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DIFFICULTIES ASSOCIATED WITH PREDICTING DEPTH OF FREEZE OR THAW

by

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ABSTRACT

Calculations using the Neumann solution (as modified by Aldrich) and thermal properties of soils (obtained by Kersten) show that the frost penetration depth for the same freezing index for essentially all soils with any moisture content and for dry sand and rock varies by a factor of about 2 to 1. The extremes calculated in this way bracket the experimentally determined design curve of the US Army Corps of Engineers and give it theoretical support. The theoretical calculations and additional experimental data are used as a basis for a small alteration in the slope of the design curve. This modified design curve is recommended for field use because of (1) inherent imperfections in existing theory and (2) practical limitations to precise specification of field conditions.

SOMMAIRE

Les calculs en se servant de la solution Neumann (et modifiée par Aldrich) et des qualités thermales du sol (obtenues par Kersten) démontrent que la pénétration de la gelée pour le même indice de gel pour presque toute sorte de sol ayant une teneur en humidité et pour du sable sec et la roche peut varier par un facteur de 2 contre 1. Les extrêmes dérivés de cette façon encadrent la courbe calculée qui était établie expérimentalement par le "U.S. Corps of Engineers" et lui donnent un appui théorique. Les calculs théoriques et autres données expérimentales sont choisis comme base pour faire un petit changement dans la pente de cette courbe. On recommande l'usage in situ de cette courbe modifiée pour les deux raisons suivantes: 1) les imperfections qui existent dans la théorie et 2) les difficultés de spécifications précises des conditions qui existent in situ.

One of the most difficult problems associated with conduction heat transfer has been that of accounting for the effect of latent heat in freezing or thawing of the ground. The fact that the latent heat of soil moisture is liberated or absorbed at a definite temperature or, more generally, over a range of temperature results in a non-linear problem with its associated mathematical difficulties. For frost-susceptible soils, which are of major interest because of frost heave, the problem is compounded by the migration of moisture to the freezing zone and its continual build-up there. A final complicating factor, which automatically places a practical limit on the precision with which frost depth can be calculated, is the usual inhomogeneity of soil, in particular with respect to its moisture content near the surface as affected by precipitation or lack of precipitation prior to the beginning of freezing.

In this paper it is intended to compare the design curve for frost penetration obtained by the US Corps of Engineers (1949) from measurements in granular base courses beneath cleared airport runways in the Northern United States (Figure 1) with the results of calculations based on a rigid mathematical solution attributed to Neumann.[1] These calculations make use of thermal conductivity data obtained by Kersten (1949) for clay and sandy soils. In carrying out the calculations, attention will be focused principally on the limits of frost penetration for any type of soil and for any moisture content.

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[1]The solution was developed about 1860 and credited by Weber-Riemann (see Ingersoll, 1954).

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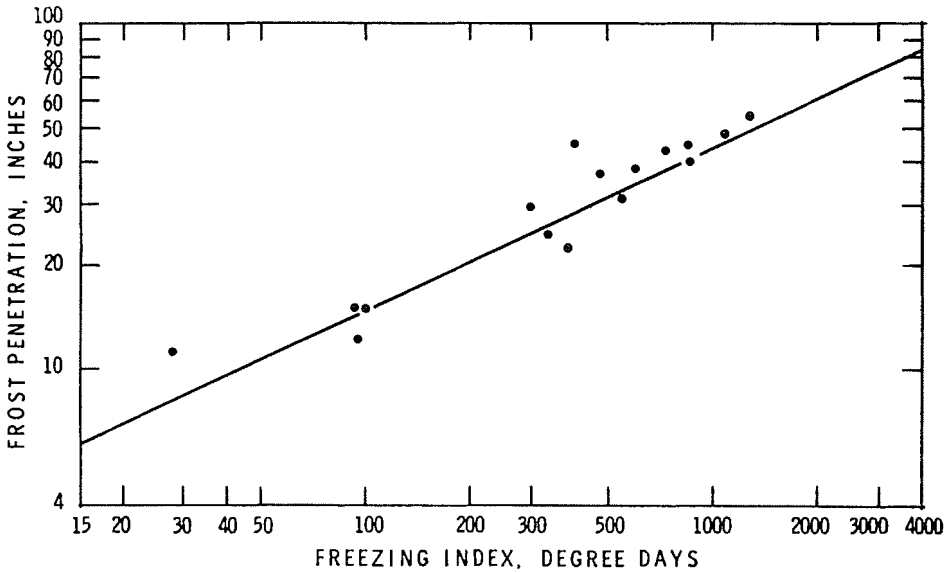


FIGURE 1. "Design curve," US Corps of Engineers. Maximum frost penetration under aircraft runways as dependent on air freezing index. (US Corps of Engineers, 1949)

Considerable attention has recently been given to the Neumann solution because it indicates that the depth of frost varies approximately inversely as the square root of the latent heat content of the soil. As the soil moisture content can have a wide range of values, it would be expected that the frost depth would also vary greatly. When different types of soils with different moisture contents are compared, however, as they are in this paper, it is found that the thermal properties of the soil also vary with moisture content in such a way as to reduce the extremes of penetration depth considerably.

