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### Hot pipelines in permafrost hydraulic: thermal and structural considerations

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NATIONAL RESEARCH COUNCIL OF CANADA  
DIVISION OF BUILDING RESEARCH

HOT PIPELINES IN PERMAFROST  
HYDRAULIC, THERMAL AND STRUCTURAL CONSIDERATIONS

by

W. A. Slusarchuk

Internal Report No. 394  
of the  
Division of Building Research

Ottawa  
January 1972

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## PREFACE

The oil and gas potential of the northern part of Canada has captured public attention in recent months. Should this potential become reality, there will be a need to transport oil by pipeline through permafrost regions. Construction in these areas poses special problems primarily because of thawing that may be caused by such activity. The Division of Building Research has developed for the construction industry a very considerable amount of knowledge over the past 20 years on building techniques appropriate for permafrost conditions. Buried hot pipelines, however, raise questions concerning design and maintenance, under thawing conditions, not unlike those associated with the construction of dykes and reservoirs in permafrost areas.

For these, there are not yet satisfactory answers. In response to this need, the Division has undertaken a study of the properties and behaviour of thawing soil. One of the first steps in the study was to give consideration to the conditions imposed on the ground by a hot pipeline. This report presents the results of the investigation.

Ottawa  
January 1972

N. B. Hutcheon,  
Director

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CONVERSION TABLE

$$1 \text{ hp} = 550 \frac{\text{ft lb}}{\text{sec}} = 2,545 \text{ Btu/hr}$$

$$1 \text{ barrel} = 42 \text{ gallons (U.S.)} = 5.63 \text{ ft}^3$$

$$1 \frac{\text{Btu in.}}{\text{ft}^2 \text{ hr } ^\circ\text{F}} = 0.173 \frac{\text{hp}}{\text{mile } ^\circ\text{F}}$$

$$1 \frac{\text{Btu}}{\text{sec}} = 1.41 \text{ hp}$$

HOT PIPELINES IN PERMAFROST  
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W. A. Slusarchuk

Significant technical problems are associated with the construction of hot ( $\sim 150^{\circ}\text{F}$ : oil) large-diameter pipelines in permafrost as evidenced by the long, costly delays of the Trans-Alaska Pipeline project. Similar problems are present for warm ( $\sim 80^{\circ}\text{F}$ : unrefrigerated natural gas) pipelines and others exist for cool ( $\sim 25^{\circ}\text{F}$ : refrigerated natural gas) and cold ( $\sim -170^{\circ}\text{F}$ : liquid natural gas) pipelines. Some of the most difficult technical problems result from a lack of knowledge of the behaviour of permafrost when it is subjected simultaneously to thermal and structural loads. If the thermal loads are sufficient to melt the permafrost, large settlements may result or thawed soil round the pipe may become unstable on slopes. Consideration must be given to these effects on the structural stability of the pipeline.

A general quantitative analysis was undertaken on the hydraulic, thermal and structural aspects of a hot pipeline in permafrost. The complex nature of the analysis quickly became apparent. Changing conditions along the pipeline (fluid and soil properties; temperature, heat flow, internal pressure, etc.) and transient factors such as ground surface temperature and rate of fluid flow make a reasonably accurate solution very difficult. Possibly the best method of obtaining a solution for any particular pipeline would be to use numerical techniques, which would involve large amounts of computer time. An insight into the interaction between the pipeline and the permafrost, however, can be obtained for general cases by selecting average values to characterize certain variables in the analysis.

Representative situations were chosen for pipelines maintained at above-freezing temperatures, i. e., oil and unrefrigerated natural gas, and do not directly pertain to pipelines maintained at below-freezing temperatures. Special consideration is given to hot, large-diameter oil pipelines similar to that proposed for transporting oil out of the Prudhoe Bay area of Alaska.

## HYDRAULIC ANALYSIS

In order to move oil through a pipeline, large amounts of energy must be put into the system. Pumps provide it by raising the pressure of the oil and this energy is converted into heat uniformly along the pipeline by the frictional resistance of the oil to viscous flow. The heat flows into the permafrost; the amount can be determined by the horsepower requirements for the rates of flow desired, since the input energy is converted into heat between pumping stations. For rates of flow of 0.5 and 2.0 million barrels per day (the minimum and maximum proposed for the pipeline from Prudhoe Bay) the required horsepower may be calculated by the following equations:

$$HP = \frac{Q_o \Delta P}{550} \quad (1)$$

and

$$\Delta P = \frac{f L \gamma V^2}{2gD} \quad (2)$$

where

- HP = required horsepower
- $Q_o$  = flow of oil,  $\text{ft}^3/\text{sec}$
- $\Delta P$  = pressure drop in length of pipe L,  $\text{lb}/\text{ft}^2$
- f = friction factor, dimensionless
- L = length of pipe, ft
- $\gamma$  = unit weight of oil,  $\text{lb}/\text{ft}^3$
- V = average flow velocity,  $\text{ft}/\text{sec}$
- g = gravitational acceleration,  $\text{ft}/\text{sec}^2$
- D = diameter of pipe, ft

For an average crude oil flowing in a 4-ft diameter pipe at approximately 150° F the horsepower requirements for the minimum and maximum proposed rates of flow for Prudhoe Bay were found to be approximately:

|   |     |                                      |
|---|-----|--------------------------------------|
| minimum rate of flow<br>(0.5 million barrels/day) | = { | 10.3 hp/mile<br>26,200 Btu/hr mile   |
| maximum rate of flow<br>(2.0 million barrels/day) | = { | 527 hp/mile<br>1,340,000 Btu/hr mile |

The thermal load exerted on permafrost will vary considerably, depending upon the rate of flow.

A better appreciation of the effects of the various factors may be obtained by substituting for  $Q_o$  in equation (1) the expression

$$Q_o = \frac{\pi D^2 V}{4} \quad (3)$$

to give for the energy dissipation (hp /mile)

$$\frac{\pi}{4400g} f\gamma DV^3 \quad (4)$$

From equation (4) it is apparent that the horsepower per mile is very sensitive to the rate of flow of the oil as it increases with velocity to the third power. Horsepower per mile also increases directly with  $\gamma$ ,  $D$  and  $f$ . The unit weight,  $\gamma$ , depends on the type of crude oil being pumped and the friction factor,  $f$ , depends on the viscosity. This, in turn, is a function of temperature. The viscosity increases exponentially with decreasing temperature and  $f$  increases with increasing viscosity (decreasing temperature). For example, if oil temperature is lowered from 150° F to 50° F, then  $f$  increases to such an extent that 50 per cent more horsepower is required at the maximum rate of flow. For pumping stations located approximately 60 miles apart the maximum pressure in the pipeline will be approximately 1,000 psi with a 35,000 hp input required at each station for maximum rate of flow.

### THERMAL ANALYSIS

#### Steady-State Condition: Buried Pipeline

The heat flowing from a buried insulated pipe for a steady-state condition may be calculated by the following formula,

$$Q_c = K_t (T_p - T_G) \quad (5)$$

where

$$Q_c = \text{heat flowing out of pipe, hp/mile}$$

$$K_t = \text{thermal transfer coefficient, hp/mile } ^\circ\text{F}$$

$T_p$  = temperature of the pipe, °F

$T_G$  = mean annual ground temperature, °F

and

$$K_t = \frac{0.346\pi K_s}{\frac{K_s}{K_i} \ln\left(\frac{r}{r-t_i}\right) + \ln\left[\frac{d}{r} + \sqrt{\left(\frac{d}{r}\right)^2 + 1}\right]} \quad (6)$$

Insulation  
effect

Soil  
effect

where

$K_s$  = thermal conductivity of soil, Btu in/ft<sup>2</sup> hr °F

$K_i$  = thermal conductivity of insulation, Btu in/ft<sup>2</sup> hr °F

$r$  = radius from centre of pipe to outside of insulation, ft

$t_i$  = thickness of insulation, ft

$d$  = distance from ground surface to centre of pipe  
(depth of pipe burial), ft.

As may be noted from equation (6), the thermal transfer coefficient is a function of five variables, insulation thickness, pipe diameter, thermal conductivity of insulation, thermal conductivity of soil, and pipe burial depth. Figures 1 to 5 provide an appreciation of the effect of the variables on the thermal transfer coefficient. The heat flow out of the pipe may be obtained by selecting the relevant thermal transfer coefficient from one of the plots in Figures 1 through 5 and using this value in equation (5).

Figure 1 shows that the thermal transfer coefficient decreases with increasing thickness of insulation. The plots indicate that relatively little benefit is derived from insulation of thickness greater than 3 to 4 in. They also show that insulation has its greatest relative effect in soils that have the highest thermal conductivity values. Figure 2 clearly illustrates how the thermal transfer coefficient increases with increasing pipe diameter; Figures 3 and 4 indicate how the thermal transfer coefficient is affected by changes in the thermal conductivity

of the soil and insulation. The importance of the thermal conductivity of the soil is shown by the plot in Figure 4; a change in conductivity value from 2.5 to 5.0 changes the transfer coefficient from 1.2 to 2.4, a 100 per cent increase in heat loss. Figure 5 shows that the thermal transfer coefficient is not significantly affected by depth of burial if some insulation is around the pipe. For an uninsulated pipe, however, the depth of burial is a factor that must be considered.

### Heat Lost versus Heat Gained in Pipeline

The steady-state heat loss may be compared with the heat generated in the pipe by viscous flow. For maximum rate of flow the heat generated per mile was calculated at 527 horsepower per mile. The net heat gain or loss per mile is presented in Table I for four cases, assuming that the oil temperature is 150° F, the average ground temperature 30° F, and that burial depth is 8 ft. For cases (a), (c) and (d) the net heat gain is positive, indicating that the temperature of the oil must increase until the temperature difference,  $T_p - T_G$ , is great enough to conduct the excess heat away. If the oil temperature must not exceed some upper value, e. g. 180 to 190° F, and if raising the oil temperature does not create a sufficiently large temperature difference before the upper limit is reached, then the rate of flow must be reduced to reduce the heat generated by viscous flow. For case (b), however, there is insufficient heat to maintain the oil temperature at 150° F and the oil temperature will drop. As it drops viscosity increases and additional horsepower is required to maintain a constant rate of flow. If sufficient extra horsepower is not available to meet the additional requirements then the rate of flow will be reduced.

