NRC Publications Archive
Archives des publications du CNRC

Uncertainty analysis of NSERC-NRC pod dynamometer system

Islam, M.

For the publisher's version, please access the DOI link below./ Pour consulter la version de l'éditeur, utilisez le lien DOI ci-dessous.

Publisher's version / Version de l'éditeur:
https://doi.org/10.4224/8895013
Technical Report (National Research Council of Canada. Institute for Ocean Technology); no. TR-2007-05, 2007

NRC Publications Archive Record / Notice des Archives des publications du CNRC :
https://nrc-publications.canada.ca/eng/view/object/?id=345b56a0-ff5a-4e39-9e09-eac699716e54
https://publications-cnrc.canada.ca/fra/voir/objet/?id=345b56a0-ff5a-4e39-9e09-eac699716e54

Access and use of this website and the material on it are subject to the Terms and Conditions set forth at
https://nrc-publications.canada.ca/eng/copyright
READ THESE TERMS AND CONDITIONS CAREFULLY BEFORE USING THIS WEBSITE.

L'accès à ce site Web et l'utilisation de son contenu sont assujettis aux conditions présentées dans le site
https://publications-cnrc.canada.ca/fra/droits
LISEZ CES CONDITIONS ATTENTIVEMENT AVANT D'UTILISER CE SITE WEB.

Questions? Contact the NRC Publications Archive team at PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca. If you wish to email the authors directly, please see the first page of the publication for their contact information.

Vous avez des questions? Nous pouvons vous aider. Pour communiquer directement avec un auteur, consultez la première page de la revue dans laquelle son article a été publié afin de trouver ses coordonnées. Si vous n'arrivez pas à les repérer, communiquez avec nous à PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca.

DOCUMENTATION PAGE

REPORT NUMBER TR-2007-05	**NRC REPORT NUMBER**	**DATE** February 2007

REPORT SECURITY CLASSIFICATION
Unclassified

DISTRIBUTION
Unlimited

TITLE

UNCERTAINTY ANALYSIS OF NSERC-NRC POD DYNAMOMETER SYSTEM

AUTHOR(S)

Mohammed Fakhrul Islam

CORPORATE AUTHOR(S)/PERFORMING AGENCY(S)
Memorial University of Newfoundland
Institute for Ocean Technology, National Research Council, St. John’s, NL

PUBLICATION

SPONSORING AGENCY(S)
Memorial University of Newfoundland
Institute for Ocean Technology, National Research Council, St. John’s, NL

IOT PROJECT NUMBER	**NRC FILE NUMBER**

KEY WORDS	**PAGES**	**FIGS.**	**TABLES**
Podded, propulsor, uncertainty, analysis	vi, 87, App. A-B	8	9

SUMMARY

The report presents a detailed uncertainty analysis for a podded propulsor open water test. A brief overview of the uncertainty analysis methodology has been provided, with a particular focus on the elements that are unique to the experiments with podded propulsors. The method used follows that of the American National Standards Institute (ANSI) and American Society of Mechanical Engineers (ASME) standard on Measurement uncertainty and the approach described by Coleman and steele in their 1999 book Experimental and Uncertainty Analysis for Engineers.

The variables of interest in the podded propulsors uncertainty analysis were propeller thrust, unit thrust, propeller torque, forces and moments on the propulsor in the three orthogonal directions, propeller shaft rotational speed, carriage advance speed, azimuthing angle, water density (function of water temperature) and propeller diameter. The uncertainty analysis results of the experiments conducted using the NSERC-NRC dynamometer system were compared to that of a very high quality, well-established equipment (tests done in the IOT towing tank) used to measure the performance of some bare podded propellers and the results from the previous similar tests using the same equipment. Comparison of the results showed that the podded propulsor tests using the NSERC-NRC pod instrumentation in the current phase of tests provided the level of accuracy comparable with the established equipment. The uncertainty levels observed in the propeller thrust and unit thrust in the podded propulsor tests were found to be higher than the thrust uncertainty for the baseline tests, but less than the corresponding uncertainties found in the previous tests done on the same podded propulsors with same operating conditions with the same equipment.

For majority of the cases, the primary element of the uncertainty of the performance coefficients was the bias error (90% or more on the total uncertainty). To reduce the overall uncertainty in the final results, the primary focuses should be to reduce the bias error in the equipment.

The uncertainty analysis results provided strong evidence that the experimental data obtained using the NSERC-NRC dynamometer system presented the true performance characteristics of the model scale podded propulsors under consideration.

ADDRESS

National Research Council
Institute for Ocean Technology
Arctic Avenue, P. O. Box 12093
St. John's, NL A1B 3T5
Tel.: (709) 772-5185, Fax: (709) 772-2462

National Research Council Canada
Conseil national de recherches Canada

Institute for Ocean Technology
Institut des technologies océaniques

UNCERTAINTY ANALYSIS OF NSERC-NRC POD DYNAMOMETER SYSTEM

TR-2007-05

Mohammed Fakhrul Islam

February 2007

Summary

The report presents a detailed uncertainty analysis for a podded propulsor open water test. A brief overview of the uncertainty analysis methodology has been provided, with a particular focus on the elements that are unique to the experiments with podded propulsors. The method used follows that of the American National Standards Institute (ANSI) and American Society of Mechanical Engineers (ASME) standard on Measurement uncertainty and the approach described by Coleman and steele in their 1999 book Experimental and Uncertainty Analysis for Engineers.

The variables of interest in the podded propulsors uncertainty analysis were propeller thrust, unit thrust, propeller torque, forces and moments on the propulsor in the three orthogonal directions, propeller shaft rotational speed, carriage advance speed, azimuthing angle, water density (function of water temperature) and propeller diameter.

The uncertainty analysis results of the experiments conducted using the NSERC-NRC dynamometer system were compared to that of a very high quality, well-established equipment (tests done in the IOT towing tank) used to measure the performance of some bare podded propellers and the results from the previous similar tests using the same equipment. Comparison of the results showed that the podded propulsor tests using the NSERC-NRC pod instrumentation in the current phase of tests provided the level of accuracy comparable with the established equipment. The uncertainty levels observed in the propeller thrust and unit thrust in the podded propulsor tests were found to be higher than the thrust uncertainty for the baseline tests, but less than the corresponding uncertainties found in the previous tests done on the same podded propulsors with same operating conditions with the same equipment.