The Neumann solution is rigidly correct for a homogeneous soil, initially at constant temperature T_1 above freezing and subjected to a sudden change at the surface to a constant temperature below freezing T_2. The thermal conductivity and diffusivity of frozen and unfrozen soil are assumed to have constant, though different, values and all moisture freezes at 32° F. With these conditions the theory can be expressed succinctly as (Ingersoll, 1954):

$$(1) \qquad \frac{-K_f(32 - T_2)e^{-b^2/4\alpha_f}}{\sqrt{\alpha_f}\,\phi\,(b/2\sqrt{\alpha_f})} + \frac{K_u(T_1 - 32)e^{-b^2/4\alpha_u}}{\sqrt{\alpha_u}\,[1 - \phi(b/2\sqrt{\alpha_u})]} = -\frac{\sqrt{\pi}}{2} L\,b,$$

where $\epsilon = b\sqrt{t}$,

and subscripts f and u are for frozen and unfrozen conditions

K is the thermal conductivity
α is the thermal diffusivity
ϵ is the frost penetration depth
T_1 is the initial soil temperature
T_2 is the temperature at the soil surface
b is a proportionality factor constant
L is the latent heat content per unit volume of soil
$\phi(b/2\sqrt{\alpha})$ is a mathematical function tabulated by Ingersoll (1954).

Among recent papers on the subject, those by Berggren (1943) and Aldrich (1956) should be mentioned. Berggren points out that the theory cannot overcome deficiencies in knowledge of field conditions of thermal properties and temperatures. He does not consider limits, however, but works several examples from the theory and points out that the depth of frost penetration is relatively insensitive to the ground temperature prior to freezing. Aldrich modifies the theory to account for the fact that temperatures in the field are not constant. In place of the term $(32 - T_2)$ in equation (1) he uses F/t where F is the freezing index at the ground surface (number of degree-days of freezing) and t is the duration of freezing in days. For T_1 Aldrich uses the mean annual temperature at the ground surface. Making these alterations to equation (1) gives:

$$\frac{-K_f(F/t)e^{-b^2/4\alpha_f}}{\sqrt{\alpha_f}\,\phi\,(b/2\sqrt{\alpha_f})} + \frac{K_u(T_m - 32)e^{-b^2/4\alpha_u}}{\sqrt{\alpha_u}\,[1 - \phi(b/2\sqrt{\alpha_u})]} = -\frac{\sqrt{\pi}}{2} L\,b. \tag{2}$$

Equation (2) might be criticized on the basis of the approximations introduced by Aldrich. It should be kept in mind, however, that the principal intent of the calculations to be carried out here is to determine relative limits of frost depth penetration rather than precise values. For this purpose equation (2) will be evaluated for all kinds of soils with dry and saturated conditions and, as well, for solid rock and pure ice over still water.

For the cleared surfaces of roads and runways the freezing index at the ground surface would not be expected to differ greatly from the air freezing index. Hence in order to compare the results of the theoretical calculations with the "design curve" of Figure 1 air freezing indexes were used in equation (2). Again, for comparative purposes, error on this account is not significant.

Concerning appropriate values of F/t and T_m for insertion into equation (2) it should be noted that in the field both of these variables are dependent, in a general way, on latitude and are consequently related to one another and to the freezing index. These relationships are shown in Figures 2 and 3, which were established by choosing data at random from maps of mean freezing index F, mean annual air temperatures T_m, and duration of normal freezing index t, as prepared by the US Corps of Engineers (1949) for the continental United States.

The thermal properties used in equation (2) were taken from Kersten's (1949) data for low and high values of soil dry density, and low and high (saturated) moisture content for both silt-clay and sandy soils. The data used in the calculations are given in Table I.

Results of the calculations are given in Figure 4. Curve a, giving the maximum frost penetration, was obtained for a saturated sandy soil of high dry density of 140 lb./cu. ft. and moisture content of 7 per cent. Curve b, giving the minimum frost penetration, was obtained for both sandy soil and silt-clay at dry densities of 90 and 80 lb./cu. ft. and moisture contents of 2 per cent for the sandy soil and 10 per cent for the silt-clay. All other combinations of dry density and moisture content fell between these limits, which differ from one another by a factor of just 2 to 1. Curve a is somewhat suspect, as the thermal conductivities given for this case by Kersten have values greater than those for solid granite.

