Constant stress rate deformation modulus of ice
Sinha, N. K.

Publisher's version / Version de l'éditeur:

POAC 81: Proceedings of the Sixth International Conference on Port Ocean Engineering under Arctic Conditions, 1, pp. 216-224, 1981-07-27

9051

National Research Council Canada
Conseil national de recherches Canada

CONSTANT STRESS RATE DEFORMATION MODULUS OF ICE

by N. K. Sinha

Reprinted from
Vol. 1, POAC 81
Proc. Sixth Int. Conf. on Port and Ocean Engineering under Arctic Conditions
Quebec City, 27 - 31 July 1981
p. 216 - 224

DBR Paper No. 1006
Division of Building Research

Price $1.00 OTTAWA NRCC 19753

Canada

SOMMAIRE

Cette étude présente une équation générale décrivant le fluage (sous une contrainte constante) de matériaux polycristallins en fonction de la grosseur des cristaux et permettant de calculer les diagrammes contrainte-déformation pour des charges entraînant des contraintes constantes. L'étude montre que le module de déformation, déterminé à partir de la première partie des diagrammes, augmente en même temps que la charge appliquée ou que la dimension des cristaux et diminue lors d'une élévation de température, comme on peut l'observer généralement avec la glace. Les essais sur la glace d'eau douce et la glace d'eau de mer permettent de comparer les résultats à ceux de la théorie. Il est prouvé que le module de déformation est moins élevé pour la glace d'eau de mer que pour la glace d'eau douce des lacs ou des rivières (pour des conditions semblables de température et de charge). Cette différence est due surtout aux différences de microstructure et non à la salinité de la glace d'eau de mer. La composition des cristaux constitue un facteur déterminant pour prévoir la déformation de la glace.

CONSTANT STRESS RATE DEFORMATION MODULUS OF ICE

N.K. Sinha
Research Officer

Geotechnical Section
Division of Building Research
National Research Council of Canada
Ottawa, Ontario, K1A 0R6 Canada

Canada

A generalized, constant-stress creep equation incorporating the effect of grain size for polycrystalline materials is presented and used in calculating ice stress-strain diagrams for constant stress rate loading conditions. It is shown that the effective modulus, determined from the initial part of these diagrams, increases with increase in rate of loading, increase in grain size, and decrease in temperature, in agreement with general observations on ice. Measurements on fresh-water and sea ice are presented and compared with the theoretical predictions. It is shown that the lower effective modulus for sea ice in comparison with that for fresh-water lake or river ice under similar conditions of temperature and rate of loading is due mainly to the differences in the microstructure and not necessarily to the salt content of sea ice. Sub-grains are shown to be a significant factor in controlling the deformation behaviour of ice.

2907/79

The material properties of most concern to engineers are strength and deformation or effective modulus, often called the apparent Young's modulus. It is common engineering practice in evaluating materials to determine the slope of the stress-strain curve; and it is quite natural that this method has been extended to ice engineering. The test performed most often on ice in both the field and the laboratory is the deformation test, under quasi-constant displacement or load rate, for estimating strength and effective modulus. Beam bending, cantilever and the uniaxial compression or tension tests all fall into this category.

The slope of the stress-strain curve gives the elastic modulus of a material for loading conditions where non-elastic deformation does not contribute noticeably to the total strain. This is essentially the situation for most structural materials when the slope is practically independent of rate of loading, for example, steel at low homologous temperatures.

Ice is a non-linear viscoelastic material like some metals and alloys and may exhibit non-elastic deformation. Most operating temperatures in ice engineering are so close to the melting point that non-elastic deformation almost always plays an important role in the deformation process. Consequently, stress-strain diagrams are sensitive to rate of loading. In addition, the deformation modulus obtained from the slope of the stress-strain curves has been found to depend not only on rate of loading and temperature but on type of ice. Fresh-water ice has considerably higher values than sea ice under similar loading conditions. Such observations may appear to be peculiar to ice and difficult to understand, and because of the apparent differences there is a tendency among ice engineers to treat sea ice as separate from fresh-water ice.

