \left(M^-_1 + \frac{2}{2-\xi} \left(M^+_3 + M^-_4\right) + M^+_5\right) \cdot \varphi \tag{79}$$

Substituting a $\leq$ for the equal sign in each equation, we get the safety inequalities of the three collapse mechanisms. In numbers they are as follows:

$$11,200 \leq M^-_2 + 2 M^+_3 + M^-_4 \tag{80}$$

$$6200 \leq M^-_1 + M^+_2 + M^-_4 + M^+_5 \tag{81}$$

$$6200 + 15{,}600 \cdot \frac{2}{2-\xi} \leq M_1^- + \frac{2}{2-\xi}(M_3^+ + M_4^-) + M_5^+ \tag{82}$$

As an approximation the quantity ξ can be replaced by 1 (exact calculation will give 0.92, which has very little effect on the inequality (82)), and we obtain three inequalities of a linear program for the values of the plastic moment:

$$M_1^- + 2 \cdot M_3^+ + M_4^- \qquad - 11{,}200 \geq 0$$

$$M_1^- + M_2^+ + M_4^- + M_5^+ \qquad - 6{,}200 \geq 0$$

$$M_1^- + 2 \cdot M_3^+ + 2 \cdot M_4^- + M_5^+ \quad - 21{,}800 \geq 0$$

From the symmetry of the system and the loading (W can come from both sides) we get the relations:

$$M_1^- = M_5^- \qquad\qquad M_1^+ = M_5^+$$

$$M_2^- = M_4^- \qquad\qquad M_2^+ = M_4^+$$

The inequalities can thus be simplified to:

$$2 \cdot M_2^- + 2 \cdot M_3^+ \qquad - 11{,}200 \geq 0$$

$$M_1^+ + M_1^- + M_2^+ + M_2^- \qquad - 6{,}200 \geq 0$$

$$M_1^- + M_1^+ + 2 \cdot M_2^+ + 2 \cdot M_2^- \quad - 21{,}800 \geq 0$$

From the linear program, therefore, five unknowns must be determined. So that in the course of optimization none of the quantities become negative or receive too great a value which would be unattainable in the design (brittle cross-sections with high steel percentage), lower and higher bounds must be introduced.

$$M_1^+ - 1000 \geq 0 \qquad\qquad - M_1^+ + 4000 \geq 0$$

$$M_1^- - 3000 \geq 0 \qquad\qquad - M_1^- + 1500 \geq 0$$

$$M^+_2 - 0 \geq 0 \qquad - M^+_2 + 1000 \geq 0$$

$$M^-_2 - 3000 \geq 0 \qquad - M^-_2 + 6000 \geq 0$$

$$M^+_3 - 2000 \geq 0 \qquad - M^+_3 + 4000 \geq 0$$

To this we add a goal function, representing more or less the relative costs resulting from the reinforcement of the different cross-sections:

$$4 \cdot M^+_1 + 3 \cdot M^-_1 + 4 \cdot M^+_2 + 7 \cdot M^-_2 + 6 \cdot M^+_3 = z$$

Table 2 - Linear Program for M_p

	M^+_1	M^-_1	M^+_2	M^-_2	M^+_3	1
f_1	0	0	0	+2	+2	- 11,200
f_2	+1	+1	+1	+1	0	- 6200
f_3	+1	+1	0	+2	+2	- 21,800
f_4	+1	0	0	0	0	- 1000
f_5	0	+1	0	0	0	- 3000
f_6	0	0	+1	0	0	- 500
f_7	0	0	0	+1	0	- 3000
f_8	0	0	0	0	+1	- 2000
f_9	-1	0	0	0	0	+ 4000
f_{10}	0	-1	0	0	0	+ 1500
f_{11}	0	0	-1	0	0	+ 1000
f_{12}	0	0	0	-1	0	+ 6000
f_{13}	0	0	0	0	-1	+ 4000
z	+4	+3	+4	+7	+6	0

The entire linear program has the form of Table 2. Its solution is:

$$M^+_1 = 3500 \text{ cmt}$$

$$M^-_1 = 1500 \text{ cmt}$$

$$M^+_2 = 0$$

$$M^-_2 = 4900 \text{ cmt}$$

$$M^+_3 = 3500 \text{ cmt}$$

Checking the plasticity conditions (principle of virtual displacements) gives:

$$f_1 = 2 \cdot M^-_2 + 2 \cdot M^+_3 = 16,700 > 11,200$$

$$f_2 = M^+_1 + M^-_1 + M^+_2 + M^-_2 = 8,550 > 6,200$$

$$f_3 = M^+_1 + M^-_1 + 2 \cdot M^-_2 + 2 \cdot M^+_3 = 21,800 = 21,800$$

Hence they are satisfied. Substituting of f_i in the first form of the plasticity condition (80 to 82), the safety factor can be calculated for each mechanism. The load must therefore be raised by this factor so that the structure collapses by the collapse mechanism in question - assuming, of course, that it has not already collapsed in some other manner. (Table 4).

As in 6.2.1. dimensioning of the individual cross-sections is now carried out, the results being represented in Table 3.

Table 3 - Required Reinforcement Fe

Cross Section	I. Calculation by the elastic method. Collapse dimensioning of the cross section		II. Dimensioning by the mechanism method.		III. Calculation and dimensioning by the elastic method.	
	Fe cm^2 internal	Fe cm^2 external	Fe cm^2 internal	Fe cm^2 external	Fe cm^2 internal	Fe cm^2 external
1	14.0	5.7	12.8	4.2	11.8	6.0
2a	4.0	18.0	0	19.0	4.0	17.1
2b	4.0	17.0	0	22.8	8.8	19.5
3	11.1	0	16.9	0	13.5	0

Table 4 - Safety Factors

Calculation and dimensioning method	Mechanism		
			
I. Elastic calculation. Collapse dimensioning of cross sections	2.15	4.5	2.3
II. Dimensioning by the mechanism method	2.7	4.0	2.5
III. Calculation and dimensioning by elastic methods	2.4	5.0	2.2

The mechanism method permits variation of the distribution of moments (force distribution) under a given load within certain limits. In this way, as shown in the example, the best arrangement of reinforcement can be can be sought.

The exact dimensioning of the cross-sections, taking the normal forces into account requires an iteration process which is not taken up here because it lies outside the scope of this work. We shall also omit any demonstration to the effect that no brittle places are among the cross-sections introduced as plastic joints (see 6.2.1.).

From the results (Table 3 and 4) obtained by the three different methods of calculation for the frame, the following conclusions may be drawn for this example:

1. The method of plastic dimensioning of cross-sections leads to about the same consumption of material as the mechanism method. Purely elastic dimensioning according to the SIA Standard calls for about 10% more reinforcement.

2. With the mechanism method it is possible to base the dimensioning on the collapse programs actually available. This is a fundamental difference from the other two methods, in which only the strength of the individual cross-sections are investigated, without any possibility of considering the behaviour of the system as a whole. This is expressed in the fact that in the results of the mechanism method all safety factors are preserved for the individual mechanisms.

With the other two methods, despite a greater consumption of material, the safety factor falls below that required by mechanism No. 3. In these structures, therefore, the over-all probability of collapse would be greater (cf. 3.6.). In the case of method III (pure elastic dimensioning) the safety factor applied to the very consequential mechanism 3 is actually the smallest; this means that if the structure should fail it would in all probability collapse in this way. This constitutes a serious objection to designing with elastic methods. The extent to which it applies apart from this particular example, of course, is still an open question.

3. Complete agreement of safety factors with all three dimensioning methods cannot, of course, be attained, even when special instructions are drawn up for each method. The proof of this statement is as follows:

In the purely elastic and the "elastic-plastic" methods the force distribution is fixed. In the mechanism method, however, it can be freely varied, within certain limits (for the collapse condition). The relationship between the various safety factors depends directly, however, on the force distribution (80 et seq). Now, if the force distribution has been determined by different methods the safety factors will be different for the different results. Thus, the designer has to be content with an approximate agreement.

The remarks about the safety factors of the different collapse mechanisms (commentary to Table 4) appears trite when seen in the light of the conventional view that the values of a structural calculation are fixed quantities. However, if we drop this assumption and consider the cross-sectional strength values in particular, as statistical variables, then the program of collapse of a structure is no longer obvious. If the safety factors applied to two mechanisms differ very little, as for example those of mechanisms 1 and 3, then there will be a probability of collapse of the same order of magnitude for both types. It is therefore entirely possible that the frame (e.g. with the dimensions of method I), although it shows a smaller safety factor than mechanism 1, may nevertheless collapse through lateral displacement - because the beam, say, was made somewhat stronger and the legs somewhat weaker than prescribed.

This is an important consequence drawn from the statistical view of structural design. It leads - as in the case of method II (mechanism method) to a new interpretation of safety. Instead of a standard criterion (cracks, deformations, plastic yielding of a cross-section, etc.), which we guard against with an equally standardized safety factor, we can now consider various possibilities of the potential course of an accident, side by side, and provide a different level of assurance against each, depending on the relative risks.

7. SUMMARY, CONCLUSIONS

The problem of safety in the designing of structures can be regarded fundamentally as a kind of prognosis. That is to say, if we succeed in arriving at conclusions about the properties of a specific structure that is still to be built, from the abundant and complex information available from research and from existing structures, then a solution to the safety problem has been solved.

In order to arrive at such a conclusion, then the reference data and properties on which it rests - in this case the totality of the structures - must be known as much as possible. Now, this is a condition which is only partially satisfied as far as structures are concerned. Apart from certain specific information about some of the properties (parameters) often only a number of uncertain estimates are available which relate not to individual parameters, but to quantities which are themselves complex functions of several properties of the structure. In the main, we can distinguish between two kinds of information.

1. Observations, test results and statistical estimates concerning individual properties of structures. These are confined to only some of all the parameters, namely the strength of the building materials and of individual structural members, as well as certain kinds of loads.

2. General experience. This has to be substituted wherever no better information is available. Experience is a form of knowledge which cannot be interpreted in a strictly scientific way, and must therefore be regarded as rather inexact. On the other hand, it is the product of a very long continuing process which embraces the observation (unsystematic, of course) of a great many structures. For this reason a great deal of emphasis must be placed on experience when discussing the safety problem. It can be applied to all quantities relating to construction, and especially to safety itself: we know approximately the number of instances of damage and accident which occur in structures, and we possess information on their causes, and thus, from experience we have an estimate for the safety of structures that have been erected to date.

The results from the two kinds of information generally do not lead to the same quantitative conclusion for the required margins of safety. They contradict each other if indeed such a conclusion is at all permissible in view of the great uncertainty and incompleteness of the two sources of information.

On the basis of the statistics which have been obtained for individual safety parameters, an estimate of the expected safety can be calculated. It lies in the order of magnitude of $1 - 10^{-6}$ to $1 - 10^{-8}$, if we start from the distribution functions which are normally applied to the scatter of structural properties. From general experience, however, the probability of failure in a given structure is about $1 - 10^{-3}$ to $1 - 10^{-5}$. There must be a reason for this discrepancy.

From considerations concerning the nature and origin of errors the following supposition has been formed as an explanation: the fact that instances of damage are more frequent than would be expected from the random deviations can be attributed to the occurrence of coarse errors, which generally cannot be included in the statistics. The coarse errors, in turn, are caused by man himself, who both as the erector and user of the structures is responsible for everything that happens to them, apart from the purely accidental and unavoidable. More precisely, carelessness and ignorance are the direct causes of coarse errors, and hence of most accidents. This supposition is also found in the legal attitude, which almost always lays the blame for an accident on some person involved either in the construction or use of the structure in question.

When we consider that a person can commit an error at any place and in any form whatsoever, this supposition is shown to be possible. No proof can be presented that the hypothesis applies to all cases. However, in specific instances it is very often possible.

If we retain this hypothesis, the question then arises whether and to what extent the safety margins must take account of the occurrence of coarse errors. Two replies are given to this question from two different points of view:

1. Since neither the type nor frequency of coarse errors can be determined beforehand, no quantitative or numerical values can be derived from them. At present, therefore, it remains impossible to present any explicit relationship between safety coefficients and errors.

2. Conversely, tolerable safety has been obtained with the safety margins customarily applied so far. Proof of this can be seen in the fact that the general tendency is not towards an increase of safety coefficients (and hence of safety itself). In other words, despite the occurrence of coarse errors, they have been adequate, and instances of damage which occur from time to time are tolerated. In the course of application, therefore, a kind of equilibrium has set in between the effects of errors (accidents, losses) and the additional expense involved in the taking of safety measures.

Since we have no detailed information on the basis of this equilibrium, there is no reason to undertake any wholesale changes of a quantitative nature in the safety margins, thereby running the risk of upsetting the equilibrium that has been attained. Of course, this conclusion does not apply strictly to all individual safety coefficients, but only to a few central guiding values, such as the general collapse safety factor for bending in reinforced concrete ($\bar{S} = 1.8$).