For majority of the cases, the primary element of the uncertainty of the performance coefficients was the bias error (90% or more on the total uncertainty). To reduce the overall uncertainty in the final results, the primary focuses should be to reduce the bias error in the equipment.

The uncertainty analysis results provided strong evidence that the experimental data obtained using the NSERC-NRC dynamometer system presented the true performance characteristics of the model scale podded propulsors under consideration.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1

Introduction

The general recommendations and guidelines provided by the International Towing Tank Conference (ITTC) for uncertainty analysis for resistance and propulsion tests are most closely aligned with the testing techniques used to study the uncertainly in the NSERC-NRC pod dynamometer (Macneill *et al.*, 2004). The methodology used in the analysis of the uncertainly in the experimentation of puller and pusher podded propulsors follows the recommended guidelines set out by the ITTC in combination with approaches described by Bose and Luznik (1996), as well as Coleman and Steele (1999).

Uncertainty in a measurement consists of two major components: bias error and precision error. Bias error is a constant, systematic error in the system or process, which may be reduced through calibration, whereas, precision error is the random contribution often referred to as repeatability error, which can be reduced through the use of multiple readings.

1-1 Bias Error:

Considering the forces and moments applied to a dynamometer during calibration and testing, analysis of the sources of bias error will depend upon the calibration as well as testing methods employed. For example, consider a calibration stand using weight pans connected to cables passing through pulleys to apply the loading to the dynamometer. If the calibration stand has not been accurately levelled, applied forces and moments will not be as expected. If various geometrical distances associated with the calibration stand are inaccurately measured, applied forces and moments will be affected by this inaccuracy. If the weights used to load the pans have drifted such that their true weight is not their stated weight, additional bias errors will creep into the calibration. If the pulleys do not have ideal, frictionless bearings, the true applied forces and moments will be altered by pulley resistance. Variation in physical constants can lead to bias errors; this is often the case when thermal variations are present. Incorrect experimental methods can lead to bias errors. For example, application of forces and moments during calibration in ascending or descending order as opposed to a random order can lead to a bias uncertainty. Identification of elemental bias errors for other types of dynamometer calibration equipment, such as load cells, would proceed in a similar fashion.

Sources of bias error are also present in the output from the dynamometer. For example, biases may be present in the A/D converter, the amplifier gain, and the applied excitation voltage. For example, a quantization bias error in the analog-to-digital converter is usually taken to be one-half of the least significant bit. Biases from the amplifier and power supply should be identified from manufacturer's specifications and should include factors such as gain, linearity and zero errors. Hess *et al.* (2000) suggested that the biases in the output from the dynamometer be calculated using equation 1-1.

$$\frac{\mu V}{V_{ex}} = bits * \left(\frac{20}{4096} \frac{V}{bit} \right) * \left(1000000 \frac{\mu V}{V} \right) * \left(\frac{1}{Gain} \right) * \left(\frac{1}{V_{ex}} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1\text{-}1)$$

Where, V_{ex} is the excitation voltage used to power the dynamometer.

A detailed accounting of the sources of bias error is often difficult and requires a careful analysis of the calibration device, physical constants and geometrical data, associated experimental equipment and calibration procedures. Because of the expense that such a thorough examination may entail, one should perform a cursory investigation to estimate the order of magnitude of the biases. If all biases are negligible when compared to precision uncertainties, then clearly precision uncertainty will dominate, and setting the biases to zero will not significantly alter the calculation.

1-2 Precision error:

Precision error is determined by repetition. For the dynamometer calibration device, the ability to *repeatedly* apply a given force or moment must be quantified. If F_i is a force component applied by the calibration device during the i^{th} repetition, and N is the total number of measurements (spots) by an independent force gauge (calibration standard), then compute a mean and a standard deviation using:

Mean of forces or moments, $\overline{F} = \frac{1}{N} \sum_{i=1}^{N} F_i$

Standard deviation of forces or moments, $S_F = \left[\frac{1}{N-1} \sum_{i=1}^{N} \left(F_i - \overline{F} \right)^2 \right]^{1/2}$

A 95% confidence estimate of the precision uncertainty at a specific magnitude of applied force is estimated as:

$$P_F = \frac{tS_F}{\sqrt{N}}$$

where, ***t*** represents a value drawn from the Student's **t**-distribution for a 95% confidence level and $v = N$ -1 degrees of freedom. The parameter v is used to extract t value from the standard t distribution table. It should be carefully noted that, if ***N*** is small, then a 95% confidence level estimate will be large due to the paucity of the data; 5 to 10 trials should be sufficient to characterize the uncertainty level for a given force magnitude. One must perform this computation for a selection of force magnitudes throughout the dynamic range of the device. Similarly, the precision uncertainty of the output from the dynamometer, expressed in $\frac{\mu V}{V_{ex}}$ (see equation 1-1), should be determined from repetitions with the same applied force or moment combination for a selection of force magnitudes throughout the dynamic range of the device. If all precision errors are negligible when compared to bias uncertainties, then clearly bias uncertainty will dominate, and setting the precision errors to zero will not significantly alter the calculation.

Chapter 2

The Methodology

2-1 The Uncertainty Expressions

The overall uncertainly in the non-dimensional performance coefficients (Table 2-1) of the podded propulsors require proper identification of all the variables contained within the data reduction expressions. Thus the variables of interest in the podded propulsors uncertainty analysis were propeller thrust, unit thrust, propeller torque, forces and moments on the propulsor in the three orthogonal directions, propeller shaft rotational speed, carriage advance speed, azimuthing angle, water density (function of water temperature) and propeller diameter. Figure 2-1 shows a block diagram for the podded propulsor open water tests including the individual measurement systems, measurement of individual variables, data reduction and experimental results.

Table 2-1. List of performance coefficients for the podded propulsor unit.

