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Publisher's version / Version de l'éditeur:

Applied Mechanics Review, 40, 9, pp. 1214-1231, 1987-09

National Research Council Canada
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Reprinted from
Applied Mechanics Reviews
Vol. 40, No. 9, September 1987
p. 1214-1231
(IRC Paper No. 1488)

Price $5.00

NRCC 28452

Canada

RÉSUMÉ

Des travaux sur la mécanique des glaces, menés sur une vaste échelle au Canada, ont donné lieu à des progrès importants au cours des dix dernières années. Les facteurs influant sur la croissance des divers types de glace marine ont fait l'objet d'une quantification fondamentale et des méthodes d'examen de la structure résultante du matériau ont été mises au point. Les caractéristiques de déformation et de résistance de la glace ont été étudiées de façon approfondie. Une étape importante des travaux a consisté à élaborer des expressions analytiques pour décrire le comportement rhéologique de la glace. Le module d'élasticité, le coefficient de Poisson et le fluage ont également été examinés. On a pris un grand nombre de mesures de résistance à la compression sur divers types de glace naturelle, et ces données ont été regroupées par la suite pour composer une description adéquate de l'enveloppe de rupture. On a également mesuré la résistance à la flexion, la résistance au cisaillement, l'adhérence et la résistance à la rupture. Des méthodes permettant d'effectuer des essais en laboratoire et de mesurer in situ les propriétés mécaniques de la glace ont été mises au point. Ce sont les efforts visant à définir l'action des glaces sur les structures qui ont donné le coup d'envoi aux recherches sur la glace, au cours desquelles on a utilisé la modélisation analytique, la modélisation physique, des études en laboratoire et des études sur le terrain très exhaustives. Les travaux effectués dans ce domaine comprenaient l'élaboration de méth[illegible]; ces travaux ont grandement tiré avan[illegible]oche et ont donné lieu à des progrès ass[illegible]et les couvertures de glace dans diver[illegible] glace flottante, stabilisation de cham[illegible]tures, et transport de grosses charges [illegible]

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Reprinted from Applied Mechanics Reviews vol 40, no 9, Sept 1987
ASME Book No. AMR022

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Work on the mechanics of ice, which has been carried forward on a broad front in Canada, has resulted in a number of significant advances in the last 10 years. The factors influencing the growth of various types of sea ice have been quantified fundamentally and methods for examining the resulting material structure have been developed. Extensive work has been done on strength and deformation characteristics of ice. A significant effort has been the development of analytical expressions to describe the rheological behavior of ice. Elastic modulus, Poisson's ratio, and creep were also treated. A great deal has been done on measuring the compressive strength of various types of naturally occurring ice and subsequently these data were combined into a suitable description of a failure envelope. Work has also been done on measuring the flexural strength, shear strength, adhesion and fracture toughness. Methods for laboratory testing and *in situ* measurements of mechanical properties have been developed. The problem of defining ice forces on structures has been the primary motivation for research on ice. Analytical modelling, physical modelling, laboratory studies and very extensive field studies have been used. Work done in this area has included development of methods and their application to actual problems and has benefitted greatly from the integration of all four approaches. Very significant progress has been made. Ice and ice covers have been successfully used to support various offshore activities: drilling off floating ice platforms, stabilizing grounded rubble fields to protect structures and transporting large loads over ice.

1. INTRODUCTION

Because of Canada's cold climate and offshore gas and oil resources in the Beaufort Sea, Sverdrup Basin, and Hibernia locations, a considerable amount of effort has been spent on research in ice mechanics. Research and funding for research is provided by government, industry, and universities. A large number of relatively small groups of researchers scattered across the country have contributed in the development of ice mechanics in this country. A great deal of work was performed by or under the auspices of the petroleum industry. To coordinate this effort and to help share the high costs associated with carrying out the necessary field projects, the Arctic Petroleum Operators Association (APOA) was established. Also, a strong and active community of consulting firms came into being and expanded to participate in many of these projects. Universities too became active in this area, some on a long standing basis of experience in ice engineering and others developing new expertise. Finally a number of government agencies became active, both in terms of their own laboratory and field programs, and in providing funding.

Canada has been involved in almost all areas related to offshore engineering in cold climates. Advances have been made in increasing the bank of experimental information on ice properties as well as in providing insight and understanding in several key areas. In this paper, the authors have attempted to report on a large number of significant advances made in many areas of ice engineering in Canada during the past ten years. In particular, there is a strong emphasis in the work *offshore* (in both the Arctic and Hibernia locations). These include the physical properties of ice-structure, strength and rheology (Section 2), ice forces on structures (Section 3), and ice as a construction material (Section 4). Due to space restrictions, several areas of ice research (such as river ice engineering, ice navigation, glaciers, etc) have been omitted. It should be kept in mind that this paper represents recent advances in ice mechanics in Canada and it is not intended to be a state-of-the-art review.

1.1. Ice: The material

Although the working temperatures in ice are low in terms of human comfort, they are very high in terms of materials. The temperature of ice in nature rarely goes below $-40°C$ or an equivalent temperature of 0.85 T_m, where T_m is the melting point in Kelvin. Its working temperatures are therefore greater than the homologous temperature of 0.85 T_m. In metals and alloys, working temperatures higher than about 0.4 T_m are considered to be elevated temperatures. At these levels polycrystalline materials, including ice, exhibit pronounced creep and grain-boundary embrittlement. A direct consequence of high-temperature deformation and failure modes is that the strength of ice is rate sensitive and loading history becomes important in ice mechanics. Recent Canadian advancement in ice mechanics is centered around this theme.

The detailed structure of ice, from the microscale to the mesoscale, has a significant influence on the properties, behavior, and interaction processes in ice.

1.1.1. Macrostructure

Ice in nature has a particular grain structure and texture which depends on the growth processes, and thermal and mechanical histories. A classification system for freshwater river and lake ice on the basis of its genesis, structure, and texture was established by Michel and Ramseier (1971). The detailed analysis that led to this classification is given in Ramseier (1976).

1.1.2. Microstructure

Traditional methods for examining the grain structure have been refined, leading ultimately to the double-microtome technique (Sinha, 1977a) that eliminates completely the use of any hot plate. This technique does not disturb the thermal state nor destroy the microscopic structure. A new method of combined cross-polarized light and scattered light technique was also developed. Used with etching and replicating methods and the scanning electron microscope (Sinha, 1977b, 1978a, 1986a) this technique permits insight into the structure and behavior of ice at the microscopic and sub-microscopic level (see Fig. 1). This procedure in conjunction with macroscopic observations and acoustic emission (Sinha, 1982c) showed that the low homologous temperature dislocation pile-up mechanism might not be the prime mechanism of crack nucleation in most engineering situations (Sinha, 1984a). An alternate crack nucleation mechanism, amenable to high temperature, was proposed by Sinha (1982b) which was expanded later to take account of the effect of temperature (Sinha, 1984a) and to model the damage accumulation in creep (Sinha, 1984b). The etching-replicating technique was also used to study the microcracks in ice, revealing the absence of any crack-tip plasticity (Sinha, 1984a).

1.1.3. Microscale

Both the thickness and quality of ice covers are influenced by the past as well as the prevailing climatological conditions. Based on a substantial volume of data on the weather and snow and ice characteristics collected by the Arctic Research Establishment in Eclipse Sound, Baffin Island, and for the winter seasons of 1977–78 and 1978–79 Sinha and Nakawo (1981) proposed a simple numerical integration method for predicting growth of ice in the sea. The method is capable of incorporating variations in snow conditions and physical properties of ice and snow during the growth period. It was shown that rapid desalination occurs within about a week after freezing. Although it decreases very slowly during the rest of the growth season, the salinity could be considered to have attained a quasi-stable value within a few weeks after ice formation. The vertical salinity profile in the ice toward the end of the season provides a record of previous climatological conditions. A relation has been shown between the predicted growth rate and the measured salinity (Nakawo and Sinha, 1981). It was shown further that the growth rate and hence the growth history determined the vertical variation of porosity and microstructure. (Nakawo, 1983; Nakawo and Sinha, 1984). The average brine layer spacing, a measure of the sub-grain structure, was shown to be inversely proportional to the growth rate as a first approximation. The existence of highly oriented sea ice was also confirmed at several locations in the Arctic (Sinha, 1983a, 1984c; Nakawo and Sinha, 1984).

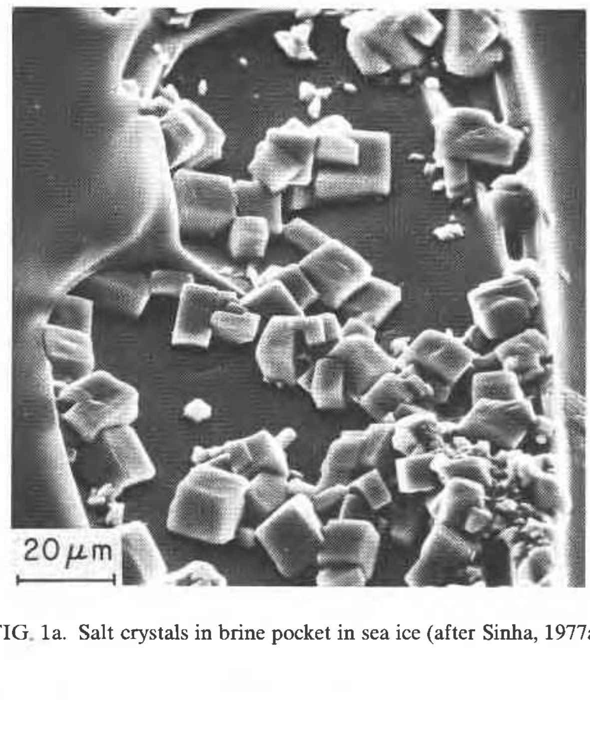

FIG. 1a. Salt crystals in brine pocket in sea ice (after Sinha, 1977a).

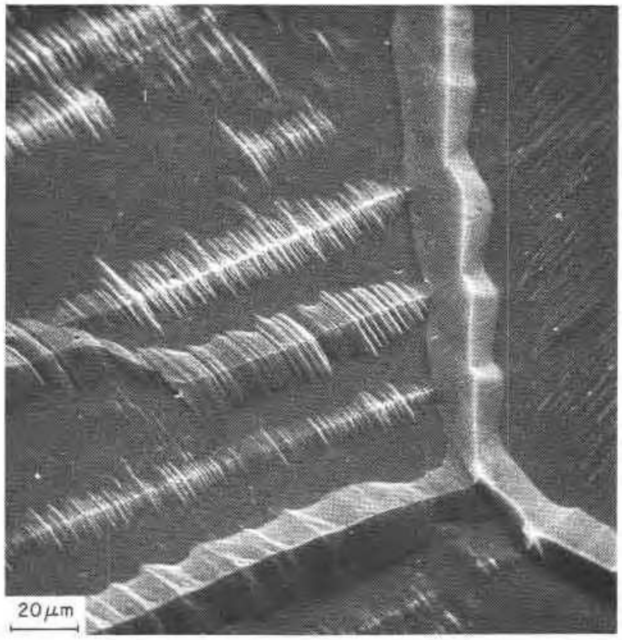

FIG. 1b. Dislocations pile ups against a grain boundary in ice (after Sinha, 1978a).