However, a variation of safety factors about this guide value with respect to special, and partially new points of view can be justified. As an example we may mention here the aspect (proportion) of consequences of an accident, i.e. the consequences which would result from the failure of the structure for its contents and surroundings: it is one thing if a subordinate structural part should collapse on stored goods of low value, and something else again if the collapse of a whole building endangers the lives of many people.

Another aspect of the safety problem can be summed up in the question of what form a system of safety margins must take in order to do complete justice, as far as possible, to all requirements of a practical and theoretical kind. The most important of these are as follows:

1. By means of a clear, codified variation, the safety margins must make allowance for the conditions of a specific structure. This means that the required safety coefficients for each design case should be obtainable in a simple manner from a compilation.

2. The relationship between the intent of the safety concept and the corresponding comparative value (safety margins) must be simple, and as far as possible, reconstructable.

3. The safety coefficients must be simple enough to handle so that their use does not complicate the problems of structural theory significantly, and their true form is not distorted.

4. The safety margins must be based on a meaningful criterion of damage, e.g. the generally accepted criterion of failure (when the structure becomes unstable). Basically, we can say that the eventuality against which we wish to guard with the safety margin must be explicitely represented in the structural calculation.

5. The application of safety margins cannot be confined to a specific theory, as is the case, for example, for the permissible stresses, which are applied on the basis of the elastic theory.

It is not possible to observe all sides of these principles completely at the same time, since to some extent they are contradictory. The solution, a system of safety coefficients, must therefore be the result of an optimization process, comparable more or less to the problems of calculating compensations, although here, of course, neither algebraic nor numerical quantities can be used and it is necessary primarily to weigh procedural and practical, formal and empirical, aspects against one another. In all compensation processes the result depends very strongly on the choice of the weights which are assigned to the individual conditions. The choice of weights, however, is a difficult one even in the simple problems of compensation calculation. Nor can any suggested solution, a system of safety factors (section 5), claim to be the best and only one. In other words, if the emphasis is shifted, another method may very well appear to be better. For example, in the more recent standards of the USA and USSR(54, 55), combined material and load factors have been introduced. This contradicts certain formal arguments, but satisfies certain other practical ones better.

The numerical suggestion for variation of the safety coefficients, (Table 1, safety factors) is based, briefly, on the following consideration:

For the quantitative principle (guide values) overwhelming weight is assigned to the experience of building practice with the safety regulations that have hitherto applied. Thus, $Y_1 = 1$ or 2 and $Y_3 = 1$ or 2, are categories embracing the most frequent structures, and have therefore been assigned the customary collapse safety factors. As far as possible, they have been taken directly from SIA Standard 162 or have been converted with reference to typical examples from the system of allowable stresses.

For the variation of the safety margins according to various different, and in some cases new, parameters ("variables") special points of view have increasingly been taken into account. Of course, in these cases one has to be satisfied with simple estimates as long as reliable structural observations remain inaccessible. In this sense, in order to demonstrate the principle, the numerical values have also been varied for cases which were not specially qualified in the previous building standards, and for which, in some cases, adequate principles of calculation were not available: $Y_1 = 3$, $Y_3 = 3$, $Y_4 = 1, 2, 3$. These should be regarded as examples; for each numerical value we cannot give specific reasons which are independent of the other values. This could not be done, of course, even for the safety regulations in force today, all of which are based on the "equilibrium" attained through long experience. The given numerical values, moreover, have been subdivided only a little in order to retain clarity; neither formally nor quantitatively, could they provide the basis for drafting a new standard.

The aim of the investigation has rather been to clarify the structure and to delineate the principal questions that go to make up the safety problem, using the available arguments and a few examples. This is an important condition for constructing safety regulations on a foundation which includes all the information available to us today and that we may acquire in future. More precisely, conditions must be created whereby we may progress step by step from purely "intuitive" safety margins to ones that have a scientific basis. No doubt it will always be necessary to employ simple estimates to considerable extent; however, they must not remain the sole basis.

For some years attempts have been repeatedly made to relate quantitative aspects of the safety problem with statistical data via individual structural properties(21, 22, 23, 24, 25, 27, 212, 215-217). The fact that no real success has been gained in this is due to a number of circumstances, including a lack of sufficiently reliable statistical information concerning all the parameters. Secondly, many properties or functions of structures depend directly on how people employ them (manufacturers, users). They are consequently non-random values and thus cannot be the subject of statistical investigation and calculation. If they are nevertheless treated by such methods, erroneous conclusions are often arrived at.

Statistical information, therefore, no matter how valuable it may be in itself, must be handled with due caution. In the present work the author has tried to indicate to what extent and in what form the presently available statistical information can be employed for the determination of safety coefficients.

8. APPENDIX, SPECIAL PROBLEM

8.1. Observations on Building with Prefabricated Elements.

Up to and including the suggested solution in section 5 we have been discussing the safety problem as it applies to the conventional methods of construction in poured-in-place concrete.

Structures are increasingly being erected from prefabricated parts. We are not concerned here with the technical advantages and disadvantages of these new methods, but only with the special aspects of the safety problem which arise from them.

The problems of prefabricated construction can be compared with those of steel construction. In both cases slender, generally unidimensional members are used, and the chief difficulty in construction and assembly is with the connections. In steel construction, also, a number of very serious accidents happened before sufficient attention was given to these problems(26).

In section 4.1. we listed the four principal groups of structural properties:

strength
geometry
load
foundation soil, joints, connections.

Let us compare the uncertainties which arise in these four fundamental parameters for the two methods of "poured-in-place concrete" and "prefabrication". This leads more or less to the following conclusions and findings, which we now discuss under the above four headings:

8.1.1 Strength properties.

Prefabricated parts are manufactured by an industrial process. It has already been indicated above that by this means, better and more uniform strength properties can be achieved, than is possible in poured-in-place concrete, which is generally produced under less favourable conditions. This affects the scatter of concrete strength values and applies both to the frequent, unavoidable random deviations and to the coarse errors, which can be eliminated much more effectively by the control methods applied at the factory than is possible on the building site. Thus, it may be assumed that the bearing capacity of parts "carefully manufactured" in a factory is more uniform than that of similar members poured in place. As a guide value approximately $v_T = \pm 10\%$ may be applied for the coefficient of variation for this quantity, a relative error that probably cannot be reduced much more, but which, in comparison with other sources of error may generally be regarded as negligible. For poured-in-place concrete, on the other hand, strength scatters of 20% and more occur.

Unfortunately, strong emphasis must be placed on the words "carefully manufactured", since a few cases have recently arisen where concrete elements had greatly reduced bearing capacity because of improper and careless manufacture, and these failed at the building site. In most cases this was due less to the strength of the concrete than to incorrect design of the part, but the effect was the same as if there had been a loss of strength.

Another danger for the prefabricated element lies in its transportation to the building site. Load-bearing elements are often damaged in the course of transportation under entirely different stress conditions, but are nevertheless assembled at the site. In many cases no serious consequences were experienced with respect to the bearing capacity of the structure as a whole. However, this does constitute an additional source of error.

8.1.2. Geometry, dimensions

Regarding inaccuracies in the dimensions of prefabricated parts, the same can be said as for the strength properties. The deviations in general are considerably less. They usually lie within the order of millimetres. For the position of reinforcement a corresponding improvement may also be reckoned with. Since the parts are frequently prestressed, this can easily be understood. The prestressing wires are usually mounted and stretched in the form with the aid of a template, so that the pouring of the concrete can scarcely move them out of position.

With due caution, therefore, it may be stated that the bearing capacity of prefabricated parts, as a function of material strength and dimensions, scatters less than the bearing capacity of poured-in-place concrete. The consequences of this fact as far as the safety problem is concerned will be discussed only later.

8.1.3 Load

Structures consisting of prefabricated parts are used for the same purposes and in the same places as similar parts from poured-in-place concrete. There is no reason, therefore, to change anything in the argument of the assumed loads.

This, however, means that the advantage which was gained owing to the smaller scatter of the bearing capacity is much less, in the end, than might be expected. This can be illustrated by a brief calculation. The scatter of bearing capacity and loading for a certain structure of poured-in-place concrete is assumed to be 15% in each case. The scatter of the safety factor, in that case, neglecting all other errors, is:

$$v_S = (2 \cdot 15^2)^{\frac{1}{2}} \cdot 100 = 21\%$$

For a prefabricated part the scatter of the bearing capacity is reduced by one half. The coefficient of variation of the safety factor is then:

$$v_S = (8^2 + 15^2)^{\frac{1}{2}} \cdot 100 = 17\%$$

The reason that the reduction of error is so small, lies in the form of the "law of propogation of errors".

For the coarse errors, similar considerations apply. However the reduction is somewhat greater here, by the amounts of the errors which can be eliminated by the improved industrial control measures.

8.1.4 Joints, connections

Prefabricated parts are usually joined together and to the rest of the structure at the building site. This has been found to be the weak point of the entire prefabricated method of construction. The joints are made of a different material than the parts themselves - generally poured-in-place concrete.

For reasons of economy prefabricated parts are usually constructed with very strong concrete. However, the tying elements are usually made of poured-in-place concrete, which owing to the conditions at the site cannot possibly be produced with the same high strength, so that there are always places in the structure which are "less strong", and also behave in a less uniformly strong manner, than the prefabricated parts. Frequently, for reasons of assembly, the tying pieces are situated at places of very high stress (nodal points, corners, etc.), so that frequently the high strength of the structural parts cannot be fully exploited.

Another consequence is that special attention has to be given to the joints, so that these will actually be capable of transmitting the forces to which they are subjected. This applies particularly to large and multi-membered building constructions. Frequent accidents of assembly have shown that in the partially assembled structures conditions may occur in which it has too few joints which are already functioning. Joint locations which are rigid in the final structure and which can transmit all six generalized forces, are not yet completely interconnected and offer no resistance to displacement in one direction or another. The structure can then become a kinematic system offering little or no resistance to certain motions and can thus be set in motion by small erection forces. This has to be prevented during the planning of the assembly process and is one of the duties of the construction engineer, who is required to ascertain whether or not the structure may become even slightly unstable during any stage in the construction.

The same applies to the finished construction. In this case a force transmission does not usually disappear altogether, as may happen during erection, but may be exhausted sooner than expected. Under extreme forces (especially wind), exhaustion of ties may even occur before the structural element has come anywhere near being loaded to its bearing capacity.

It follows from what has just been said that new problems arise with the use of prefabricated parts and construction. For the present, at least, there is no long experience to fall back on. On the basis of the principles by which safety margins are established, therefore, it is too early to formulate such margins for construction with prefabricated parts. Until this can be done, the old regulations must be applied as far as possible. Additions and modifications must come gradually from investigations and experience.

8.2. Combined Stresses with Different Sign.

As an illustration of the problem we shall begin with a simple example. It is required to determine the dimensions of a simple reinforced concrete beam. Three different loads are given (design value and scatter):

fixed load	$\bar{g} = 800$ kg/m	$v_g = \pm 0$
snow	$\bar{p} = 1400$ kg/m	$v_p = \pm 10\%$
wind (suction)	$\bar{w} = -700$ kg/m	$v_w = \pm 10\%$

The beam data are as follows:

span	$\ell = 6$ m
cross section	$d = 50$ cm
	$b = 25$ cm

Let the beam be secured adequately against uplift at its support, so that no more consideration need be given to the construction of the bearing.

We calculate (with the design values) the maximum positive and negative (or minimum positive) moment at the centre of the beam:

$$M^+_{max} = \frac{(g + p) \cdot l^2}{8} = + 1000 \text{ cmt}$$

$$M^+_{min} = \frac{(g + w) \cdot l^2}{8} = + 50 \text{ cmt}$$

As a result of the dimensioning the lower reinforcement in mid-span is determined. It is calculated from the maximum positive moment at mid-span and comes to:

$$Fe = 11.6 \text{ cm}^2$$

$$\text{with } \sigma_b = 110 \text{ kg/cm}^2 \text{ and } \sigma_e = 2000 \text{ kg/cm}^2$$

According to SIA Standard 162 no upper reinforcement is required, since no negative moments arise from the registered load.

For the sake of simplicity we hereinafter consider the strength and geometry values to be exact, i.e. there is no scatter in the bearing capacity. The only place where deviations may be expected, accordingly, is in the assumed load. For all three kinds of load let the normal distribution apply. This, of course, is a rough approximation; however, nothing will be substantially changed in the result of the consideration.

We compute the variation coefficient of the stresses:

$$v(M_{max}) = \pm\, v_p \cdot \frac{p}{g + p} = \pm\, 6.4\%$$

$$v(M_{min}) = \pm\, v_w \cdot \frac{w}{g + w} = \pm\, 70\%$$

The standard deviations in these quantities are:

$$\sigma(M_{max}) = M_{max} \cdot v(M_{max}) = \pm\, 64 \text{ cmt}$$

$$\sigma(M_{min}) = M_{min} \cdot v(M_{min}) = \pm\, 35 \text{ cmt}$$

The bearing capacity for positive moment is:

$$M_p = \sigma_F \cdot Fe \cdot h \cdot \left(1 - 0.6 \cdot \frac{\sigma_F \cdot Fe}{bh \cdot \beta}\right) = 1840 \text{ cmt}$$

The safety factor against the positive moment, therefore, is:

$$\text{for } M_{max}: \quad \bar{S} = 1.84$$

$$\text{for } M_{min}: \quad \bar{S} = 37$$

From here on we confine ourselves to the consideration of minimum moments, since the safety inequality or positive moment is sufficiently well satisfied.

