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Lie, T. T.

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#### **Publisher's version / Version de l'éditeur:**

<https://doi.org/10.4224/40001383>

*Paper (National Research Council of Canada. Institute for Research in Construction), 1994-05*

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**NRCC-36900**

Lie, T.T.

May 1994

A version of this document is published in / Une version de ce document se trouve dans:  
*Journal of Structural Engineering*, 120, (5), pp. 1489-1509, May-94, DOI:  
[10.1061/\(ASCE\)0733-9445\(1994\)120:5\(1489\)](http://doi.org/10.1061/(ASCE)0733-9445(1994)120:5(1489))

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# FIRE RESISTANCE OF CIRCULAR STEEL COLUMNS FILLED WITH BAR-REINFORCED CONCRETE

By T. T. Lie,<sup>1</sup> Member, ASCE

**ABSTRACT:** Experimental and theoretical studies have been performed to predict the fire resistance of circular hollow steel columns filled with bar-reinforced concrete. A mathematical model to calculate the temperatures, deformations, and fire resistance of the columns is presented. Calculated results are compared with those measured. The results indicate that the model is capable of predicting the fire resistance of circular hollow steel columns, filled with bar-reinforced concrete, with an accuracy that is adequate for practical purposes. The model enables the expansion of data on the fire resistance of circular concrete-filled steel columns, which at present consists predominantly of data for columns filled with plain concrete, with that for columns filled with bar-reinforced concrete. Using the model, the fire resistance of circular concrete-filled steel columns can be evaluated for any value of the significant parameters, such as load, column-section dimensions, column length, and percentage of reinforcing steel without the necessity of testing.

## INTRODUCTION

The use of hollow structural steel sections has several benefits: Such sections are very efficient structurally in resisting compression loads. By filling these sections with concrete, a substantial increase in load-bearing capacity can be achieved. Fire resistance can be obtained without the necessity of external fire protection for the steel, and eliminating steel surface protection increases the usable space in a building.

For a number of years, the National Research Council of Canada has been engaged in studies to develop methods for predicting the fire resistance of these composite columns. These studies were supported by the Canadian Steel Construction Council and the American Iron and Steel Institute. A multiphased program, which involved mathematical modeling and experiments, was established.

In the first phase, hollow steel sections filled with plain concrete were studied. These studies showed that substantial reductions in the loads on the columns have to be made to obtain reproducible and predictable fire resistances.

If the concrete is reinforced, however, the fire resistances remain predictable, even when very high loads are applied, as shown in studies on steel-bar-reinforced, concrete-filled columns (Chabot and Lie 1992). In this paper, a mathematical model for the prediction of the fire resistance of circular hollow steel columns, filled with bar-reinforced concrete, is presented, and the results produced by this model and those from tests are discussed.

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<sup>1</sup>Prin. Res. Officer, Inst. for Res. in Constr., Nat. Res. Council of Canada, Ottawa, Ontario K1A 0R6, Canada.

Note. Discussion open until October 1, 1994. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on August 5, 1993. This paper is part of the *Journal of Structural Engineering*, Vol. 120, No. 5, May, 1994. ©ASCE, ISSN 0733-9445/94/0005-1489/\$2.00 + \$.25 per page. Paper No. 6713.

## TEMPERATURES OF COLUMN DURING FIRE EXPOSURE

The calculation of the fire resistance of the column is done in various steps. It involves the calculation of the temperatures of the fire to which the column is exposed, the temperatures in the column, and its deformations and strength during the exposure to fire.

The column temperatures are calculated by a finite difference method (Dusinberre 1961). This method has been previously applied to the calculation of temperatures of various building components exposed to fire (Lie 1977). Because the method for deriving the heat transfer equations and calculating the temperatures is described in detail in those publications, it will not be discussed here; only the equations for the calculation of the column temperatures will be given.

### Division of Cross Section into Layers

The cross-sectional area of the column is subdivided into a number of concentric layers. There are  $M_1$  layers in the steel and  $M_2 - M_1 + 1$  layers in the concrete. As illustrated in Fig. 1, along any radius, a point  $P_m$ , representing the temperature of a layer ( $m$ ), is located a distance of  $(m - 1)\Delta\xi_s$  from the fire-steel boundary when the point is in the steel and a distance of  $(m - M_1)\Delta\xi_c$  from the concrete-steel boundary when the point is in the concrete. The outer layer of the steel, which is exposed to fire, has a thickness of  $1/2\Delta\xi_s$ . The layer of steel at the boundary between steel and concrete is also  $1/2\Delta\xi_s$  thick. The thickness of all other layers in the steel is  $\Delta\xi_s$ . The thickness of the layer of concrete at the boundary between steel

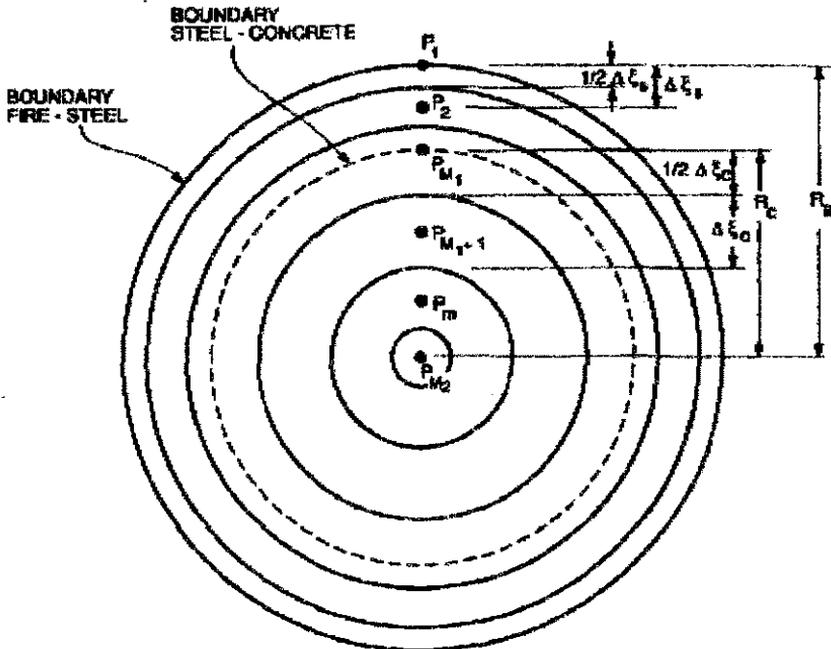


FIG. 1. Arrangement of Layers in Section of Concrete-Filled Steel Column

and concrete as well as at the center of the column is  $1/2\Delta\xi_c$ . The thickness of the other layers in the concrete is equal to  $\Delta\xi_c$ .