The aging of a sea ice cover was studied at Mould Bay, Prince Patrick Island during the 1981–82 and 1982–83 seasons. Measurements of the physical, chemical, microwave, and mechanical properties were carried out in October 1981, June–July 1982, and April 1983 on the same ice cover whose entire growth history was recorded continuously by the staff of the Weather Station run by the Atmospheric Environment Services of Canada The changes which took place in the physical, chemical and microstructural properties during the aging process have been presented by Sinha (1983b, 1984c, 1985a, 1986a), Bjerkelund et al (1985), Holt and Digby (1985), and Digby (1984).

1.1.4. Mesoscale

For a safe structural design of a structure which is to be used in ice-covered waters, it is necessary to know the maximum credible size, thickness, and concentration of any ice features in the region. This, along with a knowledge of the mechanical and rheological properties of the ice, can then be used in appropriate predictive equations for estimating both global and local loads on the structure. For several years there has been considerable effort spent on observing, recording, and predicting the large (or meso) scale ice conditions in Canadian waters. Several Canadian government agencies such as the Atmospheric Environment Service (AES) and Canada Centre for Remote Sensing (CCRS) have been instrumental in providing information on ice conditions in the Arctic, Gulf of St. Lawrence, and Great Lakes, and east Newfoundland and Southern Labrador. In addition, several private companies such as Canarctic Shipping, Nordco, INTERA, Canmar, Bercha Alberta, etc have provided ice reconnaissance information. Several sophisticated remote sensing devices are actively employed including laser profilometer, infrared line scanner, airborne radiation thermometer, synthetic aperature radar (SAR) and side-looking airborn radar (SLAR). These are used along with satellite imagery (TIROS/NOAA and LANDSAT) to develop composite charts of current ice conditions. Due to space limitations a review of this topic is not possible here. For recent work, see eg Rossiter and Bazeley, 1980; or information can be obtained from the Directors of AES in Downsview, Ontario and CCRS in Ottawa.

In addition to significant improvements in the area of remote sensing of ice, a considerable effort has been devoted to ground truthing or directly measuring the ice features in the Arctic and offshore Newfoundland. Several industry sponsored programs have documented the characteristics and statistics of extreme ice features in these areas. For the Beaufort Sea, studies on ridging include those of Wright et al (1981), Hudson (1982, 1983), Spedding (1982), and Metge et al (1982). For the east coast, a knowledge of the number, size, distribution, shape, movement, etc of icebergs are of particular importance, and several studies have documented these including Lowrey and Miller (1983) and Diemand (1983). The staff from Memorial University, C-CORE and Bedford Institute are particularly active in this area. Several models have been developed to predict the behavior of the ice on the mesoscale including models to estimate the forces involved in the ridge-building process (Sayed and Frederking, 1984, 1986), and to predict the drift trajectories of icebergs (Sodhi and El-Tahan, 1980; El-Tahan, 1980; El-Tahan et al, 1986; Smith and Banke, 1983; Lever and Sen, 1986; Chandler, 1986; Gaskill and Harris, 1986). In 1980 a workshop on ridging and pile-up summarized Canadian work in the area (ACGR, 1982). A three-day workshop on extreme ice features was held recently in Banff, Alberta under the sponsorship of the NRC Snow and Ice Subcommittee. The Proceedings of this workshop provide a good overview of the knowledge of the size, extent, and physical properties of extreme ice features (Pilkington, 1987) in Canada.

2. MECHANICAL PROPERTIES

2.1. Strength

2.1.1. Introduction

The strength of ice is the maximum stress which an ice specimen can support. It is the mechanical property which is of great importance when dealing with problems such as the determination of ice forces on a structure, load bearing capacities of ice covers, etc. To date, numerous investigators have measured the strength of ice. A comprehensive review of earlier investigations on mechanical properties of fresh water ice have been made by Gold (1977) and Michel (1978b). A compilation of test results on sea ice has been made by Lainey and Tinawi (1984). Many factors influence the response of ice including temperature, salinity, density, ice type, grain size, specimen size, loading rate, and failure mode. In spite of its complexity and the number of factors influencing it, an understanding of strength as a material property is emerging. In this section, the strength studies and subsequent understanding of ice strength by Canadians is documented for compressive strength (Section 2.1.2), shear strength (Section 2.1.3), flexural strength (Section 2.1.4), failure envelope (Section 2.1.5) and fracture toughness (Section 2.1.6).

2.1.2. Compressive strength

The strength of ice in compression is of fundamental importance in almost all aspects of ice mechanics. Studies in Canada can be categorized into three different types: *uniaxial* compression in which a uniform uniaxial stress is applied to a specimen; *multiaxial* compression in which a more complex stress state is applied to a specimen; and *in situ* where the ice is tested in its natural state in the ice cover. These tests have been performed in both freshwater ice and sea ice. Three commonly applied loading conditions for determining strengths are (a) constant displacement rate, (b) controlled constant strain rate, and (c) controlled load or stress rate. Most of the experiments have been conducted under category (a) because of the availability of universal test machines, capable of delivering precise cross-head displacement rates over a wide range. Strength values obtained under this condition at the same cross-head rate have been shown by Sinha (1981a) to vary from one machine to another. Efforts have been made in the last four years to carry out tests under categories (b) and (c) using a closed-loop test system.

Uniaxial compression: Many tests have been performed to measure the uniaxial compressive strength of freshwater ice, both laboratory grown and field ice. These studies include those of Parameswaran and Jones (1975), Ramseier (1976), Frederking (1977), Michel (1978a, c), Sinha (1981b, 1982a, 1982b, 1982c), and Jones and Chew (1983) on laboratory grown ice; Vittoratos and Kry (1979) on lake ice; Gammon et al (1983a), El-Tahan et al (1984), Nadreau (1986), and Sinha and Frederking (1987) on iceberg ice; and Frederking and Timco (1981, 1983a, 1984a), and Timco and Frederking (1980, 1984, 1986a), and Sinha (1983a, 1983b, 1984c, 1985a, 1986a, 1986b) on first-year, second-year, and multi-year ice; and Sinha et al (1986) on offshore built-up platform ice.

Strength values, ranging from 0.5 to 10 MPa, are a strong function of loading rate. Gold (1978) and Nadreau and Michel (1984a) reviewed some of the earlier published literature on the basis of nominal strain rate, $\dot{\varepsilon}_n$ and noted that the strength increases with strain rate to $\dot{\varepsilon}_n \simeq 10^{-3}\ \mathrm{s}^{-1}$ whereupon strength values generally decrease with increasing strain rate. In most uniaxial tests described in literature, the strength of the ice is obtained by loading with a constant displacement rate for which the nominal strain rate of the ice is given by $\dot{\varepsilon}_n = \dot{x}/l$, where $\dot{x}$ is the cross-head rate and l is the specimen length. These types of tests are usually referred as "constant strain rate tests" and created confusion in the literature, not only in the field of ice but also in material science as a whole. Sinha (1981b) has shown that interpretation in terms of *nominal* strain rate can be significantly in error, as the actual specimen strain rate is not constant during a test of this type. Strain rate in the specimen increases monotonically with time and approaches the nominal strain rate only after reaching maximum stress for upper yield type failure. The stiffness of the test system affects both the pre-yield behavior and failure mode such that the measured "strength" value depends upon the stiffness of the test machine if conventional analysis in terms of nominal strain rate is used for the category (a) tests (Sinha and Frederking, 1979). It is shown that a lower capacity machine yields lower strength (Sinha, 1981a). Moreover, the apparent ductile-to-brittle transition depends upon this system stiffness. The analysis by Sinha (1981b) has been instrumental in shaping the methods by which uniaxial (a) type tests are analyzed. Further, it emphasizes the need to perform these types of tests using test machines in which the strain-rate or stress-rate can be controlled accurately. Whether the tests are conducted under category (a) or under (b) there is a strong relation between upper yield failure stress, σ_f, and the corresponding failure time, t_f. Sinha [1981b, category (a); 1982b, category (b)] noted this dependence as

$$t_f/t_0 = C(\sigma_f/\sigma_0)^{-\theta}, \quad (1)$$

where C and θ are constants, t_0 is the unit of time ($=1$ s), and σ_0 is the unit stress ($=1$ MN m^{-2}). There is a remarkable similarity between this equation and the dependence of creep rupture time on stress for metals and alloys at high temperatures. Numerical values of θ at $-10°$C have been found to be in the range of 2.3 to 2.8 for columnar-grained freshwater ice [Sinha, 1981b, category (a) and 1982a, category (b)] as well as sea ice [Sinha, 1983a, 1984a, 1986b—category (a)] with load applied normal to the columns and frazil sea ice [Sinha, 1984c —category (a); Sinha 1986a—category (b)] and built-up platform ice [Sinha et al, 1986—category (a)]. Tests involved four different test machines ranging in capacity from 9 kN to 1 MN indicating large differences in stiffness. Equation (1) applies also to *in situ* borehole jack tests (Sinha, 1986b) where θ was found to be 2.1. It is also applicable to confined tests in first-year sea ice (Sinha, 1986b) and second year sea ice (Sinha, 1985a).

The simplest evaluation of the rate sensitivity of strength, as suggested by Sinha (1981b) is to be obtained by using the average stress rate to failure $\dot{\sigma}_f = \sigma_f/t_f$. It can be shown using eq. (1) that

$$\sigma_f/\sigma = M(\dot{\sigma}/\dot{\sigma}_0)^m, \quad (2)$$

where $\dot{\sigma}_0$ is the unit stress rate ($=1$ MN m^{-2} s^{-1}), $M = C^m$, and $m = 1/(1+\theta)$. Equation (2) has been successfully applied by Sinha in all his work mentioned above. Although θ and hence m does not show much sensitivity to ice types, loading direction or temperature, C and hence M does vary significantly with variations in these factors. Similar observations were also made by Frederking and Timco (1981) who measured the compressive strength of vertically-loaded land fast ice in the Beaufort Sea and the strengths of both vertically and horizontally loaded ice around Tarsiut Island (Frederking and Timco, 1983a, 1984a).