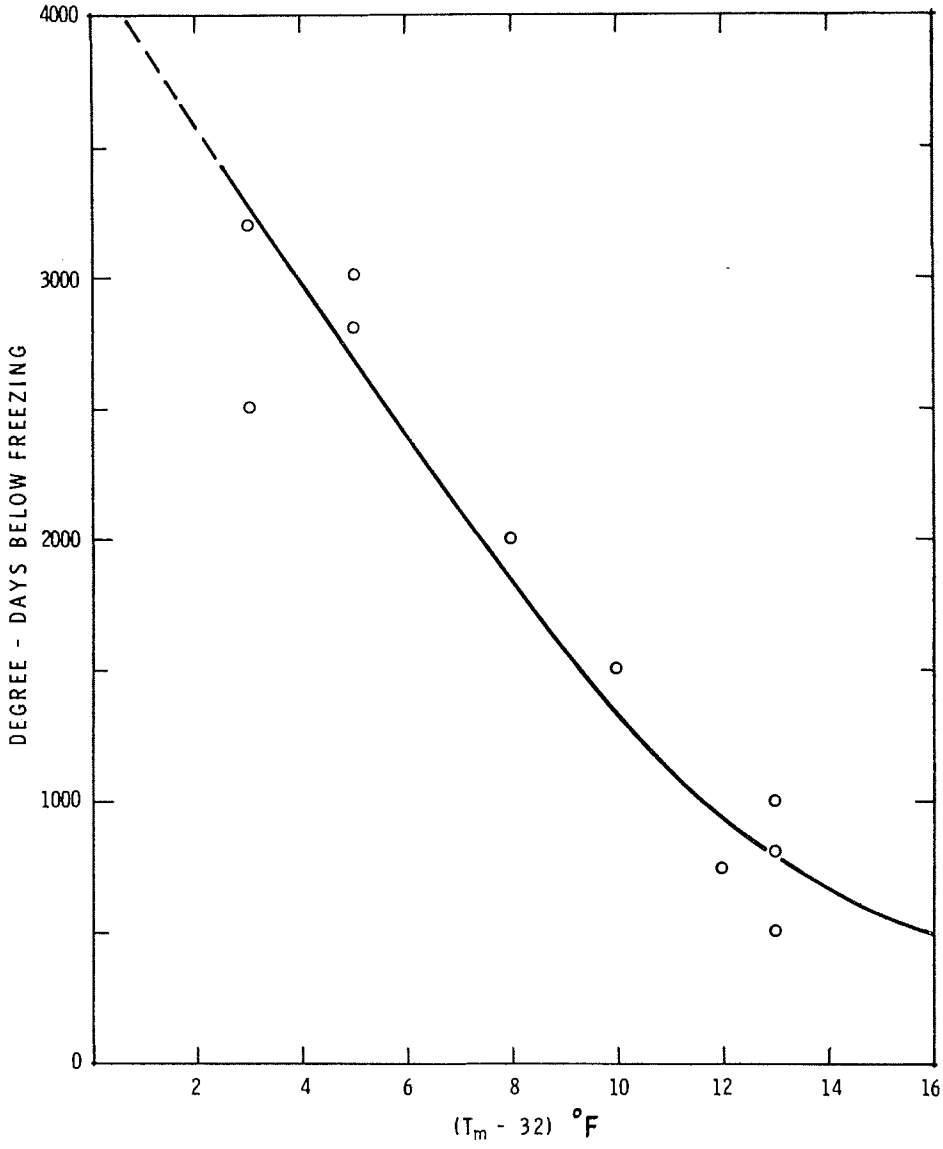


FIGURE 2. Relationship between air freezing index and the difference between mean annual temperature and the freezing temperature of water, 32° F., for Continental United States (US Corps of Engineers, 1949)

Data are not available to allow precise calculations for soils with moisture contents greater than saturation. It is clear, however, that frost penetration will be reduced below that for saturation until, in the limit, the material is essentially all water and ice, for which curve *f* applies.