### EQUATIONS FOR FIRE-STEEL BOUNDARY

It is assumed that the entire surface of the column is exposed to the heat of a fire whose temperature course follows that of the standard fire described in ASTM E119-88 (*Standard* 1988) or CAN/ULC-S101 (*Standard* 1989). This temperature course can be approximately described by the following expression:

$$T_f^j = 20 + 750[1 - \exp(-3.79553\sqrt{\tau})] + 170.41\sqrt{\tau} \quad (1)$$

where  $\tau$  = the time, in hours; and  $T_f^j$  = the fire temperature, in degrees Celsius at time  $\tau = j\Delta\tau$ .

The temperature rise in the layer can be derived by creating a heat balance for each layer. In the following, all calculations will be done for a unit length of the column. For the layer at the exterior surface of the column, the temperature at time  $\tau = (j + 1)\Delta\tau$  is given by the expression:

$$T_1^{j+1} = T_1^j + \frac{2R_s\Delta\tau}{(\rho_s c_s)_1 \left(R_s - \frac{\Delta\xi_c}{4}\right) \Delta\xi_c} \left\{ \alpha_{e,s,f} [(T_f^j + 273)^4 - (T_1^j + 273)^4] \right. \\ \left. - \frac{\Delta\tau \left(R_s - \frac{\Delta\xi_c}{2}\right)}{(\rho_s c_s)_1 \left(R_s - \frac{\Delta\xi_c}{4}\right) (\Delta\xi_c)^2} \left\{ (k_s)_1^j + (k_s)_2^j \right\} (T_1^j - T_2^j) \right\} \quad (2)$$

### Equations for Inside Steel

For the layers in the steel, except for the surface layer and the layer at the boundary of the steel and concrete, the temperature at time  $\tau = (j + 1)\Delta\tau$  is given by:

$$T_m^{j+1} = T_m^j + \frac{\Delta\tau}{2(\rho_s c_s)_m \left[R_s - (m-1)\Delta\xi_s\right] (\Delta\xi_s)^2} \left\{ \left[ R_s - \left(m - \frac{3}{2}\right) \Delta\xi_s \right] \right. \\ \cdot \left[ (k_s)_{m-1}^j + (k_s)_m^j \right] (T_{m-1}^j - T_m^j) - \left[ R_s - \left(m - \frac{1}{2}\right) \Delta\xi_s \right] \right. \\ \left. \cdot \left[ (k_s)_m^j + (k_s)_{m+1}^j \right] (T_m^j - T_{m+1}^j) \right\} \quad (3)$$

### Equations for Steel-Concrete Boundary

For the layer at the boundary of the steel and the concrete, the temperature at time  $\tau = (j + 1)\Delta\tau$  is given by:

$$\begin{aligned}
T_{M_1}^{j+1} = & T_{M_1}^j + \frac{\Delta\tau}{(\rho_w c_w)'_{M_1} \left[ R_c - \left( M_1 - \frac{5}{4} \right) \Delta\xi_c \right] \Delta\xi_c + [(\rho_w c_w)'_{M_1} + \rho_w c_w \phi'_{M_1}] \left( R_c - \frac{\Delta\xi_c}{4} \right) \Delta\xi_c} \\
& \cdot \left\{ \left[ \frac{R_c - \left( M_1 - \frac{3}{2} \right) \Delta\xi_c}{\Delta\xi_c} \right] [(k_c)'_{M_2-1} + (k_c)'_{M_1}] (T_{M_2-1}^j - T_{M_1}^j) \right. \\
& \left. - \frac{R_c - \frac{\Delta\xi_c}{2}}{\Delta\xi_c} [(k_c)'_{M_1} + (k_c)'_{M_1+1}] (T_{M_1}^j - T_{M_1+1}^j) \right\} \quad (4)
\end{aligned}$$

#### Equations for Inside Concrete

For the layers in the concrete, except for the layer at the center of the column and the layer at the boundary of the concrete and steel, the temperature at time  $\tau = (j + 1)\Delta\tau$  is given by:

$$\begin{aligned}
T_m^{j+1} = & T_m^j + \frac{\Delta\tau}{2[(\rho_w c_w)'_m + \rho_w c_w \phi'_m] [R_c - (m - M_1) \Delta\xi_c] (\Delta\xi_c)^2} \\
& \cdot \left\{ \left[ R_c - \left( m - M_1 - \frac{1}{2} \right) \Delta\xi_c \right] [(k_c)'_{m-1} + (k_c)'_m] (T_{m-1}^j - T_m^j) \right. \\
& \left. + \left[ R_c - \left( m - M_1 + \frac{1}{2} \right) \Delta\xi_c \right] [(k_c)'_m + (k_c)'_{m+1}] (T_m^j - T_{m+1}^j) \right\} \quad (5)
\end{aligned}$$

#### Equations for the Center of Concrete

For the center layer, the temperature at time  $\tau = (j + 1)\Delta\tau$  is given by:

$$\begin{aligned}
T_{M_2}^{j+1} = & T_{M_2}^j + \frac{2\Delta\tau}{[(\rho_w c_w)'_{M_2} + \rho_w c_w \phi'_{M_2}] (\Delta\xi_c)^2} \\
& \cdot [(k_c)'_{M_2-1} + (k_c)'_{M_2}] [T_{M_2-1}^j - T_{M_2}^j] \quad (6)
\end{aligned}$$

#### Effect of Moisture

The effect of moisture in the concrete on the column temperatures is taken into account by assuming that, in each layer, the moisture starts to evaporate when the temperature reaches 100°C. In the period of evaporation, all the heat supplied to a layer is used for evaporation until the layer is dry.

For the concrete layer at the boundary between steel and concrete, the initial volume of moisture is given by:

$$V_{M_1} = \pi \left( R_c - \frac{\Delta\xi_c}{4} \right) \Delta\xi_c \phi_{M_1} \quad (7)$$

From a heat-balance equation, it can be derived that, per unit length of the column, the volume  $\Delta V_{M_1}$  evaporated in the time  $\Delta \tau$  from the concrete layer at the steel-concrete boundary, is:

$$\Delta V_{M_1}^i = \frac{\pi \Delta \tau}{\rho_w \lambda_w} \left\{ \frac{1}{\Delta \xi_c} \left[ R_c - \left( M_1 - \frac{3}{2} \right) \Delta \xi_c \right] [(k_c)'_{M_1-1} + (k_c)'_{M_1}] \right. \\ \left. - (T'_{M_1-1} - T'_{M_1}) - \frac{1}{\Delta \xi_c} \left( R_c - \frac{\Delta \xi_c}{2} \right) [(k_c)'_{M_1} + (k_c)'_{M_1+1}] (T'_{M_1} - T'_{M_1+1}) \right\} \quad (8)$$