### Temperature Change Along Pipeline

One needs to be able to estimate temperature change along the length of a pipeline because temperature affects viscosity of oil and hence the horsepower requirement and rate of flow. It is necessary to know, in general terms, whether upper and lower oil temperature limits may be exceeded. A relation between oil temperature and distance along a pipeline can be developed as follows:

$$\text{Heat input} - \text{Heat output} = \text{Heat associated with a change in oil temperature,}$$

or

$$\left[ \text{HP} - K_t (T - T_G) \right] dL = 1.41CQ_o dT \quad (7)$$

where

T = temperature of the oil at any point in the pipeline, °F

L = distance along pipeline, miles

C = volumetric heat capacity, Btu/ft<sup>3</sup> °F

HP = average heat input per mile, hp/mile

Separating the variables and integrating equation (7) gives an expression for T:

$$T = \frac{\text{HP}}{K_t} + T_G + (T_i - T_G - \frac{\text{HP}}{K_t}) \exp \left( - \frac{LK_t}{1.41CQ_o} \right) \quad (8)$$

where  $T_i$  = initial temperature of oil at  $L = 0$ , °F.

For example, cases listed in Table II were investigated using equation (8) and are plotted in Figure 6. It shows that oil temperature may increase or decrease along the pipeline, depending upon the rate of flow and the thermal transfer coefficient which reflects the amount of insulation present and the thermal conductivity of the soil.

Differentiating equation (8) with respect to distance gives the temperature gradient along the pipeline. This can be described by the following equation,

$$\frac{dT}{dL} = \left( T_i - T_G - \frac{\text{HP}}{K_t} \right) \left( - \frac{K_t}{1.41CQ_o} \right) \exp \left( - \frac{LK_t}{1.41CQ_o} \right) \quad (9)$$

where

$\frac{dT}{dL}$  = temperature gradient, °F/mile.

The results of equation (9) are plotted in Figure 7 for cases 1, 2 and 3; the plots indicate that the temperature of the oil will not generally rise or fall more than 1° F in 10 miles, and in most instances that the temperature change in the oil will be much less at the higher rates of flow.

### Temperature Change After Shutdown

Information on the rate of change in oil temperature after shutdown is required to estimate how much time is available for repair or maintenance operations before the lower oil temperature limit is reached. An equation for temperature change after shutdown can be developed by equating heat loss from the pipe to that associated with change in oil temperature.

Heat flow out of pipe = Heat from change in temperature of oil,

or

$$2545 K_t (T - T_G) dt = C V_o dT \quad (10)$$

where

T = temperature of oil at any time after shutdown, ° F

t = time after shutdown, hr

V<sub>o</sub> = volume of oil, ft<sup>3</sup>/mile.

Separating the variables and integrating gives the following expression:

$$T = T_G + (T_i - T_G) \exp \left( - \frac{2545K_t}{CV_o} t \right) \quad (11)$$

where

T<sub>i</sub> = initial temperature at shutdown ( t = 0 ), ° F.

Equation (11) is plotted in Figure 8 for four situations. If a lower temperature limit of 50° F is used, for example, Figure 8 shows that (a) for the uninsulated pipe all repair work would have to be

completed within 5 days to a week, depending upon the initial temperature of the oil, and that (b) for an insulated pipe the time for repair work could be extended to 3 or 4 weeks, or longer in some cases.

### Steady-State Condition: Elevated Pipeline

The ice and water content is very high in some permafrost areas and because of stability or settlement considerations a pipeline would probably have to be built above ground. The heat flow out of an elevated pipe is governed by equation (5), with the thermal transfer coefficient defined as

$$K_t = \frac{0.346\pi K_i}{\ln \left( \frac{r}{r-t_i} \right)} \quad (12)$$

The results of a parametric study based on equation (12) are presented in Figures 9 through 11. Figure 9 shows that the thermal transfer coefficient decreases with increasing insulation thickness, and Figures 10 and 11 illustrate the coefficient increase with increasing pipe diameter and thermal conductivity of the insulation.

### Transient Condition: Buried Pipeline

It is extremely difficult, if not impossible, to obtain a closed form solution for the transient heat flow problem round a buried pipeline for most natural conditions in permafrost. Consequently numerical methods must be used to estimate the extent of the thaw zone and how quickly the thaw front is advancing. The amount and rate of thawing round a pipeline and the amount of water liberated significantly influences the settlement, rate of settlement and stability of the melted permafrost and hence the safety of the pipeline. Figures 12 and 13 graphically illustrate how the 32° F isotherm advances round a pipeline and how large thaw depths occur in relatively short times.

As well as the extent of the thaw bulb, Figure 12 shows how quickly equilibrium above the pipe is reached, i. e., within one year. Below the pipe the thaw bulb continues to enlarge for many years, and only after approximately 15 years does the 32° F isotherm tend to approach equilibrium. Both Figures 12 and 13 show that greater thaw depths occur in the warmer permafrost regions. Figure 13 also indicates the effect of water content on the penetration rate of the 32° F isotherm.

Although the frozen water content is a major factor in determining the rate at which the 32° F isotherm advances, the ratio of the thermal conductivity of the unfrozen soil to the frozen soil is a determining factor. The greater the ratio of K unfrozen / K frozen the greater the equilibrium thawing depth will be, because an increase in the ratio of K unfrozen / K frozen is equivalent to increasing the pipeline temperature for a homogeneous soil. Figure 14 (Lachenbruch, 1970) shows how the thaw bulb increases in size with increase in pipeline temperature.

A parametric study should be carried out for the transient condition in order that a better appreciation may be obtained of the time dependence of the amount and rate of thawing round a hot pipeline in permafrost. Such parameters as water content, thermal conductivity of frozen and unfrozen soil, pipe and average ground temperature, insulation thickness round pipe, geometry as an initial condition (e. g. berm) or as a time condition (e. g. settlement) could be investigated to advantage. The numerical analysis should be expanded to give heat flux at the ground and pipe boundaries and amount of water liberated upon thawing, as well as the temperature profile.

## STRUCTURAL ANALYSIS

An elementary analysis only was undertaken of the structural aspects and consequently the picture of some of the loads and stresses is a general one in this Section. Stresses due to torsion, local buckling, vibration, earthquakes, crack propagation or plastic strain are considered to be beyond the scope of this report.

### Circumferential Stress

The circumferential stress induced in a pipe by internal pressure may be calculated by the following formula

$$\sigma_c = \frac{PD}{2t_p} \quad (13)$$

where

$\sigma_c$  = circumferential stress, lb/in.<sup>2</sup>

P = internal pipe pressure, lb/in.<sup>2</sup>

D = diameter of pipe, in.

$t_p$  = thickness of pipe, in.

For maximum rate of flow the greatest internal pressure was calculated to be approximately 1,000 psi. For a 48-in. diameter pipe with 60,000 psi strength steel the thickness of pipe may be calculated by equation (13) to be 0.4 in. With a safety factor of 1.25 the pipe should be approximately  $\frac{1}{2}$  in. thick.

### Flexural Stress

The flexural stress of bending is given by

$$\sigma_f = \frac{12Mc}{I} \quad (14)$$

where

$\sigma_f$  = flexural stress, lb/in.<sup>2</sup>

M = moment at a section, ft-lb

c = distance from neutral axis to extreme fibre, in.

I = moment of inertia of pipe, in.<sup>4</sup>

For a 48-in. diameter pipe with a  $\frac{1}{2}$ -in. wall thickness and a yield strength of 60,000 psi the maximum moment may be calculated by equation (14) to be  $4.37 \times 10^6$  ft-lb. Using this value for the maximum moment, values may be obtained for the maximum length and deflection of a pipe for various support conditions, assuming that the steel is not strained beyond the elastic limit. The equations for maximum span length and for maximum deflection at maximum span length are listed in Table III; the pipeline loadings are listed in Table IV.

It may be noted (Table IV) that the greatest downward loads on the pipeline are present when the water table is below the pipe. If the water table is above the bottom of the pipe, then less downward load per linear foot is exerted on the pipe, with the least downward load being present when the water table is at the ground surface. When the soil is a slurry, the resultant forces act vertically upward.

In Figure 15 the maximum span length and deflection for the simple support case are plotted as a function of pipeline loading based upon the equations and values in Tables III and IV. The plots indicate how the maximum allowable span length and maximum deflection decrease with increasing pipeline load.

### Thermal Stress

The stresses resulting from thermal expansion or contraction may be calculated by the following equation,

$$\sigma_{th} = En\Delta T \quad (15)$$

where

$\sigma_{th}$  = thermal stress, lb/in.<sup>2</sup>

n = thermal coefficient of expansion, 1/°F

$\Delta T$  = temperature change, °F.

For  $\Delta T = 100^\circ\text{F}$  and  $n = 6.5 \times 10^{-6}$ , the axial thermal stress is 19,500 psi. Such a thermal stress would produce an axial thrust in the pipeline of approximately 1.5 million pounds if restrained or an expansion of 3.4 ft per mile if unrestrained.

### CONCLUSION

A preliminary investigation of the hydraulic, thermal and structural aspects of hot pipelines in permafrost has been undertaken to provide general background information for future geotechnical analysis. In this context, it is concluded that additional information on the factors affecting the transient heat flow problem round a pipeline in permafrost is required.