If we plot the probability function of the minimum moments we realize (Fig. 17) that there is clearly an appreciable probability of negative values also arising. This probability is represented by the hatched region and comes to:

$$W(M_{min} \leq 0) = F(0) = 7.6\%$$

For negative moments, however, the value of our calculated bearing capacity (M_p) is incorrect. In reinforced concrete construction it obviously depends on the sign of the stress and, if we neglect the tensile strength of the concrete - is given approximately by

$$M^{-}_{p} = 0$$

For a negative moment, therefore, the safety factor also vanishes.

The hatched region therefore simultaneously represents the probability that the safety factor is smaller than the critical value of unity. It corresponds approximately to the probability of collapse. We can plot the probability function of the safety factor (Fig. 18) and at $S = 1$ the collapse probability intersects at approximately 7.6%. The safety is thus only 92.4%, which is obviously quite unsatisfactory.

It may be concluded from this that caution is required in applying the usual methods of limiting values from design values of the loading(35).

This could be formulated somewhat better as follows:

If a given stress is the sum of components of different sign stemming from different loads, there will be discontinuity in the distribution function of the safety factor if the stress is applied to a cross-section the strength of which depends on the sign of the stress. If this is not taken into account, the safety will be over-estimated.

The example is chosen very simply so that in construction practice it would immediately be recognized that the beam must also be reinforced against negative moments. There are other cases where the play of forces is not so clear, and the true character of the problem is much more difficult to realize. The question arises as to how a system of safety coefficients should appear in order to be able to meet such a danger automatically.

One possibility is offered in the use of modified load factors, as follows:

Coefficients are applied to every load component which in their sign correspond to the expected deviations upwards and downwards and containing the safety margins of the design case in question. Every load must therefore be multiplied with two different factors and must be investigated for each of the two. The most unfavourable combinations of these modified load components then become the basis of dimensioning and from this a kind of expanded limit value procedure is obtained which we shall now carry out for the original example.

We form load factors for the maximum values of the loading components:

$$\lambda^+_g = 1.2 \quad \lambda^+_p = 2.1 \quad \lambda^+_w = 1.8$$

and for the minimum values:

$$\lambda^-_g = 0.8 \quad \lambda^-_p = 0 \quad \lambda^-_w = -0.5$$

These are applied to the design values of the loads, and we obtain the "largest" and the "smallest" stress for each load separately:

	M_{max}	M_{min}
g:	$\lambda^+_g \cdot \dfrac{g \cdot \ell^2}{8} = +\ 4.3$ mt	$\lambda^-_g \cdot \dfrac{\ell^2 \cdot g}{8} = +\ 2.6$ mt
p:	$\lambda^+_p \cdot \dfrac{p \cdot \ell^2}{8} = +\ 13.2$ mt	$\lambda^-_p \cdot \dfrac{\ell^2 \cdot p}{8} = 0$
w:	$\lambda^-_w \cdot \dfrac{w \cdot \ell^2}{8} = +\ 1.6$ mt	$\lambda^+_w \cdot \dfrac{w \cdot \ell^2}{8} = -5.7$ mt

It must be emphasized here that "minimum value" and "maximum value" are not to be taken in a statistical sense. The figures for the load factors include all the safety margins and basically, therefore, they have nothing to do with a statistical variation of the assumed load. Division into load factors is carried out here solely in order to prevent the errors described above. Because as was shown for the material factors in 4.5.1., no algebraic relation between distribution functions and safety margins can be constructed as soon as these are divided into partial factors, these load factors are merely comparative values. As such they cannot be brought into agreement with the statistical laws of safety, but can no doubt be used as an approximation for the single safety factor.

We now calculate the extreme values of the load considering the various load cases. In order not to calculate all variations separately, but instead to determine the extreme values directly, the following rule concerning the signs of the load factors to be chosen may be observed:

Let

$$\text{sign}\,(\lambda_i - 1) \cdot \text{sign}\,(q_i) = \text{sign}\,(B_{max} - \bar{B})$$

always be satisfied, where q_i is a specific load component, λ_i the corresponding load factor, B_{max} and $\bar{B}$ the extreme values sought and the design value of the load respectively. The case $\lambda_i = 1$ is not embraced in this simple rule. This is the case where the extreme is exactly equal to the design value, and as such can be used in determining the stress. It is also assumed that

$$\lambda^-_i \leq 1 \leq \lambda^+_i$$

This means that the design value of any load must lie between the two "extremes", as will generally be the case.

A few additional remarks are required on the use of the two-fold load factors(214).

1. The formal difficulty which was noted in connection with the load factors in 4.5. is equivalent to saying that the "extreme values" just calculated are not true extreme values, but are comparative values used for dimensioning purposes. This is in conflict with the principle that the true form of the structural problem should not be modified by the safety margins. This is a serious objection. It may be illustrated as follows: because of the safety margins which must be introduced into the load factors, we get for the calculated "extreme values of the dimensioning" values which no longer bear any relation to reality.

2. The advantage that is gained with the method of two-fold load factors is that the often obscure and difficult problem of combined stresses, can be reliably taken into account without greatly adding to the work of calculation.

3. The method assumes the superposition of the forces in the stress. Since this is correct only for truly elastic problems of the first order and in the broader sense for all statically determinate problems, the method cannot be applied in this form, for example, to plastic methods of design.

4. A system of two-fold load factors would have to be directed towards the assumed load alone. Admittedly one would also have to consider variations of the other structural properties. This can be avoided by a combination with material factors, as is already done in some places (see 4.5.2.) where, of course, two-fold load factors are not prescribed.

BIBLIOGRAPHY

1. STATISTICS, MATHEMATICS

(11) B. L. Van der Waerden, Mathematische Statistik, Springer-Verlag, Berlin, 1957.

(12) Emil J. Gumbel, Statistical Theory of Extreme Values and Some Practical Applications, National Bureau of Standards, Applied Mathematics Series No. 33, Washington D. C., 1954.

(13) W. A. Weibull, Statistical Theory of the Strength of Materials, Ingeniørsvetenskapsakademiens Handlingar, No. 151, Stockholm, 1939.

2. SAFETY

(21) A. M. Freudenthal, Safety and Probability of Structural Failure. Proc. of the ASCE, vol. 80, sep. No. 468, 1954.

(22) A. M. Freudenthal, Safety, Reliability and Structural Design, Proc. ASCE, Struct Div., März 1961.

(23) E. Basler, Untersuchungen über den Sicherheitsbegriff von Bauwerken, Schweizer Archiv für angewandte Wissenschaften, 27. Jahrgang, 1961.

(24) W. R. Schriever, Loads and Load Factors, Technical Paper No. 106, of the Division of Building Research, Ottawa, Okt. 1960.

(25) C. B. Brown, Concepts of Structural Safety, Proc. ASCE, Struct. Div., Dez. 1960.

(26) C. Stamm, Brückeneinstürze und ihre Lehren, Mitteilungen aus dem Institut für Baustatik, ETH. Zürich.

(27) A. G. Pugsley, Concepts of Safety in Structural Engineering, Journal of the Inst. of Civ. Eng., März 1951.

(28) A. H. Chilver, Some Problems of Structural Safety, British Welding Journal, 1955.

(29) William I. Stieglitz, Safety and Reliability as Design Parameters, Aeronautical Engineering Review, Sept. 1955.

(210) A. G. Pugsley, Design for Safety and Efficiency, The Structural Engineer, Jan. 1957.

(211) A. G. Pugsley, Current Trends in the Specifications of Structural Safety, Engineer, Juni 1961.

(212) Paul Weidlinger, A new Approach to Safety of Buildings, Architectural Record, Okt. 1952.

(213) M. Délesques, Ce qui représente le coefficient de la sécurité, Annales de l'Institut Technique du Bâtiment et des Travaux Publics.

(214) R. Wolfensberger, Bemerkungen über die Sicherheit, Kolloquium zum Kurs über plastische Berechnungsmethoden, ETH Zürich, Institut für Baustatik, 1963.

(215) E. Torroja, Philosophy of Structures, University of California Press, Berkeley 1960.

(216) E. Torroja, A. Paez, Calcul du Coefficient de sécurité, IABSE, 4, Kongress, Vorbericht, 1952.

(217) E. Torroja, Load Factors, Report ICBR, Journal of the ACI, 1958.

(218) A. L. L. Baker, The Work of the European Committee on Concrete, Structural Engineer, Volume 36, No. 1, Jan. 1958.

(219) S. O. Asplund, The Risk of Failure, Structural Engineer, vol. 36, No. 8, Aug. 1958.

(220) Hajnal-Konyi, Reinforced Steel in Concrete and the Concept of Safety, Proc. ACI, vol. 48, 1952.

(221) F. G. Thomas, Load Factor Methods of Designing Reinforced Concrete, Reinforced Concrete Review, vol. 13, No. 8, 1955.

(222) S. C. C. Bate, Discussion on Prestressed Units for Shortspan Highway Bridges, Proc. Inst. Civ. Engineers, Juni 1955.

(223) A. M. Freudenthal, Load Factors, Jour. ACI, März 1960.

(224) Rosenblueth, Discussion to (217).

(225) M. Tichy, M. Vorliček, Safety of Eccentrically Loaded Reinforced Concrete Columns, Jour. ASCE, Struct. Div., Okt. 1962.

(226) N. C. Lind, C. J. Turkstra, D. T. Wright, Sicherheit, Wirtschaftlichkeit und Aufwand in der Tragwerksberechnung, IVBH-Vorbericht zum VII, Kongress, 1964.

3. STRUCTURAL ANALYSIS

(31) B. Thürlimann, H. Ziegler, Plastische Berechnungsmethoden, Fortbildungskurs für Bau- und Maschineningenieure, ETH Zürich, 1963.

(32) B.G. Neal, Die Verfahren der plastischen Berechnung biegesteifer Stahlstabwerke, Springer-Verlag, Berlin, 1958.

(33) L.B. Kriz, Ultimate Strength Criteria for Reinforced Concrete, Proc. ASCE, Eng. Mech. Div., Juli 1959.

(34) F. Knoll, Tabellen für die Traglast doppelt armierter Stahlbetonquerschnitte bei Druck mit kleiner Exzentrizität, Schweiz, Bauzeitung, Dez. 1965.

(35) K. Beyer, Die Statik im Eisenbetonbau, Band 2, Springer-Verlag, Berlin 1934.

4. STATISTICAL DATA ON PROPERTIES OF MATERIAL

(41) G. Winter, Properties of Steel and Concrete and the Behaviour of Structures, Proc. ASCE, Struct. Div., Febr. 1960.

(42) O.G. Julian, Synopsis of the first Report of Committee on Factors of Safety, Proc. ASCE, Struct. Div., Juli 1957.

(43) H. Blaut, Über den Zusammenhang von Qualität und Sicherheit im Betonbau, Deutscher Ausschuss für Stahlbeton, Heft 149, Berlin, 1962.

(44) Stahlton AG, Die statistische Festigkeitsprüfung des Standardbetons B im Werk III, Zürich, Nov. 1963.

(45) AG der von Moss'schen Eisenwerke, « Spannstähle », Berichte, Luzern, Jan. 1964.

5. STANDARDS

(51) SIA-Norm 160, Normen für die Belastungsannahme, die Inbetriebnahme und die Überwachung der Bauten, Zürich, 1956.