Most recently, Timco and Frederking (1986a) analyzed many of these data for category (a) tests and found that the uniaxial strength of columnar ice could be related to the average stress rate by

$$\sigma = 8.4(\dot{\sigma}_a)^{0.22}\left(1 - \sqrt{v_T/320}\right) \quad (3)$$

for horizontally-loaded samples, and

$$\sigma = 32.6(\dot{\sigma}_a)^{0.22}\left(1 - \sqrt{v_T/280}\right) \quad (4)$$

for vertically-loaded samples, where σ is in MPa, $\dot{\sigma}$ is in MPa s^{-1} ($10^{-3} \le \dot{\sigma}_a \le 10^0$) and v_T is the total porosity (brine + air) in parts per thousand. Similar expressions have been determined in terms of nominal strain rate. These equations relate the uniaxial compressive strength of columnar sea ice explicitly in terms of loading direction, loading strain rate, loading stress rate, and total porosity of the ice, and implicitly in terms of ice salinity, temperature, and density.

Justification for the above approach of using stress rate comes from the fact that load and time can both be measured readily and accurately without much difficulty. Moreover, the response of a test system is reflected in the measured loading rate and consistent results, useful for the purpose of comparison, are obtained using stress rate. These results should not, however, be considered the material property under truly constant stress rate. This is primarily because stress rate is not constant in these tests but approaches zero at upper yield and then reverses. Category (c) tests conducted by Sinha (1982a), using a closed-loop controlled system, indicated that the stress-rate dependence of upper yield stress, as derived from category (a) tests and eq. (2), underestimates strength under truly constant stress-rate. Stress-rate analyses could, under certain circumstances depending on the load capacity of test system, lead to erroneous results and hence erroneous conclusions (Sinha, 1983a).

There is no substitute for measuring both the load and specimen deformation. With known stress and strain histories, analyses can be made simpler, more straightforward and devoid of misinterpretation (Sinha, 1981b, 1982a, 1983a,b, 1984c, 1986a; Timco and Frederking, 1983a). For category (b) tests under controlled constant strain rate, $\dot{\varepsilon}_c$, the dependence of upper yield σ_f on $\dot{\varepsilon}_c$ can be presented as

$$\sigma_f/\sigma_0 = P(\dot{\varepsilon}_c/\dot{\varepsilon}_0)^p, \quad (5)$$

where P is the failure stress at unit strain rate, $\dot{\varepsilon}_0$ ($=1$ s^{-1}) and p gives the strain rate sensitivity. Equations (5) and (1) give all the interdependence between σ_f, t_f and failure strain ε_f.

To date, only two sets of data from Canada for category (b) tests are available in the open literature. From true constant strain-rate ($\dot{\varepsilon}_c$) tests, the uniaxial upper yield strength (σ_f) at $-10°$C of horizontally loaded columnar grained freshwater ice is given by $\sigma_f = 212(\dot{\varepsilon}_c)^{0.34}$, where σ_f is in MPa and $\dot{\varepsilon}_c$ is in s^{-1} such that $1\times10^{-7} \le \dot{\varepsilon} \le 1\times10^{-4}$ s^{-1} (Sinha, 1982a). The corresponding relation for congealed frazil sea ice (salinity: 5‰) at $-10°$C is found to be $\sigma_f = 70(\dot{\varepsilon}_c)^{0.34}$ for $\dot{\varepsilon}_c \le 2\times10^{-3}$ s^{-1} (Sinha, 1986a). The strains at failure are less than 1×10^{-3} for freshwater ice and less than 2×10^{-3} for sea ice.

Equation (5) applies well to category (a) tests also, giving comparable values for P and p, if $\dot{\varepsilon}_c$ is replaced by the average strain rate to failure, $\dot{\varepsilon}_{af}$. However, the customary use of $\dot{\varepsilon}_n$ gives only a good estimation of p. In this case P value depends on stiffness of the machine and usually a higher value is obtained with stiffer machines (Sinha, 1981a). Detailed comparison between the two common types of tests can be seen in Sinha (1981b, 1982a) for freshwater ice and in Sinha (1984c, 1986a) for sea ice. The latter papers also compare field tests with laboratory tests and examine the suitable methods of transportation and storage of sea ice. These tests also assisted in drawing a conclusion that the strain rate sensitivity, as given by the value of p in the previous paragraph, is the same for freshwater as well as sea ice.

One to one numerical correspondence between the rate effect on strength in controlled constant strain rate, category (b) test, and the rate effect on viscous flow in a constant load or stress creep test, to be discussed in Section 2.2.2, were established for pure freshwater S-2 ice by Sinha (1982a), who showed that the two sets of results are related numerically by

$$\dot{\varepsilon}_{v_0} = P^{-1/p} \text{ and } n = 1/p, \quad (6)$$

in which $\dot{\varepsilon}_{v_0}$ is the viscous flow rate for unit stress and n is the stress exponent. The applicability of eq. (6) for sea ice is still an open question, primarily because of lack of constant stress creep data on sea ice.

Michel (1978a) has proposed a phenomenological model to describe the crushing strength of freshwater ice in both the brittle and ductile range of ice behavior. In the brittle regime, Michel proposed that the crushing strength σ_f(Pa) is

$$\sigma_f = 9.4\times10^4\left(d^{-1/2} + |\theta|^{0.78}\right), \quad (7)$$

where d is the grain size (in m), and θ is the ice temperature (in °C). In the range of ductile behavior, Michel proposed an equation of the form

$$\dot{\varepsilon}_n = A\sigma_f^n \exp(-Q/R\theta^*), \tag{8}$$

where $\dot{\varepsilon}_n$ is the nominal strain rate, A and n are constants, Q is the activation energy, R is the universal gas constant, and θ^* is the absolute temperature. These equations yield a relatively good fit to the strength data reported by Michel.

Both eqs. (7) and (8) should be used with caution. Equation (7) bears similarity to Hall–Petch relation developed for yield strengths in metals and alloys at low temperatures. Jones and Chew (1983) did not find any grain size effect on compressive strength for granular ice. While trying to find an explanation for the observed differences, Sinha (1983c) noted that in all of these studies on ice, including those carried out elsewhere, the dependence of strength on grain size was examined by conducting tests in which other parameters, as well as the grain size, varied. Moreover, in Michel's tests various grain sizes actually represent different types of ice with different textures and fabric such as frazil, granular and columnar-grained. It should also be pointed out that eq. (8) suffers from the fact that the strain rate is nominal strain rate and that Q, in this case, depends on σ_f (Sinha, 1984d) which, on the other hand, depends on the stiffness of the test system. Whether the strength of ice depend on grain size and how Q depends on σ_f are still open questions.

Multiaxial compression: The strength of ice under a multiaxial stress state is of interest for the development of failure criteria appropriate to the complex stress states associated with field problems. Confining sub-presses have been used to do biaxial tests and conventional triaxial cells for triaxial tests. All the tests available now in the literature are of category (a). Recently a test system has been developed for doing proportionate loading triaxial tests (Smith et al, 1986). For freshwater ice Frederking (1977) and Croasdale et al (1977) have performed strength measurements on columnar-grained S-2 ice for the case of confined (biaxial) compression. In this case, for both confinement and loading in the plane of the ice cover, the biaxial strength is from 2–4 times higher than the uniaxial strength measured under similar conditions, with the difference being smaller at higher strain rates. Jones (1978), in tests carried out in a triaxial cell on granular T1 ice at -11°C found that at a confining pressure of 30 MPa and a nominal strain rate of 5.4×10^{-4} s^{-1}, the yield strength increased to 12 MPa from about 6 MPa for the unconfined loading case. More recently, Nadreau and Michel (1986a) report on triaxial tests on laboratory made freshwater ice and iceberg ice and on low-salinity granular ice.

For sea ice, there have only been a few multiaxial tests. Timco and Frederking (1983a, 1984, 1986a) report on confined compression tests on granular, discontinuous columnar and columnar ice over a range of loading rates, confinement conditions, and temperatures. Blanchet and Hamza (1983) measured the confined compressive strength with horizontal loading at one temperature and a very limited range of strain rates. Nawwar et al (1983) performed triaxial tests on laboratory-grown saline ice as a function of loading rate, confining pressure, temperature, and loading direction. Sinha (1986b and 1985a) reports respectively on the biaxial confined strength of oriented first-year and second-year columnar-grained sea ice in Mould Bay. He found that for oriented ice, the confined strength could be 10 times greater than the uniaxial strength. He found also that the strength and its rate sensitivity of second-year sea ice was comparable to the strength of laboratory-made freshwater ice tested under ideal conditions. Strength values up to 15 MPa were measured in these field tests.

In situ *field tests*: Measuring the strength of ice *in situ* offers a number of advantages including testing the ice in a relatively undisturbed condition, maintaining ambient temperatures, avoiding transportation problems and having a large scale sample. In addition, in situ testing usually requires a minimum of sample preparation. Several types of *in situ* measuring devices have been developed and tested in Canada. The Fenco borehole jack (Kivisild, 1975; Masterson, 1983) consists of two circular cylindrical surfaces which are jacked horizontally outward into the surrounding ice while the plate pressure is recorded against relative displacement. The device has been used extensively in many field evaluations of ice quality for assessing bearing capacity of ice covers, ice loads on structures, etc. Usually pressure increases monotonically with displacement in a borehole jack test. Assessing a strength index is, therefore, problematic if consideration is given only to the pressure–displacement curve. This explains, probably, why no data on borehole jack tests on different types of ice are available in the open literature. Recent tests by Sinha (1986b) on first-year and multi-year sea ice and Sinha et al (1986) on offshore built-up platform ice have verified that this device, with some modifications in the method of analysis, is very useful in characterizing ice strengths. It was pointed out that analyses must include the loading histories. Sinha has modified the basic borehole jack and a test program with this new version is in progress. Another borehole device is the pressuremeter (Ladanyi and Saint Pierre, 1978; Murat et al, 1986; Ladanyi and Huneault, 1987) which uniformly expands a cylindrical cavity, generating cylindrical stress and deformation fields which are amenable to analysis. More recently, Michel and Hodgson (1986) developed a relatively high-pressure borehole pressure meter to measure the crushing strength of ice. Preliminary results are encouraging. Flat jacks are another type of device used to carry out *in situ* loading in an ice cover (Vittoratos and Kry, 1979).

A test series was performed by Imperial Oil Ltd. (Esso) using the "nutcracker" technique to measure the crushing strength of Arctic ice. In these tests flat and cylindrical indentors up to 1.5 m across were pushed through the ice. Crushing strengths from 4 to 6 MPa were measured (Croasdale, 1974).

2.1.3. Shear strength

Shear is characterized by lateral movement within a material, ie, angular distortion or change in shape. It is considered a separate type of material response to load, although it is really only a special case of multiaxial loading. In many engineering problems there is a need to know the shear strength of ice and, because the complete multiaxial failure behavior is not yet known, shear tests are performed to obtain strength values.