Performance Characteristics	**Data Reduction Equation**
$K_{T\mathrm{Prop}}$ – propeller thrust coefficient	$T_{\mathrm{Prop}} / \rho n^2 D^4$
$K_{T\mathrm{Unit}}$– unit thrust coefficient, $K_{T\mathrm{x}}$ *or Longitudinal force coefficient,* $K_{F\mathrm{X}}$	$T_{\mathrm{Unit}} / \rho n^2 D^4$ or $F_X / \rho n^2 D^4$
$10K_Q$ – propeller torque coefficient	$10Q / \rho n^2 D^5$
J – propeller advance coefficient	V_A / nD
η_{Prop} – propeller efficiency	$J / 2\pi \times \left(K_{T\mathrm{Prop}} / K_Q\right)$
η_{Unit} – unit efficiency	$J / 2\pi \times \left(K_{T\mathrm{Unit}} / K_Q\right)$
K_{FZ} – transverse force coefficient	$F_Y / \rho n^2 D^4$
K_{FZ}– vertical force coefficient	$F_Z / \rho n^2 D^4$
K_{MX}– moment coefficient around x axis	$10 M_X / \rho n^2 D^5$
K_{MY}– moment coefficient around y axis	$10 M_Y / \rho n^2 D^5$
K_{MZ}– moment coefficient around z axis (Steering moment)	$10 M_Z / \rho n^2 D^5$

Where,

T_{Prop} - propeller thrust
T_{Unit} - unit thrust
Q - propeller torque
V_A - propeller advance speed, in the direction of carriage motion

ρ – water density
n – propeller rotational speed
D – propeller diameter
$F_{X,\,Y,\,Z}$ - components of the hydrodynamic force on the pod
$M_{X,\,Y,\,Z}$ - components of the hydrodynamic moment on the pod

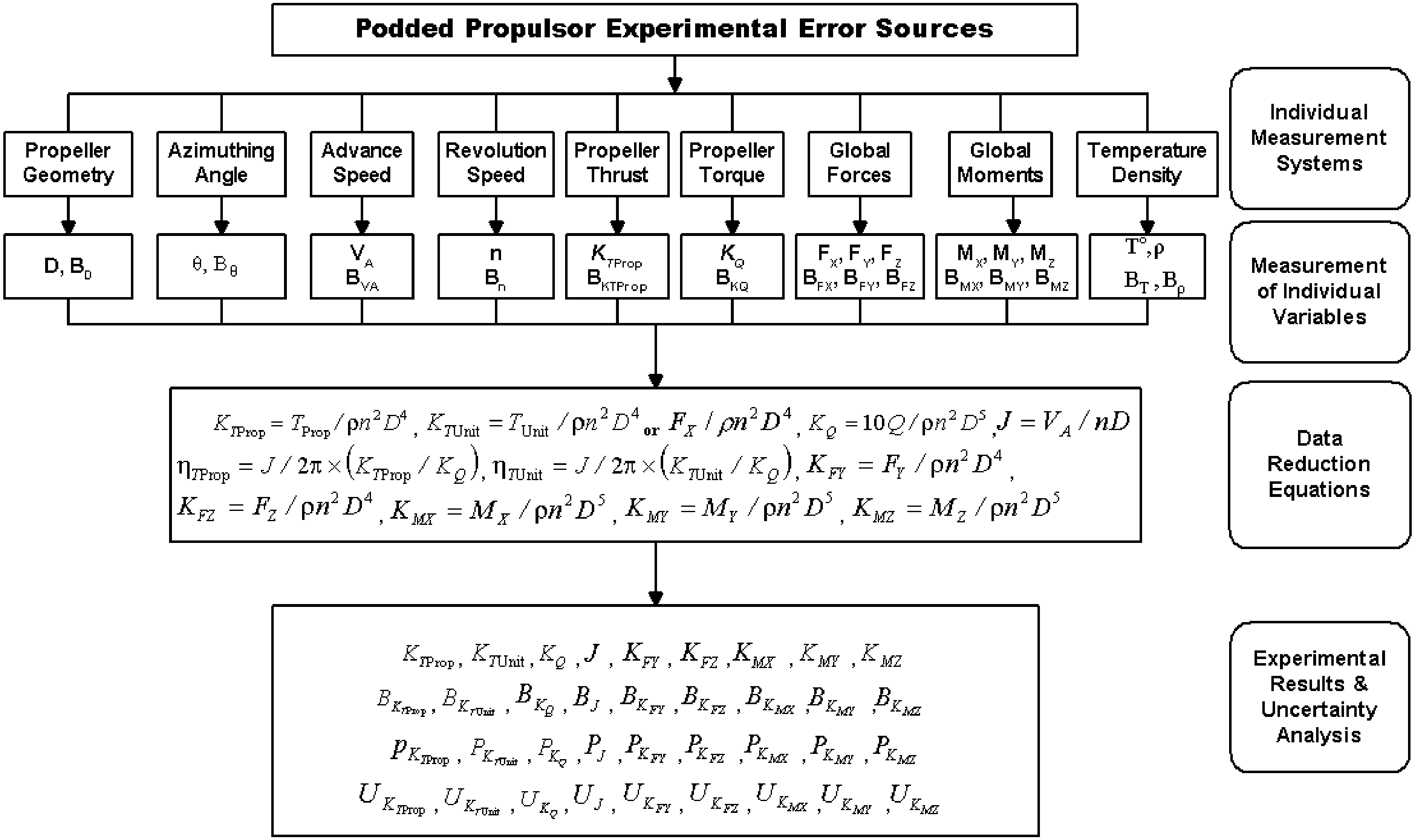


Figure 2-1: Block diagram for podded propulsor open water tests and uncertainty analysis.

The experimental approaches used to obtain the data for each of these variables were influenced by a variety of elemental sources of error. These elemental sources were estimated, as detailed in Appendix B, and combined using the root-sum-square (RSS) method to give the bias and precision limits for each of the variables. The bias errors were consisted of many elemental sources of error, which basically depended on the approaches followed to measure the variables. However, for the precision error estimates of most variables, only one source of error (repeatability) was considered significant.