This paper briefly describes the micromechanics that control the behaviour of polycrystalline materials including ice at high homologous temperatures. A phenomenological creep equation incorporating grain size is presented; and a numerical integration method is described for predicting the stress-strain diagram, using this creep equation and assuming conditions of constant stress rate. It is shown as well that the observed dependence of the initial slope of the stress-strain curve on stress rate, temperature and type of ice can be predicted. The substantially lower values of the initial effective modulus for sea ice in comparison with that for fresh-water ice under similar conditions of temperature and rate of loading are shown to be due mainly to difference in microstructure.

Deformation Processes

A minimum of three macroscopically observed strain components describe the deformation of any material irrespective of operational conditions. These are: a pure elastic and instantaneously recoverable deformation, ε_e, a delayed elastic or time-dependent recoverable strain, ε_d, and a permanent or viscous strain, ε_v. The total strain, ε_t, is given by the sum of ε_e, ε_d and ε_v.

Uniaxial, constant-load creep of polycrystalline ice was used by Sinha [1] to demonstrate a method of examining the three strain components. A phenomenological viscoelastic creep model was proposed. Pure elastic deformation was assumed to be related to lattice deformation and the viscous component was attributed to intragranular deformation processes, particularly to the movement of dislocations. Delayed elasticity was hypothesized to be associated with intergranular sliding phenomena. These physical processes were later considered in the creep model, allowing the incorporation of the effect of grain size [2]. The total strain, ε_t, at time, t, in pure randomly oriented, polycrystalline material of grain size, d, subjected to a uniaxial stress, σ, at a temperature, T, was given by

$$\varepsilon_t = \varepsilon_e + \varepsilon_d + \varepsilon_v$$

$$= \frac{\sigma}{E} + c_1 \left(\frac{d_1}{d}\right) \left(\frac{\sigma}{E}\right)^s \left[1 - \exp\{-(a_T t)^b\}\right] + \dot{\varepsilon}_{v_1} t \left(\frac{\sigma}{\sigma^1}\right)^n \tag{1}$$

where E is Young's modulus; $\dot{\varepsilon}_{v_1}$ is the viscous strain rate for unit or reference stress σ^1; c_1 is a constant corresponding to the unit or reference grain size, d_1; b, n and s are constants; and a_T is the inverse relaxation time. Both $\dot{\varepsilon}_{v_1}$ and a_T vary with temperature and were shown to have the same value for the activation energy.

$$\dot{\varepsilon}_{v_1}(T_2) = \dot{\varepsilon}_{v_1}(T_1)\, S_{1,2}$$

$$\text{and } a_T(T_2) = a_T(T_1)\, S_{1,2} \tag{2}$$

where T_1 and T_2 are two temperatures in Kelvin and $S_{1,2}$ is a shift function [1].

Engineering Test

Common engineering tests involve deforming a specimen until it fails. Load rates are usually monitored, particularly in the field, because it is difficult to measure strain and even more difficult to control it. It is relatively straightforward, however, to present results as a function of load rate or, if possible, stress rate.

Presentation of strength results on the basis of stress rate during loading, or average stress rate to failure, was recommended by Sinha [3] after he pointed out that conventional laboratory tests conducted under constant displacement rate are better presented as a function of stress rate rather than of nominal strain rate. It was shown later [4] that this method of presentation also removes some of the ambiguities in the results, caused by the effect of the stiffness of the test system. Stress rate analysis of strength is, in fact, extremely well suited to field tests, as may be seen in the work of Frederking and Timco [5] on sea ice. Conventional laboratory results can be compared more easily with conventional field results if they are presented on the basis of stress rate. The dependence of strength on the stiffness of the testing machine can also be handled more conveniently by this approach [6]. Ice has proved to be stronger in this decade than previously supposed because of the use of bigger testing machines. In the following it is shown how the proposed creep equation can be used to predict the stress-strain diagram under constant stress-rate conditions. Analysis is limited to the initial period of loading that is usually used for the determination of effective modulus.