For the concrete layers inside the column, except for the layer at the boundary between the steel and concrete and the center layer, the initial volume of moisture is given by:

$$V_m = 2\pi [R_c - (m - M_1) \Delta \xi_c] \Delta \xi_c \phi_m \quad (9)$$

Similarly, as for the boundary concrete layer, it can be derived that, per unit length of the column, the volume  $\Delta V_m^i$  evaporated in time  $\Delta \tau$  from these layers, is:

$$\Delta V_m^i = \frac{\pi \Delta \tau}{\rho_w \lambda_w \Delta \xi_c} \left\{ \left[ R_c - \left( m - M_1 - \frac{1}{2} \right) \Delta \xi_c \right] [(k_c)'_{m-1} + (k_c)'_m] \right. \\ \left. - (T'_{m-1} - T'_m) \left[ R_c - \left( m - M_1 + \frac{1}{2} \right) \Delta \xi_c \right] [(k_c)'_m + (k_c)'_{m+1}] (T'_m - T'_{m+1}) \right\} \quad (10)$$

For the concrete center layer, the initial volume of moisture is:

$$V_{M_2} = \pi \frac{(\Delta \xi_c)^2}{4} \phi M_2 \quad (11)$$

From a heat-balance equation, it can be derived that, per unit length of the column, the volume  $\Delta V_{M_2}$  evaporated in the time  $\Delta \tau$  from the center layer, is:

$$\Delta V_{M_2}^i = \frac{\pi \Delta \tau}{2 \rho_w \lambda_w} [(k_c)'_{M_2-1} + (k_c)'_{M_2}] [T'_{M_2-1} - T'_{M_2}] \quad (12)$$

### Stability Criteria

To ensure that any error existing in the solution at some time will not be amplified in subsequent calculations, a stability criterion has to be satisfied which, for a selected value of  $\Delta \xi$ , limits the maximum time step  $\Delta \tau$ . Following the method described by Dusenberre (1961), it can be derived that, for the fire-exposed columns, the criterion of stability is given by the smallest of the following three criteria of stability.

- At the fire-steel boundary:

$$\Delta \tau_1 = \frac{(\rho_w c_w)_{\min} (\Delta \xi_c)^2}{2 [h_{\max} \Delta \xi_c + (k_c)_{\max}]} \quad (13)$$

- At the steel-concrete boundary:

$$\Delta\tau_2 = \frac{(\rho_s c_s)_{\min} \left[ R_s - \left( M_s - \frac{5}{4} \right) \Delta\xi_s \right] \Delta\xi_s + (\rho_c c_c)_{\min} \left( R_c - \frac{\Delta\xi_c}{4} \right) \Delta\xi_c}{2 \left\{ \frac{\left[ R_s - \left( M_s - \frac{3}{2} \right) \Delta\xi_s \right]}{\Delta\xi_s} (k_s)_{\max} + \frac{\left( R_c - \frac{\Delta\xi_c}{2} \right)}{\Delta\xi_c} (k_c)_{\max} \right\}} \quad (14)$$

- At the center:

$$\Delta\tau_3 = \frac{(\rho_c c_c)_{\min} (\Delta\xi_c)^2}{4(k_c)_{\max}} \quad (15)$$

where  $(\rho_s c_s)_{\min}$  and  $(\rho_c c_c)_{\min}$  = the minimum values of the heat capacity of the steel and concrete;  $(k_s)_{\max}$  and  $(k_c)_{\max}$  = the maximum values of the thermal conductivity of steel and concrete; and  $h_{\max}$  = the maximum value of the coefficient of heat transfer to be expected during the exposure to fire. For exposure to the standard fire, the maximum value of the coefficient of heat transfer,  $h_{\max}$ , is approximately 675 W/m<sup>2</sup>°C.

#### Procedure for Calculation for Column Temperatures

With the aid of (1)–(15), and the thermal properties for carbonate-aggregate concrete given in Appendix I (Lie 1992), the temperature distribution in the column and on its surface can be calculated for any time,  $\tau = (j + 1)\Delta\tau$ , if the temperature distribution at time  $j\Delta\tau$  is known. Starting from an initial temperature of 20°C, the temperature history of the column can be calculated by repeated application of (1)–(15).

### STRENGTH OF COLUMN DURING FIRE

#### Division of Cross Section into Annular Elements

To calculate the deformations and stresses in the column and its strength, the cross-sectional area of the column is subdivided into a number of annular elements. In Fig. 2, the arrangement of the elements is shown in a quarter section of the column. The arrangement of the elements in the three other quarter sections is identical to this. In the radial direction, the subdivision is the same as that shown in Fig. 1, where the cross section is divided into concentric layers. In the tangential direction, each quarter-section layer is divided into  $N$  elements. The temperature, representative of that of an element, is assumed to be equal to the temperature at its center. It is obtained by taking the average of the temperatures at the tangential boundaries of each element, previously calculated with the aid of (1)–(15).

Thus, for an element,  $P_{m,n}$ , the representative temperature is:

$$(T_{m,n}^l)_{\text{annular}} = \left( \frac{T_m^l + T_{m+1}^l}{2} \right)_{\text{layer}} \quad (16)$$

where the subscripts *annular* and *layer* refer to the annular elements shown in Fig. 2 and the element layers shown in Fig. 1, respectively.

For the steel reinforcing bars, a representative bar temperature can also be indicated. Measurements at various locations in steel bar sections during fire tests showed that the differences in temperature in the bar sections are

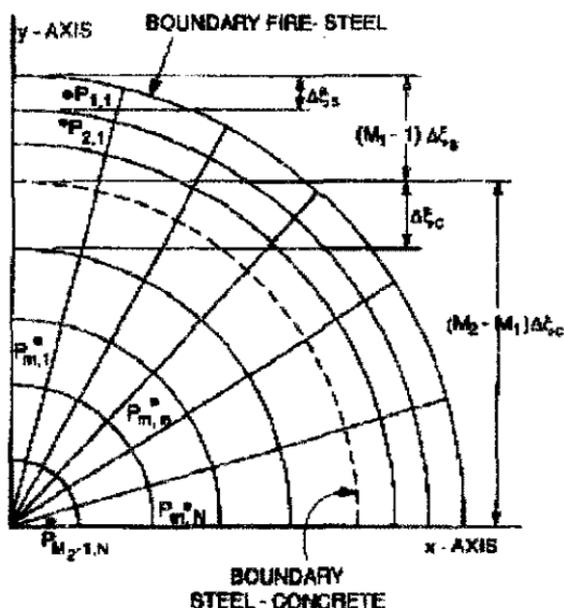


FIG. 2. Arrangement of Elements in Quarter Section

small (Lie et al. 1984). A close approximation of the average bar temperature is obtained by considering the column as consisting entirely of concrete and selecting the temperature at the location of the center of the bar section as the representative bar temperature. Thus, for a steel reinforcing bar, the center of whose section is located in an element  $P_{m,n}$ , the representative temperature is equal to that of  $P_{m,n}$ , which is given by (10).

Similarly, it is assumed that the stresses and deformations at the center of an element are representative of those of the whole element.