In interpreting Figure 4 it should be noted that extremes of dry density have been used in the calculations. Actual field soils will not differ from one another

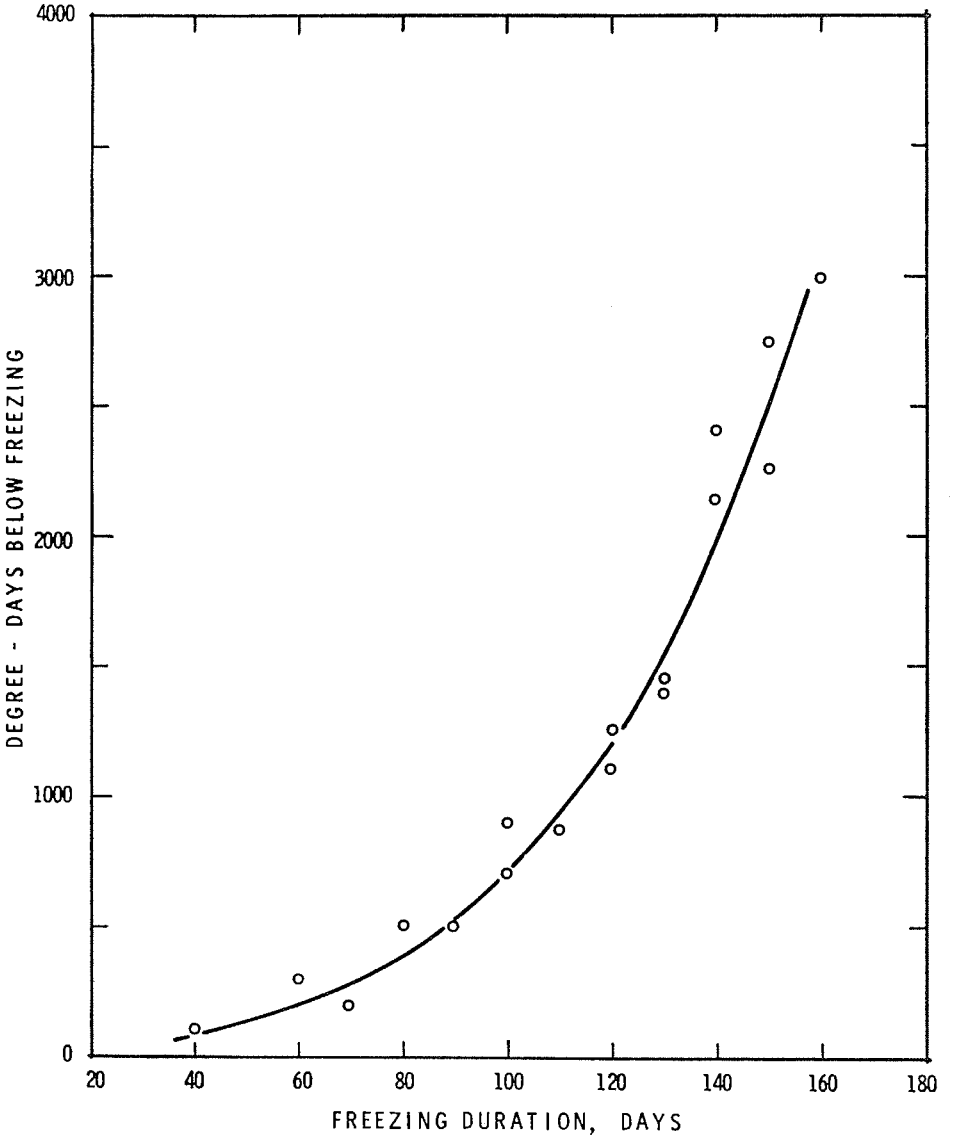


FIGURE 3. Relationship between air freezing index and normal freezing duration for Continental United States (US Corps of Engineers, 1949)

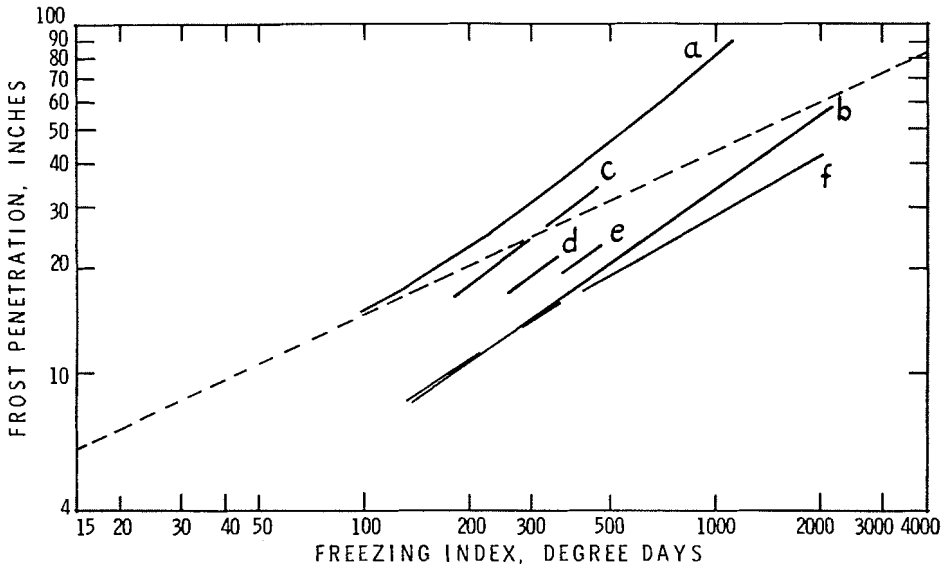
to this extent, and it is to be expected that the spread represented by curves *a* and *f* will be excessive for field use.