Theory

A constantly increasing stress path can be represented by a series of positive stress steps, $\Delta\sigma$, each acting during an equal interval of time, Δt, and approximating the stress rate $\dot{\sigma} = \Delta\sigma/\Delta t$. Stress $\Delta\sigma$ applied at $t = 0$ produces, according to the first term of equation (1), an elastic strain $\Delta\sigma/E$ immediately after loading ($t = 0^+$), with negligible contributions from the second and third terms. At $t = 1\Delta t^+$ there will be an elastic strain due to the total stress of $2\Delta\sigma$, whereas the delayed elastic and viscous strains (according to equation (1)) will be the amounts produced by $\Delta\sigma$ applied for the first period. At $t = 2\Delta t^+$ elastic strain will correspond to the stress $3\Delta\sigma$. For ice exhibiting $s = 1$ [1], delayed elastic strain at $t = 2\Delta t^+$ is given by the sum of the strains produced by $\Delta\sigma$ applied for $2\Delta t$ and $\Delta\sigma$ for Δt. Viscous strain will be the sum of the strain produced by $\Delta\sigma$ applied for the first increment of time Δ_t and the component produced by $2\Delta\sigma$ applied for the second increment of time Δt. Thus, at $t = N\Delta t^+$, equation (1) gives

$$\varepsilon_t = \frac{(N+1)\Delta\sigma}{E} + \frac{c_1}{E}\left(\frac{d_1}{d}\right)\sum_{i=1}^{N+1}\Delta\sigma\left[1 - \exp\{-(a_T\,[N+1-i]\,\Delta t)^b\}\right] + \dot{\varepsilon}_{v_1}\,\Delta t\sum_{i=1}^{N}\left(\frac{i\Delta\sigma}{\sigma^1}\right)^n \qquad (3)$$

The above analysis is based on the assumption that the principle of superposition associated with a standard linear solid is applicable to delayed elasticity for the class of polycrystalline materials having $s = 1$. The analysis also assumes the applicability of the commutative law of creep for viscous flow. Successful application of the above treatment to the more general case of variable load has been discussed by Sinha [7].

The constants in equation (1) were determined from creep experiments to be [1,2]: $E = 9.5\ \mathrm{GN \cdot m^{-2}}$; $c_1 = 9$; $n = 3$; $b = 0.34$; $s = 1$; a_T (-10°C) $= 2.5 \times 10^{-4}\ \mathrm{s^{-1}}$ and $\dot{\varepsilon}_{v_1}$ (-10°C) $= 1.76 \times 10^{-7}\ \mathrm{s^{-1}}$ for the system using $\sigma^1 = 1\ \mathrm{MN \cdot m^{-2}}$ and $d_1 = 1$ mm. Activation energy was found to be 67 kJ/mole. These values were used for the calculations.

Figure 1 presents a family of stress-strain curves computed from equation (3) for several stress rates at constant temperature and grain size. The temperature and grain size dependence of stress-strain diagrams are shown, respectively, in Figures 2 and 3, which show that the slope of the early portion of stress-strain curves, and hence the effective modulus, increases with increase in loading rate and decrease in