#### Assumptions in Calculation of Strength during Fire

During exposure to fire, the strength of the column decreases with the duration of exposure. The strength of the column can be calculated by a method based on a load-deflection or stability analysis (Allen and Lie 1974).

In this method, the columns are idealized as pin-ended columns of effective length  $KL$  (Fig. 3). The load on the column is intended to be concentric. Due to imperfections of the columns and the loading device, some eccentricity exists. The loading system and the test columns were made with high precision, however. Therefore, in the calculations, a very small arbitrary load eccentricity of 0.2 mm, reflecting a nearly concentric load, has been selected for the initial eccentricity.

The curvature of the column is assumed to vary from pin ends to midheight according to a straight-line relation, as illustrated in Fig. 3. For such a relation, the deflection at midheight  $Y$ , in terms of the curvature  $\chi$  of the column at this height, can be given by:

$$Y = \chi \frac{(KL)^2}{12} \quad (17)$$

For any given curvature, and thus for any given deflection at midheight,

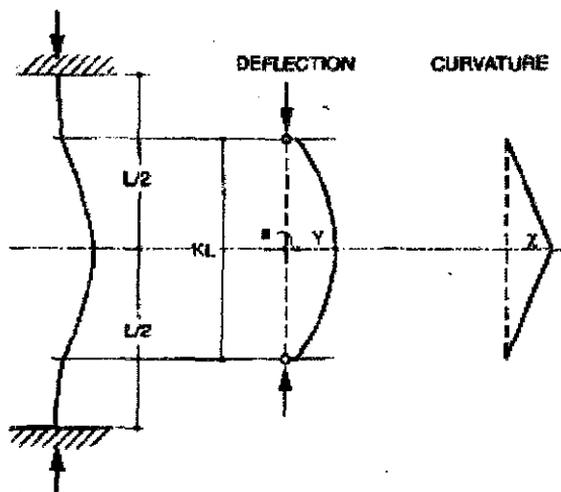


FIG. 2. Load Deflection Analysis

the axial strain is varied until the internal moment at the midsection is in equilibrium with the applied moment, i.e.

$$\sum_{m,n} f_{m,n} A_{m,n} x_{m,n} = \sum_{m,n} f_{m,n} A_{m,n} (Y + \epsilon) \quad (18)$$

In this way, a load-deflection curve can be calculated for any specific time during the exposure to fire. From these curves, the strength of the column, i.e., the maximum load that the column can carry, can be determined for each time. In the calculation of column strength, the following assumptions were made:

1. The properties of the steel and concrete are those described in Appendix I.
2. Concrete has no tensile strength.
3. Plane sections remain plane.
4. There is no slip between steel and concrete.
5. There is no composite action between the steel and concrete.
6. The reduction in column length before exposure to fire (consisting of free shrinkage of the concrete, creep, and shortening of the column due to load) is negligible. This reduction can be eliminated by selecting the length of the shortened column as initial length from which the changes during exposure to fire are determined.

Based on these assumptions, the column strength during exposure to fire was calculated. In the calculations, the network of annual elements shown in Fig. 2 was used. Because the strains and stresses of the elements are not symmetrical with respect to the y-axis, the calculations were performed for both the network shown and an identical network to the left of the y-axis. The load that the column can carry and the moments in the section were obtained by adding the loads carried by each element and the moments contributed by them.

### Equations for Concrete

The strain in the concrete for the elements to the right of the  $y$ -axis can be given by:

$$(\epsilon_c)_R = -(\epsilon_c)_T + \epsilon + \frac{x_c}{\rho} \quad (19)$$

and for the elements to the left of the  $y$ -axis by:

$$(\epsilon_c)_L = -(\epsilon_c)_T + \epsilon - \frac{x_c}{\rho} \quad (20)$$

where  $(\epsilon_c)_T$  = thermal expansion of concrete, in millimeters<sup>-1</sup>;  $\epsilon$  = axial strain of the column, in millimeters<sup>-1</sup>;  $x_c$  = horizontal distance from the center of the element to a vertical plane through  $y$ -axis of the column section, in meters; and  $\rho$  = radius of curvature, in meters.

The stresses in the elements are calculated using the stress-strain relations for carbonate-aggregate concrete given in Appendix I (Lie 1992).

### Equations for Steel

The strain in an element of the steel can be given as the sum of the thermal expansion of the steel  $(\epsilon_s)_T$ , the axial strain of the column  $\epsilon$ , and the strain due to bending of the column  $x_s/\rho$ , where  $x_s$  is the horizontal distance of the steel element to the vertical plane through the  $y$ -axis of the column section and  $\rho$  is the radius of curvature. For the steel to the right of the  $y$ -axis (Fig. 2), the strain  $(\epsilon_s)_R$  is given by:

$$(\epsilon_s)_R = -(\epsilon_s)_T + \epsilon + \frac{x_s}{\rho} \quad (21)$$

For the steel elements to the left of the  $y$ -axis, the strain  $(\epsilon_s)_L$  is given by:

$$(\epsilon_s)_L = -(\epsilon_s)_T + \epsilon - \frac{x_s}{\rho} \quad (22)$$

The stresses in the steel are calculated using the steel-strain relations for steel given in Appendix I (Lie 1992).

### Equations for Steel Reinforcement

The strain in the steel reinforcing bars can be given as the sum of the thermal expansion of the steel  $(\epsilon_B)_T$ , the axial strain of the column  $x_B/\rho$  where  $x_B$  is the horizontal distance of the center of the section of steel bar to the vertical plane through the  $y$ -axis of the column section, and  $\rho$  is the radius of curvature. For the steel bars at the right of the  $y$ -axis, the strain  $(\epsilon_B)_R$  is given by:

$$(\epsilon_B)_R = -(\epsilon_B)_T + \epsilon + \frac{x_B}{\rho} \quad (23)$$

For the steel bars to the left of the  $y$ -axis, the strain  $(\epsilon_B)_L$  is given by:

$$(\epsilon_B)_L = -(\epsilon_B)_T + \epsilon - \frac{x_B}{\rho} \quad (24)$$

The stresses in the steel are calculated using the stress-strain relations for steel given in Appendix I (Lie 1992).

#### Procedure for Calculation of Column Strength

With the aid of (19)–(24) and equations in Appendix I, the stresses at midsection in the steel and concrete elements can be calculated for any value of the axial strain  $\epsilon$  and curvature  $1/\rho$ . From these stresses, the load that each element carries and its contribution to the internal moment at midsection can be derived. By adding the loads and moments, the load that the column carries and the total internal moment at midsection can be calculated.

The fire resistance of the column is derived by calculating the strength of the column as a function of time of fire exposure. This strength reduces gradually with time. At a certain point, the strength becomes so low that it is no longer sufficient to support the load. At this point, the column becomes unstable and is assumed to have failed. The time to reach this failure point is the fire resistance of the column.