The error estimates used in the determination of the bias and precision errors in this study were considered to be a 95% coverage estimates. The bias uncertainly and the precision uncertainly were combined using the root-sum-square (RSS) method shown below to provide estimates of overall uncertainty levels in these variables. The overall uncertainly was thus considered to be a 95% coverage estimate.

$$U_v = \sqrt{B_v^2 + P_v^2}$$

The final step in the methodology of uncertainly analysis was to determine how uncertainties in each of the variables propagate through the data reduction equations (see Table 2-1). Using the approaches described by Bose and Luznik (1996), and Coleman and Steele (1999) the uncertainty expressions for each set of experiments were developed as shown in equation 2-1 to 2-9.

$$\left(\frac{U_{K_{T\mathrm{Prop}}}}{K_{T\mathrm{Prop}}}\right)^2 = \left(\frac{U_{T\mathrm{Prop}}}{T_{\mathrm{Pr}\,op}}\right)^2 + \left(\frac{U_\rho}{\rho}\right)^2 + 4\left(\frac{U_n}{n}\right)^2 + 16\left(\frac{U_D}{D}\right)^2 \text{ ....(2-1)}$$

$$\left(\frac{U_{K_Q}}{K_Q}\right)^2 = \left(\frac{U_Q}{Q}\right)^2 + \left(\frac{U_\rho}{\rho}\right)^2 + 4\left(\frac{U_n}{n}\right)^2 + 25\left(\frac{U_D}{D}\right)^2 \text{ ....(2-2)}$$

$$\left(\frac{U_{K_{TUnit}}}{K_{T\mathrm{Unit}}}\right)^2 = \left(\frac{U_{T\mathrm{Unit}}}{T_{\mathrm{Unit}}}\right)^2 + \left(\frac{U_\rho}{\rho}\right)^2 + 4\left(\frac{U_n}{n}\right)^2 + 16\left(\frac{U_D}{D}\right)^2 \text{ ....(2-3)}$$

$$\left(\frac{U_{F_Y}}{K_{FY}}\right)^2 = \left(\frac{U_{FY}}{F_Y}\right)^2 + \left(\frac{U_\rho}{\rho}\right)^2 + 4\left(\frac{U_n}{n}\right)^2 + 16\left(\frac{U_D}{D}\right)^2 \text{ ....(2-4)}$$

$$\left(\frac{U_{F_Z}}{K_{FZ}}\right)^2 = \left(\frac{U_{FZ}}{F_Z}\right)^2 + \left(\frac{U_\rho}{\rho}\right)^2 + 4\left(\frac{U_n}{n}\right)^2 + 16\left(\frac{U_D}{D}\right)^2 \text{ ....(2-5)}$$

$$\left(\frac{U_{K_{MX}}}{K_{MX}}\right)^2 = \left(\frac{U_{MX}}{M_X}\right)^2 + \left(\frac{U_\rho}{\rho}\right)^2 + 4\left(\frac{U_n}{n}\right)^2 + 25\left(\frac{U_D}{D}\right)^2 \quad \text{(2-6)}$$

$$\left(\frac{U_{K_{MY}}}{K_{MY}}\right)^2 = \left(\frac{U_{MY}}{M_Y}\right)^2 + \left(\frac{U_\rho}{\rho}\right)^2 + 4\left(\frac{U_n}{n}\right)^2 + 25\left(\frac{U_D}{D}\right)^2 \quad \text{(2-7)}$$

$$\left(\frac{U_{K_{MZ}}}{K_{MZ}}\right)^2 = \left(\frac{U_{MZ}}{M_Z}\right)^2 + \left(\frac{U_\rho}{\rho}\right)^2 + 4\left(\frac{U_n}{n}\right)^2 + 25\left(\frac{U_D}{D}\right)^2 \quad \text{(2-8)}$$

$$\left(\frac{U_J}{J}\right)^2 = \left(\frac{U_{V_A}}{V_A}\right)^2 + \left(\frac{U_n}{n}\right)^2 + \left(\frac{U_D}{D}\right)^2 \quad \text{(2-9)}$$

In the expressions for the podded propulsors' tests, it should be noted that for both thrust and torque coefficient uncertainties, the tare thrust and frictional torque were imbedded in the corresponding measurements. Since the tare thrust and frictional torque were part of the same data stream as the thrust and torque readings, it had not been treated as an independent contributor of error to the corresponding coefficients, but rather has been treated as a bias error on the static-zero value of the thrust and the torque measurements.

2-2 Uncertainly in the Six-Component Global Dynamometer:

The calibration of the six-degree-of-freedom NSERC-NRC global dynamometer yielded an N x M array of applied force and moment components, **F**, and an N x M array of corresponding output voltages, **V** (Islam 2006c). To each element of the **F** and **V** arrays there was a corresponding bias and precision uncertainty that had been determined as described in Appendix B.

In order to calculate uncertainty in a six-component dynamometer measurement, one must determine how the uncertainties in the calibration data propagates into each element of the interaction matrix and into future measured forces and moments. As described in Hess *et al.* (2000), the solution for interaction matrix is given as:

$$\mathbf{C}=(\mathbf{V}^T\mathbf{V})^{-1}\mathbf{V}^T\mathbf{F}$$

If we consider one dynamometer axis at a time (one column of **C** and **F**), then the equation can be rewritten as

$$\mathbf{V}^{\mathbf{T}}\mathbf{V}\ \mathbf{C}_i=\mathbf{V}^{\mathbf{T}}\mathbf{F}_i$$

which is a classic form of the normal equations for a least squares fit problem. To determine the bias and precision uncertainty propagated into each column of the interaction matrix, one must determine how uncertainty in input data propagates into the coefficients of a least squares fit. Similarly, to determine the uncertainty present in future measured forces and moments from the fit (**F=VA**), one must understand how uncertainty propagates through a least squares fit into the output.

As given in Hess *et al.* (2000), the uncertainty propagated into the slope, $\boldsymbol{m}$, for a linear least square fit of the form, $\boldsymbol{y=mx+b}$ is of the form:

$$U_m=\left[\begin{gathered}\left(\frac{\partial m}{\partial x_1}U_{x_1}\right)^2+\left(\frac{\partial m}{\partial x_2}U_{x_2}\right)^2+\left(\frac{\partial m}{\partial x_3}U_{x_3}\right)^2+\ldots\ldots\ldots\ldots+\left(\frac{\partial m}{\partial x_N}U_{x_N}\right)^2\\+\left(\frac{\partial m}{\partial y_1}U_{y_1}\right)^2+\left(\frac{\partial m}{\partial y_2}U_{y_2}\right)^2+\left(\frac{\partial m}{\partial y_3}U_{y_3}\right)^2+\ldots\ldots\ldots\ldots+\left(\frac{\partial m}{\partial y_N}U_{y_N}\right)^2\end{gathered}\right]^{1/2}\quad\ldots\ldots\ldots\ldots(2\text{-}10)$$