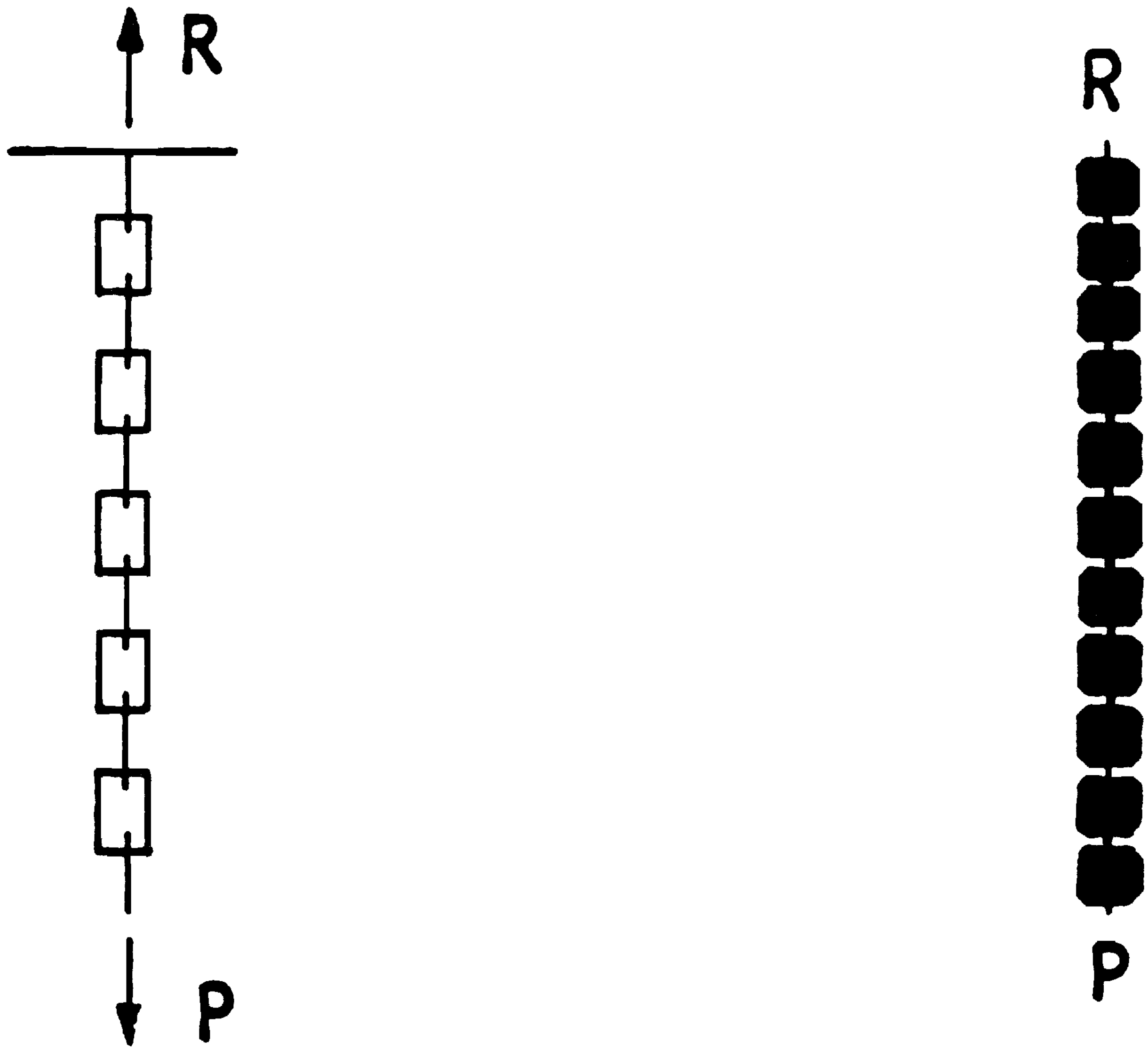
(52) SIA-Norm 161, Normen für die Berechnung und Ausführung der Stahlbauten, Zürich, 1956.

(53) SIA-Norm 162, Normen für die Berechnung und Ausführung der Beton- und Eisenbetonbauten, Zürich, 1956.

(54) Building Code Requirements for Reinforced Concrete, ACI Standard 318-63, Detroit 19, Juni 1963.

(55) UdSSR-Norm für Eisenbeton, N und TU 123-55, Moskau, 1955.

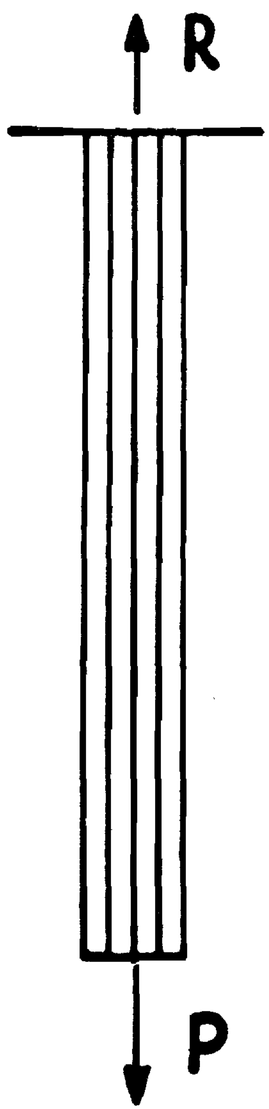
(56) Dansk Ingeniørforenings Normer for Bygningskonstruktioner, Beton- og Jernbetonkonstruktioner, Dansk Standard DS 411, Kopenhagen, 1949.