#### TEST SPECIMENS

Two specimens, consisting of hollow steel columns filled with reinforced carbonate-aggregate concrete, were tested and used to verify the model given in this paper. The test specimens are described in detail by Chabot and Lie (1992) and are illustrated in Fig. 4.

The columns were 3,810 mm long from end plate to end plate. The outside diameter was 273 mm and the steel wall thickness was 6.35 mm.

The steel columns were fabricated by cutting the steel to appropriate lengths. Steel end plates were then welded to the column extremities. Centering and perpendicularity of the end plates were given special attention to ensure a high degree of accuracy. Before welding the end plates, a hole, with a diameter 26 mm smaller than the inner diameter of the hollow steel section, was cut in each plate. Because of the smaller diameter of the holes in the end plates, a lip of 13 mm to transfer the load from the steel plate to the concrete filling was created after welding, as shown in Fig. 5.

Four small holes were also drilled in the steel wall to provide vent holes for water vapour produced during the experiment. Two holes were located opposite one another at 1,448 mm above midheight of the column; the other two were located opposite one another at 1,448 mm below midheight of the column.

The steel of the columns had a specified yield strength of 350 MPa. Deformed bars, with a minimum yield strength of 400 MPa, were used for the main reinforcing and tie bars. The diameter of reinforcing bars was 19.5 mm and that of the ties was 6.4 mm.

The main reinforcing bars were tied together to complete the steel cage and were cut 10 mm shorter than the column length (Figs. 4 and 5). The steel cage was then placed into the column with special care to ensure appropriate centering.

The concrete was poured in the column through the top opening. Its composition, per cubic meter of concrete mix, was as follows:

- Cement, 439 kg
- Water, 161 kg
- Fine aggregate, 621 kg
- Coarse aggregate, 1,128 kg

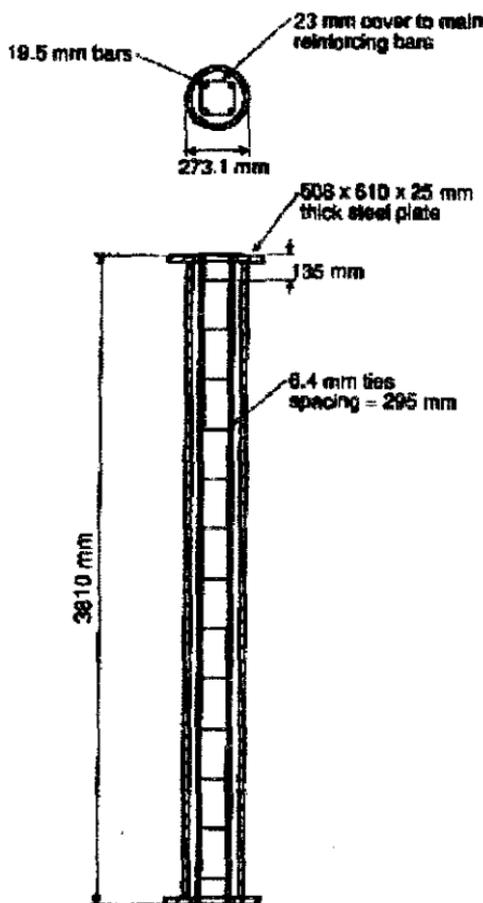


FIG. 4. Elevation and Cross Section of Columns

The 28-day cylinder strength was approximately 42 MPa. The average cylinder strength at the time of testing was approximately 47 MPa.

Chromel-alumel thermocouples with a thickness of 0.91 mm were installed at the midheight of the column for measuring the temperatures of the steel reinforcement and concrete at different locations in the cross section. The locations of the thermocouples are described in detail in a report by Chabot and Lie (1992).

#### TEST APPARATUS

The tests were done by exposing the columns to heat in a column test furnace. The test furnace was designed to produce the conditions to which a member might be subjected during a fire. It consists of a steel framework supported by four steel columns, with the furnace chamber inside the framework. The characteristics and instrumentation of the furnace, which has a loading capacity of 1,000 t, are described in detail in Lie (1980).

#### TEST CONDITIONS AND PROCEDURES

The tests were done with both ends of the columns fixed, i.e., restrained against rotation and horizontal translation.

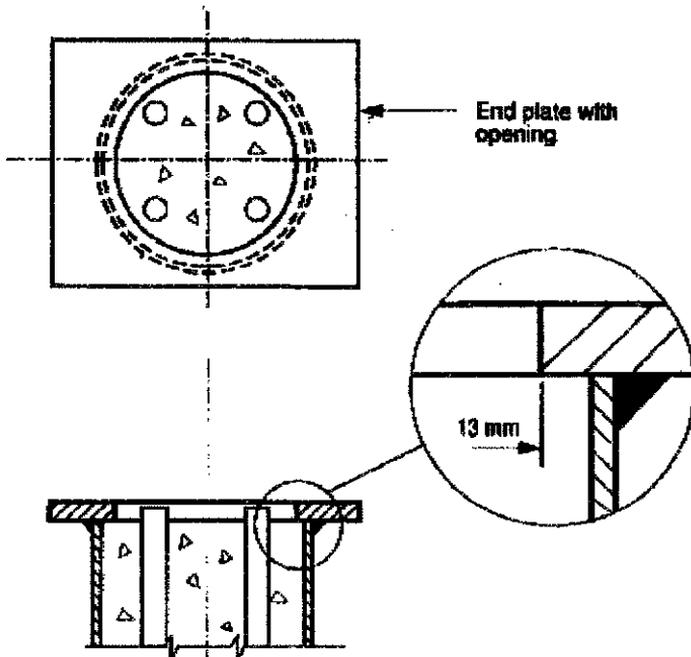


FIG. 5. End-Plate Connection Details

The columns were tested under a concentric load. The applied loads were 37% and 67% of the factored compressive resistance of the columns ( $C_{rc}$ ) or 95% and 171% of the factored compressive resistance of the concrete core ( $C_c$ ), determined according to the Canadian Standards Association standard CSA/CAN-S16.1-M89 ("Limit States" 1989). The factored compressive resistances of each column, as well as the applied loads, are given in Table 1. The effective length factor  $K$  used in the calculation of the factored compressive resistances was that recommended in CSA/CAN-S16.1-M89 for the given end condition, i.e., 0.65. The effective length of the columns,  $KL$ , was thus assumed to be 2.48 m.

During the test, the column was exposed to heating controlled in such a way that the average temperature in the furnace followed, as closely as possible, the ASTM E119-88 or CAN/ULC-S101 standard temperature-time curve.

## RESULTS AND COMMENTS

Using the mathematical model described in this paper, the temperatures, axial deformations, and strengths of the columns were calculated. In the calculations, the thermal and mechanical properties of the carbonate-aggregate concrete and steel, given in Lie (1992), were used. These properties and the specifics of the columns and the furnace are given in Appendix II.