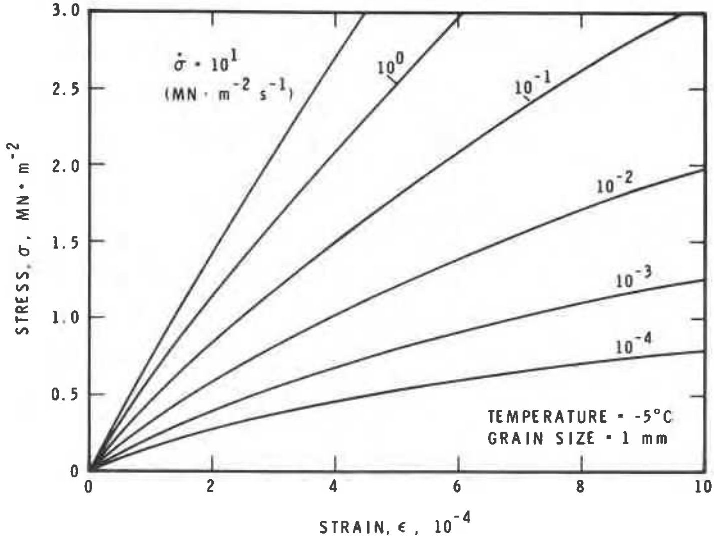

Figure 1. Dependence of stress-strain behaviour on stress rate (MN·m^{-2} s^{-1})

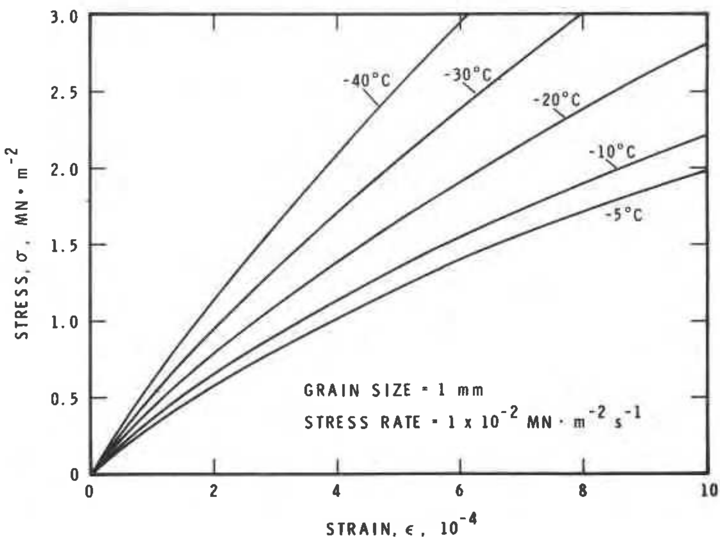

Figure 2. Dependence of stress-strain behaviour on temperature

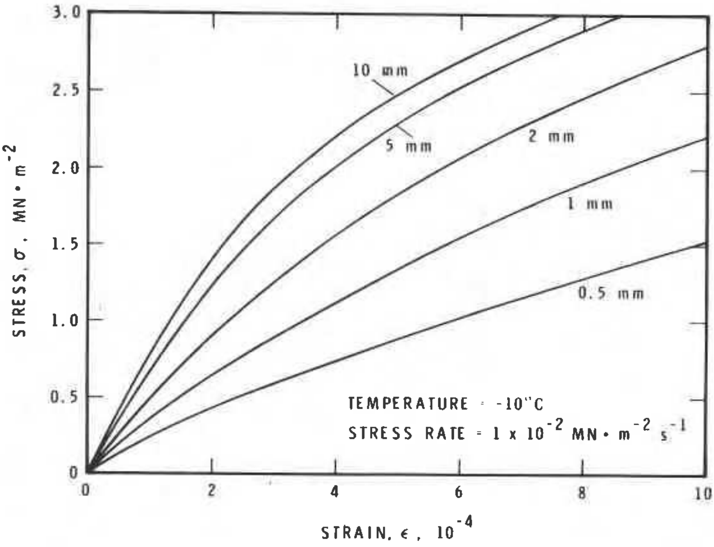

Figure 3. Dependence of stress-strain behaviour on grain size

temperature. A decrease in the effective modulus with decrease in grain size is also evident.

Grain size (in the conventional sense) could be quite large. Sea ice, however, has sub-grains (often called platelets) owing to the high salt content of the water from which it is formed. There is a general belief that the sea ice substructure controls its mechanical properties. Weeks and Assur [8] reported that average plate spacing varied in the range of 0.3 to 0.6 mm in laboratory-made saline ice 30 cm thick. Nakawo and Sinha [9] found that the width of the platelets varied from 0.4 mm to about 1.0 mm in natural first-year sea ice from the High Arctic. They give information on the dependence of the microstructure on weather conditions. If deformation behaviour depends on the sub-grain size, sea ice could be considered as fine-grained ice. It would be expected, therefore, to have a lower effective modulus than the larger grain size fresh-water ice.