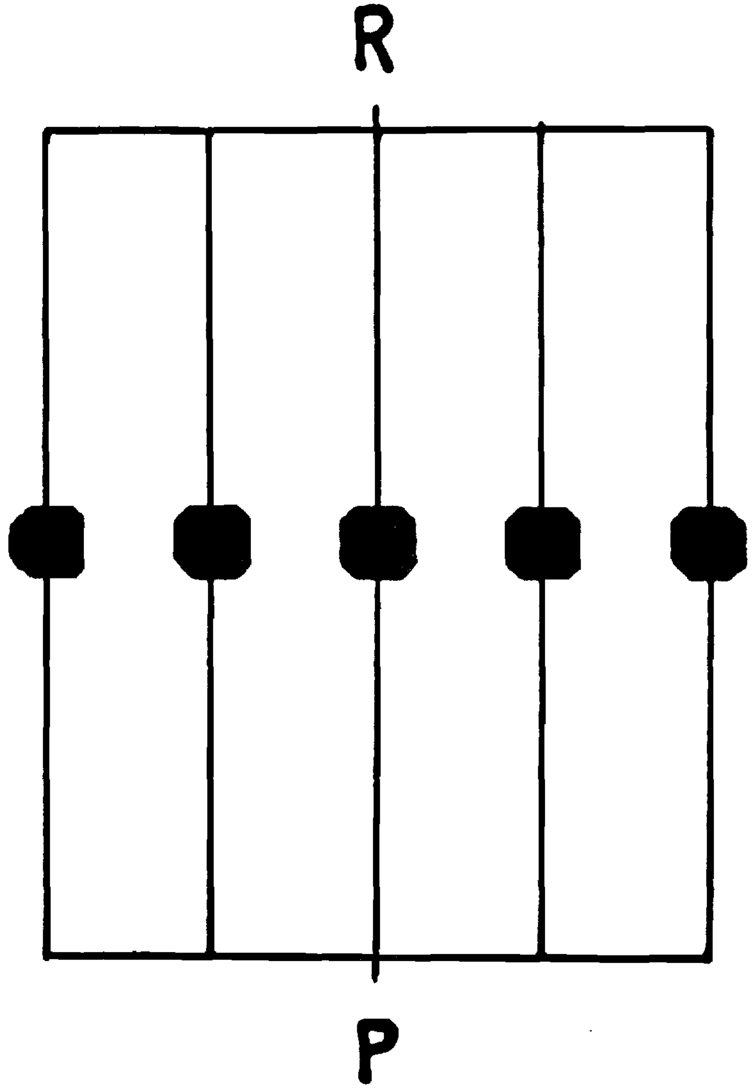
Load-Bearing Structure

Circuit Diagram

FIGURE 1

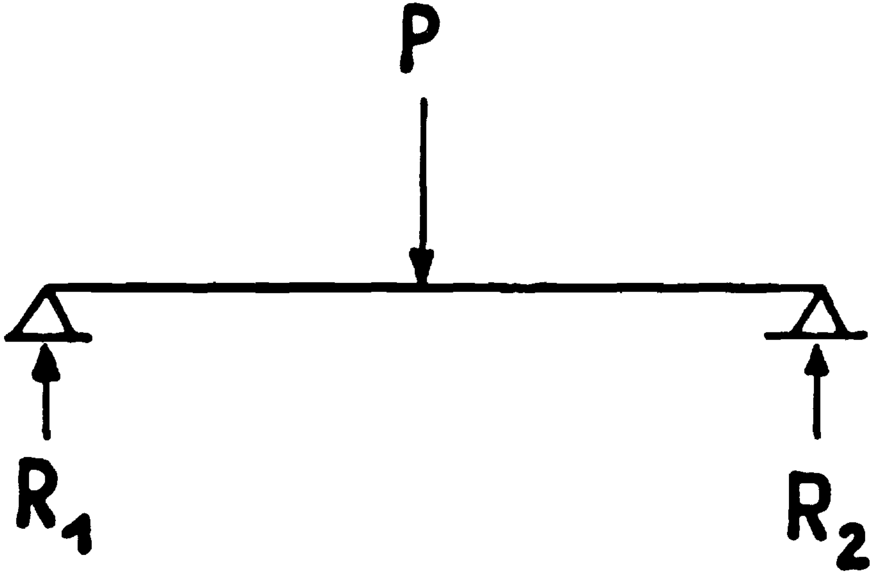


Load-Bearing Structure

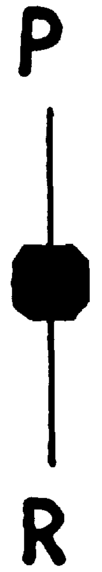


Circuit Diagram

FIGURE 2



Load-Bearing Structure



Circuit Diagram

FIGURE 3

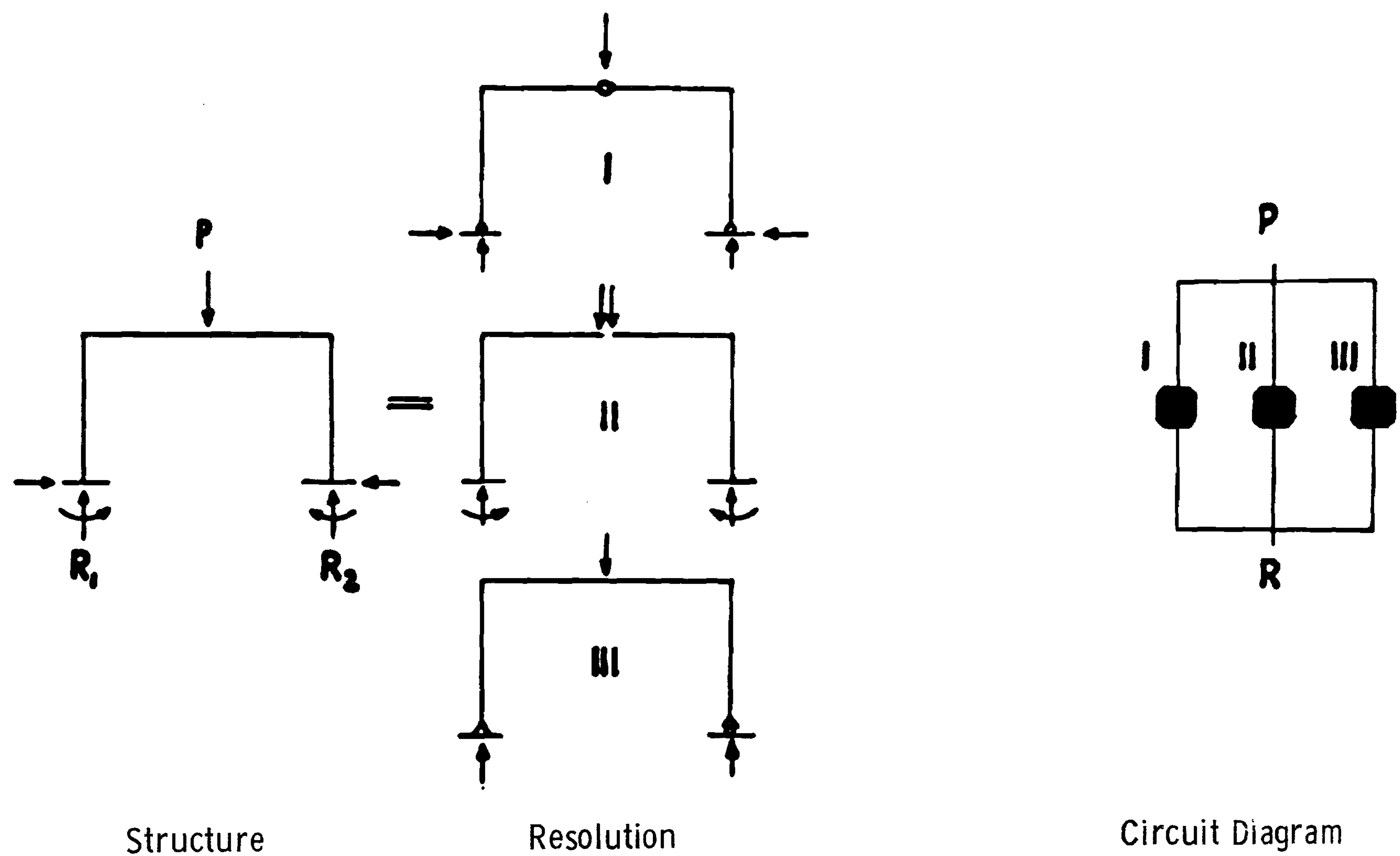
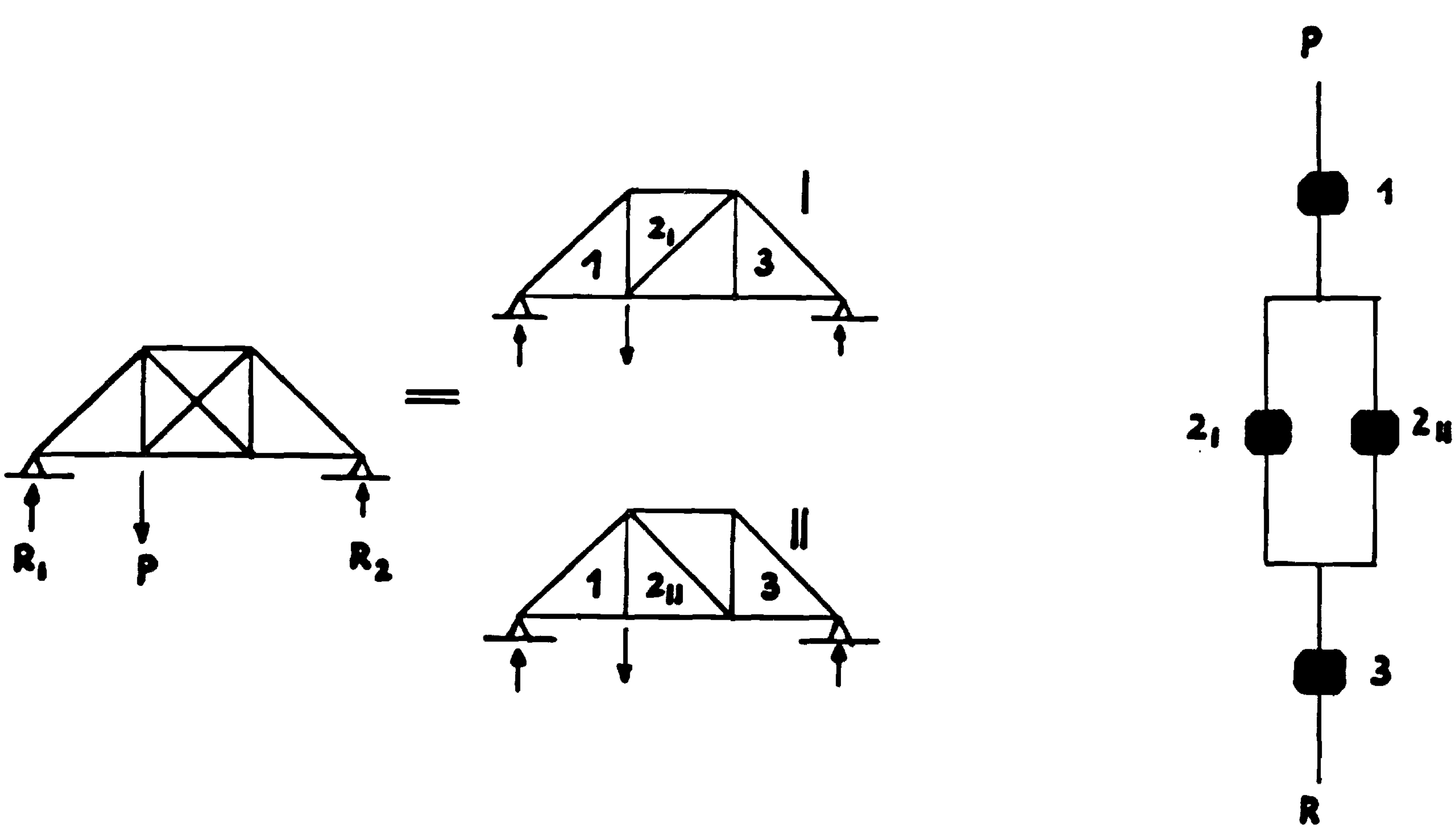
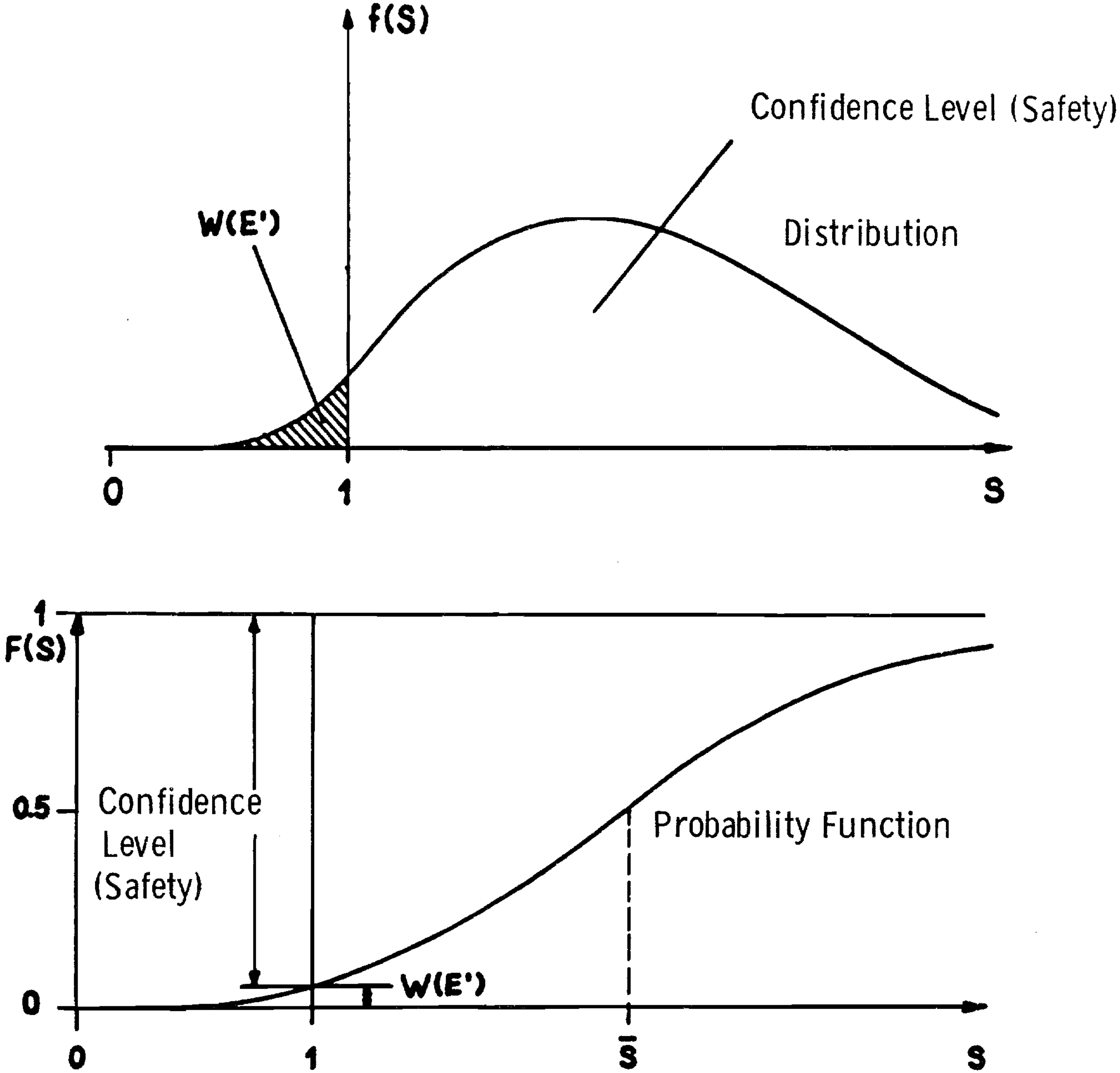

P
R₁
R₂
I
II
III
P
I
II
III
R
Structure
Resolution
Circuit Diagram

FIGURE 4



Structure Resolution Circuit Diagram

FIGURE 5



f(S)
Confidence Level (Safety)
W(E')
Distribution
0
1
S
1
F(S)
0.5
Confidence
Level
(Safety)
Probability Function
W(E')
0
0
1
$\bar{S}$
S

FIGURE 6

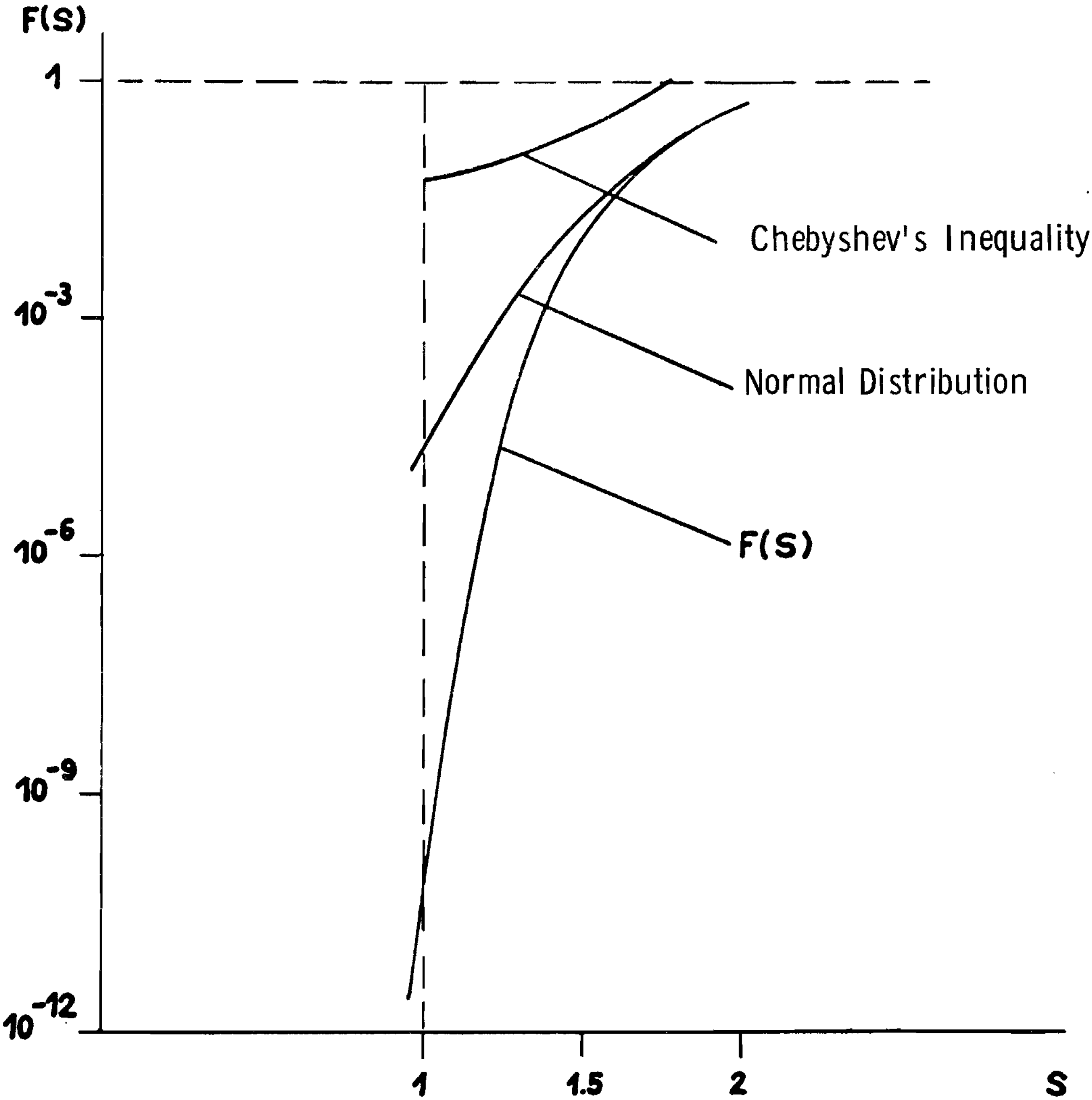

F(S)
1
10^-3
10^-6
10^-9
10^-12
Chebyshev's Inequality
Normal Distribution
F(S)
1
1.5
2
S