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TABLE I

## NET HEAT GAINED OR LOST

| <u>Case</u> | $K_s$<br>(Btu in. /<br>ft <sup>2</sup> hr °F) | $K_i$<br>(Btu in. /<br>ft <sup>2</sup> hr °F) | <u>Insulation<br/>Thickness<br/>(in.)</u> | $K_t$<br>(hp/mile °F) | $Q_c$<br>(hp/mile) | Heat in-<br>Heat out<br>(hp/mile) |
|-------------|---|---|---|-----------------------|--------------------|-----------------------------------|
| a           | 5.0   | -   | -   | 2.3                   | 276                | +251                              |
| b           | 15.0  | -   | -   | 7.2                   | 864                | -337                              |
| c           | 5.0   | 0.15  | 2   | 1.0                   | 120                | +407                              |
| d           | 15.0  | 0.15  | 2   | 1.4                   | 168                | +359                              |

TABLE II  
 TRANSFER COEFFICIENTS FOR VARIOUS RATES OF FLOW  
 AND INSULATION THICKNESS

| <u>Case</u> | <u>Rate of Flow<br/>(barrels/day)</u> | <u>Insulation<br/>Thickness<br/>(in.)</u> | <u>Transfer<br/>Coefficient<br/><math>K_t</math><br/>(hp/mile °F)</u> | <u>Temperature<br/><math>T_i</math><br/>(°F)</u> | <u>Temperature<br/><math>T_G</math><br/>(°F)</u> |
|-------------|---------------------------------------|---|---|--|--|
| 1           | 2,000,000                             | 2   | 1.2   | 150  | 30   |
| 2           | 2,000,000                             | -   | 7.2   | 150  | 30   |
| 3           | 500,000                               | 2   | 1.2   | 150  | 30   |
| 4           | 500,000                               | -   | 7.2   | 150  | 30   |

TABLE III

## EQUATIONS FOR MAXIMUM SPAN LENGTH AND DEFLECTION OF PIPELINE

| <u>Support Type</u>                           | <u>Equation for l max</u>                           | <u>Equation for Δ max</u>                    |
|---|---|--|
| Simple  | $\sqrt{8} \left( \frac{M_{\max}}{W} \right)^{1/2}$  | $1440 \left( \frac{M_{\max}^2}{WEI} \right)$ |
| Cantilever                                    | $\sqrt{2} \left( \frac{M_{\max}}{W} \right)^{1/2}$  | $864 \left( \frac{M_{\max}^2}{WEI} \right)$  |
| Cantilever (free but<br>guided at free end)   | $\sqrt{3} \left( \frac{M_{\max}}{W} \right)^{1/2}$  | $648 \left( \frac{M_{\max}^2}{WEI} \right)$  |
| Cantilever (simple<br>support under free end) | $\sqrt{8} \left( \frac{M_{\max}}{W} \right)^{1/2}$  | $598 \left( \frac{M_{\max}^2}{WEI} \right)$  |
| Fixed at both ends                            | $\sqrt{12} \left( \frac{M_{\max}}{W} \right)^{1/2}$ | $648 \left( \frac{M_{\max}^2}{WEI} \right)$  |

W = load per linear foot, lb/ft

E = Young's modulus, lb/in.<sup>2</sup>

l = span length, ft

Δ = deflection for maximum span length, in.

TABLE IV  
DESCRIPTION AND VALUES OF LOADING

| Load                                 | Load with<br>Water Table<br>below Pipe<br><u>(lb/lin ft)</u> | Load with<br>Water Table<br>at Ground<br>Surface, but<br>Soil not a<br>Slurry<br><u>(lb/lin ft)</u> | Load with<br>Water Table<br>at Ground<br>Surface and<br>Soil is a<br>Slurry<br><u>(lb/lin ft)</u> |
|--------------------------------------|--|---|---|
| Pipe only                            | 250↓   | -530↑   | -1250↑  |
| Pipe + oil                           | 930↓   | +150↓   | -570↑   |
| Pipe + oil<br>+ 1 ft over-<br>burden | 1330↓  | 310↓  | -570↑   |
| Pipe + oil<br>+ 4 ft over-<br>burden | 2530↓  | 790↓  | -570↑   |
| Pipe + oil<br>+ 8 ft over-<br>burden | 4130↓  | 1430↓   | -570↑   |

For these calculations the following values were assumed,

$$\delta_{\text{soil}} = 100 \text{ lb /ft}^3$$

$$\delta_{\text{water}} = 62.4 \text{ lb/ft}^3 \text{ and}$$

$$\text{S. G. of slurry} = 1.9$$

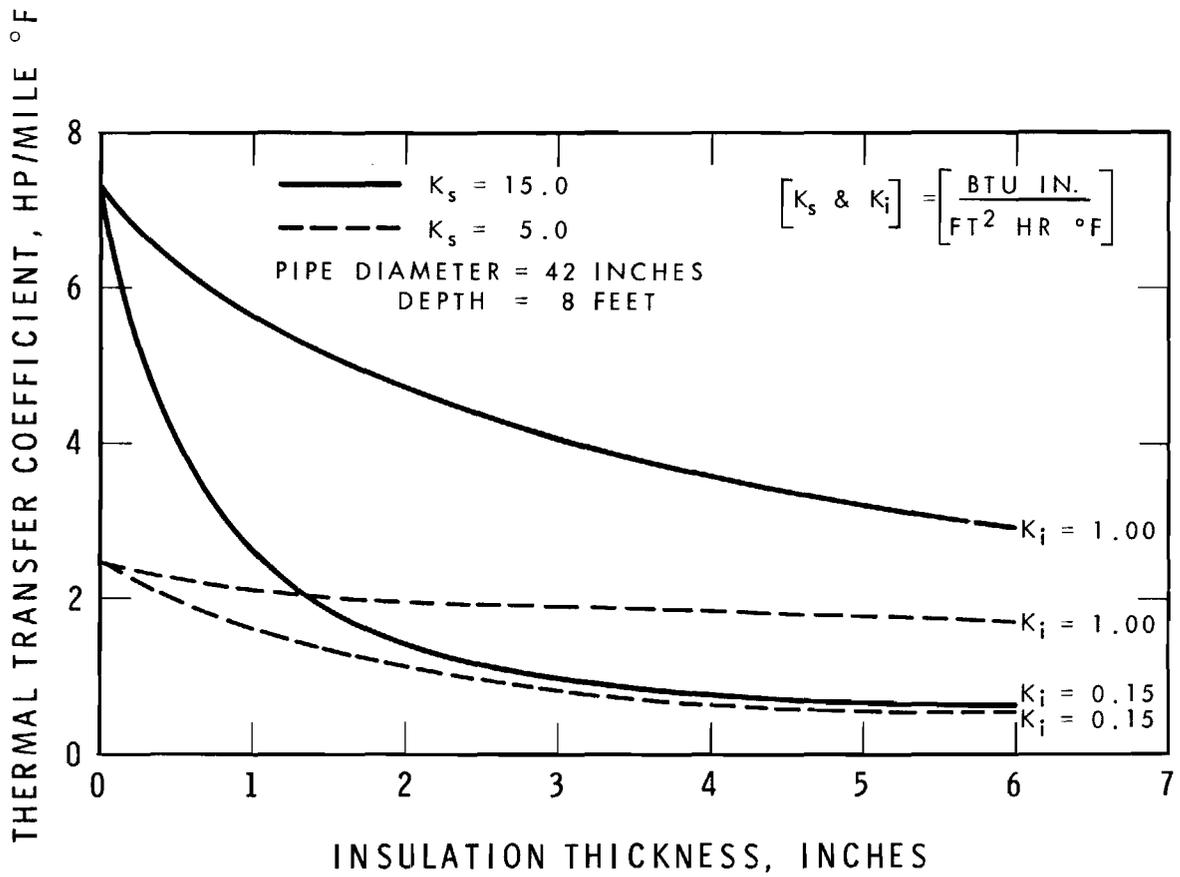


FIGURE 1

THERMAL TRANSFER COEFFICIENT AS A FUNCTION OF INSULATION THICKNESS (BURIED PIPE)

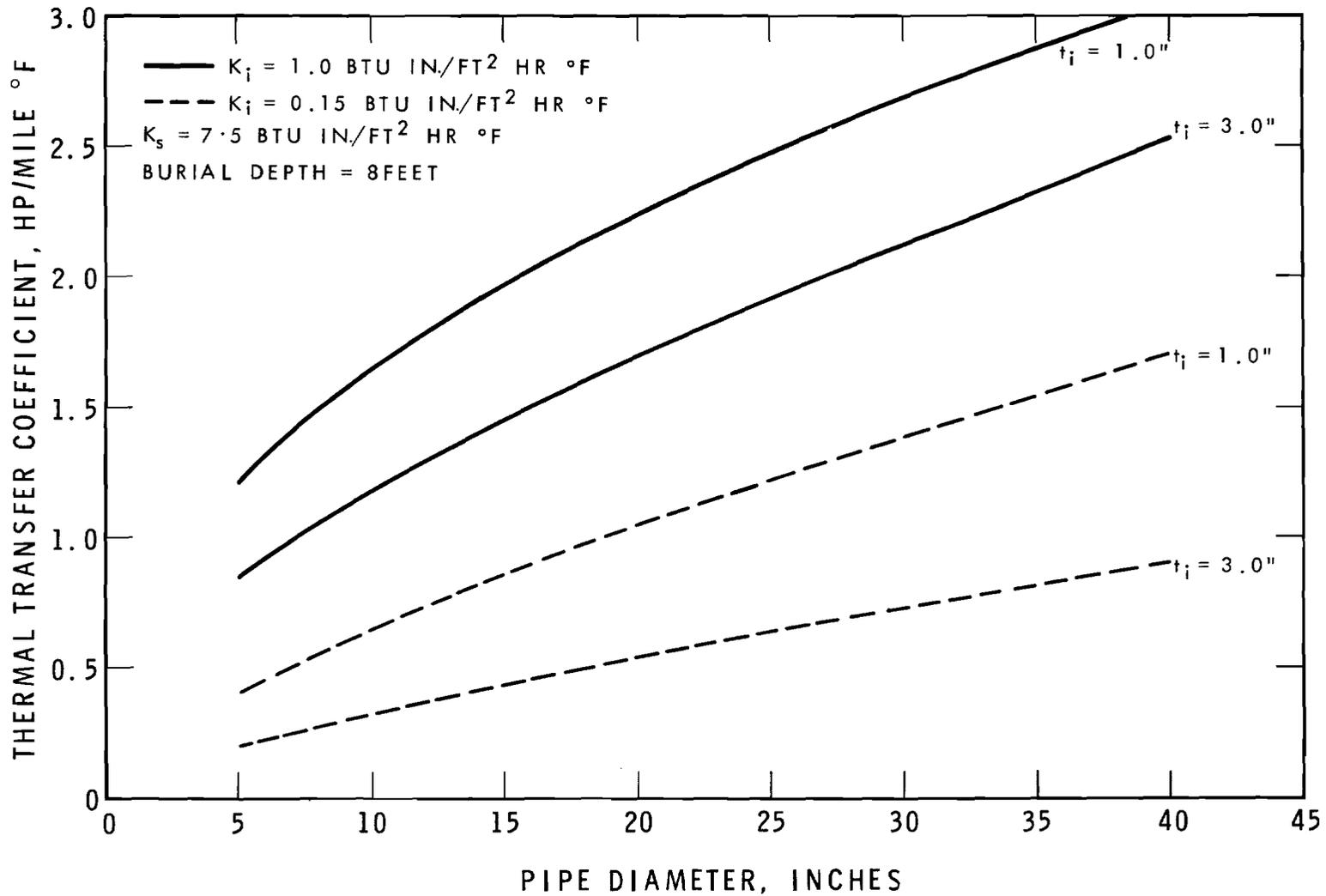


FIGURE 2

THERMAL TRANSFER COEFFICIENT AS A FUNCTION OF PIPE DIAMETER (BURIED PIPE)