Thus, the uncertainty in the slope depends upon the uncertainties in each of the abscissas and ordinates of the raw data used to construct the fit. Thus, one must determine the partial derivatives, $\frac{\partial m}{\partial x_i}$ and $\frac{\partial m}{\partial y_i}$, which are found to be of the form:

$$\frac{\partial m}{\partial x_i}=\frac{Ny_i-\sum y_i-2m\left(Nx_i-\sum x_i\right)}{N\sum x_i^2-\left(\sum x_i\right)^2}$$

and

$$\frac{\partial m}{\partial y_i}=\frac{Nx_i-\sum x_i}{N\sum x_i^2-\left(\sum x_i\right)^2}$$

where,

$$m=\frac{N\sum_{i=1}^{N}x_iy_i-\sum_{i=1}^{N}x_i\sum_{i=1}^{N}y_i}{N\sum_{i=1}^{N}x_i^2-\left(\sum_{i=1}^{N}x_i\right)^2}$$

For a six-degree-of –freedom dynamometer system, the uncertainty in each of the element of the interaction matrix is obtained using equation 2-10 where U_{x_1} and U_{y_1} are the uncertainties in the voltage and applied load measurements for each of the ***N*** loading conditions.

In the present case, a total loading cases of ***N***=195 was obtained. The applied loads to the calibration frame designed specifically for the dynamometer system (Islam 2006c) were converted to forces and moments in the three coordinate directions. In the matrix form, the forces and moments and the corresponding voltage output from the six-component dynamometer are expressed as:

$$F = \begin{bmatrix} F1,1 & - & - & - & - & F1,6 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ F195,1 & - & - & - & - & F195,6 \end{bmatrix} \text{ and } V = \begin{bmatrix} V1,1 & - & - & - & - & V1,6 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ V195,1 & - & - & - & - & V195,6 \end{bmatrix}$$

The ***F*** and ***V*** matrices used to calculate the uncertainties in the global components of the dynamometer are given in Appendix B. The uncertainties in the interaction matrix were then obtained using the equation 2-10. Each of the 36 elements of the interaction matrix had corresponding uncertainties that were calculated using equation 2-10. It is to be noted that the uncertainties in each elements in the ***F*** and ***V*** matrix is assumed to be equal for simplicity, as described in Appendix B. The Interaction matrix and the uncertainties in each of the 36 elements are given in the following two matrices.

Interaction/Calibration Matrix=

$$C = \begin{vmatrix} -718.3000 & 107.9000 & -195.6000 & 249.0000 & -79.8000 & 11.9000 \\ -23.7000 & 42.2000 & 519.2000 & 48.8000 & -15.5000 & 1.7000 \\ 7.3000 & -491.1000 & 246.9000 & -131.6000 & 12.1000 & 156.6000 \\ 1380.6000 & 978.0000 & -1421.4000 & -248.2000 & 414.6000 & -755.6000 \\ -2552.9000 & -80.0000 & -165.5000 & 866.7000 & -714.2000 & 773.9000 \\ 1333.1000 & -1666.2000 & 2303.9000 & -1178.7000 & 455.1000 & 514.9000 \end{vmatrix}$$

Bias Uncertainties in the Interaction/Calibration Matrix=

$$U_C = \begin{vmatrix} 0.1043 & 0.1508 & 0.4143 & 0.1109 & 0.1171 & 0.0882 \\ 0.0537 & 0.0767 & 0.2070 & 0.0570 & 0.0601 & 0.0458 \\ 0.0513 & 0.0732 & 0.1969 & 0.0544 & 0.0573 & 0.0438 \\ 0.2223 & 0.3224 & 0.8899 & 0.2363 & 0.2498 & 0.1876 \\ 0.2364 & 0.3428 & 0.9464 & 0.2513 & 0.2656 & 0.1995 \\ 0.2050 & 0.2972 & 0.8201 & 0.2179 & 0.2303 & 0.1730 \end{vmatrix}$$

Next thing to do in the uncertainty analysis of the dynamometer is to consider how the uncertainties in the calibration matrix propagate into a future calculation. A general formula for the uncertainty, U_R, which propagates into a results, R, from uncertainties in M different variables, X_i; i=1,2,……,M, where $R=R(X_1, X_2,\ldots\ldots X_M)$ is given by (Hess *et al*. 1999):

$$U_R = \left[\left(\frac{\partial R}{\partial X_1}U_{X_1}\right)^2 + \left(\frac{\partial R}{\partial X_2}U_{X_2}\right)^2 + \left(\frac{\partial R}{\partial X_3}U_{X_3}\right)^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \left(\frac{\partial R}{\partial X_n}U_{X_n}\right)^2\right]^{1/2} \quad \text{(2-11)}$$

In the present case of the six component dynamometer, the defining equation, **F=VA,** where $F=F(F_X, F_Y, F_Z, M_X, M_Y, M_Z)$ gives us:

$$\begin{aligned} F_X &= C_{11}V_1 + C_{12}V_2 + C_{13}V_3 + C_{14}V_4 + C_{15}V_5 + C_{16}V_6 \\ F_Y &= C_{21}V_1 + C_{22}V_2 + C_{23}V_3 + C_{24}V_4 + C_{25}V_5 + C_{26}V_6 \\ F_Z &= C_{31}V_1 + C_{32}V_2 + C_{33}V_3 + C_{34}V_4 + C_{35}V_5 + C_{36}V_6 \\ M_X &= C_{41}V_1 + C_{42}V_2 + C_{43}V_3 + C_{44}V_4 + C_{45}V_5 + C_{46}V_6 \\ M_Y &= C_{51}V_1 + C_{52}V_2 + C_{53}V_3 + C_{54}V_4 + C_{55}V_5 + C_{56}V_6 \\ M_Z &= C_{61}V_1 + C_{62}V_2 + C_{63}V_3 + C_{64}V_4 + C_{65}V_5 + C_{66}V_6 \end{aligned}$$