Roggensack (1975) carried out direct shear tests on columnar grained S-2 ice with loading direction perpendicular to and shear plane surfaces parallel to growth direction. The loading rate was 0.1 mm/s and the test temperature -2.5°C. This set of experiments investigated the effect of normal stress, σ_n, on shear strength, τ_f, and found an expression of the form

$$\tau_f = 0.7 + 0.47\sigma_n, \tag{9}$$

where stresses are in MPa and the maximum normal stress in the range 0.5 to 1.4 MPa. Extrapolating eq. (9) to the case of zero normal stress, a shear strength of 0.7 MPa is obtained.

The asymmetric four-point bending method which has been developed for determining the shear strength of fiber reinforced polymers and ceramics has been applied to ice. The technique produces more consistent results than the single or double direct shear methods traditionally used. In tests on granular/discontinuous columnar sea ice of salinity 4‰ (parts per thousand) and at a temperature of -13 ± 2°C, the average shear strength obtained was 550 kPa with a standard deviation of 120 kPa (Frederking and Timco, 1984b). No effect of loading direction or shear plane orientation with respect to ice sheet growth direction was observed. A further test program was carried out on columnar grained sea ice at temperatures of -2 and $-12°C$ (Frederking and Timco, 1986). It yielded an average shear strength of 600 kPa at $-2°C$ independent of loading direction or shear plane orientation. At $-12°C$ shear strength was 700 kPa irrespective of whether the shear plane was parallel to or perpendicular to the ice growth direction and 900 kPa when the shear plane was orientated 45° to the growth direction.

2.1.4. Flexural strength

Information on the flexural strength of ice has application to such problems as measurements of icebreaker performance, determination of ice forces on inclined structures, ridge building and rubble building processes, and establishing safe bearing capacity of ice covers. Flexural testing has the disadvantage of creating a nonuniform complex stress field. It is also an indirect test since the interpretation of the results requires some knowledge of material behavior. On the other hand it has the advantage of simulating, at least in an analogous fashion, a representative loading condition in the ice cover. There are several basic types of flexural tests; ie, cantilever, three-point loading (also called simple beam), four-point loading, and plate loading.

The cantilever beam test has been examined as a method for determining *in situ* flexural properties of floating ice covers. Field tests and finite element analysis of cantilever beams in a 0.4 m thick columnar-grained freshwater ice cover indicated that the application of simple beam theory could result in significant errors in interpreting flexural strength and elastic modulus (Svec and Frederking, 1981) because of stress concentrations and rotations at the root of the beam. This lead to a photo-elastic and three-dimensional finite element analysis of stress concentrations at the root of a cantilever beam (Svec, Thompson, and Frederking, 1985). It was shown that circular holes at the root of the beam effectively reduced stress concentrations. A follow-up field program was carried out to evaluate these techniques for reducing stresses in a 0.35 m thick freshwater ice cover (Frederking and Svec, 1985). The conclusion of all this work was that for the usual cantilever beam tests with a saw-cut root and length 7–10 times thickness and width 1–2 times thickness flexural strength determined from simple beam theory can be corrected by applying a factor of 1.35 to eliminate stress concentration effects. This factor applies to freshwater ice. The elastic modulus determined from simple beam theory can be corrected by applying a factor of 1.2 to account for rotation at the root of the beam. Corrected values of flexural strength and elastic modulus obtained in a 0.3 thick freshwater ice cover were 700 and 6 GPa, respectively.

Frederking and Timco (1983b) determined the flexural strength and elastic modulus of cantilever beams. The tests were performed in a model basin on 40 to 70 mm thick fine grained, columnar freshwater ice covers. If simple beam theory was used, the effect of beam width and length in relation to ice thickness had a significant effect on flexural properties. An analytical model developed for beam deflections taking into account effects of buoyancy, shear, rotation and deflection at the root satisfactorily explained the observed behavior. The corrected elastic modulus determined was 6 GPa. The average flexural strength obtained was 800 kPa. In a concurrent test series on the same ice, simple beams removed from the ice cover and tested at $-10°C$ had a flexural strength of 2200 kPa (Timco and Frederking, 1982). Sinha (1982c) investigated acoustic emission and microcracking in the same ice under compressive loading and proposed a method of estimating the tensile strength by giving consideration to the observed time of formation of cracks and their sizes. He estimated the tensile strength of this ice at $-10°C$ to increase from about 900 to 2000 kPa for an increase in the strain rate from about 1×10^{-7} to 2×10^{-5} s^{-1} or a stress rate of 3×10^{-3} to 1×10^{-1} MPa s^{-1}. The effective modulus at the highest rate was found to be about 5 GPa.

Michel (1978b) developed an analytical method to take into account the effects of temperature gradients and development of a plastic moment in a beam. This approach showed the apparent flexural strength (resisting moment at failure) to be a function of strain rate, but actually controlled by the tensile strength of the extreme fiber of the beam which is relatively strain rate and temperature independent.

Studies on the loading rate and temperature dependence of the flexural strength of small beams of laboratory grown columnar sea ice were carried out by Lainey and Tinawi (1981) using four-point loading. Flexural strength increased from about 500 kPa at $-5°C$ to about 2000 kPa at $-40°C$. Stress rates covered the range 10^{-1} to 600 kPa s^{-1}. For temperature higher than $-20°C$, strength decreased monotonically by 20% with increasing stress rate in the range studied, but for lower temperature there was an intermediate peak strength between 50 and 100 kPa s^{-1}.

Timco and Frederking (1983b) carried out flexural tests of small beams of sea ice sampled from the Beaufort Sea using four-point loading. Salinity of the ice was 5‰. At $-20°C$ the flexural strength was 1160 kPa for granular (1 mm diam) ice and 860 kPa for columnar (3 mm diam) ice. Flexural strength of the columnar sea ice tested varied from 900 kPa at $-20°C$ to 500 kPa at $-40°C$, comparable to results obtained by Lainey and Tinawi (1981).

Acoustic emission studies carried out in a field station at $-10°C$ on multi-year columnar grained sea ice with salinities in the range of 0.5 to 1.5‰ led Sinha (1985b) to estimate the tensile strength. He noted the values to increase from about 500 to 1200 kPa for an increase in the stress rate from 3×10^{-4} to 1×10^{-1} MPa s^{-1}. These numbers agree well with those obtained from direct flexural tests discussed above.

2.1.5. Failure envelope

The failure envelope is a description of the stress levels at which a material yields for any combination of compressive and tensile stress states. For ice, the failure envelope has been determined by using the combined results of uniaxial compression tests, confined compression tests and shear strength tests both for mostly granular sea ice (Timco and Frederking, 1983a, 1984), and columnar sea ice (Nadreau and Michel, 1984; Timco and Frederking, 1986a) over a range of loading rates and temperatures. Due to the hexagonal structure of the ice lattice and the horizontal alignment of the *c*-axis in the columnar ice, there is a marked difference in the failure envelope between the two ice structures (see Fig. 2). This figure compares the failure envelope for sea ice with that of freshwater ice for both granular and columnar sea ice for plane stress conditions. It is evident that the void volume (brine and air) in sea ice significantly reduces the size of the failure envelope compared to that for freshwater ice at the same loading rate and temperature. For columnar ice, the failure envelope extends much further into the compression–compression quadrant than that for granular ice. This implies that columnar structured ice in a confined state (such as that in an ice cover) will sustain much higher stresses before failure than granular structured ice. This has clear implications when viewed in terms of the forces which an ice sheet can exert on a structure. A mathematical description of the three-dimensional failure envelope based on a modified *n*-type yield function has been given for both granular and columnar

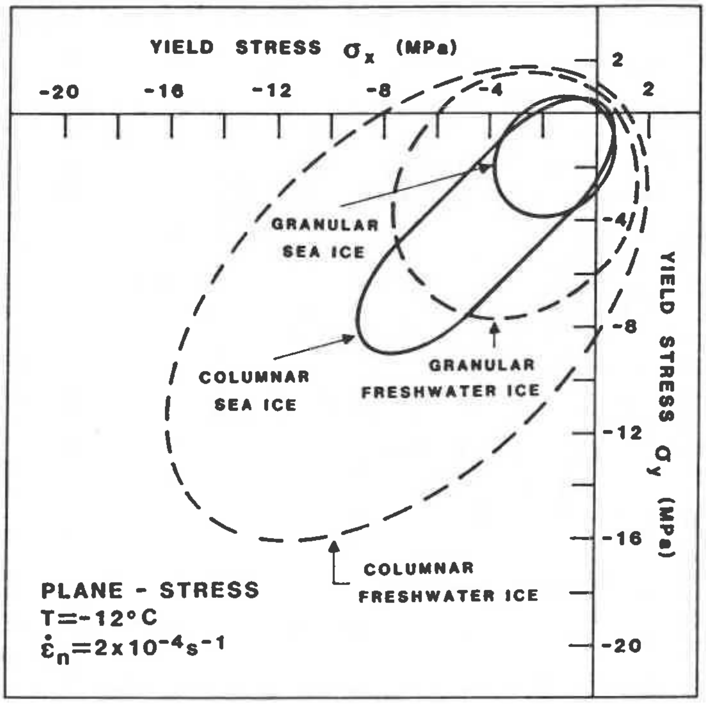

FIG. 2. Comparison of the failure envelopes for both granular and columnar sea ice and freshwater ice in the horizontal plane (after Timco and Frederking, 1984).

structured ice (Timco and Frederking, 1984, 1986a), allowing its use in analytical models. The importance of using the correct yield function in these types of analyses has been demonstrated by McKenna et al (1983).

Using triaxial test results of Jones (1982) and Nadreau and Michel (1985) on granular freshwater ice, Nadreau and Michel (1986b) proposed a new formulation for describing the failure envelope. The proposed yield surface models the pressure melting, sensitivity of the ice at high confining pressures. It is a cubic function of the invariants of the stress tensors. The three-dimensional surface defined by this criterion is teardrop-shaped and symmetrical around the hydrostatic line. The volume of this teardrop envelope is rate sensitive and increases with the increase in strain-rate.

2.1.6. Fracture toughness

The fracture toughness (or critical stress intensity factor (K_{1c})) has been measured by a few investigators for both freshwater ice and sea ice. The understanding of this property as a material property of ice is, however, still in its infancy.

For freshwater ice, measurements of K_{1c} have been performed by Timco and Frederking (1982) in a test series comparing several mechanical properties measured under comparable conditions. Hamza and Muggeridge (1983) measured the nonlinear fracture toughness of small ice beams while investigating the effects of loading rate, temperature, and specimen size. A field test series measuring the fracture toughness of large beam specimens was performed by Parsons and Snellen (1985). They measured the highest K_{1c} values reported to date with mean values of K_{1c} approximately 200–250 kPa m$^{1/2}$ depending upon the loading direction. Finally, Timco and Frederking (1986b) investigated the effects of both anisotropy and microcracks on the fracture toughness of fine-grained columnar ice. They found that the presence of microcracks causes a decrease in the apparent fracture toughness of the ice. Based on these tests, Cormeau et al (1986) performed a finite element (FE) analysis of this process. They found that depending upon the position of a microcrack relative to the main crack, there can be an amplifying or shielding effect on the stress intensity factor. Overall, the FE analysis showed excellent agreement with the test results, and provided valuable insight into the mechanics of fracturing in ice.