FIGURE 7

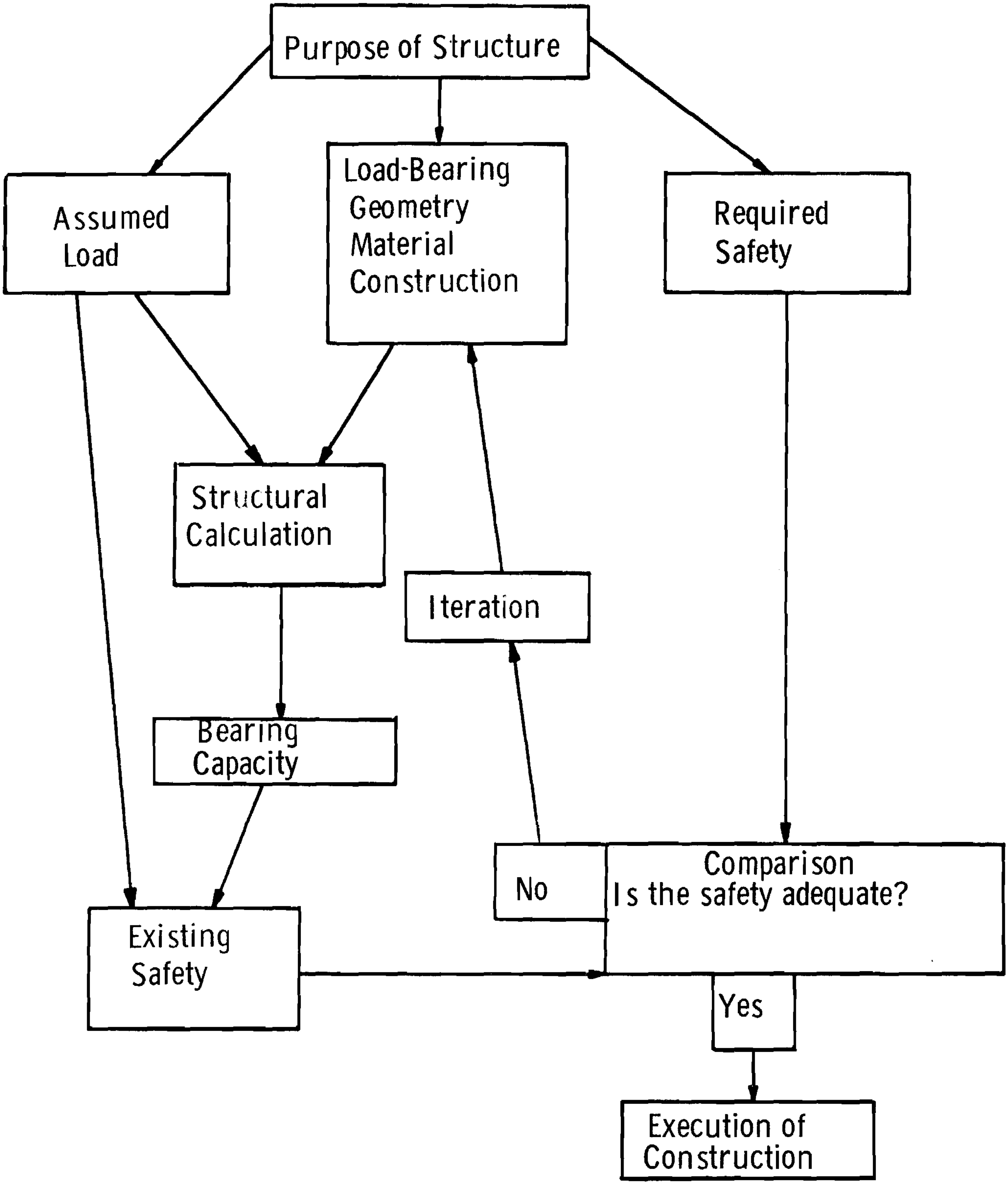

Purpose of Structure
Assumed Load
Load-Bearing Geometry Material Construction
Required Safety
Structural Calculation
Iteration
Bearing Capacity
No
Comparison
Is the safety adequate?
Existing Safety
Yes
Execution of Construction

FIGURE 8

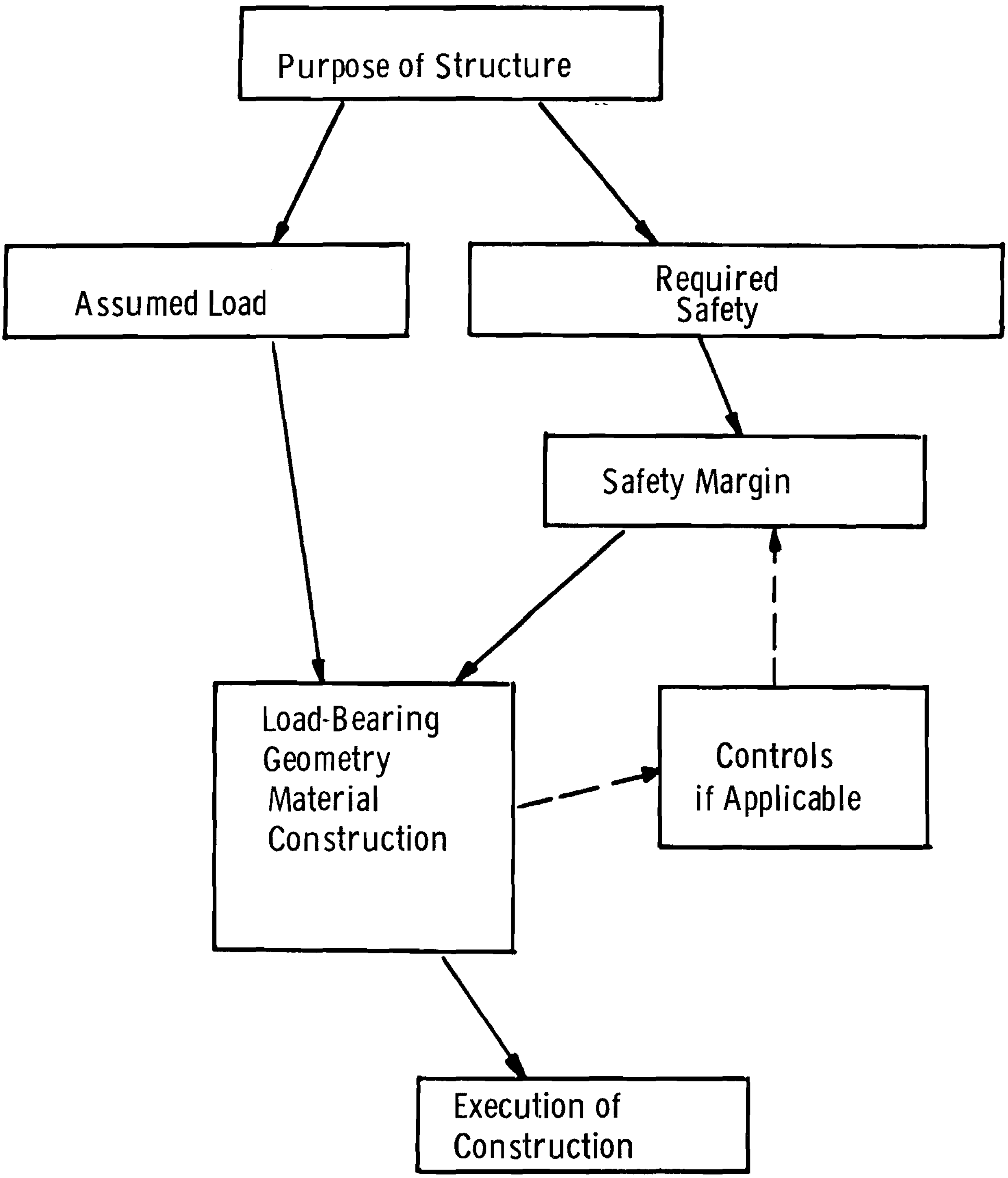

Purpose of Structure
Assumed Load
Required
Safety
Safety Margin
Load-Bearing
Geometry
Material
Construction
Controls
if Applicable
Execution of
Construction

FIGURE 9

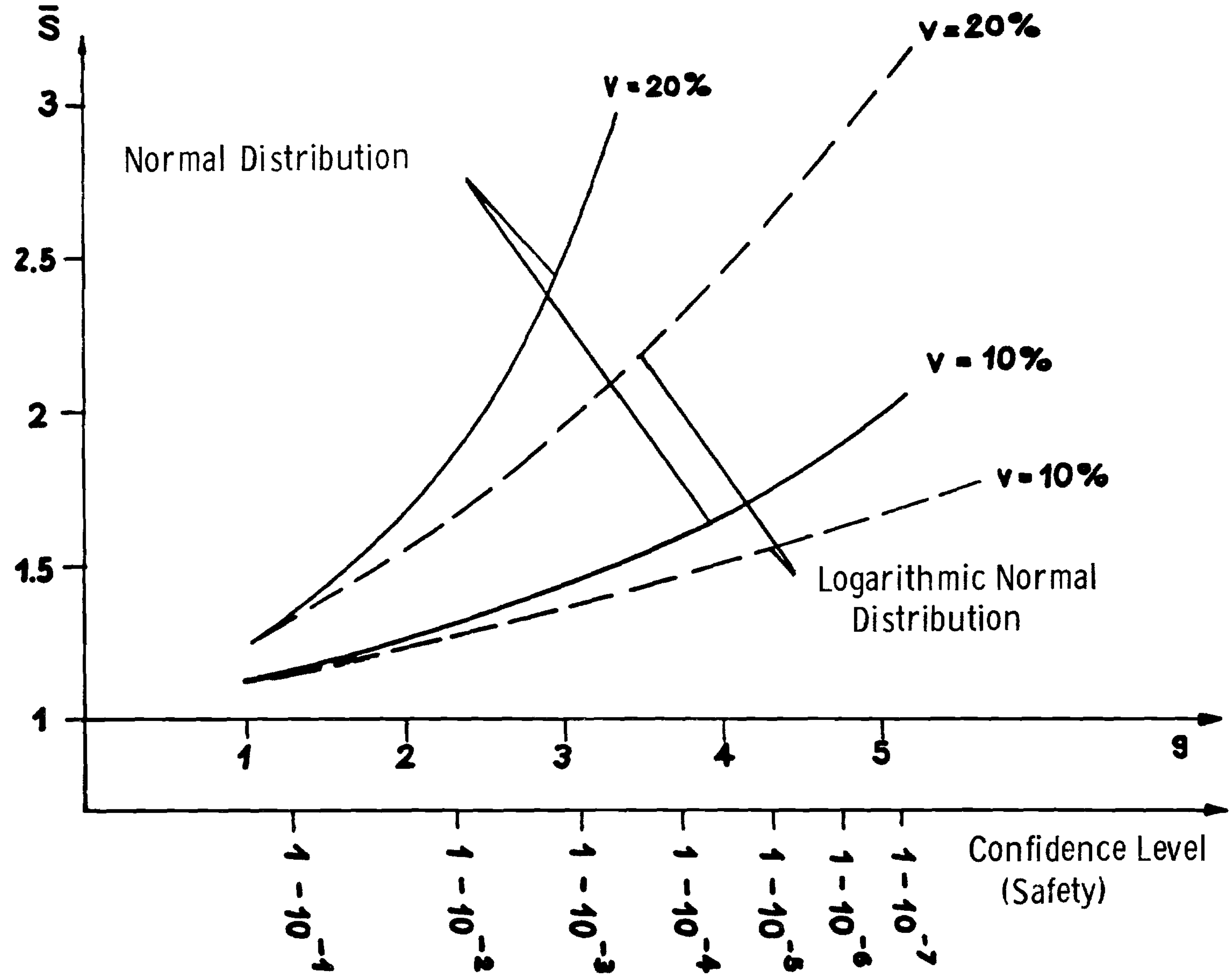

S̄
3
2.5
2
1.5
1
v = 20%
v = 20%
Normal Distribution
v = 10%
v = 10%
Logarithmic Normal Distribution
1
2
3
4
5
g
1 - 10^-1
1 - 10^-2
1 - 10^-3
1 - 10^-4
1 - 10^-5
1 - 10^-6
1 - 10^-7
Confidence Level (Safety)