Now applying equation 2-2 to these equations yielded:

$$U_{F_X} = \begin{bmatrix} \left(\frac{\partial F_X}{\partial C_{11}}U_{C_{11}}\right)^2 + \left(\frac{\partial F_X}{\partial C_{12}}U_{C_{12}}\right)^2 + \left(\frac{\partial F_X}{\partial C_{13}}U_{C_{13}}\right)^2 + \left(\frac{\partial F_X}{\partial C_{14}}U_{C_{14}}\right)^2 + \left(\frac{\partial F_X}{\partial C_{15}}U_{C_{15}}\right)^2 + \left(\frac{\partial F_X}{\partial C_{16}}U_{C_{16}}\right)^2 \\ + \left(\frac{\partial F_X}{\partial V_1}U_{V_1}\right)^2 + \left(\frac{\partial F_X}{\partial V_2}U_{V_2}\right)^2 + \left(\frac{\partial F_X}{\partial V_3}U_{V_3}\right)^2 + \left(\frac{\partial F_X}{\partial V_4}U_{V_4}\right)^2 + \left(\frac{\partial F_X}{\partial V_5}U_{V_5}\right)^2 + \left(\frac{\partial F_X}{\partial V_6}U_{V_6}\right)^2 \end{bmatrix}^{1/2}$$

Which reduces to

$$U_{F_X} = \begin{bmatrix} \left(V_1 U_{C_{11}}\right)^2 + \left(V_2 U_{C_{12}}\right)^2 + \left(V_3 U_{C_{13}}\right)^2 + \left(V_4 U_{C_{14}}\right)^2 + \left(V_5 U_{C_{15}}\right)^2 + \left(V_6 U_{C_{16}}\right)^2 + \\ \left(C_{11} U_{V_1}\right)^2 + \left(C_{12} U_{V_2}\right)^2 + \left(C_{13} U_{V_3}\right)^2 + \left(C_{14} U_{V_4}\right)^2 + \left(C_{15} U_{V_5}\right)^2 + \left(C_{16} U_{V_6}\right)^2 \end{bmatrix}^{1/2}$$

In a similar fashion we get,

$$U_{F_Y} = \begin{bmatrix} \left(V_1 U_{C_{21}}\right)^2 + \left(V_2 U_{C_{22}}\right)^2 + \left(V_3 U_{C_{23}}\right)^2 + \left(V_4 U_{C_{24}}\right)^2 + \left(V_5 U_{C_{25}}\right)^2 + \left(V_6 U_{C_{26}}\right)^2 + \\ \left(C_{21} U_{V_1}\right)^2 + \left(C_{22} U_{V_2}\right)^2 + \left(C_{23} U_{V_3}\right)^2 + \left(C_{24} U_{V_4}\right)^2 + \left(C_{25} U_{V_5}\right)^2 + \left(C_{26} U_{V_6}\right)^2 \end{bmatrix}^{1/2}$$

These equations and the rest four components when put in the matrix form yielded:

$$\begin{vmatrix} U_{F_X} \\ U_{F_Y} \\ U_{F_Z} \\ U_{M_X} \\ U_{M_Y} \\ U_{M_Z} \end{vmatrix} = \begin{vmatrix} U_{C_{11}} & U_{C_{12}} & U_{C_{13}} & U_{C_{14}} & U_{C_{15}} & U_{C_{16}} & C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ U_{C_{21}} & U_{C_{22}} & - & - & - & U_{C_{26}} & C_{21} & - & - & - & - & C_{26} \\ - & - & - & - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - & - & - & - \\ U_{C_{61}} & U_{C_{62}} & - & - & - & U_{C_{66}} & C_{61} & - & - & - & - & C_{66} \end{vmatrix} \times \begin{vmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ U_{V_1} \\ U_{V_2} \\ U_{V_3} \\ U_{V_4} \\ U_{V_5} \\ U_{V_6} \end{vmatrix}$$

2-3 Results on Uncertainties

The details of the uncertainty calculation for each component of the dynamometer are provided in Appendix B.

Table 2-2 summarizes the bias and precision limit estimated for the podded propulsors obtained using the approach described in Appendix B.

Table 2-2. List of performance coefficients for the podded propulsor unit.

Variable		**Bias Error**		**Bias Limit**		**Precision error**	**Precision Limit**
Temp	Calibration	+/- 0.2000 °C	Fossilized into for Temp	+/- 0.2828 °C	Scale	+/- 0.0005 °C	
					Range	+/- 0.2 °C	
Density	Temp related Errors	+/- 0.0441 Kg/m^3	Overall Limit	+/- 0.0940 Kg/m^3			
	Density Equation related errors	+/- 0.0830 Kg/m^3					
Propeller Diameter	CNC Machining Errors	+/- 0.0001 m	Overall Limit	+/- 0.0001 m			
	Hnad Polishing Errors	+/- 0.0001 m					
Azimuthing angle	CNC Machining Errors	+/- 0.0002 rad	Overall Limit	+/- 0.6920 Deg			
	Hnad Polishing Errors	+/- 0.0020 rad					
	Hole Allowance	+/- 0.0081 rad					
	Equipment Allign	+/- 0.0087 rad					

