Empirical corrections to snow creep pressure equations

McClung, D. M.

Publisher’s version / Version de l'éditeur:

Canadian Geotechnical Journal, 21, 1, pp. 191-193, 1984-02

National Research Council Canada | Conseil national de recherches Canada

EMPIRICAL CORRECTIONS TO SNOW CREEP PRESSURE EQUATIONS

by D. M. McClung

Reprinted from
Canadian Geotechnical Journal
Vol. 21, No. 1, February 1984
pp. 191–193

DBR Paper No. 1155
Division of Building Research

Price $1.25 | OTTAWA | NRCC 22884

Publications of the Division may be obtained by mailing the appropriate remittance (a Bank, Express, or Post Office Money Order, or a cheque, made payable to the Receiver General of Canada, credit NRC) to the National Research Council of Canada, Ottawa, K1A 0R6. Stamps are not acceptable.

A list of all publications of the Division is available and may be obtained from the Publications Section, Division of Building Research, National Research Council of Canada, Ottawa, K1A 0R6.

Empirical corrections to snow creep pressure equations

D. M. McClung

Division of Building Research, National Research Council of Canada, British Columbia Regional Station, 3904 West Fourth Avenue, Vancouver, B.C., Canada V6R 1P5

Received April 21, 1983

Accepted October 13, 1983

A one-dimensional analytical model has been proposed for the average quasi-static snow creep pressure parallel to a slope at the centre of a long rigid structure. The model includes forces due to interruption of creep (internal deformation) and glide (slip of the snow cover over the ground). In reality, the problem is two-dimensional. By a series of two-dimensional finite element calculations, an empirical analytical correction has been found to improve the model. This yields a simple analytical model that provides results within 6% or less of the more complex two-dimensional problem.

Keywords: snow mechanics, creep, glide, snow pressure, snow-pack, finite element.

Un modèle analytique uni-dimensionnel a été proposé pour le calcul de la pression quasi-statique moyenne exercée par un couvert de neige en fluage parallèle à une pente sur une structure rigide longue. Ce modèle inclu les forces dues à l'interruption du fluage (déformation interne) et au glissement (glissement du couvert de neige sur le terrain). En réalité le problème est bi-dimensionnel. Par une série de calculs en éléments finis à deux dimensions, une correction analytique empirique a été trouvée pour améliorer le modèle. Ceci conduit à un modèle analytique simple qui donne des résultats, semblables à 6% ou moins près à ceux du problème bi-dimensionnel plus complexe.

Mots-clés: mécanique des neiges, fluage, glissement, pression de neige, banc de neige, éléments finis.

[Traduit par la revue]

Can. Geotech. J. 21, 191–193 (1984)

Background

Haefeli (Bader *et al.* 1939) provided the first formulation of the plane-strain snow pressure problem; it consists of calculation of the average pressure due to interruption of creep and glide processes at the centre of a long avalanche defence structure on a snow-covered slope. His model was later extended (Haefeli 1948), and although it was not derived from continuum mechanics, it constitutes essentially a one-dimensional treatment of the two-dimensional problem.

McClung (1982) gave a continuum mechanical one-dimensional model of the plane-strain problem, assuming the snow to deform as a linear, Newtonian viscous fluid with neglect of the static fluid pressure term. This model was shown by comparison with two-dimensional finite element results to be a more accurate predictor for the linear problem than Haefeli's formulation. The continuum model has a tendency to underestimate the pressures at low slope angles (near 25°) in comparison with finite element results. This was attributed to the two-dimensional character of the problem and an *ad hoc* empirical correction was proposed to improve the model prediction. Subsequent study with a more comprehensive set of finite element calculations has provided a much improved set of equations to replace the *ad hoc* empirical correction.

One-dimensional continuum model

The one-dimensional model for the average pressure, $\bar{\sigma}$ (more precisely $\bar{\sigma}$ is the average value of the maximum principal stress), across the face of a long rigid structure perpendicular to a snow-covered slope is given by

McClung (1982) as:

$$[1] \quad \frac{\bar{\sigma}}{\bar{\rho}gH} = \left[\left(\frac{2}{1-\nu}\right)\left(\frac{D}{H} + \frac{L}{H}\right)\right]^{1/2} \sin\psi + \frac{1}{2}\left(\frac{\nu}{1-\nu}\right)\cos\psi$$

where ν is the viscous analog of Poisson's ratio, ψ is the slope angle, $\bar{\rho}$ is depth-averaged snow density, g is acceleration due to gravity, and H is snow depth measured perpendicular to the ground surface. The parameter D is the stagnation depth for glide (McClung 1981) and L is a length scale derived from a creep stiffness equation. For the one-dimensional model, it was proposed that $L/H \simeq 1/2$.

Plane-strain finite element calculations presented by McClung (1982) showed that [1] is not completely accurate, particularly at slope angles near 25°. In reality L/H is a function of ν and ψ. The representation

$$\frac{L}{H} \approx \frac{1}{2}\left[1 + (\cot\psi - 1)\left(\frac{1-2\nu}{1-\nu}\right)\right]$$

for $\psi \leq 45°$ was proposed.