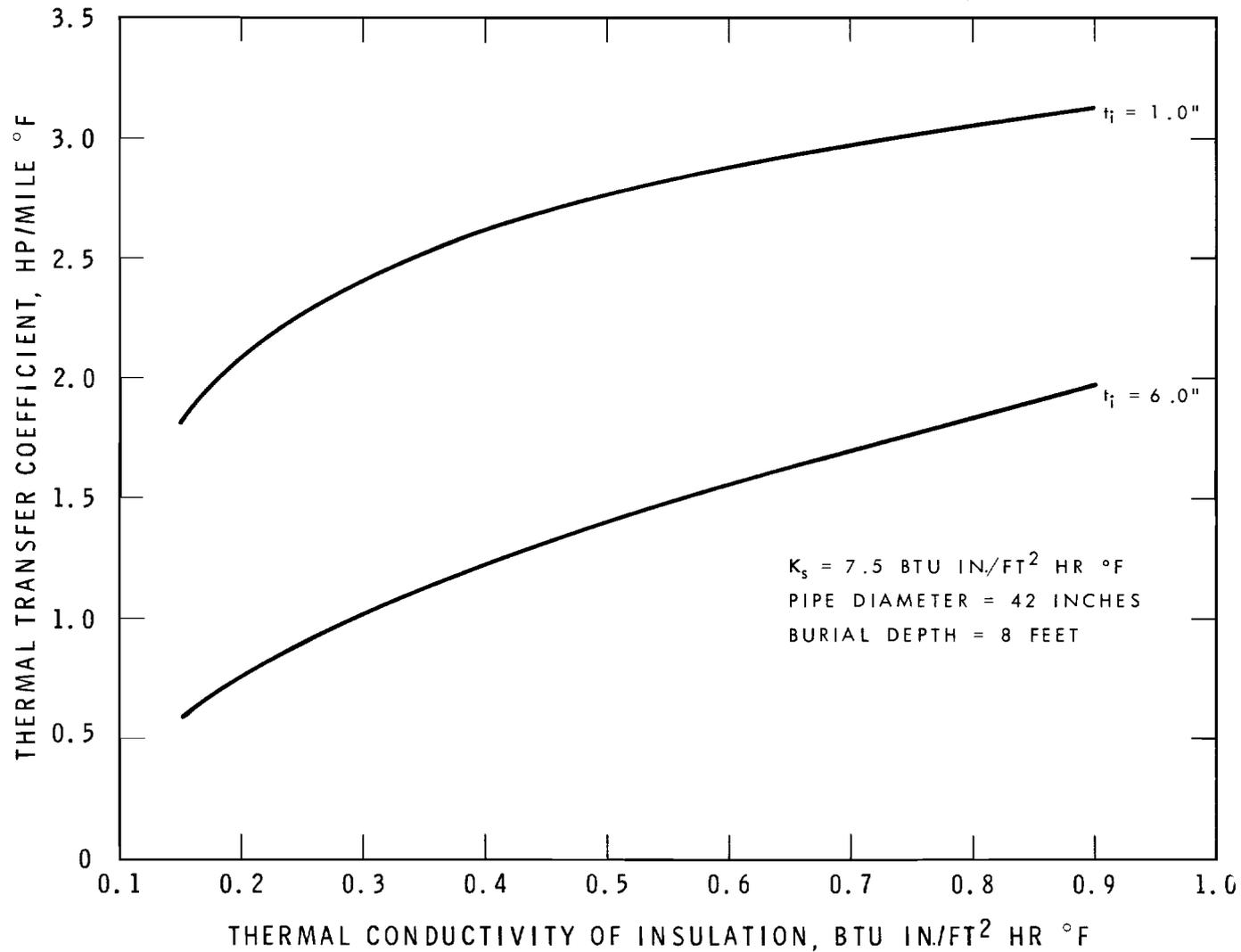


FIGURE 3

THERMAL TRANSFER COEFFICIENT AS A FUNCTION OF INSULATION CONDUCTIVITY  
(BURIED PIPE)

BR 4758-3

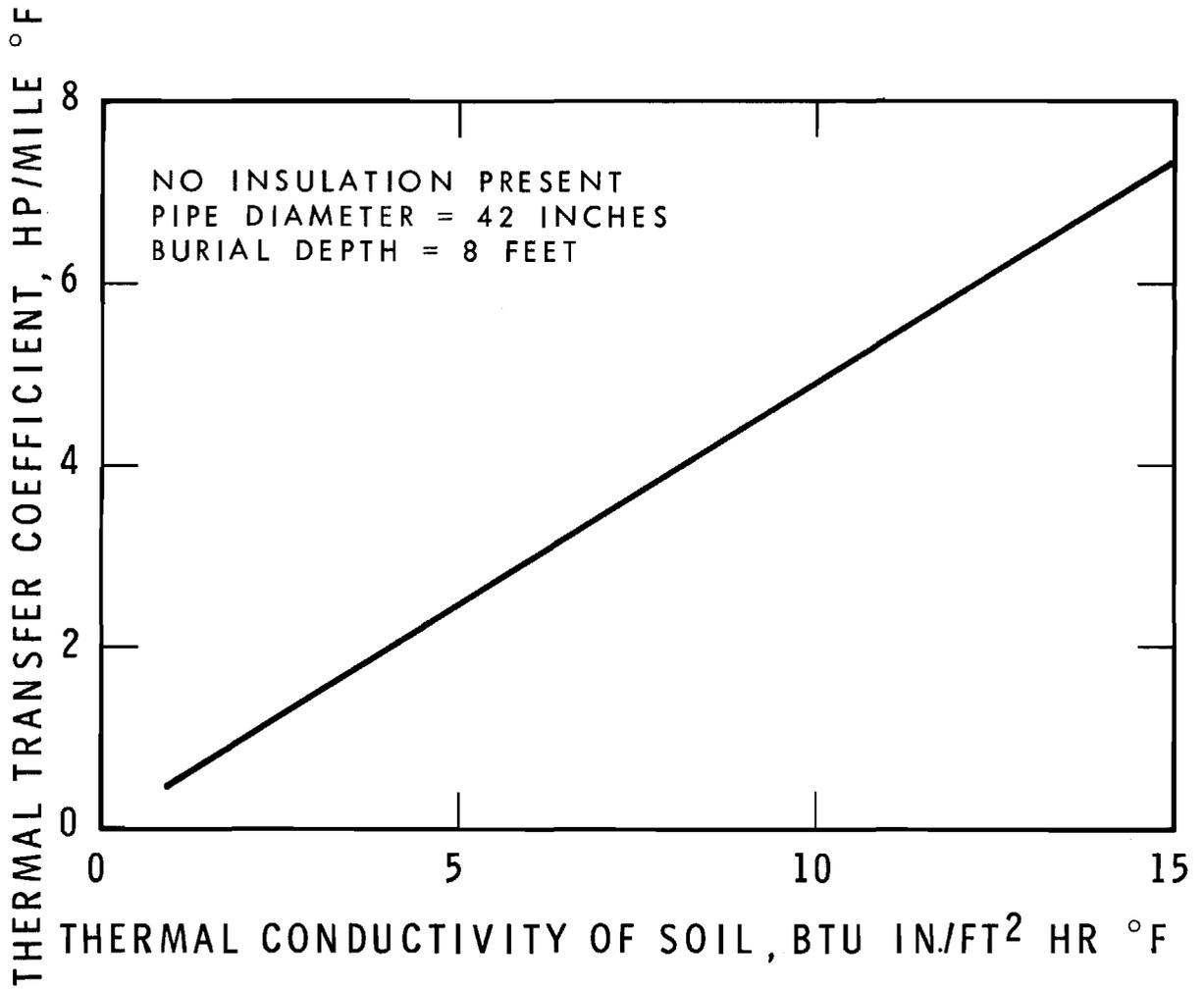


FIGURE 4

THERMAL TRANSFER COEFFICIENT AS A FUNCTION OF SOIL CONDUCTIVITY (BURIED PIPE)