Variable		**Bias Error**	**Bias Limit**	**Precision error**	**Precision Limit**
					J=0.0 +/- 0.0100 rps
					J=0.1 +/- 0.0132 rps
					J=0.2 +/- 0.0124 rps
					J=0.3 +/- 0.0098 rps
Shaft	Techometer				J=0.4 +/- 0.0118 rps
Speed	Reading Error	+/- 0.0080 rps			J=0.5 +/- 0.0117 rps
	A/D Error	+/- 0.0079 rps	Overall		J=0.6 +/- 0.0129 rps
	Curve Fit Error	+/- 0.0320 rps	Limit +/- 0.0500 rps		J=0.7 +/- 0.0124 rps
	Static Zero Error	+/- 0.0241 rps			J=0.8 +/- 0.0114 rps
	A/D Error	+/- 0.0079 rps			J=0.9 +/- 0.0111 rps
					J=1.0 +/- 0.0097 rps
					J=1.1 +/- 0.0126 rps
					J=1.2 +/- 0.0122 rps
					J=0.0 +/- 0.0001 m/s
					J=0.1 +/- 0.0000 m/s
					J=0.2 +/- 0.0001 m/s
	Calibration Error	+/- 0.0050 m/s			J=0.3 +/- 0.0002 m/s
	A/D Error	+/- 0.0010 m/s			J=0.4 +/- 0.0000 m/s
	Wheel Dia Error	+/- 0.0001 m/s	Overall		J=0.5 +/- 0.0001 m/s
Advance	Tide Error	+/- 0.0010 m/s	Limit +/- 0.0154 m/s		J=0.6 +/- 0.0003 m/s
Speed	Curve Fit Error	+/- 0.0143 m/s			J=0.7 +/- 0.0008 m/s
	A/D Error	+/- 0.0010 m/s			J=0.8 +/- 0.0005 m/s
	Static Zero Error	+/- 0.0024 m/s			J=0.9 +/- 0.0008 m/s
					J=1.0 +/- 0.0006 m/s
					J=1.1 +/- 0.0012 m/s
					J=1.2 +/- 0.0012 m/s
					J=0.0 +/- 0.3824 N
					J=0.1 +/- 0.1408 N
					J=0.2 +/- 0.4024 N
	Weights Error	+/- 0.0003 N			J=0.3 +/- 0.5376 N
	Load Angle Error	+/- 0.0114 N			J=0.4 +/- 0.3261 N
	Load Cell Align	+/- 0.0005 N			J=0.5 +/- 0.5898 N
Propeller	Static Zero Error	+/- 0.0356 N			J=0.6 +/- 0.5319 N
Thrust	A/D Card Error	+/- 0.6041 N	Overall		J=0.7 +/- 0.3637 N
	Curve Fit Error	+/- 2.0439 N	Limit +/- 2.2159 N		J=0.8 +/- 0.1126 N
	Equipment Align	+/- 0.0114 N			J=0.9 +/- 0.0239 N
	Load Cell Align	+/- 0.0005 N			J=1.0 +/- 0.1432 N
	Static Zero Error	+/- 0.0356 N			J=1.1 +/- 0.5068 N
	A/D Card Error	+/- 0.6041 N			J=1.2 +/- 0.1053 N

Variable	**Bias Error**				**Bias Limit**				**Precision error**	**Precision Limit**			
										J=0.0	+/-	0.0017	Nm
										J=0.1	+/-	0.0362	Nm
										J=0.2	+/-	0.0247	Nm
	Calibration Error	+/-	0.0008	Nm						*J*=0.3	+/-	0.0191	Nm
	Static Zero Error	+/-	0.0217	Nm						*J*=0.4	+/-	0.0048	Nm
	A/D Card Error	+/-	0.0171	Nm	Overall					*J*=0.5	+/-	0.0066	Nm
Propeller	Curve Fit Error	+/-	0.0557	Nm	Limit	+/-	0.0662	Nm		*J*=0.6	+/-	0.0419	Nm
Torque	Equipment Align	+/-	0.0005	Nm						*J*=0.7	+/-	0.0126	Nm
	Static Zero Error	+/-	0.0152	Nm						*J*=0.8	+/-	0.0361	Nm
	A/D Card Error	+/-	0.0171	Nm						*J*=0.9	+/-	0.0211	Nm
										J=1.0	+/-	0.0589	Nm
										J=1.1	+/-	0.0216	Nm
										J=1.2	+/-	0.0096	Nm
					Overall Limit								
					J=0.0	+/-	0.6391	N		*J*=0.0	+/-	1.2450	N
					J=0.1	+/-	0.6391	N		*J*=0.1	+/-	0.7886	N
					J=0.2	+/-	0.6391	N		*J*=0.2	+/-	0.9099	N
	Weights Error	+/-	0.0014	N	*J*=0.3	+/-	0.6392	N		*J*=0.3	+/-	0.8143	N
	Load Angle Error	+/-	0.0114	N	*J*=0.4	+/-	0.6392	N		*J*=0.4	+/-	0.9623	N
	Load Cell Align	+/-	0.0114	N	*J*=0.5	+/-	0.6394	N		*J*=0.5	+/-	0.9832	N
Unit	Static Zero Error	+/-	0.0356	N	*J*=0.6	+/-	0.6395	N		*J*=0.6	+/-	0.7479	N
Thrust	A/D Card Error	+/-	0.2014	N	*J*=0.7	+/-	0.6397	N		*J*=0.7	+/-	0.8920	N
	Curve Fit Error	+/-	-0.0559	N	*J*=0.8	+/-	0.6401	N		*J*=0.8	+/-	0.8374	N
	Equipment Align	+/-	0.0114	N	*J*=0.9	+/-	0.6403	N		*J*=0.9	+/-	1.2828	N
	Load Cell Align	+/-	0.0005	N	*J*=1.0	+/-	0.6406	N		*J*=1.0	+/-	0.9188	N
	Static Zero Error	+/-	0.0356	N	*J*=1.1	+/-	0.6411	N		*J*=1.1	+/-	1.0128	N
	A/D Card Error	+/-	0.6041	N	*J*=1.2	+/-	0.6415	N		*J*=1.2	+/-	0.5676	N
					Overall Limit								
					J=0.0	+/-	0.7626	N		*J*=0.0	+/-	0.8558	N
					J=0.1	+/-	0.7626	N		*J*=0.1	+/-	1.0702	N
					J=0.2	+/-	0.7591	N		*J*=0.2	+/-	0.8474	N
	Weights Error	+/-	0.0014	N	*J*=0.3	+/-	0.7530	N		*J*=0.3	+/-	0.6915	N
	Load Angle Error	+/-	0.0114	N	*J*=0.4	+/-	0.7516	N		*J*=0.4	+/-	1.1776	N
	Load Cell Align	+/-	0.0114	N	*J*=0.5	+/-	0.7476	N		*J*=0.5	+/-	0.9410	N
Side	Static Zero Error	+/-	0.0356	N	*J*=0.6	+/-	0.7456	N		*J*=0.6	+/-	0.8047	N
Force	A/D Card Error	+/-	0.2014	N	*J*=0.7	+/-	0.7429	N		*J*=0.7	+/-	1.0712	N
	Curve Fit Error	+/-	0.4162	N	*J*=0.8	+/-	0.7397	N		*J*=0.8	+/-	1.1341	N
	Equipment Align	+/-	0.0114	N	*J*=0.9	+/-	0.7374	N		*J*=0.9	+/-	0.9027	N
	Load Cell Align	+/-	0.0005	N	*J*=1.0	+/-	0.7353	N		*J*=1.0	+/-	1.0157	N
	Static Zero Error	+/-	0.0356	N	*J*=1.1	+/-	0.7318	N		*J*=1.1	+/-	0.8930	N
	A/D Card Error	+/-	0.6041	N	*J*=1.2	+/-	0.7294	N		*J*=1.2	+/-	1.1445	N