The fracture toughness of sea ice has been measured by Timco and Frederking (1983b) on small beam samples and by Parsons et al (1986) on large beams. The latter study investigated the effects of anisotropy and temperature and showed that K_{1c} increases with decreasing temperature with mean values of the order of 500 kPa m$^{1/2}$ at -25°C and 150 kPa m$^{1/2}$ at -10°C.

2.2. Rheology: Introduction

Rheology in ice is part of a broad subject that involves the stress–time–temperature dependent deformation and fracture of polycrystalline materials at elevated temperatures. Polycrystalline ice in nature is generally an anisotropic, nonlinear viscoelastic solid and its mechanical behavior is described by the generalized Hooke's Law relating strains ε_i to stress σ_j:

$$\varepsilon_i = S_{ij}\sigma_j;\ i, j = 1,2,3,4,5,6, \tag{10}$$

where S_{ij} are the compliances. Granular snow ice, for all practical purposes, may be considered as an isotropic material and its deformation properties may be described by $S_{11} = S_{22} = S_{ii}$ and $S_{12} = S_{21} = S_{ij}$ $(i \neq j)$. For commonly observed transversely isotropic (or orthotropic) columnar grained lake, river or sea ice, classified as S-2 ice, we are concerned usually with the compliances $S_{11} = S_{22} \neq S_{33}$, in which the Cartesian axes are chosen in order to have the 3-axis along the axis of the columnar grains, ie, the vertical (growth) direction, and the 1-axis and 2-axis in the plane of the ice cover perpendicular to the growth direction, hence in the horizontal plane. For stresses applied along 1-direction, which is the case in many engineering situations, the important lateral compliances are S_{21} and S_{31}.

A minimum of three macroscopically observed strain components describe the deformation of any material irrespective of operational conditions. Regardless of loading conditions, the deformation, ε_i, of any polycrystalline material (including ice), at high homologous temperatures can be described phenomenologically as (Sinha, 1978b)

$$\varepsilon_i = S_{ij}\sigma_j = \varepsilon_{ie} + \varepsilon_{id} + \varepsilon_{iv}, \tag{11}$$

where ε_{ie} is pure elastic and immediately reversible, ε_{id} is delayed elastic and recovers with time, and ε_{iv} is the viscous or permanent strain. Delayed elasticity is particularly noticeable in ice because it is always, in nature, at high temperatures.

The simplest method for bringing out the macroscopic behavior is the uniaxial tensile or compressive test. Two commonly applied experiments to determine material properties for engineering applications are the uniaxial "constant stress" creep and the "constant strain rate" deformation tests. Ideally a certain stress is suddenly imposed and held constant in the creep tests; in the other tests, a certain strain rate is suddenly imposed and held constant.

2.2.1. *Elasticity*

Unless low amplitude and very high frequency loading, in the order of MHz, is involved, it is almost impossible to have only elastic deformation with no contributions due to grain-boundary sliding or delayed elastic and viscous deformation. This observation is applicable to deformation parallel as well as normal to the axis of loading. Consequently, the compliances S_{ij} given by the observed ratio, ε_i/σ_j or the corresponding moduli and the ratio between lateral strain and longitudinal strain reflects the contribution due to nonpure elastic deformation. It is preferable, therefore, to use the term "effective modulus" for the reciprocal of compliance or σ_j/ε_i except where Young's modulus and other elastic moduli, in the formal sense, are intended.

The dependence of the effective moduli, for freshwater ice, on the rate of loading and temperature and grain size, for loads less than 0.5 MN m^{-2} in uniaxial loading, has been studied experimentally by Gold and Traetteberg (1975) and Traettenberg et al (1975). These along with many previous results, were compiled by Gold (1977). Figure 3, taken from Sinha (1979a), shows that the effective modulus increases with increase in rate of loading and grain size. Similar rate sensitivity was noted in saline water ice by Murat (1980) and Murat and Lainey (1982). The illustration also shows that the experimental observations, however complex, are reasonably predicted by a simple micromechanically based rheological model to be discussed later. The applicability of this model in the case of saline ice can be seen in Sinha (1981d), which also explains why sea ice appears to be more ductile than freshwater ice.

Shear modulus has also received some attention. While examining the bearing capacity of saline ice covers, Lainey (1982), and Lainey and Tinawi (1983) realized and emphasized the importance of shear deformation. Selvadurai (1981) modelled an ice cover, containing a primary snow ice layer and secondary columnar zone, as a composite structure assuming the secondary ice as a Pasternak–Vlazov-type elastic layer which possesses only shear interaction. Murat and Degrange (1986) examined the relative influence of shear and flexural deformation on the deflection of laboratory made transversely isotropic, saline ice beams. They concluded that the shear deflections could be 2 to 3 times greater than those expected for an isotropic material.

2.2.2. *Creep*

The term "creep" is defined as "the slow deformation of a material" according to BS 5168 or British Standard Glossary of Rheological terms (1975). This definition is broad enough to be universally acceptable. One might object to the term "slow"—however, slow or fast are only relative terms. It has been pointed out by Sinha (1979b) that a very short creep time of 10 s at a homologous temperature of 0.96 T_m is equivalent to a creep time of 12 days at 0.7 T_m, 36 years at 0.6 T_m and about half a million years at 0.5 T_m. It should be pointed out here, to give a feeling of the actual thermal state, a homologous temperature of 0.96 T_m in ice is a temperature of -10°C and a temperature of 0.5 T_m in austenitic stainless steel could be as high as 700°C.

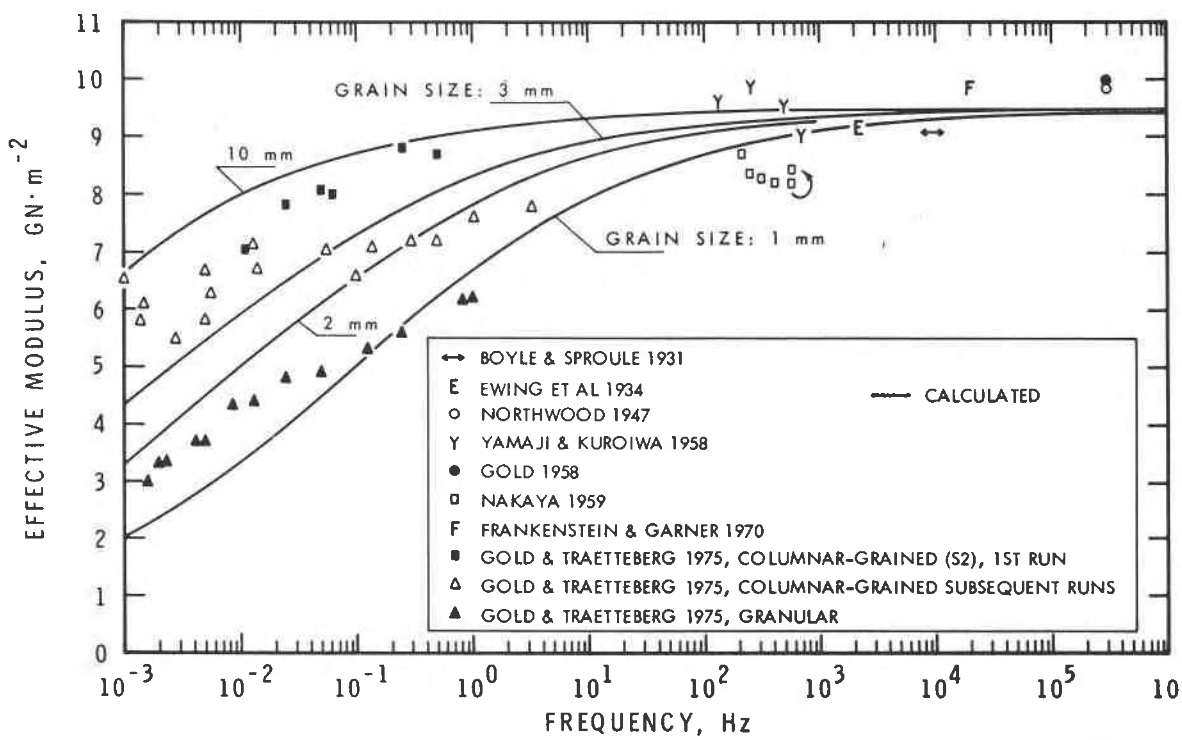

FIG. 3. Frequency dependence of effective modulus for polycrystalline ice at -10°C. Experimental data are from Gold (1977). Calculated results are shown by the solid lines for stress amplitude of 0.3 MN m^{-2} (after Sinha, 1979a).

Creep is usually measured under constant stress or rather constant load. Three distinct types of constant load tests have been conducted in Canada in the last decade reviewed here. Among these, two belong to the laboratories and the third belongs to the field. Laboratory tests usually involve uniaxial deformation of prismatic or cylindrical specimens (Sinha, 1978b; Nadreau and Michel, 1985; Gold, 1983) or bending of beams or plates (Murat, 1978; Tinawi and Murat, 1979; Lainey, 1982; Lainey and Tinawi, 1981; Nadreau and Michel, 1981, 1986a). Field tests involve deflections of ice covers under slowly moving or static loads (Eyre and Hesterman, 1976; Eyre, 1977; Frederking and Gold, 1976; Beltaos, 1978, 1981).

Bending of beams or plates is relatively simple to perform in the laboratory and in the field. However simple these experiments may be, results are difficult to analyze (Williams, 1976; Murat, 1978; Liu and Hsu, 1982; Vinogradov, 1984) because stress levels vary with depth and the viscoelastic response of ice is not only nonlinearly stress dependent but also depends on temperature and the history of loading. Consequently, the significance of beam or plate bending tests lies in the fact that they give strength indices or bearing capacity indices for applications to real life engineering problems.

Efforts have also been made to estimate *in situ* creep properties using borehole pressuremeters or jacks (Ladanyi et al, 1978; Sinha, 1986b; Sinha et al, 1986; Michel and Hodgson, 1986; Murat et al, 1986). Again the tests are simple but results are difficult to use to develop constitutive equations for creep.