Figure 4 presents an example of the relative contributions of the elastic, delayed-elastic and viscous components during the initial 10^{-3} strain. It shows that the slope in the apparently linear part at the beginning of the stress-strain curve is governed substantially by the delayed-elastic effect; the viscous component becomes significant only at the higher strains. As the first and the last terms in

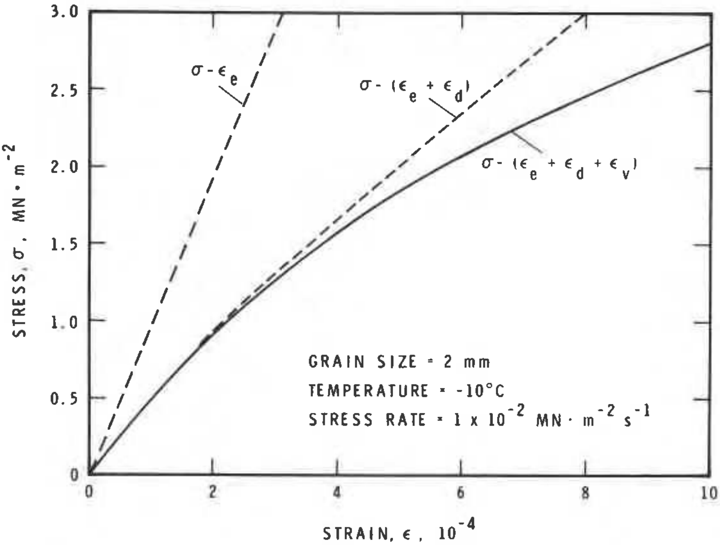

Figure 4. Stress versus strain showing strain components

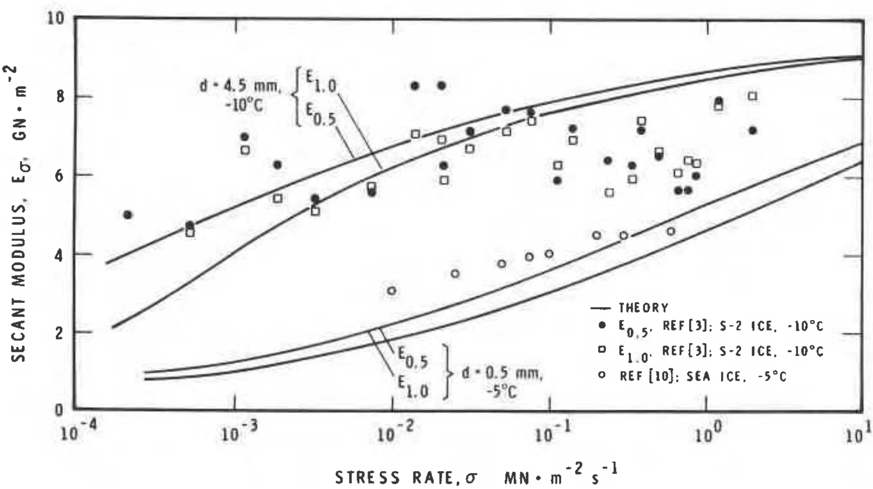

Figure 5. Comparison of theory and measurement of the stress rate dependence of the effective modulus

equation (3) do not have any grain size effect, it is the second term that determines the grain size dependence presented in Figure 3. Finer grained ice is shown to have more delayed-elastic strain and hence more recoverable strain. This explains why sea ice is apparently more "rubber-like" than fresh-water lake or river ice.