Proposed empirical correction and comparison with other predictions

By performing a comprehensive set of plane-strain finite element calculations (e.g. McClung 1982) for slope angles in the range of interest for avalanche defense structures (25–55°) it has been found that an accurate representation for L/H is given by:

$$[2] \quad \frac{L}{H} \approx 0.3\sqrt{2\cot\psi}\left(\frac{1-\nu}{1-2\nu}\right)^{1/4}$$

Equation [2] when used with [1] constitutes the proposed model.

For comparison, Haefeli's (1948) formulation of this problem is given by McClung (1982) in the present notation:

$$[3] \quad \frac{\bar{\sigma}}{\bar{\rho}gH} = \frac{2}{3}\left(\frac{1-\nu}{1-2\nu}\right)^{1/2}\left(1 + \frac{3D}{H}\right)^{1/2}\tan\psi + \frac{1}{2}\left(\frac{\nu}{1-\nu}\right)\cos\psi$$

Figure 1 shows a comparison of $\bar{\sigma}/\bar{\rho}gH$ as a function of ν from finite element results with the one-dimensional model with empirical corrections ([1] and [2]) and Haefeli's eq. [3] for $\psi = 25°$ and $D/H = 0.0$. Also shown in Fig. 1 is the one-dimensional model without empirical correction for which $L/H = 1/2$ in [1]. Figure 2 displays a similar comparison for $\psi = 55°$ and $D/H = 0.0$.

Figure 3 depicts the ratio $\bar{\sigma}/\bar{\rho}gH$ as a function of D/H for $\psi = 45°$ and $\nu = 0.25$. In this figure the one-dimensional model without empirical correction is not shown because in this case it is almost identical to the model with empirical corrections.

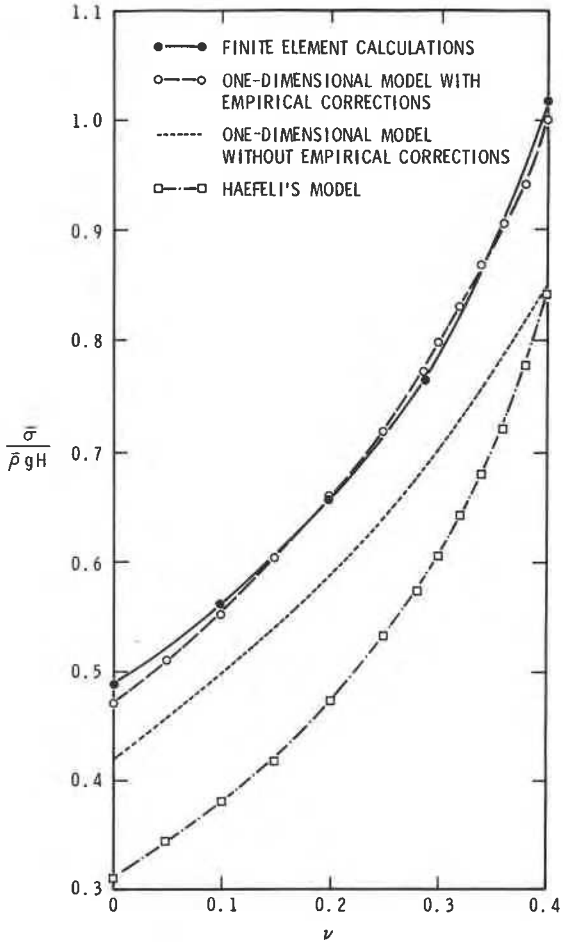

FIG. 1. The ratio $\bar{\sigma}/\bar{\rho}gH$ versus ν for $\psi = 25°$ and $D/H = 0.0$.

A number of other examples have been studied in addition to those presented in Figs. 1–3 and the maximum deviation of [1] and [2] from the two-dimensional finite element calculations found so far is 6% for parameters in the range $0 \leq \nu \leq 0.4$, $0 \leq D/H \leq 3$, and $25° \leq \psi \leq 55°$. These are the ranges of parameters of interest for the prediction of pressures at the centre of avalanche defence structures. It is felt, therefore, that the model provides prediction of creep pressures for the problem in which snow is modeled as a linear material with an accuracy better than the measurement accuracy of existing field measurements of the pressures (McClung *et al.* 1984).