Other limits can be put on the range of frost penetration depth by determining the maximum depth of the 32° F. isotherm penetration for perfectly dry sand and solid granite. In this case the Neumann solution depends only on the thermal diffusivity of the material. Ingersoll (1959) gives for granite, $\alpha = 0.05$, and for dry sand, $\alpha = 0.008$ sq. ft./hr. (An independent check on

TABLE I

Thermal Properties of Soils (after Kersten (1949))

Soil type	Moisture content per cent of dry weight	Density lb./cu. ft.	Thermal conductivity Btu/(hr)(°F.)(ft)		Thermal diffusivity sq. ft./hr		Specific heat Btu/cu. ft.		Maximum latent heat Btu/cu. ft.
			Frozen	Unfrozen	Frozen	Unfrozen	Frozen	Unfrozen	
Silt, clay dry density = 80 lb./cu. ft.	10	88	0.34	0.36	0.0193	0.0167	17.6	21.6	1150
	30	104	1.0	0.60	0.0391	0.0160	25.6	37.6	3450
Silt, clay dry density = 120 lb./cu. ft.	10	132	1.0	0.90	0.0379	0.0277	26.4	32.5	1730
	20	144	1.75	1.35	0.0455	0.0240	38.4	56.3	3460
Sandy soil dry density = 90 lb./cu. ft.	2	91.8	0.2	0.4	0.0124	0.0235	16.2	17.1	259
	30	117	1.7	0.94	0.0590	0.0222	28.8	42.4	2880
Sandy soil dry density = 140 lb./cu. ft.	2	142.8	1.0	1.3	0.0396	0.049	25.2	26.6	403
	7	149.8	2.5	2.2	0.087	0.0656	28.7	33.5	1410



Curve a - Sandy soil, dry density = 140 lb/ft^3, saturated, moisture content = 7 %
b - Silt, clay, dry density = 80 lb/ft^3, unsaturated, moisture content = 10 % and sandy soil, dry density = 90 lb/ft^3, unsaturated, moisture content = 2 %
c - Sandy soil, dry density = 140 lb/ft^3, unsaturated, moisture content = 2 %
d - Silt, clay, dry density = 120 lb/ft^3, moisture content 10 to 20 % (saturated)
e - Silt, clay, dry density = 80 lb/ft^3, saturated, moisture content = 30 %
f - Pure ice over still water

FIGURE 4. Comparison of theoretical frost penetration depths for different soils and for pure ice over still water with the design curve

the value for sand is available from Brown (1963) where α was found to be 0.01 sq. ft./hr.) The two limits are given in Figure 5. It will be noted that by chance they differ very little from curves *a* and *f* of Figure 3.

Several conclusions can now be drawn. For any soil or ground material including dry rock and sand, and clays and silts with varying moisture content, the extremes of frost penetration for the same value of freezing index are about 2 to 1. For most actual soils the range of variation will be less than 2 to 1, because high density materials, e.g. rock, will be excluded. The design curve is moderately accurate in the range of freezing index from 100 to 1500 degree-days, but its shape is probably in error. Indeed, it is apparent from Figure 2 that for a degree-day value of about 4000 the mean annual air temperature is 32° F. Thus the mean annual ground temperature for about the same value of the freezing index is also 32° F. and this requires "infinite" frost penetration depth. Consequently, the design curve should bend up on this account alone. For freezing index values less than about 100 no proper correlation can be expected between cumulative freezing index and frost depth because occasional cold snaps of a few days' duration will overshadow any seasonal effects.