In Figs. 6 and 7, the calculated temperatures are compared with the temperatures measured at the external surface of the steel section and at various depths in the concrete. With the exception of the temperatures measured at an early stage, there is good agreement between calculated

TABLE 1. Summary of Test Parameters and Results

Column (1)	Dimensions of steel section (mm) (2)	Steel bars (%) (3)	Concrete Strength		Factored Resistance <sup>a</sup>		Test load C (kN) (8)	Load Intensity		Fire Resistance (hr:min)	
			29 days (Mpa) (4)	Test data (Mpa) (5)	C <sub>c</sub> (kN) (6)	C <sub>cr</sub> (kN) (7)		C/C <sub>c</sub> (9)	C/C <sub>cr</sub> (10)	Calculated (11)	Measured (12)
1	273 diameter × 6.35	2.3	42.3	46.7	1,110	2,851	1,050	0.95	0.37	2:27	3:08
2	273 diameter × 6.35	2.3	42.3	47	1,110	2,851	1,900	1.71	0.67	1:28	1:36

<sup>a</sup>Factored resistance: C<sub>c</sub> = factored compressive resistance of concrete core of column according to CAN3-S16.1-M89; and C<sub>cr</sub> = factored compressive resistance of concrete-filled steel column according to CAN3-S16.1-M89.

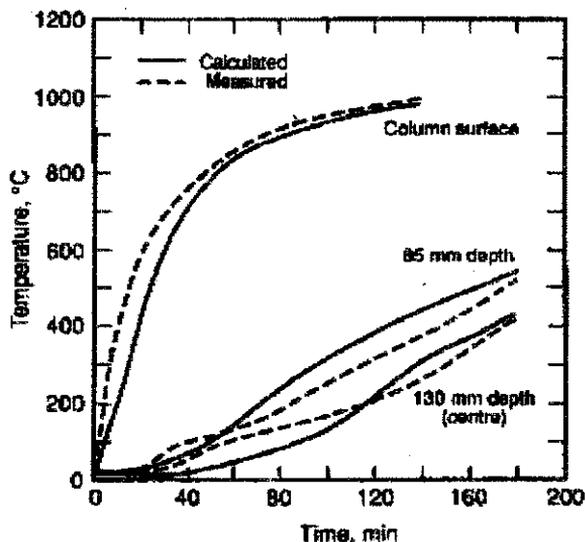


FIG. 6. Temperatures at Various Depths of Column 1 as Function of Exposure Time

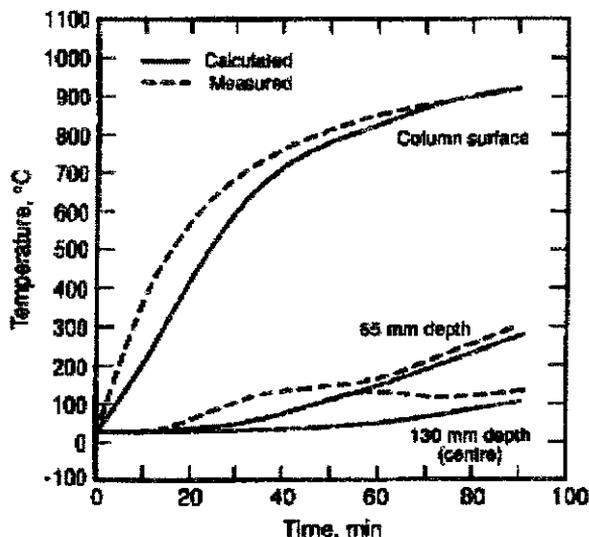
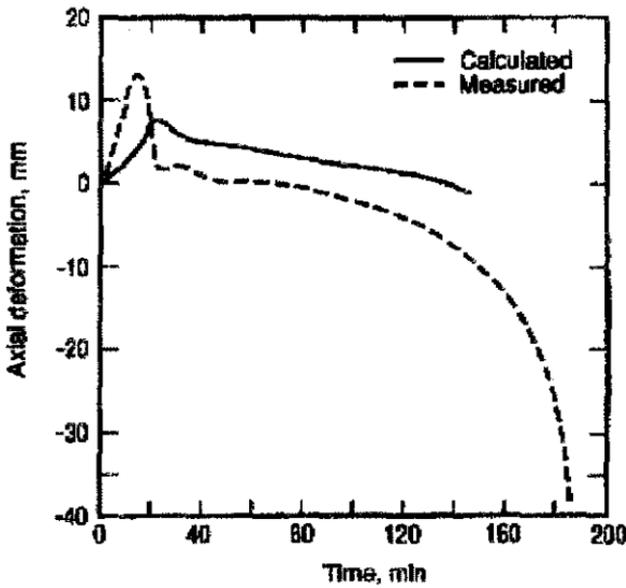
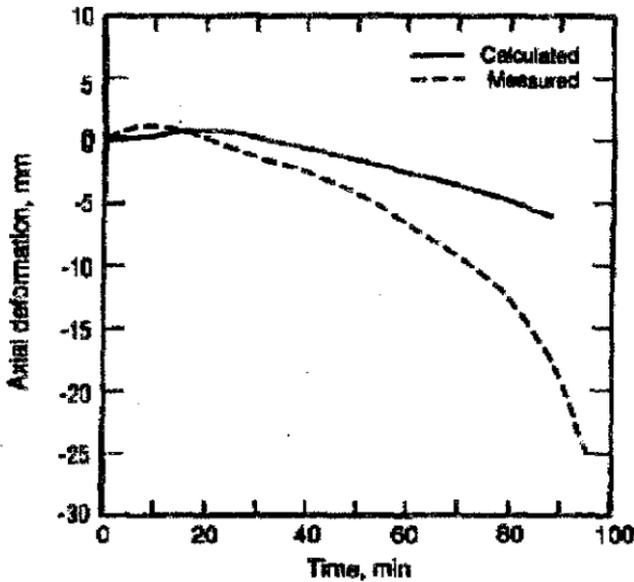


FIG. 7. Temperatures at Various Depths of Column 2 as Function of Exposure Time

and measured column temperatures. The temperatures measured deeper inside the column show initially a relatively rapid rise up to temperatures of approximately 100°C, followed by a period of relatively slow rate of temperature rise. This temperature behavior may be the result of thermally induced migration of the moisture toward the center of the column where,



**FIG. 8. Calculated and Measured Axial Deformations of Column 1 as Function of Exposure Time**



**FIG. 9. Calculated and Measured Axial Deformations of Column 2 as Function of Exposure Time**

as shown in previous tests (Chabot and Lie 1992), the influence of migration is most pronounced. Although the model takes into account evaporation of moisture, it does not take into account the migration of the moisture toward the center. That migration appears to account for the deviation between calculated and measured temperatures at the earlier stages of fire exposure.