Of further interest, the finite element results show that the zone of influence of the structure, termed by Haefeli the backpressure zone length, is given accurately by:

$$[4] \quad \chi' = 5H\left[\left(\frac{2}{1-\nu}\right)\left(\frac{D}{H} + \frac{L}{H}\right)\right]^{1/2}$$

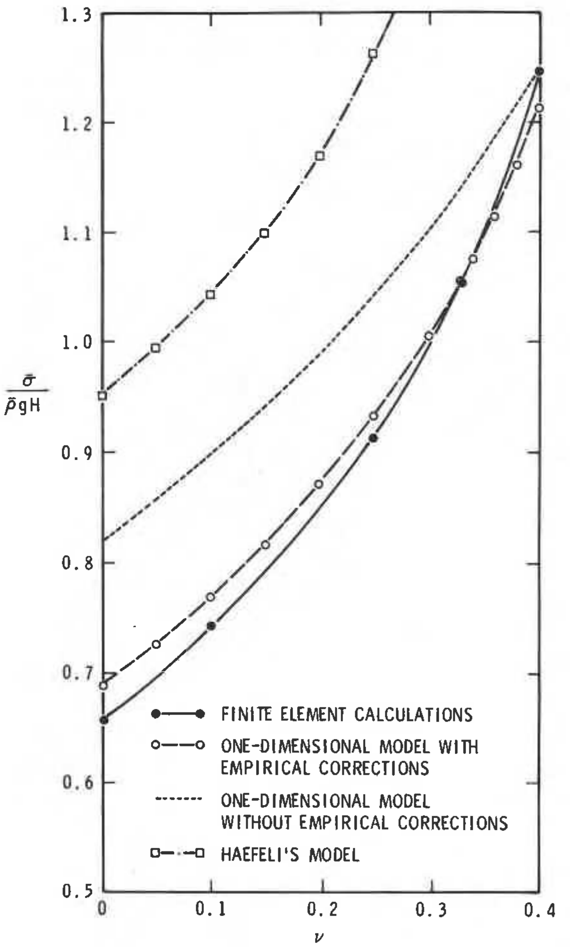

FIG. 2. The ratio $\bar{\sigma}/\bar{\rho}gH$ versus ν for $\psi = 55°$ and $D/H = 0.0$.

with L/H expressed by [2]. Equation [4] follows logically from the theory given by McClung (1982) with L/H given by [2] rather than $L/H = 1/2$ as used in the earlier one-dimensional model.

Concluding remarks

An analytical model has been presented that gives the average pressure for the plane-strain problem. This was shown by comparison with a two-dimensional finite element analysis. The main disadvantage of the model may be that the snow is modeled as a linear, Newtonian viscous fluid, which it certainly is not. The model can be extended to the simplest visco-plastic model by introducing a time-dependent Poisson ratio. Recent field measurements from western Norway (Larsen and McClung 1982) show that average pressures are within the range predicted by the linear model extended for time dependence, which provides support for its use in engineering calculations. These measurements will be reported in detail in a future publication (McClung *et al.* 1984).

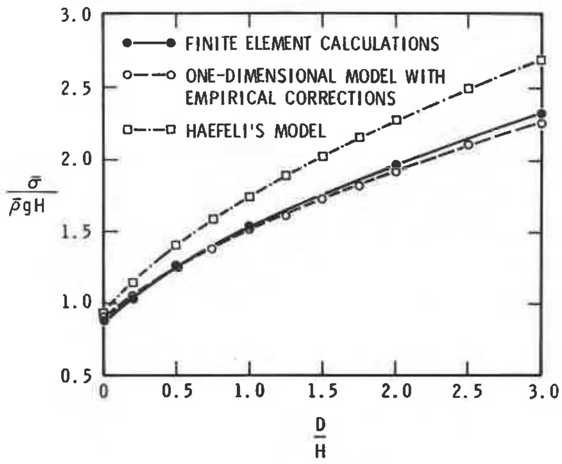

FIG. 3. The ratio $\bar{\sigma}/\bar{\rho}gH$ versus D/H for $\psi = 45°$ and $\nu = 0.25$.

Acknowledgement

This note is a contribution from the Division of Building Research of the National Research Council of Canada and is published with the approval of the Director of the Division.

BADER, H., HAEFELI, R., BUCHER, E., NEHERER, J., ECKEL, O., THAMS, C., and NIGGLI, P. 1939. Der Schnee und seine Metamorphose, Beiträge zur Geologie der Schweiz. Geotechnische Serie, Hydrologie, Lief. 3. (German, English translation, U.S. Snow, Ice and Permafrost Establishment, Trans. 14, 1954.)

HAEFELI, R. 1948. Schnee, Lawinen, Firn und Gletscher. *In* Ingenieur—Geologie. 2 Bd. *Edited by* L. Bendel. Springer-Verlag, Vienna, pp. 663–735.

LARSEN, J. O., and MCCLUNG, D. M. 1982. Measurements of snow creep pressures on an avalanche defense structure. Paper presented at Second Symposium on Applied Glaciology, Hanover, NH, August 23–27, 1982.

MCCLUNG, D M. 1981. A physical theory of snow gliding. Canadian Geotechnical Journal, **18**(1), pp. 86–94.

——— 1982. A one-dimensional analytical model for snow creep pressures on rigid structures. Canadian Geotechnical Journal, **19**(4), pp. 401–412.

MCCLUNG, D. M., LARSEN, J. O., and HANSEN, S. B. 1984. Comparison of snow pressure measurements and theoretical predictions. Canadian Geotechnical Journal, in press.