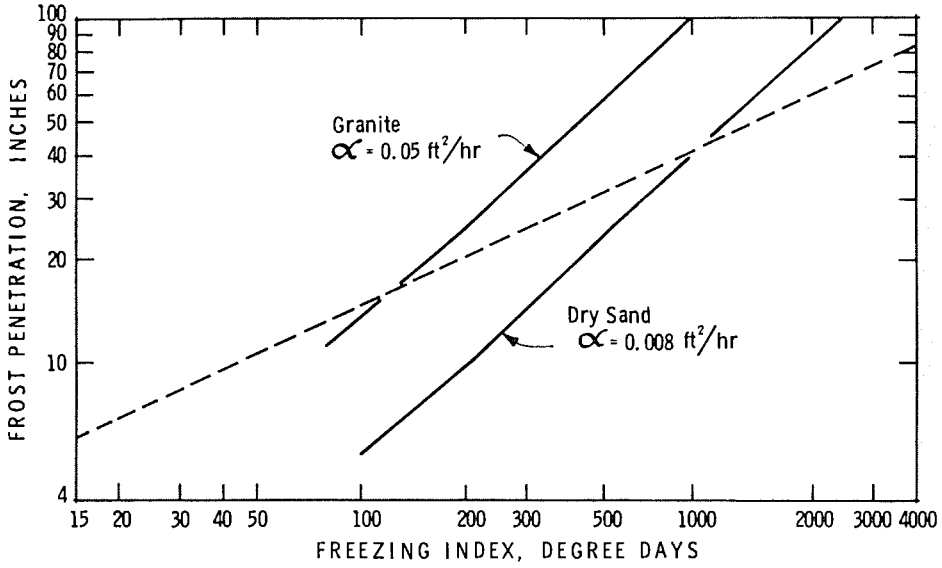


FIGURE 5. Comparison of the theoretical depth of the 32° F. isotherm for solid granite and dry sand with the design curve

INACCURACIES ASSOCIATED WITH THE NEUMANN SOLUTION

There are several factors that severely limit the precision of the Neumann solution for field use. The first of these is that the moisture content usually varies with depth and time. These variations also mean that thermal properties are not constant. Examples of measured moisture content obtained by Crawford (1961) are given in Figure 6. Secondly, for many if not most soils, not all moisture freezes at 32° F.; thus the latent heat is a variable quantity. Examples for clay and fine sand obtained by Penner (1963) are given in Figure 7. Thirdly, if all moisture does not freeze then thermal properties of the soil are in doubt for temperatures below 32° F. and presumably vary with the actual temperature. In addition to these difficulties it is not probable that a highly detailed field investigation will be carried out to determine the soil properties with precision for the purpose of a frost depth calculation. For field use, then, it would be misleading to expect the Neumann calculation to give results any more accurate than those to be obtained by an experimental curve similar to the design curve.

REVISION OF THE DESIGN CURVE

Figures 4 and 5 indicate that the slope of the design curve should be altered somewhat to be more consistent with the slope of the calculations using the Neumann solution. Some additional data obtained by Crawford (1955) are available to support this contention (Figure 8). In this case, for sandy and unclassified soils under cleared streets at Ottawa, Canada, the measured depths of frost penetration at different times during the freezing period are plotted against the accumulated freezing index. The Neumann solution shows the