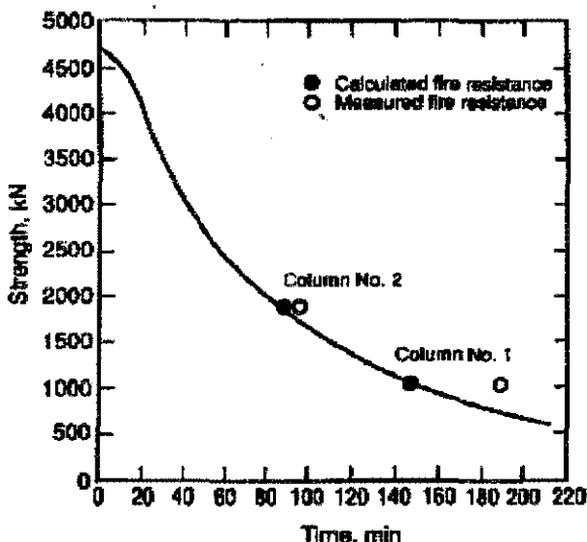


FIG. 10. Calculated Column Strength as Function of Time and Calculated and Measured Fire Resistances

At the later stages, however, which are important from the point of view of predicting the fire resistance of the columns, there is a good agreement between calculated and measured temperatures.

In Figs. 8 and 9, the calculated and measured axial deformations of the columns during exposure to fire are shown. There is reasonably good agreement in the trend of deformations between calculated and measured results. There are some differences, however, between the actual values of the calculated and measured deformations.

It must be noted that the column deforms axially as a result of several factors—namely, load, thermal expansion, bending, and creep—that cannot be completely taken into account in the calculations. Since the axial deformations, which are in the order of 20 mm, are for columns with a length of about 3,800 mm, small inaccuracies in these factors may cause noticeable differences between calculated and measured axial deformations. A difference of 10% between the theoretical and actual coefficients of thermal expansion of steel, for example, will cause a difference of approximately 5 mm in the axial deformations.

The effect of creep, which is more pronounced at the later stages of fire exposure, may be even greater. The model defines the failure point as the point at which the column can no longer support the applied load and assumes that failure at this point is instantaneous. During the tests, failure was not instantaneous but the columns contracted considerably, apparently as a result of continued loss of strength and creep, before they were crushed.

In Fig. 10, the calculated column strengths, as a function of the fire-exposure time, are shown together with the calculated and measured fire resistances for the test loads given in Table 1. The strength decreases with time until it becomes so low that the column can no longer support the load. The time to reach this point is the fire resistance of the column. The results show that the calculated fire resistance of column 1 is about 20% lower and

that of column 2 about 10% lower than the measured fire resistances. The differences are probably caused mainly by the considerable contraction of the columns, which the model can only partly take into account. For practical purposes, however, the calculated fire resistances, which lie on the safe side, are reasonably accurate.

## CONCLUSIONS

Based on the results of this study, the following conclusions can be drawn:

The mathematical model employed in this study is capable of predicting the fire resistance of circular columns, made of hollow structural steel filled with bar-reinforced concrete, with an accuracy that is adequate for practical purposes. The results indicate that the model is conservative in its predictions.

The model will enable the expansion of data on the fire resistance of circular concrete-filled steel columns, which at present predominantly consists of data for columns filled with plain concrete, with that for columns filled with bar-reinforced concrete.

Using the model, the fire resistance of circular concrete-filled steel columns can be evaluated for any value of the significant parameters—such as load, column-section dimensions, column length, and percentage of reinforcing steel—without the necessity of testing.

The model can also be used for the calculation of the fire resistance of columns made with concretes other than those investigated in this study—for example, lightweight or siliceous aggregate concretes that were not tested—if the relevant material properties are known.

## ACKNOWLEDGMENTS

This work was carried out at the National Fire Laboratory of the Institute for Research in Construction, National Research Council of Canada, with the support of the Canadian Steel Construction Council and the American Iron and Steel Institute. The writer would like to thank Martin Chabot for his contribution in processing the theoretical and experimental results, and John MacLaurin and John Latour for their assistance with the experiments.

## APPENDIX I. MATERIAL PROPERTIES AND SPECIFICS OF COLUMNS AND FURNACE

### Concrete Properties

#### Stress-Strain Relations

For  $\epsilon_c \leq \epsilon_{max}$

$$f_c = f'_c \left[ 1 - \left( \frac{\epsilon_{max} - \epsilon_c}{\epsilon_{max}} \right)^2 \right] \quad (25)$$

For  $\epsilon_c > \epsilon_{max}$

$$f_c = f'_c \left[ 1 - \left( \frac{\epsilon_c - \epsilon_{max}}{3\epsilon_{max}} \right)^2 \right] \quad (26)$$

where

$$\epsilon_{\max} = 0.0025 + (6.0T + 0.047^2) \times 10^{-6} \quad (27)$$

and

$$f_c = f'_{co} \quad \text{for } 0^\circ\text{C} < T < 450^\circ\text{C} \quad (28)$$

$$f_c = f'_{co} \left[ 2.011 - 2.353 \left( \frac{T-20}{1,000} \right) \right] \quad \text{for } 450^\circ\text{C} \leq T \leq 874^\circ\text{C} \quad (29)$$

$$f_c = 0 \quad \text{for } T > 874^\circ\text{C} \quad (30)$$

### Thermal Capacity

$$\rho_c c_c = 2.566 \times 10^6 \text{ J/(m}^3\text{C)} \quad \text{for } 0 \leq T \leq 400^\circ\text{C} \quad (31)$$

$$\rho_c c_c = (0.1765T - 68.034) \times 10^6 \text{ J/(m}^3\text{C)} \quad \text{for } 400 < T \leq 410^\circ\text{C} \quad (32)$$

$$\rho_c c_c = (-0.05043T - 25.00671) \times 10^6 \text{ J/(m}^3\text{C)} \quad \text{for } 410 < T \leq 445^\circ\text{C} \quad (33)$$

$$\rho_c c_c = 2.566 \times 10^6 \text{ J/(m}^3\text{C)} \quad \text{for } 445 < T \leq 500^\circ\text{C} \quad (34)$$

$$\rho_c c_c = (0.01603T - 5.44881) \times 10^6 \text{ J/(m}^3\text{C)} \quad \text{for } 500 < T \leq 635^\circ\text{C} \quad (35)$$

$$\rho_c c_c = (0.16635T - 100.90225) \times 10^6 \text{ J/(m}^3\text{C)} \quad \text{for } 635 < T \leq 715^\circ\text{C} \quad (36)$$

$$\rho_c c_c = (-0.22103T + 176.07343) \times 10^6 \text{ J/(m}^3\text{C)} \quad \text{for } 715 < T \leq 785^\circ\text{C} \quad (37)$$

$$\rho_c c_c = 2.566 \times 10^6 \text{ J/(m}^3\text{C)} \quad \text{for } T > 785^\circ\text{C} \quad (38)$$