BR 4758-4

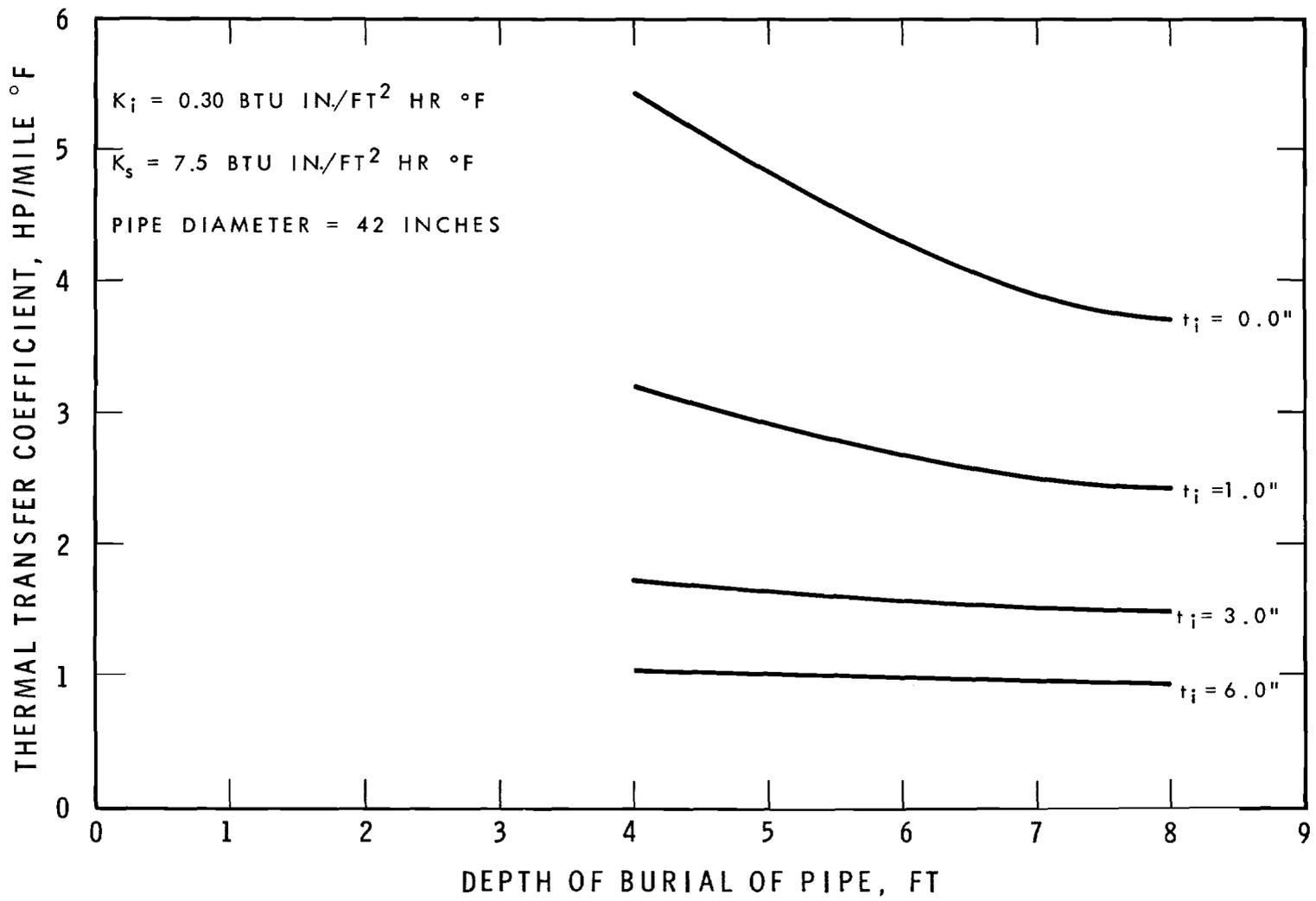


FIGURE 5

THERMAL TRANSFER COEFFICIENT AS A FUNCTION OF PIPE BURIAL DEPTH (BURIED PIPE)