Phenomenological observations, expressed in a general sense in eq. (11), describe ice as a viscoelastic material. Experimental observations discussed in Section 2.2.1 show that ice does not behave as an ideal elastic material even at relatively high loading rates. Its behavior, on the other hand, cannot be described by the classical elastic-plastic model, because Gold and Traetteberg (1975) noted that for a significant time following the initiation of loading the strain imposed on ice is essentially recoverable and grain size dependent. Many problems involve only this initial creep—a range referred by Sinha (1979a) as "elasto-delayed elastic." A very significant number of applied mechanics problems for ice involve elastic and primary deformation or the transient creep behavior. They include most ice-structure interaction problems, all bearing capacity problems for moving loads and static bearing capacity problems for which the maximum deflection is limited to the free board (Gold, 1977). The level of strains involved in these problems is indeed very small (Gold and Sinha, 1980). Measurements carried out by Allan (1979) and Masterson et al (1979) show that maximum surface strain actually measured in a floating platform toward the end of a drilling season of more than 3 months was only about 1.5×10^{-3}. This is due to a combined effect of creep and consolidation, and the average strain rate of about 10^{-10} s^{-1} indicates little cracking activity. Failure strains involving severe cracking activity, for uniaxial upper-yield type compressive failures have also been measured to be small—about 2×10^{-3} for both freshwater ice (Sinha, 1981b, 1982a) and sea ice (Sinha, 1983b; 1984c, 1986a; Frederking and Timco, 1984). In case of problems involving tensile fracture, such as in bending, the fracture strains are less than 5×10^{-4} and 95% of these are recoverable (Sinha, 1984d). Compressive brittle type failures at high rates also involve strains of these magnitudes (Sinha, 1981b). Upper yield failure strains under confined compression loading conditions are relatively large but still only in the range of about 1×10^{-2} (Sinha, 1985a). To be applicable to most ice engineering problems, a constitutive model must describe the rheological behavior at small strain. Constitutive equations developed in Canada in the last decade seem to focus strongly in this direction (Sinha, 1978b,c, 1979b; Michel, 1978a,b, 1981; Gopal et al, 1984; Szyszkowski et al, 1985; Vinogradov and Bakalchuk, 1986).

Uniaxial, constant-load creep and recovery on unloading, of polycrystalline ice was used by Sinha (1978b) to show that ice can be treated as a nonlinear thermorheologically simple material. He also demonstrated a method of examining experimentally the three strain components in eq. (11) and in developing a phenomenological equation capable of explaining inconsistencies in the results of earlier creep investigations of ice (Sinha, 1978c). It was shown that pure elastic deformation was related to lattice deformation and the viscous component could be attributed to intragranular deformation processes, particularly to the movement of dislocations for the level of stresses and rates of loading encountered in engineering situations. Delayed elasticity was hypothesized to be associated with intergranular sliding phenomena. These physical processes were considered in modifying the phenomenological creep model of Sinha (1978b) to what might be called a micromechanically based model, allowing the incorporation of the effect of grain size (Sinha, 1979b). Key assumptions in developing this model are (a) the primary creep depends on grain size and (b) the viscous creep rate is independent of grain size for conditions where grain boundary diffusional creep does not play the dominating role and where the microstructure has not deteriorated by internal cracks or voids. Data published in the literature since then from France and Australia (Duval and LeGac, 1980; Jacka, 1984) support these assumptions. The axial strain, ε_1, at time t, in pure randomly oriented polycrystalline material of grain size, d, subjected to a uniaxial stress, σ_1, at a temperature, T, was given by

$$\varepsilon_1 = (\sigma_1/E) + c_1(d_1/d)(\sigma_1/E)^s\left[1 - \exp\{-(a_T t)^b\}\right] + \dot{\varepsilon}_{v_0} t(\sigma_1/\sigma_0)^n, \qquad (12)$$

where E is Young's modulus; $\dot{\varepsilon}_{v_0}$ is the viscous strain rate for unit or reference stress σ_0; c_1 is a constant, corresponding to the unit or reference grain size, d_1, that depends on grain boundary structure and texture; b, n and s are constants; a_T is the inverse relaxation time. Both $\dot{\varepsilon}_{v_0}$ and a_T vary with temperature T in Kelvin and were shown to have the same value for the activation energy as follows,

$$\dot{\varepsilon}_{v_0}(T_2) = \dot{\varepsilon}_{v_0}(T_1)F_{1,2}$$

and

$$a_T(T_2) = a_T(T_1)F_{1,2}, \qquad (13)$$

where T_1 and T_2 are two temperatures and $F_{1,2}$ is a shift function given by $F_{1,2} = \exp\{(Q/R)[(1/T_1) - (1 - T_2)]\}$ in which Q and R are the activation energy and gas constants, respectively. Note that both delayed elasticity and viscous flow have the same activation energy. Recovery, on unloading, is described as the mirror image of the delayed elastic term.

Equation (12) was developed not just for ice but for also metals, alloys and ceramics. Experiments on ice indicated that $s = 1$ and $b = 1/n$ (Sinha, 1978b). Consequently, the creep equation in ice is relatively simple and is described in terms of only five material constants, E, c_1, a_T, $\dot{\varepsilon}_{v_0}$, and n. Since E can be estimated fairly well from available single crystal elastic moduli (Parameswaran, 1982, 1987), the unknown material constants reduce to only four. With the assumption of $n = 3$, because of intragranular dislocation mechanisms, the unknown material constants reduce to only three. For the system in which grain size, d, is expressed in meters (ie, $d_1 = 1$ m), stress, σ_1, in MPa (ie, $\sigma_0 = 1$ MPa), and time, t, in seconds, experimental observations in pure ice (Sinha, 1979b) provided the following values: $E = 9.5$ GN m^{-2}, $c_1 = 9 \times 10^{-3}$, a_T ($T = 263$ K) $= 2.5 \times 10^{-4}$ s^{-1}, $\dot{\varepsilon}_{v_0}$ ($T = 263$ K) $= 1.8 \times 10^{-7}$ s^{-1}, and $n = 3$. The activation energy, Q, was found to be 67 kJ mol^{-1} (16 kcal mol^{-1}) taking $R = 8.32$ J mol^{-1} K^{-1}. It should be pointed out that the E value chosen above agrees well with the calculated by Parameswaran (1987) from single crystal elastic constants determined by Gammon et al (1980, 1983a, b) using the method of Brillouin spectroscopy. Direct application of eq. (12) with the above values of material constants, to predict effective modulus (Fig. 3) and to a wide range of independent experimental observations including conditions for superplasticity, creep rupture, cracking activities, and damage accumulation in constant stress creep can be seen in Sinha (1978c, 1979a, b, 1982b, 1984a, b). A simple method for prediction of strain, and hence a stress–strain diagram, corresponding to a monotonically increasing stress history, such as encountered in constant cross-head rate tests and constant stress rate loading, were then developed and applied successfully to both compressive and tensile experimental data (Sinha, 1981c, 1983d, 1984d) on the basis of eq. (12) and the material constants given above. This model has now been used also to predict the rate sensitivity of Poisson's ratio (Sinha, 1987), tertiary creep, and anisotropic deformation of common ice types.

Basal slip is comparatively easier than slip on other planes in a single crystal of ice with hexagonal structure. This is the essence of the model proposed by Michel (1978a). He assumes that grains best oriented for basal slip deform first, on application of a load, and grain boundary sliding occurs to accommodate the total deformation. An equation for this model which, for uniaxial constant stress, σ, and temperature, T, is given by

$$\dot{\varepsilon} = A\left[1 + \alpha(\varepsilon/\varepsilon_t)^m\right]\left\{1 + \beta(1 - \varepsilon/\varepsilon_r)^n\right\} \times (\sigma/\sigma_0)^n \exp(-Q/RT), \tag{14}$$

in which n, A, α, m, β, ε_r, and ε_t are material constants; σ_0 is the unit stress, and Q and R are activation energy and gas constant, respectively. Note that the above equation combines both delayed elastic and viscous flow. Consequently calculating the recovery curve, on unloading, is to be performed by another equation.

It has been demonstrated by Michel (1978a, 1981) that the model can be used accurately fit experimentally observed creep and recovery curves. The model has also been shown to be capable of fitting the stress–strain relation obtained under constant cross-head rate loading. Fitting of experimental curves is done by empirically adjusting the numerical values of the constants. This is a complex procedure which may require different values of the constants from one test to another for similar ice.

An empirical nonlinear spring/dashpot model that requires six parameters was proposed by Szyszkowski et al (1985) to describe the primary and secondary stage of creep behavior. An improved phenomenological model capable of describing tertiary creep was then proposed by Szyszkowski and Glockner (1985). This later model, which requires eight parameters, is promising but rather lengthy, cumbersome, and difficult to use. Numerical techniques, in conjunction with trial and error, are required to solve even the simplest case of a constant stress unconfined creep test. Although excellent experimental data from Canadian laboratories was available to these authors, no effort has been made so far to examine the predictive capabilities of their model.

2.2.3. *Poisson's ratio*

The ratio of the lateral strain to the longitudinal strain in a homogeneous material for a uniaxial elastic loading condition is defined as Poisson's ratio and is often denoted by μ. It is a manifestation of the factors that play a central role in three-dimensional constitutive formulations. Poisson's ratio is an important property which has received surprisingly little attention.

The complexities in the interdependence between the axial and the lateral strain can be seen in the experimental work of Murat and Lainey (1982) on laboratory made saline ice beams for loading conditions involving no cracking activity. While examining the rate sensitivity of compressive strength of congealed frazil sea ice, Sinha (1986a) observed that the ratio depends not only on the rate of loading or loading history but also on stress and strain level and the damage state of the material. He coined the term "strain ratio" and suggested that the use of Poisson's ratio should, in a formal sense, be restricted to loading conditions in which only elastic responses related to lattice deformations are involved.

The contribution of the grain-boundary or interplatelet sliding strain to the total strain, in addition to pure elastic and viscous flow, influences both effective modulus and strain ratio. These mechanisms were proposed briefly by Sinha (1981e) in explaining the experimental work on strain ratio in sea ice by Wang (1981). This model has now been formalized for estimating lateral strain and simple equations have been developed to predict the complex response of strain ratio in ice. Application of the theory to granular ice is presented in Sinha (1987). Experimental and theoretical work carried out at NRCC in anisotropic ice indicates that the strain ratio could be significantly higher than 0.5 even for conditions with no cracking activities.

2.3. Interface properties

This class of property which relates to the behavior at the interface between ice and a substrate material involves mechanical and thermal processes. The properties are commonly known as adhesion, friction, icing, etc. Friction is not included because there has been no recent work on the topic in Canada.