FIGURE 10

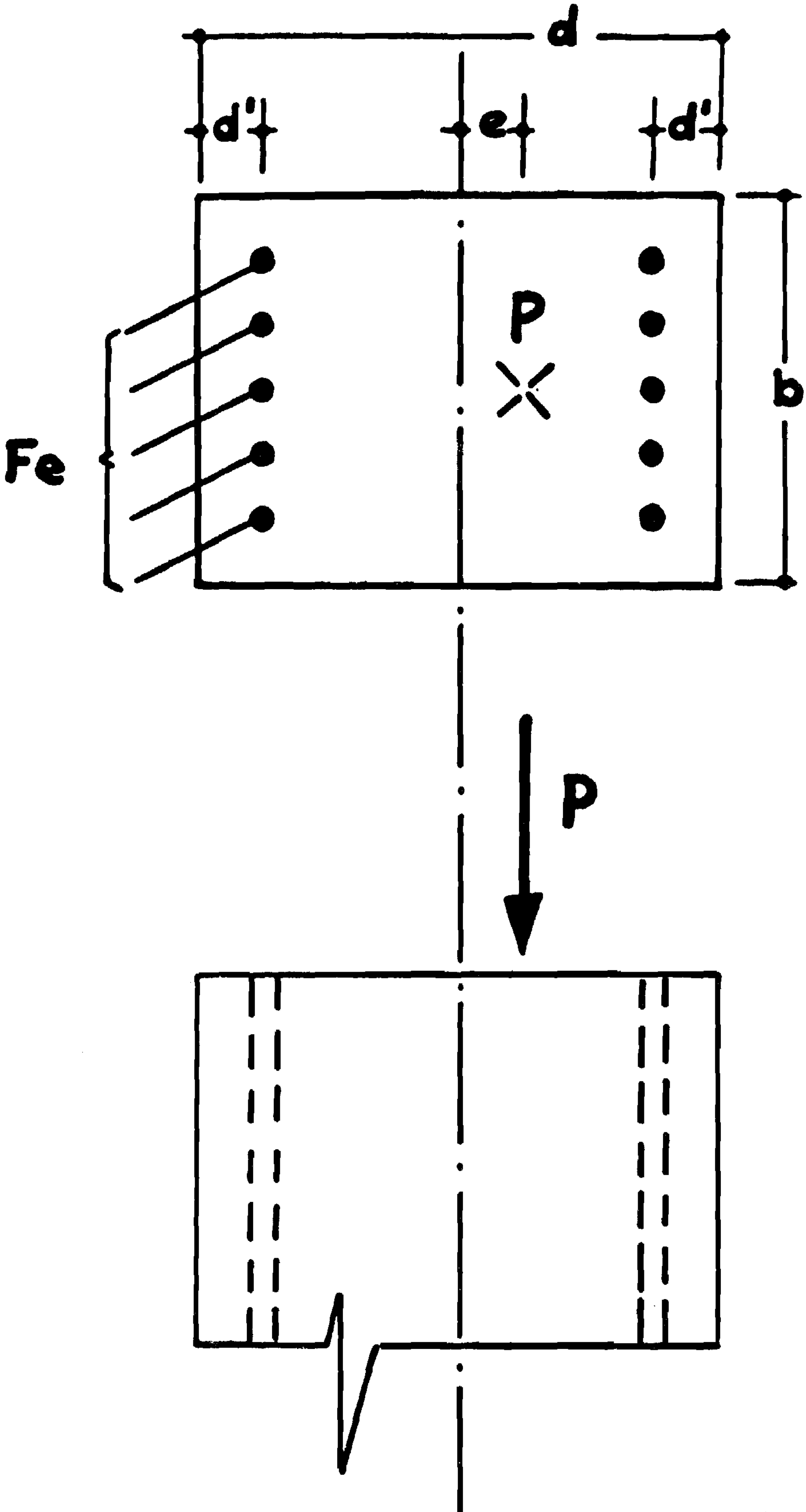


FIGURE 11

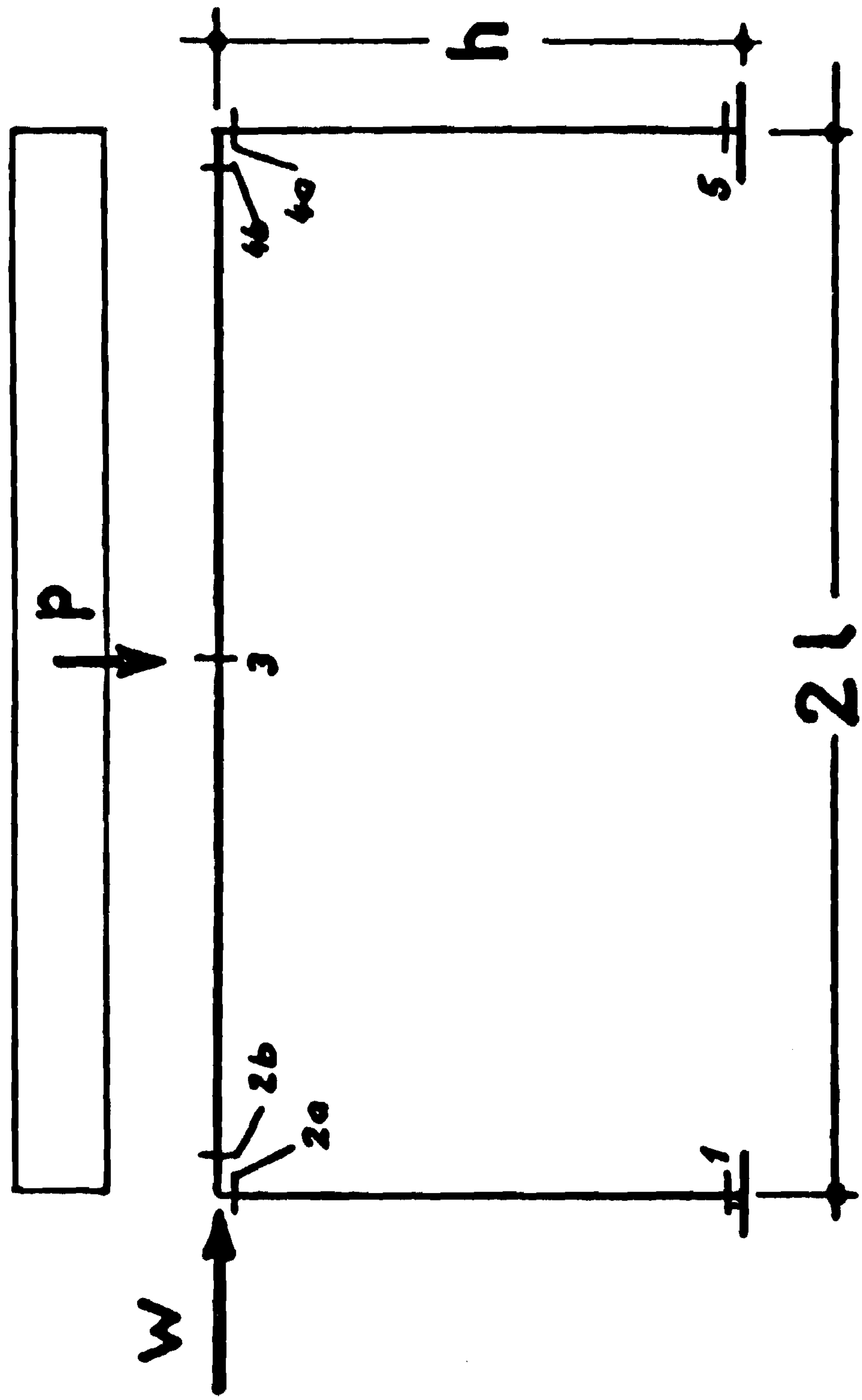


FIGURE 12

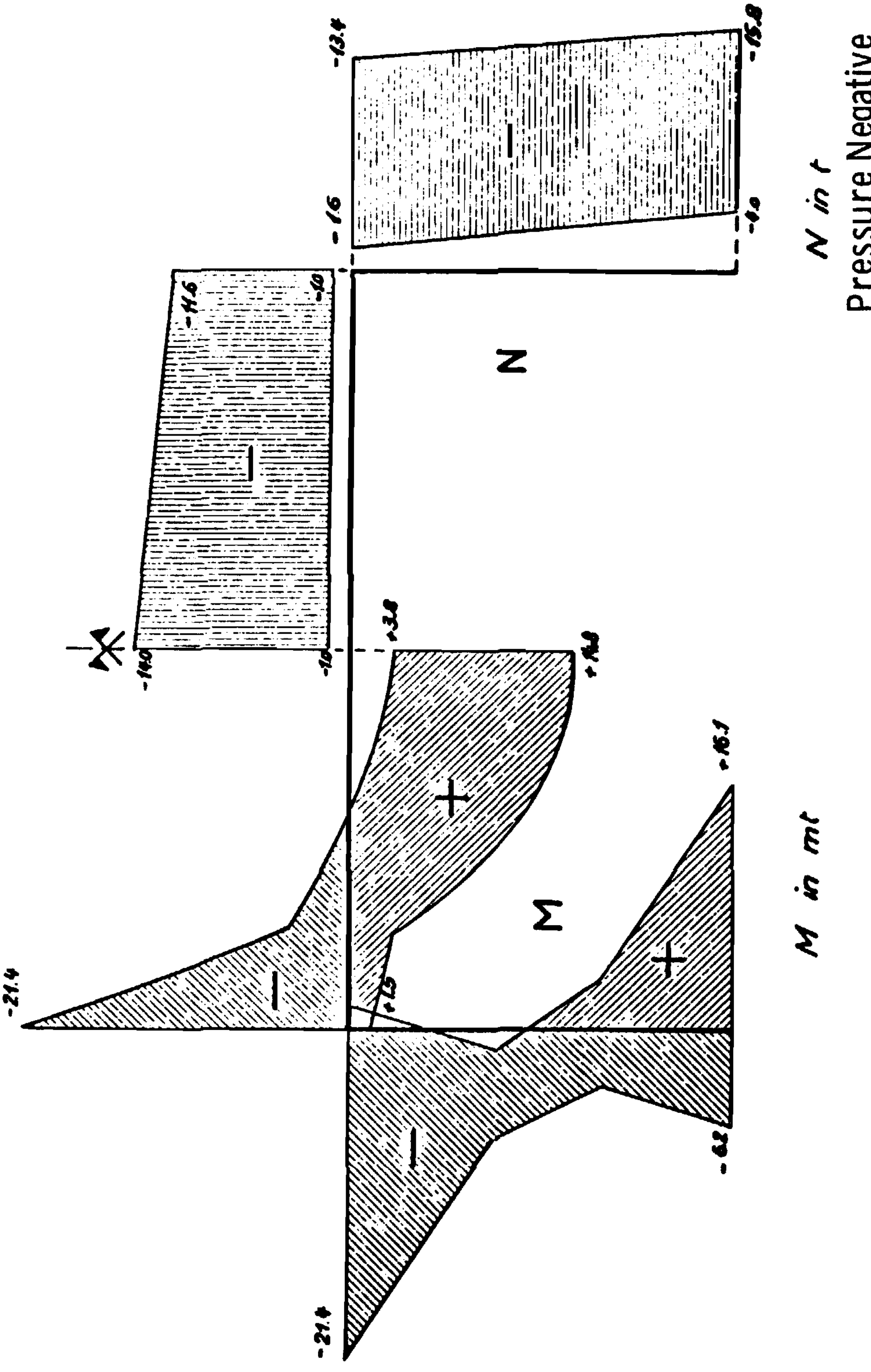

-21.4
-21.4
+1.5
-6.2
+16.1
+3.8
-14.0
-11.6
-1.6
-13.4
-15.8
M
N
M in mt
N in t
Pressure Negative