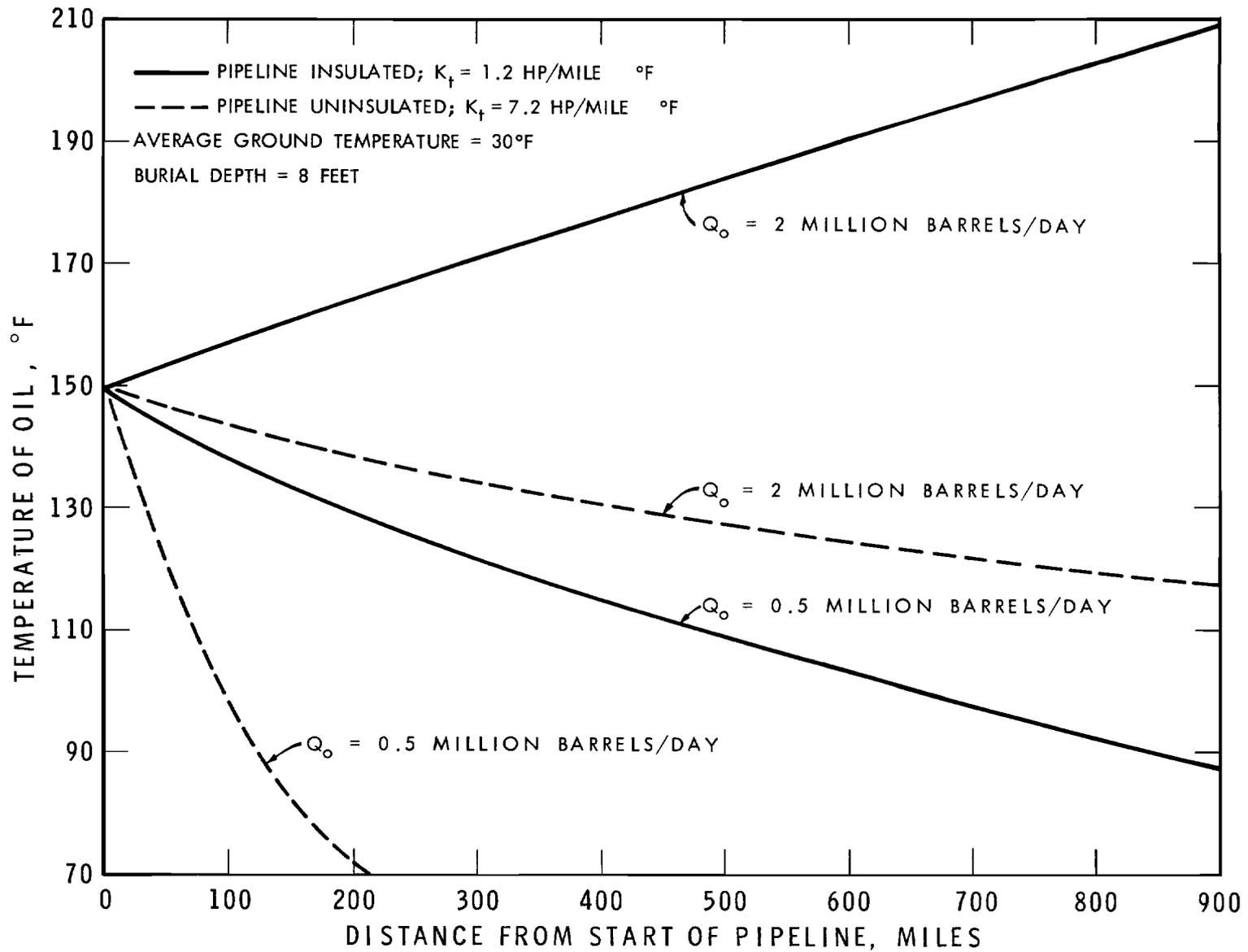


FIGURE 6 OIL TEMPERATURE CHANGE ALONG PIPELINE GR 4758-6

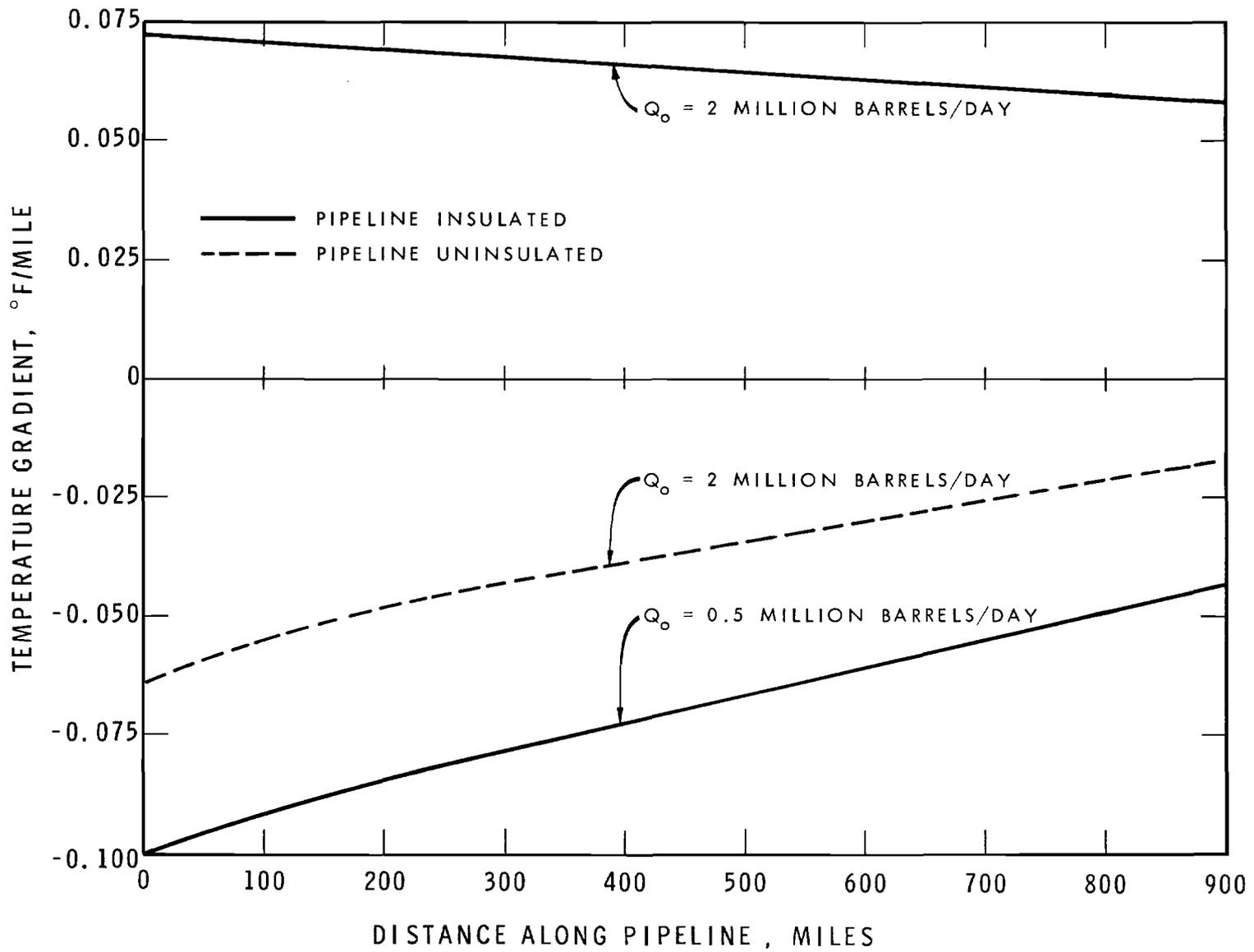


FIGURE 7 TEMPERATURE GRADIENT ALONG PIPELINE

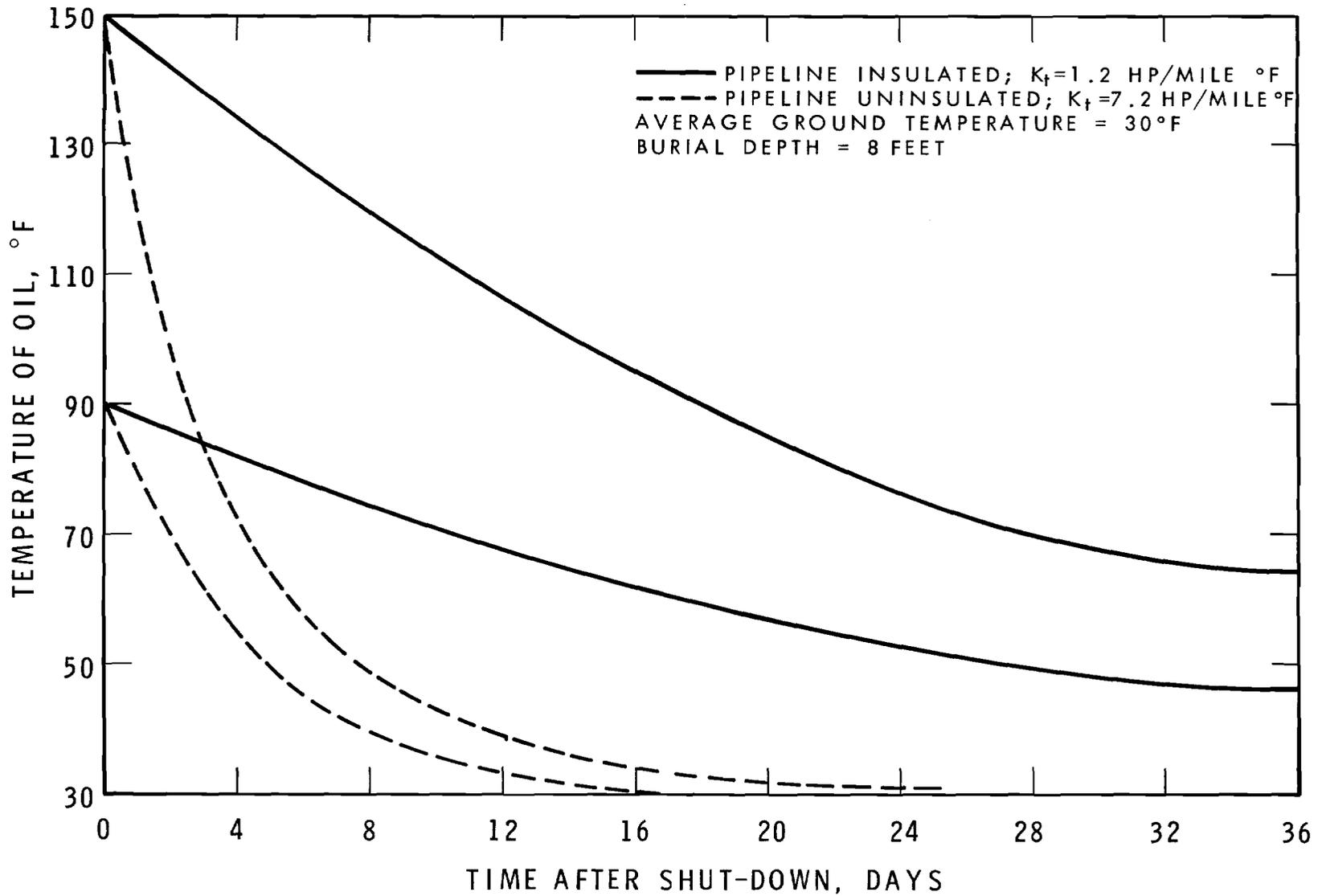


FIGURE 8 OIL TEMPERATURE CHANGE AFTER SHUT-DOWN

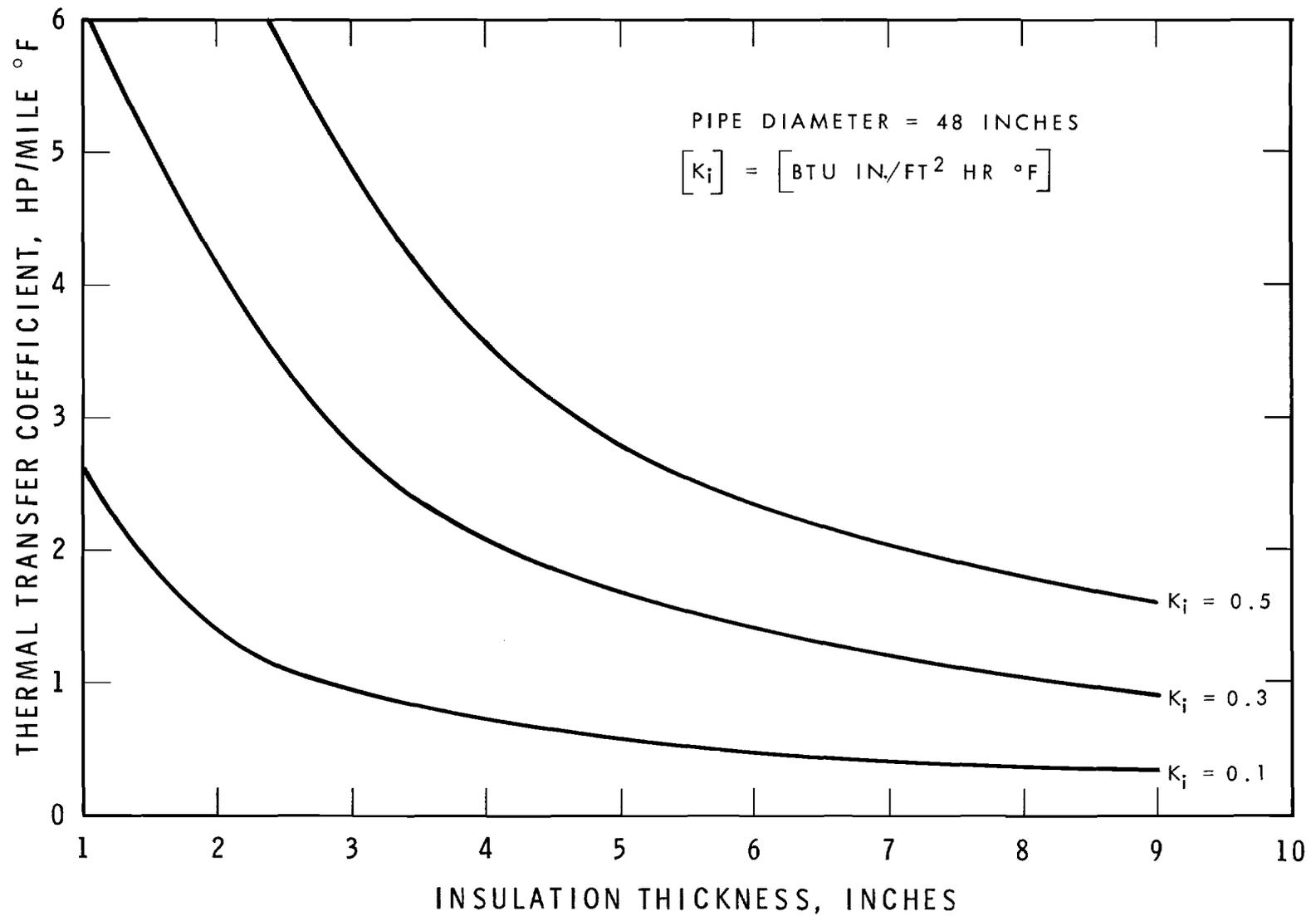


FIGURE 9

THERMAL TRANSFER COEFFICIENT AS A FUNCTION OF INSULATION THICKNESS  
 (ELEVATED PIPE)

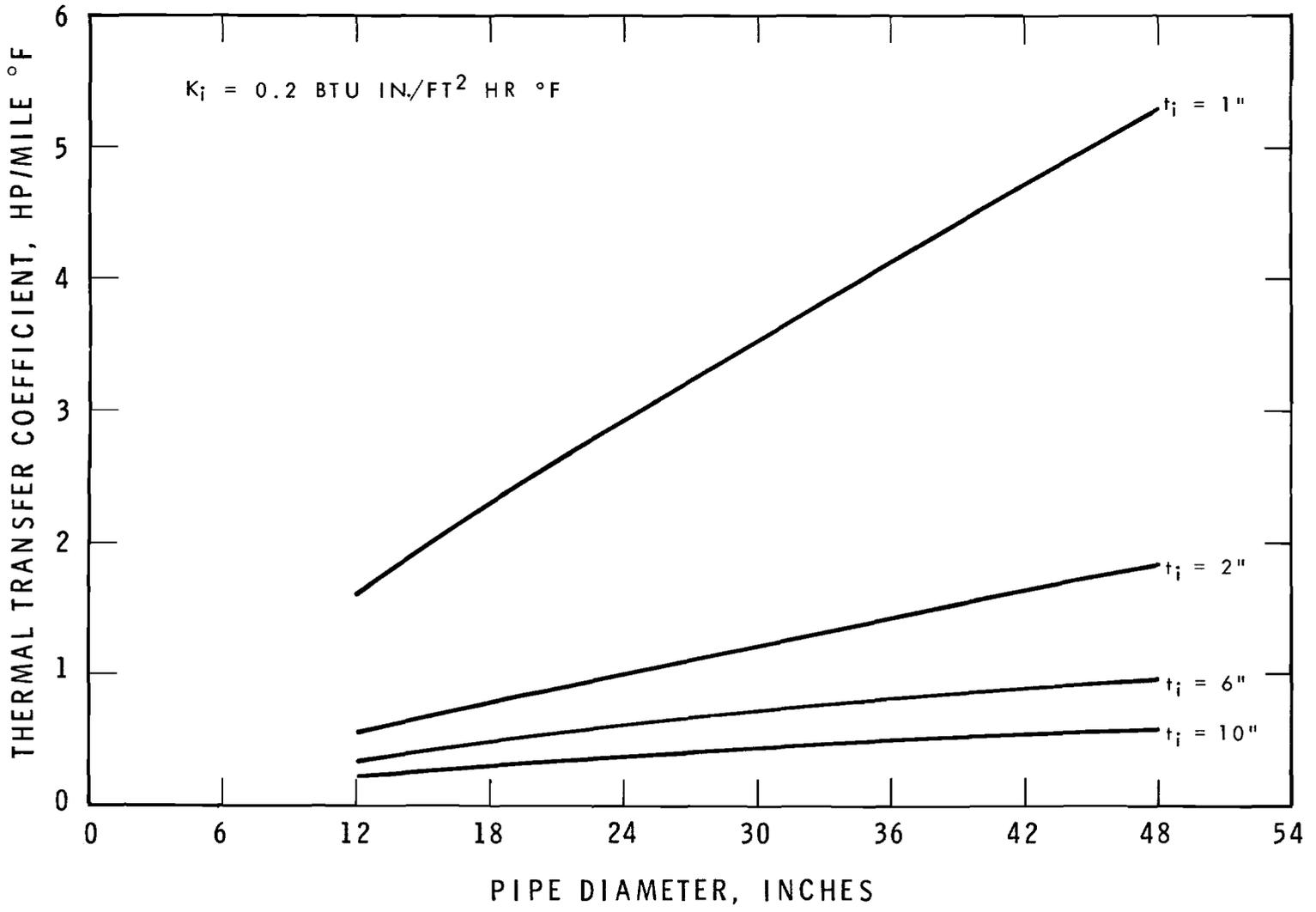


FIGURE 10  
 THERMAL TRANSFER COEFFICIENT AS A FUNCTION OF PIPE DIAMETER (ELEVATED PIPE)