2.3.1. *Adhesive strength*

A floating ice cover can develop substantial vertical loads on a structure to which it is frozen as a result of water level changes. Because of this, a knowledge of the adhesive strength of ice to various materials is important. A number of tests have been performed in the laboratory to measure adhesive strength. Frederking (1979) determined the effects of deflection rate, ice thickness, and pile diameter on vertically acting ice loads on wooden piles. Parameswaran (1981) measured the adfreeze bond strengths of wood, concrete and steel H-section piles embedded in freshwater ice under constant displacement rates for rates between 10^{-4} and 10^{-1} mm/min. He measured adhesive strengths of 0.6–1.8 MPa for wood, 0.8 MPa for concrete and 0.2–0.6 MPa for steel. Frederking and Karri (1981, 1983) carried out laboratory tests on piles of six different materials: polyethylene, polyvinylchloride, steel, wood, concrete, and steel coated with Inerta 160 marine coating. Typical adhesive strengths were ≈ 0.05–0.07 MPa for the PE and PVC piles, 0.25 MPa for the Inerta coated piles, and 0.4–0.5 MPa for the others. Most recently, Cammaert and others (1986) have reviewed the published information on adhesive strength and have developed an analytical approach for calculating adfreeze loads on conical structures.

2.3.2. *Icing*

Spray icing can cause considerable loads and inconvenience to helicopters, icebreakers and offshore platforms. For many years, Canada has been involved in investigating the growth, properties, detection and methods of alleviation of spray ice. Three groups in particular have been active: the Low Temperature Laboratory of the National Research Council of Canada, the Université du Québec à Chicoutimi, and the University of

Alberta in Edmonton. These groups have produced numerous contributions and the reader is referred to the *Proceedings of the international workshop on atmospheric icing on structures* (CRREL Report 83-17, 1983) for representative examples. More recently, Lozowski and Gates (1985), and Lozowski and others (1986) present overviews of marine icing research, especially as applied to mobile offshore drilling units.

3. ICE FORCES ON STRUCTURES

A considerable amount of work has been done in Canada to measure and predict the forces which ice features can exert on a structure. There have been several approaches to this problem including analytical predictions (Section 3.1), physical modelling (Section 3.2), laboratory studies (Section 3.3), and field studies (Section 3.4). A great deal of exchange of information, insights, and ideas amongst these approaches has gone on within the ice community.

3.1. Analytical predictions

Work in Canada on analytical predictions of ice forces on structures has benefited from and been guided by access to extensive field experience (Gold, 1978). This has provided the opportunity of calibrating prediction methods with actual field data and observations. Both deterministic and probabilistic approaches have been used and they have been integrated together in certain cases. The subject of analytical predictions has addressed both narrow and wide structures and rigid and flexible structures. A general review of mathematical modelling was made by Kivisild and Iyer (1978), examining the suitability of various methods (finite element or finite difference) for various ice–structure interaction problems. A detailed study on the physics of ship/ice interaction processes and energies, based primarily on recent measurements from ramming by icebreakers (Ghoneim and Keinonen, 1983; Ghoneim et al, 1983; Daley, 1984; Daley et al, 1984) has been presented by Kivisild et al (1987).

Ice loading on conical structures was examined analytically by Danys and Bercha (1976). The ice was treated as an elastic brittle material and loads were determined as a function of ice properties, structure geometry and ice–structure friction. The work showed the importance of ice–structure friction in determining ice forces and failure mode.

Kry (1980) effectively summarized the state of the art of predicting global ice forces on wide structures. Four failure modes (flexure, rubble formation, buckling, and crushing) were identified and the likelihood of occurrence related to ice thickness—going progressively from flexure for thin ice to crushing for thick ice. Analytical prediction equations were developed for each failure mode. Of particular interest was the proposal of multiple zones of failure in the case of the crushing mode, which provided a means of introducing the statistical influence of nonsimultaneous failures across a wide structure. This approach leads to the so-called "limit-stress" global ice forces. Closed-form solutions for ice loads on several categories of rigid structures (narrow, wide, vertical, and sloped) were also compiled by Croasdale (1980). Sodhi and Hamza (1977) developed expressions for loads associated with ice sheets failing in buckling against a structure. For wide structures, aspect ratio and size effect are important factors. Iyer (1983) has examined these factors, distinguished between them and presented expressions for each. Blanchet (1986) has developed a method for relating local failure pressures to ice property variations with depth.

In examining the approaches to calculating global ice forces, Croasdale (1984) has identified three limits to global ice forces: "limit-stress," "limit-momentum," and "limit-force." The identification of these conditions has helped provide a rational framework within which various loading scenarios could be examined. The maximum force is given by the limit-stress case; ie, the ice cover or feature fails against the full width of the structure. It is possible, however, that the environment cannot generate sufficient load to reach the limit-stress condition. In this case either the limit-momentum or limit-force would apply. In the limit-momentum approach the kinetic energy of the ice feature is absorbed as the ice fails against the structure until such time it is brought to rest. Depending on the size or velocity of the floe the final contact area could be significantly less than the area assumed in the limit-stress case. In the limit-force approach the environmental forces which can be generated on a large ice feature lodged against a structure are examined. These include wind and current drag and ridge building forces. Work is still required on establishing the level of ridge building forces.

Various scenarios of ice loading have been incorporated into computer models for predicting ice loads. These have been developed for use in the design process where structure characteristics can be changed (Bruce and Allyn, 1983) and for making probabilistic predictions of ice loads (Marcellus and Croasdale, 1984).

The problem with many prediction methods is either the absence of data on ice mechanical properties to insert in the model and/or ice force data to calibrate the method. One case where some of these shortcomings have been addressed is an analysis of failure modes and damage processes for indentation tests in freshwater ice (Tomin et al, 1986). The three modes of global fracture (radial, spalling and flexural cracking) were analyzed using the finite element and the J-integral methods to determine the crack extension force for each mode. Local fracture, which leads to pulverization of the ice, was examined with Sinha's criterion for internal crack initiation and a continuous damage model for the following progressive microcracking. The combined approaches provide insight into the mechanics of ice failure in the indentation process.

The finite element method potentially provides a more precise method for calculating ice loads given the complex boundary conditions which apply for various ice loading scenarios. Cormeau et al (1984) have used the method in an energy-based approach to study ice–structure interaction. Energy sinks such as fracture, creep, and fragment rotations were identified as being significant in ice load determination. Recently, Jordaan (1986) has reviewed the use of FE techniques for predicting ice loads on structures.

Probabilistic methods have also been used in predicting ice loads. These include statistical characterization of ice kinematic and mechanical data, probabilistic analysis of ice–structure interaction, and simulations of interaction scenarios. Bercha (1984) prepared a review of the above statistical methods and their application to ice loading. Blanchet and Metge (1984) used a probabilistic approach with classical statistical data on ice strength, thickness, and floe size to arrive at global load predictions for the case of multi-year ice floe impacts ("limit momentum") in summer.

The case of iceberg impact on a gravity structure has been treated by Cammaert et al (1983). The analysis pointed up the importance of structure shape and mechanical properties in predicting ice loads. Bergybit impacts on semi-submersible structures were modeled numerically using conventional structural dynamics and showed local denting of columns to be likely (Swamidas et al, 1983).

In the dynamic interaction between structures and ice the flexibility of the structure has a significant effect on the ice load generated on the structure. This has been examined in the case of offshore structures (Reddy et al, 1979) and bridge piers (Montgomery et al, 1980).

3.2. Ice forces on structures: Physical modelling

Physical modelling is a technique which has been used to a large extent to study the problem of ice interacting with structures. In these model tests, the forces involved in the interaction process are reduced but maintained in the same ratio as those in the full scale. Special techniques and facilities are required for accurate results.

In Canada, there are five laboratories which have the necessary equipment for performing model tests. These include the Esso test basin in Calgary (Robbins et al, 1975), the Arctec test basins in Calgary and Kanata, the NRC tank in Ottawa (Pratte and Timco, 1981), and the new NRC facility in St. John's (Jones, 1986). The latter facility, which opened in late 1985, is the largest in the world comprising a refrigerated ice tank $80 \times 12 \times 3$ m. Great strides have been made in the development and understanding of the "model ice" which is an integral part of the modelling process. The model ice must be an accurate representation of the full scale sea ice in all of its mechanical properties. Michel (1969) developed a synthetic model ice which is proprietary in nature and used by a private consulting company in both Canada and the United States. Timco (1979) developed a refrigerated model ice grown from an aqueous solution containing carbamide (or urea) as a chemical dopant. This type of model ice is now used in the majority of ice modelling facilities in the world. More recently, Timco (1986a) developed a new type of model ice—termed EG/AD/S ice—which is grown from an aqueous solution containing ethylene glycol, aliphatic detergent, and sugar. Analysis of the structure of this ice indicates that it is single-layered, fine-grained, and strictly columnar. This ice has excellent scaling of the mechanical properties of sea ice, including the flexural strength, uniaxial and confined compressive strength, strain modulus, and critical stress intensity factor.

To date, there have been a large number of model test studies measuring the ice forces on several Arctic structures (Hnatiuk and Felzein, 1986; Pilkington et al, 1986). At this time, however, the results of many of these are proprietary. Studies have elucidated the forces on more basic shaped structures and these have been used to verify analytical models of the interaction process. These include studies on conical and sloping structures (Abdelnour, 1981; Frederking and Schwarz, 1982; Timco, 1984a; Frederking and Timco, 1985), on vertical-sided piles (Frederking et al, 1982), arrays of piles (Timco and Pratte, 1985), and on ice-breakers (Nawwar et al, 1984). A recent review article has been written by Timco (1984b).

3.3. Ice forces on structures: Laboratory studies

A number of tests have been performed in the laboratory to study ice–structure interactions at a small scale (as compared to the field situation). In these tests, freshwater ice is used. Because the properties of this type of ice have been studied to a large extent, this allows an analysis of a tractable experimental situation, which can provide good insight into the physics of the interaction process.

To date, most studies of this type have looked at the horizontal loads which an ice sheet can exert on a structure. Tests of this type have been performed over a wide range of loading rates covering the range from ductile indentation (Frederking and Gold, 1975; Croasdale et al, 1977; Michel and Toussaint, 1977) to the transition (Michel and Jolicoeur, 1986) and brittle regions (Michel and Blanchet, 1983; Timco, 1986b). These studies have been concerned with many aspects of the interaction problems including loading rate effects, aspect ratio effects, peak and average pressures, failure modes in the ice, etc. Most of the test results have been interpreted in terms of the Kořzhavin equation $p = C_i mk\sigma$ which gives the pressure (p) on the structure in terms of the strength of the ice (σ), and empirical coefficients for indentation (C_i), structure shape (m), and degree of contact (k). A compilation of the test results which gives the variation of the indentation coefficient for various aspect ratios (structure width to ice thickness) for various loading rate regimes can be found in Michel and Jolicoeur (1986).