Comparison with Experiments

Observations by Sinha [3] during compressive strength tests on S-2 ice of average cross-sectional grain diameter of 4 to 5 mm at -10°C are presented in Figure 5. Secant moduli to 0.5 and 1.0 MPa are given, as well as some measurements for sea ice made by Murat [10] during four-point beam bending tests at -5°C. The ice had a salinity of 5 to 7‰. Strain gauges were attached to the compression as well as tension surfaces of the beam. Both sets of measurements were in good agreement; the average values are given in Figure 5. Levels of stress were low (< 0.5 MPa) and varied during each test (private communication).

A quick glance at Figures 1 to 4 shows that the stress-strain curves do not have a linear section in the strict sense. It would be more realistic, therefore, to calculate the secant modulus E_σ to some chosen stress level σ. Figure 5 gives examples of calculations relevant to the experimental observations for both coarse-grained and fine-grained ice. The grain diameter of 0.5 mm for the fine-grained ice was chosen because Murat's [10] measurements were on saline ice made in the laboratory and should have a platelet spacing similar to that of the ice used by Weeks and Assur [8].

Agreement between the theoretical predictions for fine-grained ice and the measurements by Murat can be considered as fair. The salinity of the ice used is similar to that usually observed for first-year sea ice. The presence of brine at grain and sub-grain boundaries could certainly influence the deformation properties, but it appears that the lower value of the elastic modulus for sea ice, and hence its greater ductility, can be predicted reasonably well on the basis of grain size alone.

Acknowledgement

The author wishes to thank Professor J.R. Murat for providing the experimental data on sea ice and for discussing the experimental details.

This paper is a contribution from the Division of Building Research, National Research Council of Canada, and is published with the approval of the Director of the Division.

References

[1] Sinha, N.K. Rheology of columnar-grained ice. Experimental Mechanics, 18(12), 1978, p. 464-70.

[2] Sinha, N.K. Grain-boundary sliding in polycrystalline materials. Philosophical Magazine A, 40(6), 1979, p. 825-842.

[3] Sinha, N.K. Rate sensitivity of compressive strength of columnar-grained ice. Presented at Spring Meeting, Society of Experimental Stress Analysis, 20-25 May 1979. San Francisco, California. (In press.)

[4] Sinha, N.K., and Frederking, R. Effect of test system stiffness on strength of ice. Proceedings, 5th International Conference on Port and Ocean Engineering under Arctic Conditions, Trondheim, Norway, V.I, 1979, p. 708-717.

[5] Frederking, R., and Timco, G. Mid-winter mechanical properties of ice in the Southern Beaufort Sea. To be presented, POAC 81, International Conference on Port and Ocean Engineering under Arctic Conditions, Quebec, Canada, 27-31 July 1981.

[6] Sinha, N.K. Comparative study of some ice strength data. To be presented at International Symposium on Ice, IAHR, Quebec, Canada, 27-31 July 1981.

[7] Sinha, N.K. Deformation behaviour of ice-like materials in engineering applications. To be presented at International Symposium on the Mechanical Behaviour of Structured Media, Carleton University, Ottawa, Ontario, 18-21 May 1981.

[8] Weeks, W.F., and Assur, A. Structural control of the vertical variation of the strength of sea and salt ice. In "Ice and Snow - Processes, Properties and Applications," (edited by W.D. Kingery). Cambridge, Mass., MIT Press, 1963, p. 258-276.

[9] Nakawo, M., and Sinha, N.K. Brine layer spacing of first-year sea ice. To be published.

[10] Murat, J.R. Small-scale surface strain measurement on sea ice. Presented at Workshop on Sea Ice Field Measurements, St. John's, Newfoundland, 29 April - 1 May 1980.

Publications of the Division may be obtained by mailing the appropriate remittance (a Bank, Express, or Post Office Money Order, or a cheque, made payable to the Receiver General of Canada, credit NRC) to the National Research Council of Canada, Ottawa. K1A 0R6. Stamps are not acceptable.

A list of all publications of the Division is available and may be obtained from the Publications Section, Division of Building Research, National Research Council of Canada, Ottawa. KIA 0R6.