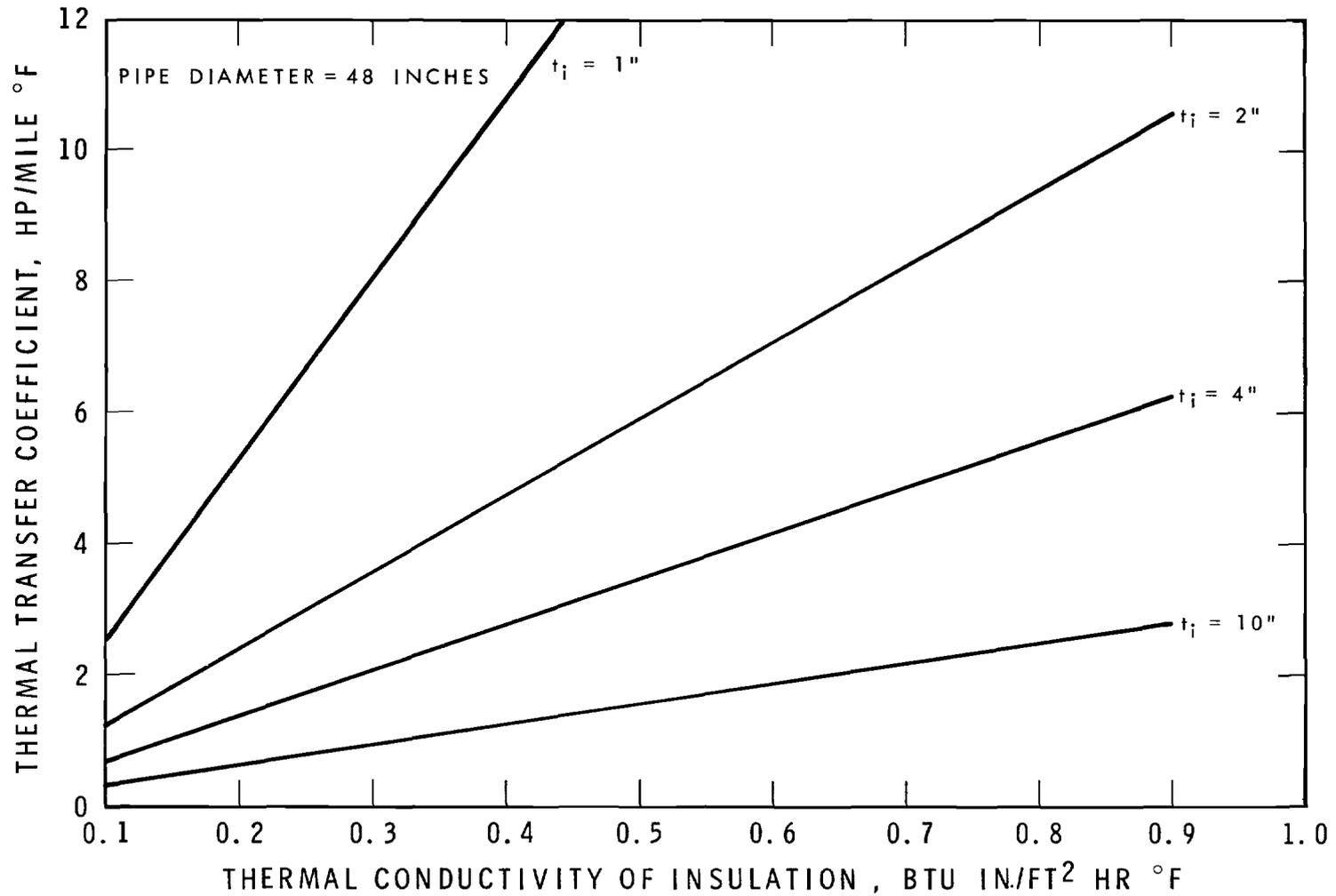


FIGURE 11  
 THERMAL TRANSFER COEFFICIENT AS A FUNCTION OF INSULATION CONDUCTIVITY  
 (ELEVATED PIPE)

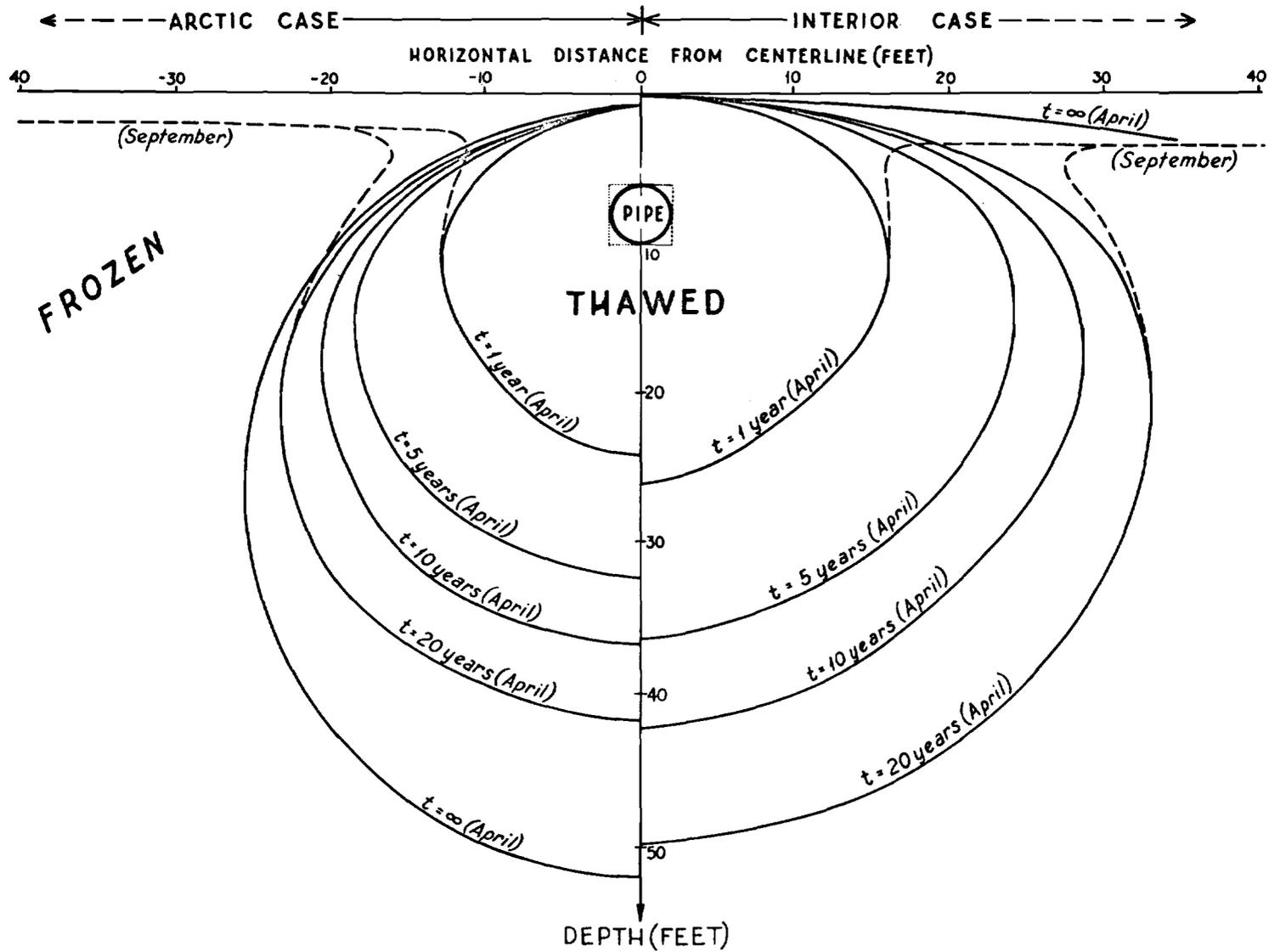


FIGURE 12  
 ADVANCING THAW ZONE AROUND A HOT PIPELINE ( After A.H. Lachenbruch, 1970)