3.4. Field studies

Canada has benefited from having many structures placed in an offshore environment exposed to ice and the opportunity of monitoring ice behavior and forces around these structures. This has been a key element in the rapid advances in ice mechanics and in improving confidence in the design of offshore structures for ice covered areas. The results of field measurements are used to verify design loads and also as an integral part of operational alert procedures. There are two basic approaches to estimating the total ice force on a structure; either from *in situ* pressure measurements in the ice cover around the structure or else measuring pressures on or response of the structure. An extensive review of techniques was made by Croasdale and Frederking (1986).

For determination of forces on the artificial islands constructed in the land fast ice region of the Beaufort Sea, the approach taken was to place transducers in the level ice cover around the island to measure *in situ* ice pressures. Large thin panel type transducers were initially developed (Metge et al, 1975) and successfully used at a number of locations (Metge, 1976; Strilchuk, 1977; Semeniuk, 1977). They were relatively temperature sensitive and so were subsequently replaced by MEDOF panels which were used around the Tarsiut Island location (Pilkington et al, 1983). More recently Arctec Canada has built a wide thin metal sensor called the Hexpac (Graham et al, 1983) and Weir–Jones a similar transducer called the Ideal panel (Witney et al, 1986). Also a hydraulic flat-jack type transducer (Masterson, personal communication) and a stiff biaxial transducer (CMEL 1984) have been developed.

In a recent experiment ten different sensors used for determining ice pressures were tested in a large outdoor ice basin under controlled load conditions (Croasdale et al, 1986b). The results of the experiment showed that all the sensors predicted stresses within $\pm 30\%$ of the applied ones. A good level of confidence has been established in the ability to accurately measure *in situ* ice stresses.

A number of projects have been carried out to measure ice pressures around caisson structures, and natural islands. The results tabulated in Croasdale and Frederking (1986) show that global loads can be satisfactorily estimated from measurements made *in situ* in the ice cover.

Another approach to measuring ice forces during ice–structure interaction is to deduce the ice forces from floe motions during an impact. Such an approach was used at Hans Island in 1980, 1981, and 1983. At Hans Island the decelerations of multi-year floes impacting the island were measured. By also estimating the mass of the floes, the ice forces acting between floe and island were deduced. These experiments have been described by Metge et al (1981) and Danielewicz et al (1983).

There are three means of measuring ice loads from structure response: (i) measure strains, deformations or movements of the structure itself, (ii) addition of load sensing panels to the structure, and (iii) in cases of dynamic loads, measure accelerations of the structure, which, when combined with a knowledge of the dynamic response of the structure, allows loads to be determined. All three approaches, but particularly the first two, generate information relevant to both local and global ice loads.

Canadian measurements of direct structure response to ice loads are summarized in Croasdale and Frederking (1986). Over the years, there has been considerable refinement in the instrumentation. Measurements started with work on bridge piers in the 1960s and lightpiers in the 1970s (Danys, 1975). Extensive instrumentation to measure ice forces was built into the caisson structures deployed into the Beaufort Sea into the 1980s; eg, Tarsiut, Dome's SSDC, Esso's CRI, and Gulf's Molikpaq. The transducers used ranged from large panels 4×4 m down to 50 mm diam pressure sensors. Panel type transducers with sensing areas of the order of 1 m^2 have proved most successful. The information obtained from these transducers has been used to develop a better understanding of the nature of ice loads and as a primary real-time indicator of ice load levels. This knowledge of loads when combined with interpretation methods is used to establish alert levels which govern the drilling operation (Wright and Weaver, 1983).

4. ICE AS A CONSTRUCTION MATERIAL

4.1. Ice roads and airstrips

Many areas in Canada depend heavily on winter roads and ice crossings across rivers, lakes, and marshy areas. Experimental and theoretical studies on the bearing capacity of freshwater ice covers have been covered well by Eyre and Hesterman (1976), Eyre (1977), Frederking and Gold (1976), Beltaos (1978, 1981). A workshop was held in 1980 in Ottawa to bring together a small group of people with a common interest and involvement in these areas and to exchange ideas and information. The proceedings of this meeting (ACGR-1980) provides valuable information on the practical aspects of constructing and maintaining these important communication links. The papers by Betteridge and Clift (1980) and Haspel and Masterson (1980) provide important data on the construction of floating ice roads in support of offshore drilling activities. Betteridge and Clift presented a detailed study on the successful completion of a 200 km river ice road from Tuktoyaktuk to the middle delta of the Mackenzie River. This road was used to haul the heavy equipment required for Esso Resources Canada Limited's Beaufort exploration drilling program for the winter of 1978–1979.

4.2. Floating ice platform

Sea ice remains relatively stationary during the winter months within the high Arctic islands in Canada. Exploratory offshore drilling has been successfully performed in this area for over 12 years by Panarctic Oils Ltd., from artificially thickened floating ice platforms in water depths up to 450 m (Baudais et al, 1974; Kivisild, 1975; Masterson and Kivisild, 1980). The drilling pads usually cover a 150×300 m area and are from 4.5 to 7.5 m thick. The pads are made by successive flooding around the clock, an existing ice cover thick enough to support men and machines. Submersible pumps lowered to sea water through holes drilled through the ice covered are used to draw the water. An average of five layers every 24 h are frozen during very cold weather giving almost 8 cm of new ice being added each day. Construction of a platform usually begins in November when the ice is thick enough to allow to the location. It takes about 2 months to build a platform.

Heat exchanges at a flooded ice surface during construction of a platform were studied by Nakawo (1980) primarily to determine the dependence on meteoroligical variables of the freeze time taken to flooded layer and hence to establish a basis for determining the optimum build-up rate for an ice platform. He concluded that sensible heat loss accounts for most of the latent heat released during freezing. Combined evaporation and sensible heat losses were noted to be about four times larger for this flooding situation than expected in neutral conditions.

Both vertical and horizontal migration of brine occurs in a platform during construction. Detailed observations on salinity of flooded water and built-up ice were carried out by Nakawo and Frederking (1981). The bulk salinity of built-up ice has been observed to be about half that of sea water which is about 30‰ in the area. The technique of freeflooding also allows considerable entrapment of air in the ice. The ice exhibits layered structure with fine grained regions at the top and bottom of each layer. Finer grains developed as a result of rapid freezing when the sea water, just above freezing point, comes in contact with the cold ice surface and air. *In situ* strength and deformation of this layered ice at various depths and temperatures have been studed by Sinha et al (1986) using a borehole jack developed earlier (Kivisild, 1975). Uniaxial tests were also conducted on this ice in the field. Both confined and uniaxial strengths show similar rate of loading and temperature sensitivity. The rate sensitivity of this ice has been noted to be also similar to other types of ice including freshwater ice.

Ice platforms have been used to support drilling rigs up to the size of 1 or 2 million kg for periods up to about 3 months. Finite element as well as probabilistic techniques have been used to analyze the long-term response (Masterson and Strandberg, 1979; Hamza and Muggeridge, 1983). Long term deformation processes were also studied directly by installing gauges in the ice (Masterson et al, 1979; Allan, 1979). Embedded wire strain gauges were used by the former group whereas the latter investigator used wire strain meters consisting of displacement gauges. Surface strains in the range of 1.5×10^{-3} at about the end of the loading period were reported by both the groups.

4.3. Grounded ice pads

Grounded ice pads have been used to support construction activities in winter for many decades. The technique used has been to pump water from under the ice cover up onto the surface where it freezes. This is done in successive layers or lifts until the ice cover grounds on the bottom. This technique was used in the construction of the Eagle River Bridge (McCutcheon, 1979).

In exploratory drilling in the Canadian Beaufort Sea grounded ice pads have been constructed to provide a site for relief well drilling. This was generally done by spray flooding over existing rubble adjacent to the caisson. With spray flooding heat is extracted from the water droplets while they are being projected through the air with the result that higher build up rates can be achieved than with simple surface flooding. The objective was to produce a level area with sufficient freeboard and surcharge load to resist horizontal ice forces. The grounded ice pad produced at Tarsiut had a freeboard of 8 m. Its design, construction, and stability analysis were described by Neth et al (1983). Since the prime objective was to produce a large mass of ice quickly, it soon became apparent that this could be achieved best by the "spray ice" technique. Water is sprayed into the air where the droplets cool and freeze, falling to the surface as ice crystals. The ice formed is more porous than sea ice and thus has lower density and strength. In a trial conducted over the winter 1983/84 production rates from 20 cm/day up to 2 m/day were achieved depending on the capacity (power) of the spraying system (Goff and Masterson, 1986). Esso Resources Canada has also carried out work on the production and properties of spray ice for grounded ice pads (Kemp, 1984) and will be constructing a pad as a primary drilling site for the winter 1986/87.

5. CONCLUDING REMARKS

The contributions made by Canadian researchers in the field of ice mechanics are certainly impressive and significant. In this review paper the authors have covered a wide range of topics in this area and have presented highlights of these achievements. In summary, some of the salient advances are:

1. The development of microstructural observation techniques including: (a) double microtoming technique for making thin sections, (b) combined cross polarized-scattered light technique, (c) etching-replicating technique, and (d) scanning electron micrography of ice. These methods have made the metallographical studies in ice comparable to those in metals and other materials.

2. The study of growth and aging processes, in conjunction with observations on texture and structure, in natural sea ice in the Arctic are considered to be the best in the world. These assisted in bringing out the major factors influencing the mechanical properties of sea ice.

3. The detailing of the extreme ice features in the Beaufort Sea and Hibernia locations giving valuable information on the geometry, size and strength of pressure ridges, ice islands, icebergs, etc.

4. The understanding of the mechanical behavior of ice. Refined laboratory and field techniques have provided more accurate and detailed information and insight into the strength of ice. The effects of loading rate, temperature, confinement, texture, structure, etc are now much better understood, especially for all types of sea ice.

5. Study of deformation properties in the transient range—the range important to most engineering situations. Great progress has been made in the understanding of crack nucleation and damage accumulation in ice. In fact, these studies helped to understand the mechanical properties of polycrystalline materials, in general, at elevated temperatures.

6. Measurement of the forces which an ice sheet can exert on different types of structures in both the laboratory and the field. Comparisons have shown that there is a definite "size effect" and a pronounced discrepancy between the global pressures. Both types of measurements have proved invaluable in understanding how ice interacts with a structure.

7. Analytical and numerical methods for calculating ice forces on offshore structure. The importance of providing good input information has been highlighted.

8. The construction and use of floating ice covers in the High Arctic as offshore drilling platform.

9. The development of physical model testing. Significant progress has been made particularly in developing "model ice" especially refrigerated urea ice, which is used worldwide, and more recently EG/AD/S model ice. The world's largest ice tank has recently been constructed in St. John's, Newfoundland.

10. In the area of remote sensing of ice, particularly in the application of SAR, impulse radar, and thermal scanning. This research has helped greatly in advancing many areas including navigation through ice infested waters and in the safe use of ice covers.

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