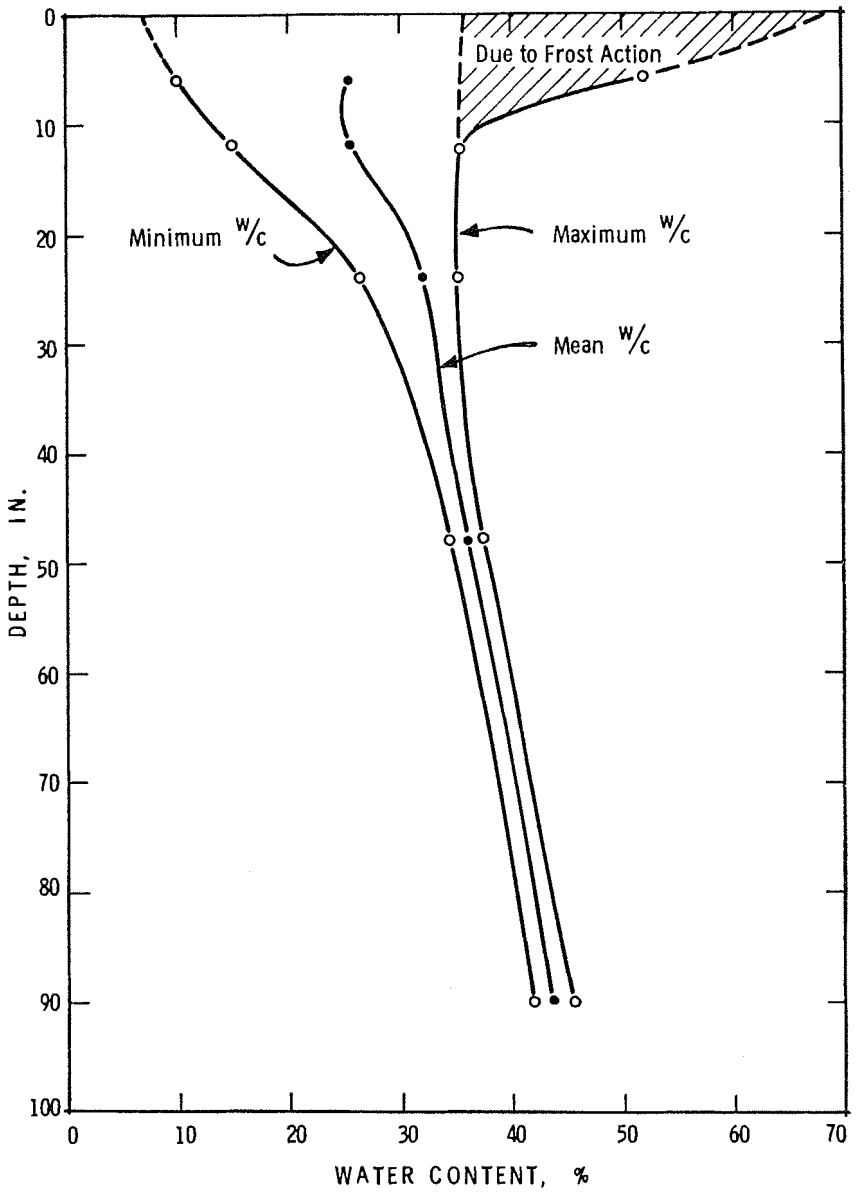


FIGURE 6. Seasonal variation in soil water content of Leda clay (after Crawford, 1961)

validity of this procedure (see Aldrich, 1956). Also shown in Figure 8 are the limiting curves *a* and *b* from Figure 4 and the design curve with its data replotted from Figure 1. The measured points indicate a slope more nearly like that of curves *a* and *b*, which enclose essentially all the data.

In view of the above arguments, the tentative, revised design curve (curve *g*, Figure 8) has been drawn and is repeated in Figure 9 for clarity.

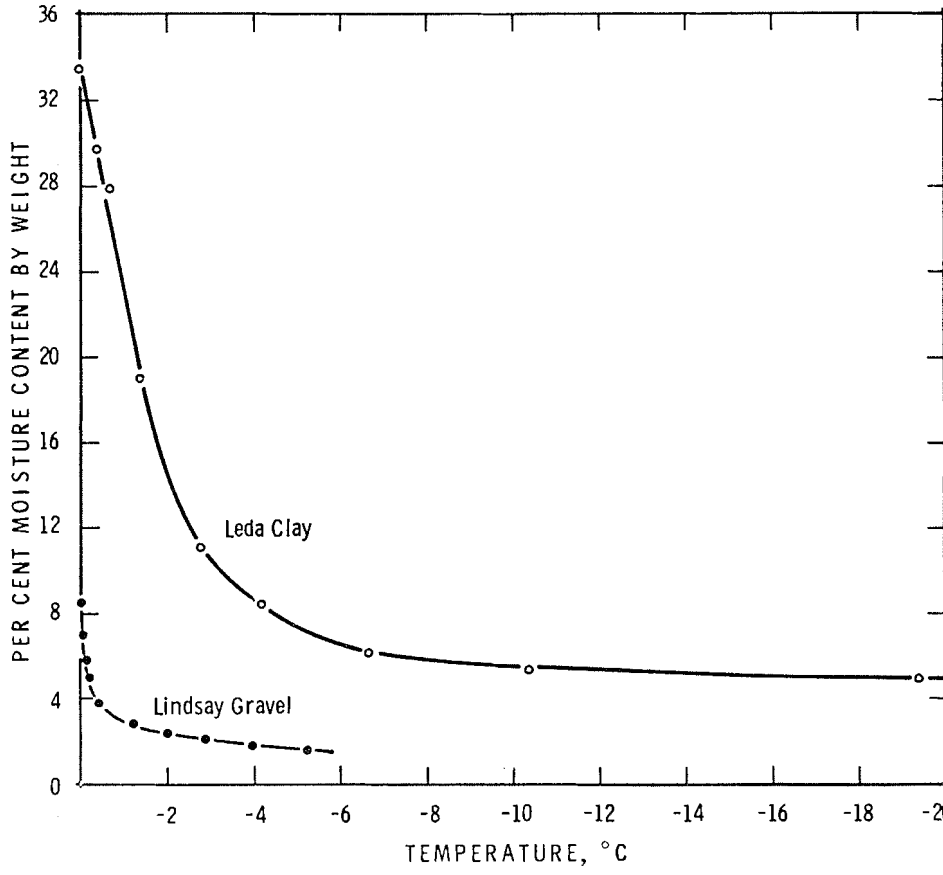


FIGURE 7. Unfrozen moisture content of two soils as a function of temperature (after Penner, 1963)