FIGURE 13

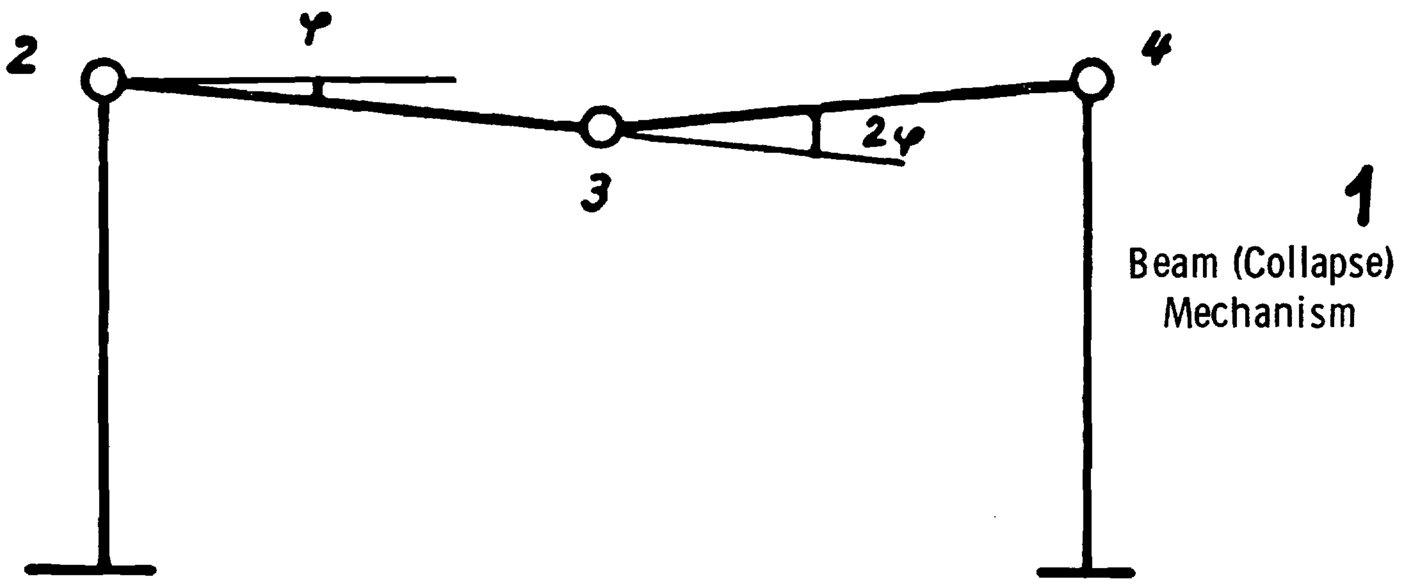


FIGURE 14

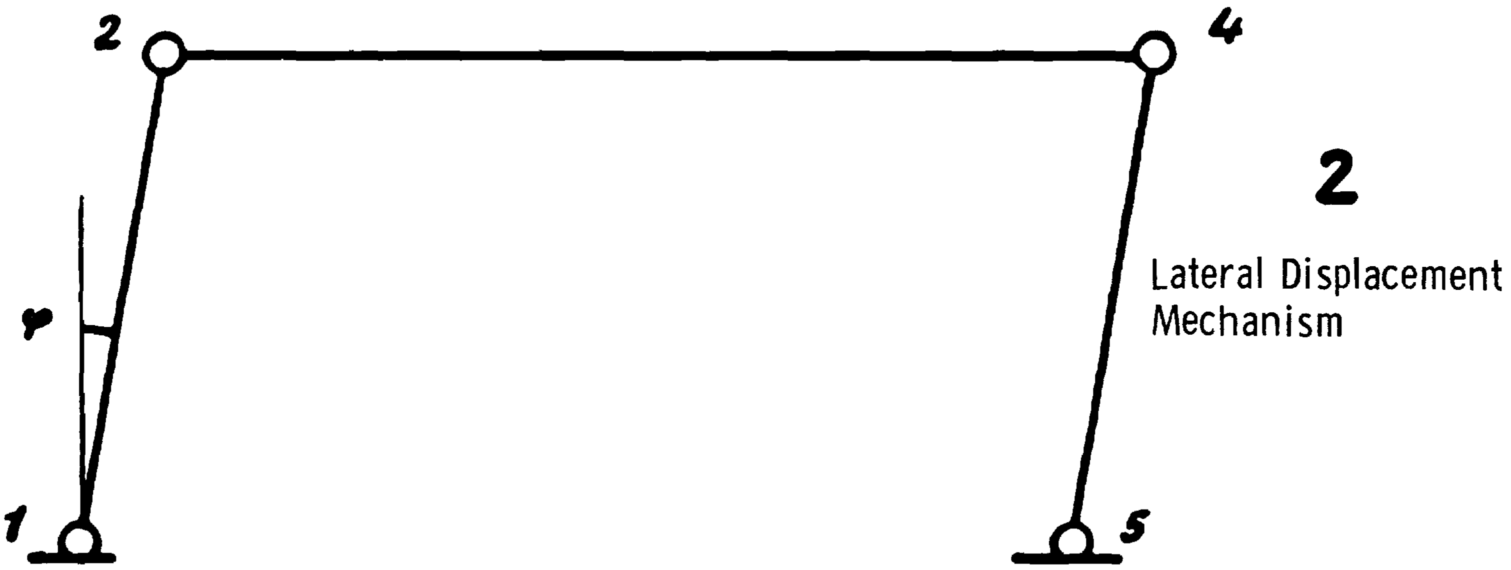
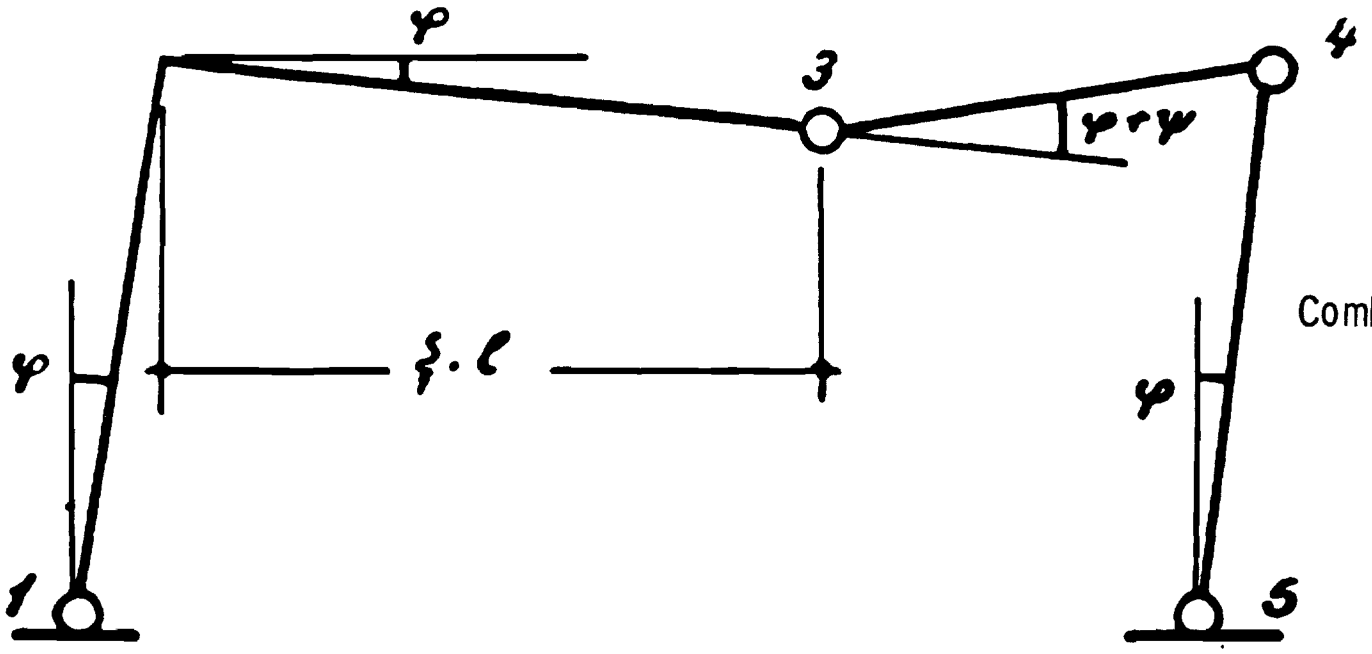


FIGURE 15



3

Combined Mechanism

$$\varphi + \psi = \varphi \cdot \frac{2}{2-\xi}$$

FIGURE 16

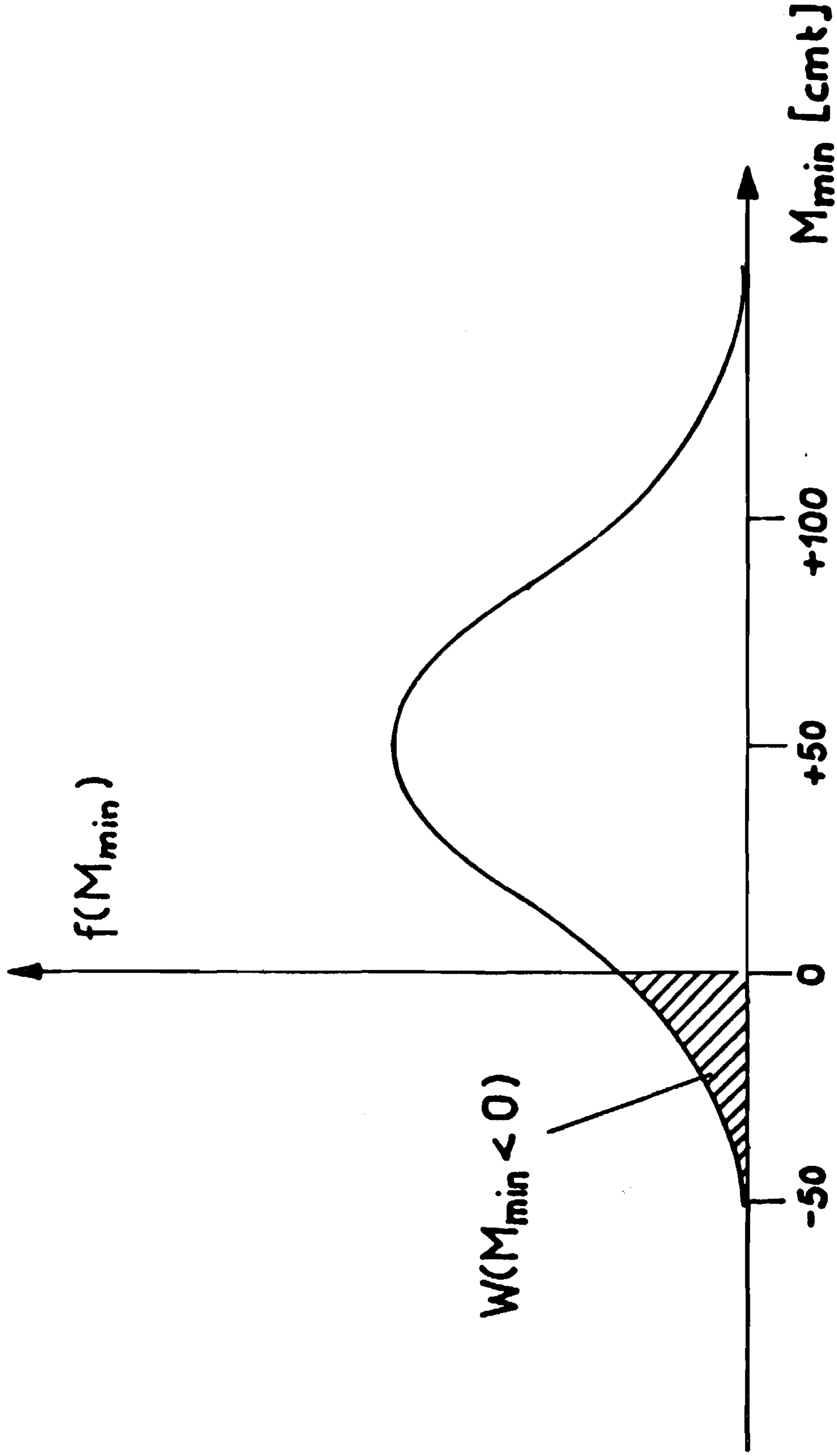


FIGURE 17

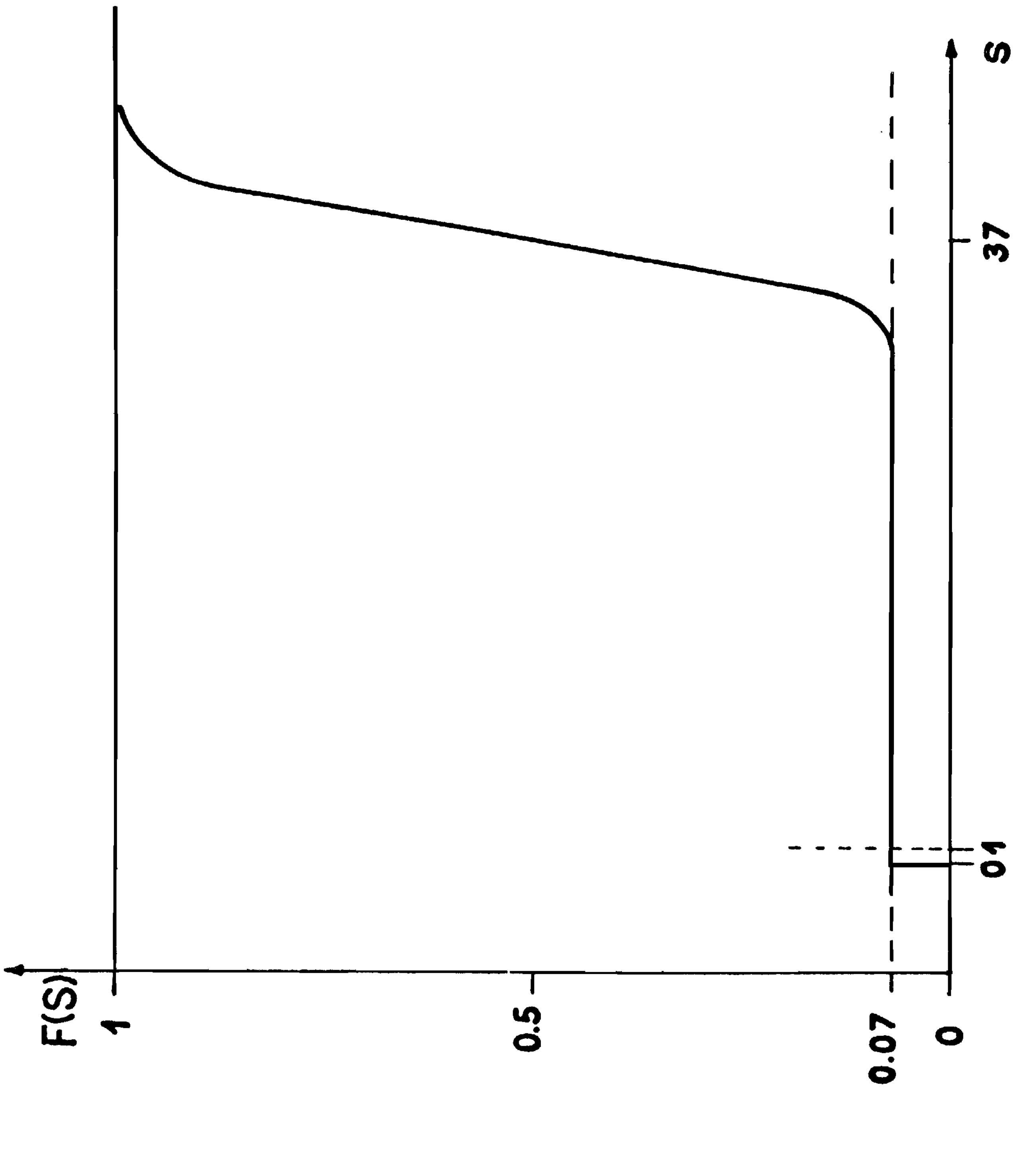


FIGURE 18