### Thermal Conductivity

$$k_c = 1.355 \text{ W/(m}^\circ\text{C)} \quad \text{for } 0 \leq T \leq 293^\circ\text{C} \quad (39)$$

$$k_c = -0.001241T + 1.7162 \text{ W/(m}^\circ\text{C)} \quad \text{for } T > 293^\circ\text{C} \quad (40)$$

### Coefficient of Thermal Expansion

$$\alpha_c = (0.008T + 6) \times 10^{-6} \text{ m/(m}^\circ\text{C)} \quad (41)$$

### Steel Properties

#### Stress-Strain Relations

For  $\epsilon_s \leq \epsilon_p$

$$f_s = \frac{f(T, 0.001)}{0.001} \epsilon_s \quad (42)$$

where

$$\epsilon_p = 4 \times 10^{-6} f_{yo} \quad (43)$$

and

$$f(T, 001) = (50 - 0.047) \times [1 - \exp((-30 + 0.037)\sqrt{0.001})] \times 6.9 \quad (44)$$

For  $\epsilon_s > \epsilon_p$

$$f_y = \frac{f(T, 0.001)}{0.001} \epsilon_p + f(T, (\epsilon_s - \epsilon_p + 0.001)) - f(T, 0.001) \quad (45)$$

where

$$f(T, (\epsilon_s - \epsilon_p + 0.001)) = (50 - 0.04T) \times [1 - \exp((-30 + 0.03T)\sqrt{(\epsilon_s - \epsilon_p + 0.001)})] \times 6.9 \quad (46)$$

#### Thermal Capacity

$$\rho_c c_c = (0.004T + 3.3) \times 10^6 \text{ J/(m}^3\text{C)} \quad \text{for } 0^\circ\text{C} \leq T \leq 650^\circ\text{C} \quad (47)$$

$$\rho_c c_c = (0.068T + 38.3) \times 10^6 \text{ J/(m}^3\text{C)} \quad \text{for } 650^\circ\text{C} < T \leq 725^\circ\text{C} \quad (48)$$

$$\rho_c c_c = (-0.086T + 73.35) \times 10^6 \text{ J/(m}^3\text{C)} \quad \text{for } 725^\circ\text{C} < T \leq 800^\circ\text{C} \quad (49)$$

$$\rho_c c_c = 4.55 \times 10^6 \text{ J/(m}^3\text{C)} \quad \text{for } T > 800^\circ\text{C} \quad (50)$$

#### Thermal Conductivity

$$k_x = -0.022T + 48 \text{ W/(m}^\circ\text{C)} \quad \text{for } 0^\circ\text{C} \leq T \leq 900^\circ\text{C} \quad (51)$$

$$k_x = 28.2 \text{ W/(m}^\circ\text{C)} \quad \text{for } T > 900^\circ\text{C} \quad (52)$$

#### Coefficient of Thermal Expansion

$$\alpha_x = (0.004T + 12) \times 10^{-6} \text{ m/(m}^\circ\text{C)} \quad \text{for } T < 1,000^\circ\text{C} \quad (53)$$

$$\alpha_x = 16 \times 10^{-6} \text{ m/(m}^\circ\text{C)} \quad \text{for } T \geq 1,000^\circ\text{C} \quad (54)$$

#### Water Properties

##### Thermal Capacity

$$\rho_w c_w = 4.2 \times 10^6 \text{ J/(m}^3\text{C)} \quad (55)$$

##### Heat of Vaporization

$$\lambda_w = 2.3 \times 10^6 \text{ J/kg} \quad (56)$$

## APPENDIX II. SPECIFICS OF COLUMNS AND FURNACE

The specifics of columns and furnaces are:  $\epsilon_f$  = emissivity of column furnace fire, 0.75;  $\epsilon_s$  = emissivity of steel, 0.8;  $KL$  = effective length of columns, 2.0 m for fire-resistance calculations;  $l$  = length of column that contributes to axial deformation, 3.5 m; and  $\phi$  = concentration of moisture in concrete by volume, 0.10.

## APPENDIX III. REFERENCES

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#### APPENDIX IV. NOTATION

The following symbols are used in this paper:

- $A$  = area of element ( $m^2$ );
- $C$  = load intensity;
- $C_c$  = factored compressive resistance of concrete core;
- $C_{rc}$  = factored compressive resistance of concrete-filled steel columns;
- $c$  = specific heat [ $J/(kg^\circ C)$ ];
- $e$  = eccentricity (m);
- $f$  = stress (MPa);
- $f_c$  = cylinder strength of concrete at temperature  $T$  (MPa);
- $f_{co}$  = cylinder strength of concrete at room temperature (MPa);
- $f_s$  = strength of steel at temperature  $T$  (MPa);
- $h$  = coefficient of heat transfer at fire-exposed surface [ $W/(m^2^\circ C)$ ];
- $j$  = 0, 1, 2, . . . ;
- $K$  = effective length factor;
- $k$  = thermal conductivity [ $W/(m^\circ C)$ ];
- $L$  = unsupported length of column (m);
- $l$  = length of column that contributes to axial deformation (m);
- $M_1$  = number of points  $P$  in steel section in radial direction;
- $M_2$  = total number of points  $P$  in column section in radial direction;
- $N_1$  = number of elements in tangential direction;
- $P$  = point;
- $R_c$  = radius of concrete core (m);
- $R_s$  = radius of steel column (m);
- $T$  = temperature ( $^\circ C$ );
- $V$  = volume of moisture in an element ( $m^3$ );
- $x$  = coordinate (m);
- $Y$  = lateral deflection of column at midheight (m);
- $y$  = coordinate (m);
- $\alpha$  = coefficient of thermal expansion ( $1/^\circ C$ );

- $\Delta$  = increment or difference;
- $\Delta\xi$  = mesh width in radial direction (m);
- $\epsilon$  = emissivity, strain (m/m);
- $\lambda$  = heat of vaporization (J/kg);
- $\rho$  = density (kg/m<sup>3</sup>);
- $\sigma$  = Stefan-Boltzmann constant [W/(m<sup>2</sup>K<sup>4</sup>)];
- $\tau$  = time (h);
- $\phi$  = concentration of moisture; and
- $\chi$  = curvature of column at midheight (1/m).

#### Subscripts

- $B$  = steel reinforcement;
- $c$  = concrete;
- $f$  = fire;
- $L$  = left of y-axis;
- $m, M_1, M_2$  = points  $m, M_1$ , and  $M_2$  in radial direction;
- $n, N_1$  = points  $n$ , and  $N_1$  in tangential direction;
- $o$  = room temperature;
- $p$  = proportional stress-strain relation;
- $R$  = right of y-axis;
- $s$  = steel;
- $T$  = temperature; and
- $w$  = water.

#### Superscripts

- $j$  = location at  $\tau = j\Delta\tau$ .