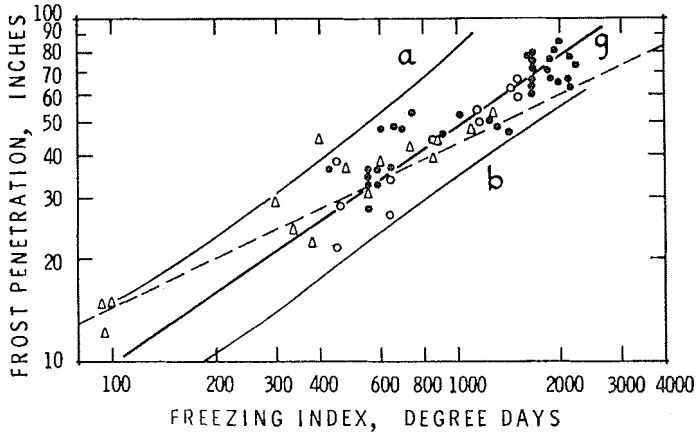


FIGURE 8. Comparison of theoretical frost penetration extremes (curves *a* and *b*) with experimental data of US Corps of Engineers (1949) and Crawford (1955)

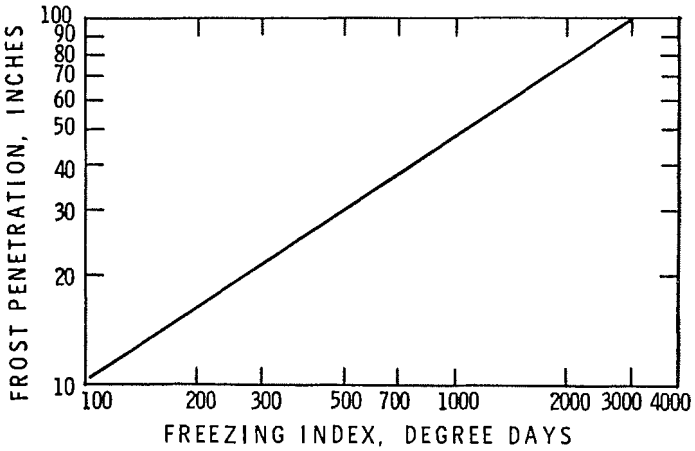


FIGURE 9. Modified design curve

DISCUSSION AND CONCLUSION

By considering the possible extremes of soil types, densities, and moisture content it is possible to show that frost penetration variations of less than 2 to 1 are obtained for the same freezing index. The calculated frost penetration range is consistent with the field records upon which the design curve of the US Army Corps of Engineers was based and with other records obtained by Crawford.

Difficulties associated with precise determination of thermal properties and moisture contents for field conditions indicate that little practical benefit can be obtained by attempting frost penetration calculations using the Neumann theory. On the other hand, the Neumann theory has been useful in establishing more clearly the general shape the design curve should have. On this basis the design curve has been altered slightly (Figure 9) to agree more closely with theory and with the additional experimental results of Crawford.

It should be borne in mind that the revised design curve of Figure 9 applies only to bare surfaces free of vegetation and snow cover. For other than bare surfaces, radiation, transpiration, and snow cover will have marked effects. It is to be hoped that future investigations of frost penetration can be concentrated on these aspects of the problem.

It would be expected that Figure 9 would be applicable to predicting depth of thaw in permafrost regions. In this case the thawing index would replace the freezing index and depth of thaw would replace frost depth.

In connection with the prediction of frost depth one further point should be made. For a frost-susceptible soil, that is, a soil subject to heaving, knowledge of the maximum frost depth is usually required only in order that the ground can be excavated and back-filled with a non-susceptible material. To avoid heaving with a non-susceptible soil of negligible moisture content, the 32° F. temperature line should not penetrate below the non-susceptible material; and

to calculate the depth of backfill for this condition a different approach neglecting latent heat is needed and is available in the recent work of Lachenbruch (1959). This is the problem of a layer of one material overlying another where the surface temperature varies periodically under the influence of the annual climate cycle. Lachenbruch gives a simple set of curves from which, knowing thermal properties, the depth of backfill required to prevent freezing in the sublayer is obtained directly.

The above considerations indicate that there is no great problem inherent in estimating frost penetration in the ground and that the design curve is supported by theory and experiment and gives moderately good results.

Acknowledgments

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