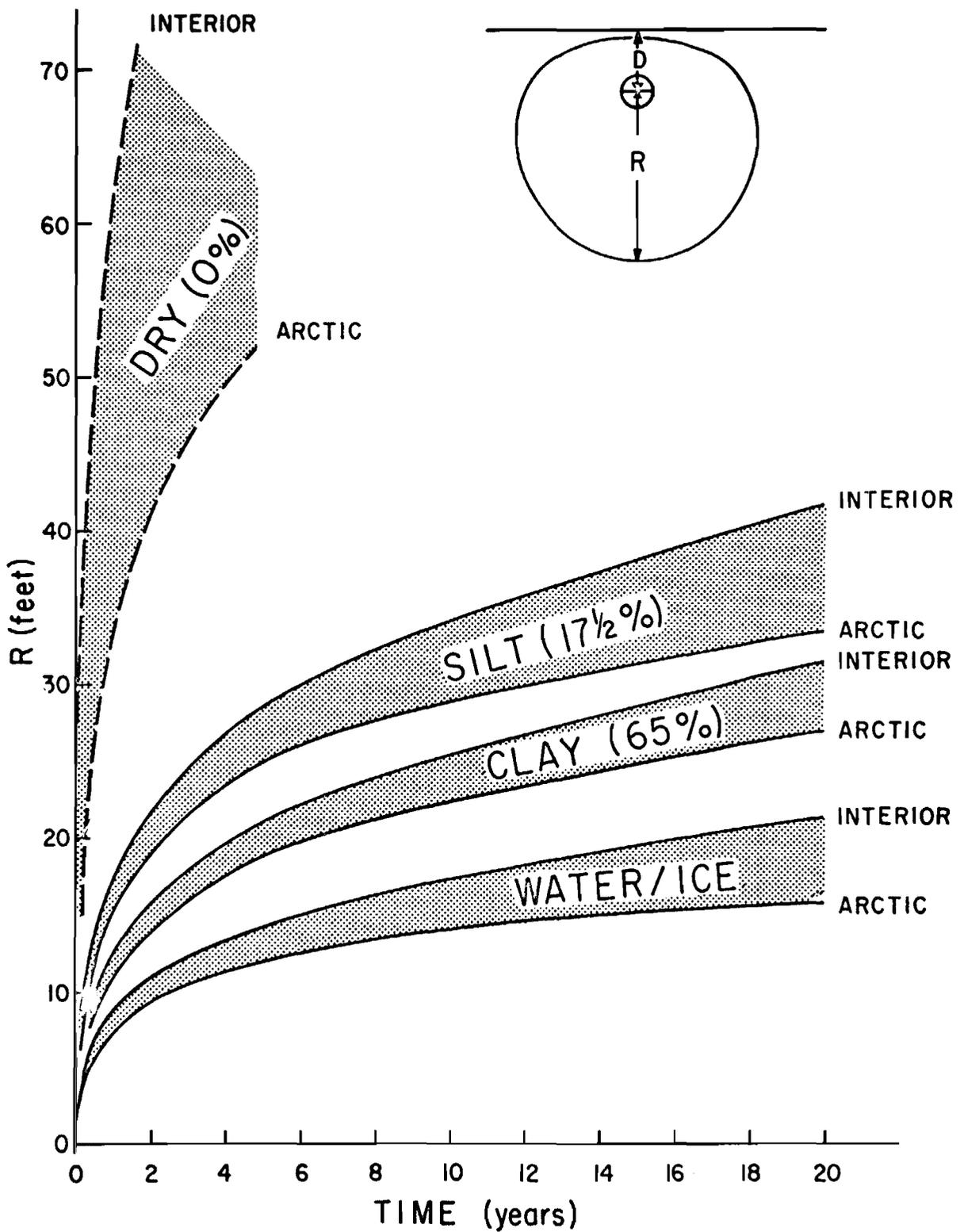


FIGURE 13

DEPTH OF THAW ZONE UNDER A HOT PIPELINE  
 ( After A.H. Lachenbruch, 1970)

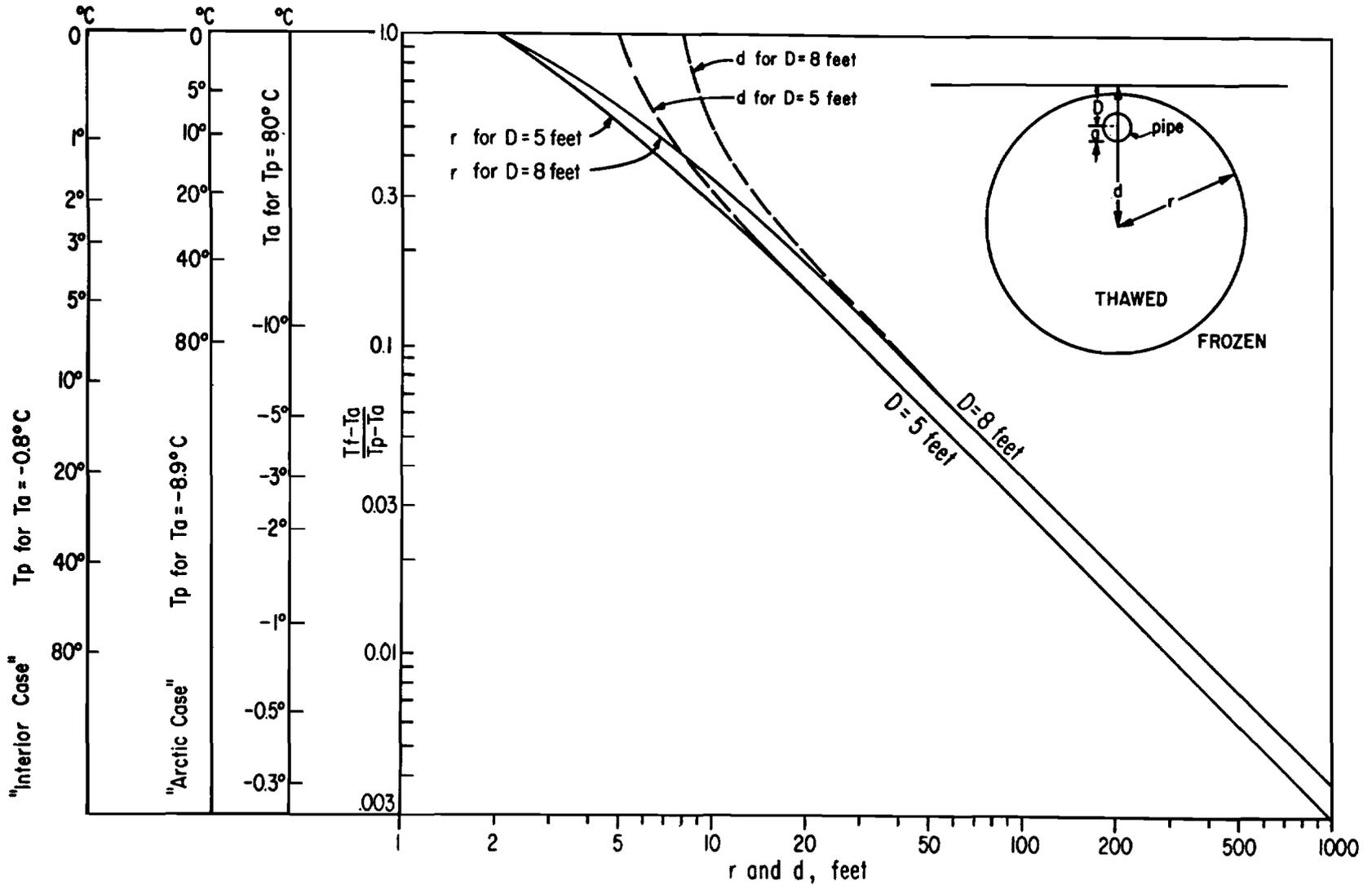


FIGURE 14

EQUILIBRIUM POSITION OF  $32^\circ\text{F}$  ISOTHERM AROUND A HOT PIPELINE ( After A.H. Lachenbruch, 1970 )

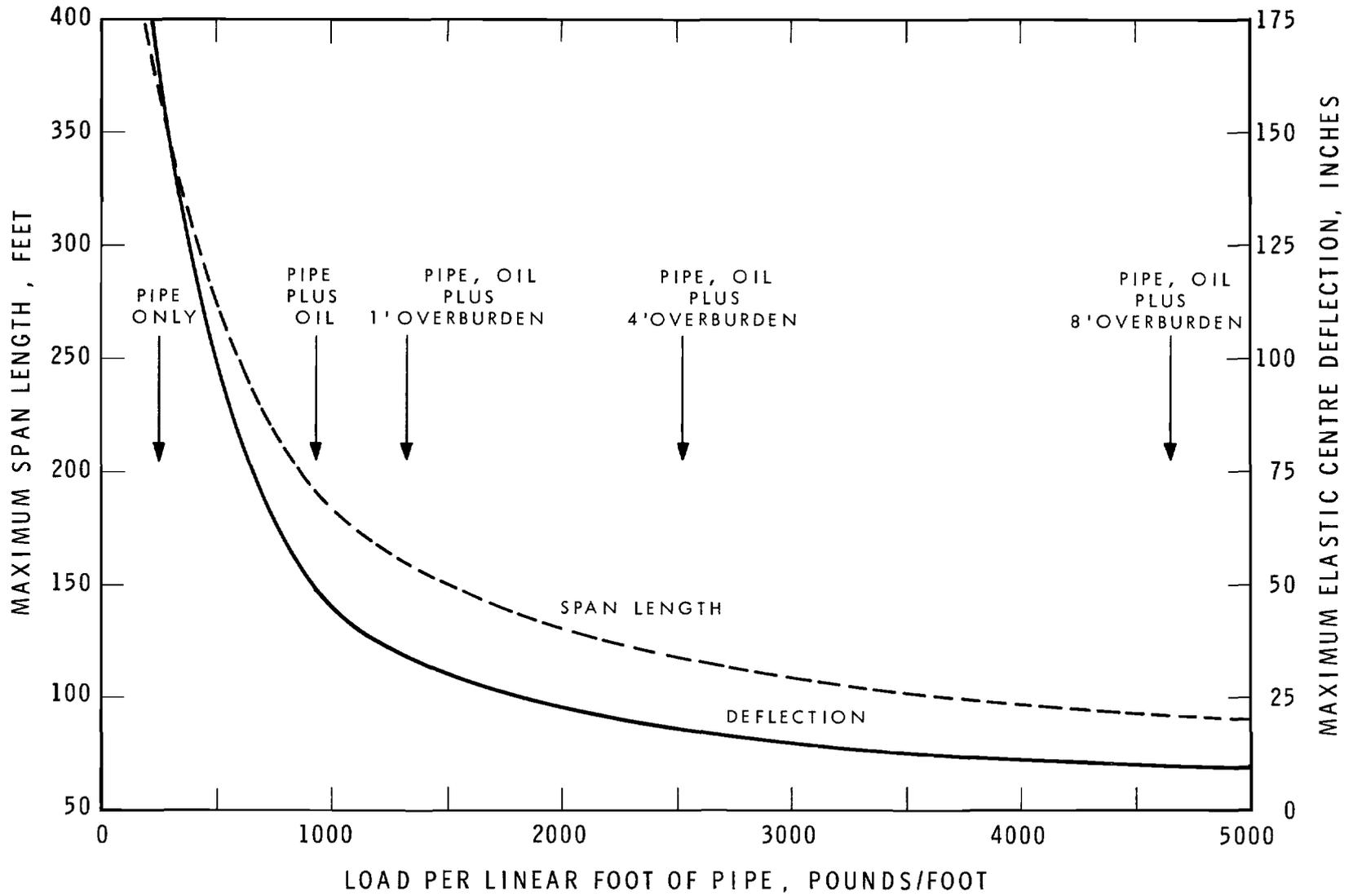


FIGURE 15 MAXIMUM LENGTH AND DEFLECTION OF PIPE