Variable			Bias Error				Bias Limit		Precision error		Precision Limit	
					Overall Limit							
					J=0.0	+/-	1.8743	N	*J*=0.0	+/-	0.4463	N
					J=0.1	+/-	1.8747	N	*J*=0.1	+/-	0.5393	N
					J=0.2	+/-	1.8368	N	*J*=0.2	+/-	0.4013	N
	Weights Error	+/-	0.0014	N	*J*=0.3	+/-	1.7673	N	*J*=0.3	+/-	0.4272	N
	Load Angle Error	+/-	0.0114	N	*J*=0.4	+/-	1.7529	N	*J*=0.4	+/-	0.6584	N
	Load Cell Align	+/-	0.0114	N	*J*=0.5	+/-	1.7076	N	*J*=0.5	+/-	0.6634	N
Vertical	Static Zero Error	+/-	0.0356	N	*J*=0.6	+/-	1.6865	N	*J*=0.6	+/-	0.6921	N
Force	A/D Card Error	+/-	0.2014	N	*J*=0.7	+/-	1.6560	N	*J*=0.7	+/-	0.7357	N
	Curve Fit Error	+/-	1.7620	N	*J*=0.8	+/-	1.6193	N	*J*=0.8	+/-	0.6908	N
	Equipment Align	+/-	0.0114	N	*J*=0.9	+/-	1.5930	N	*J*=0.9	+/-	0.7486	N
	Load Cell Align	+/-	0.0005	N	*J*=1.0	+/-	1.5695	N	*J*=1.0	+/-	0.6885	N
	Static Zero Error	+/-	0.0356	N	*J*=1.1	+/-	1.5286	N	*J*=1.1	+/-	0.5180	N
	A/D Card Error	+/-	0.6041	N	*J*=1.2	+/-	1.5016	N	*J*=1.2	+/-	0.5110	N

					Overall Limit							
					J=0.0	+/-	0.5243		*J*=0.0	+/-	2.3265	Nm
					J=0.1	+/-	0.5240		*J*=0.1	+/-	1.2123	Nm
					J=0.2	+/-	0.5130		*J*=0.2	+/-	0.9070	Nm
	Calibration Error	+/-	0.0081	Nm	*J*=0.3	+/-	0.4931		*J*=0.3	+/-	0.9807	Nm
	Static Zero Error	+/-	0.0217	Nm	*J*=0.4	+/-	0.4885		*J*=0.4	+/-	0.8905	Nm
	A/D Card Error	+/-	0.0098	Nm	*J*=0.5	+/-	0.4752		*J*=0.5	+/-	0.8255	Nm
Axial	Curve Fit Error	+/-	-0.5235	Nm	*J*=0.6	+/-	0.4688	Nm	*J*=0.6	+/-	0.9849	Nm
Moment	Equipment Align	+/-	0.0005	Nm	*J*=0.7	+/-	0.4597		*J*=0.7	+/-	1.1349	Nm
	Static Zero Error	+/-	0.0087	Nm	*J*=0.8	+/-	0.4487		*J*=0.8	+/-	0.5787	Nm
	A/D Card Error	+/-	0.0098	Nm	*J*=0.9	+/-	0.4407		*J*=0.9	+/-	0.4877	Nm
					J=1.0	+/-	0.4335		*J*=1.0	+/-	0.1552	Nm
					J=1.1	+/-	0.4211		*J*=1.1	+/-	0.4179	Nm
					J=1.2	+/-	0.4126		*J*=1.2	+/-	0.4890	Nm

					Overall Limit							
					J=0.0	+/-	2.0743		*J*=0.0	+/-	1.2884	Nm
					J=0.1	+/-	2.0743		*J*=0.1	+/-	0.9576	Nm
					J=0.2	+/-	2.0765		*J*=0.2	+/-	1.0572	Nm
	Calibration Error	+/-	0.0202	Nm	*J*=0.3	+/-	2.0809		*J*=0.3	+/-	0.8891	Nm
	Static Zero Error	+/-	0.9536	Nm	*J*=0.4	+/-	2.0818		*J*=0.4	+/-	0.9239	Nm
	A/D Card Error	+/-	1.0742	Nm	*J*=0.5	+/-	2.0848		*J*=0.5	+/-	0.5615	Nm
Vertical	Curve Fit Error	+/-	0.4192	Nm	*J*=0.6	+/-	2.0863	Nm	*J*=0.6	+/-	0.2293	Nm
Moment	Equipment Align	+/-	0.0005	Nm	*J*=0.7	+/-	2.0884		*J*=0.7	+/-	0.8934	Nm
	Static Zero Error	+/-	0.9536	Nm	*J*=0.8	+/-	2.0910		*J*=0.8	+/-	0.8517	Nm
	A/D Card Error	+/-	1.0742	Nm	*J*=0.9	+/-	2.0929		*J*=0.9	+/-	0.7744	Nm
					J=1.0	+/-	2.0946		*J*=1.0	+/-	0.5021	Nm
					J=1.1	+/-	2.0977		*J*=1.1	+/-	0.2432	Nm
					J=1.2	+/-	2.0998		*J*=1.2	+/-	0.4441	Nm

Variable		Bias Error			Bias Limit				Precision error		Precision Limit	
					Overall Limit							
					J=0.0	+/-	0.1398		*J*=0.0	+/-	1.2725	Nm
					J=0.1	+/-	0.1398		*J*=0.1	+/-	0.9869	Nm
					J=0.2	+/-	0.1398		*J*=0.2	+/-	0.8850	Nm
	Calibration Error	+/-	0.0081	Nm	*J*=0.3	+/-	0.1396		*J*=0.3	+/-	1.0819	Nm
	Static Zero Error	+/-	0.0037	Nm	*J*=0.4	+/-	0.1396		*J*=0.4	+/-	0.9617	Nm
	A/D Card Error	+/-	0.0042	Nm	*J*=0.5	+/-	0.1396		*J*=0.5	+/-	0.9373	Nm
Vertical	Curve Fit Error	+/-	0.1394	Nm	*J*=0.6	+/-	0.1395	Nm	*J*=0.6	+/-	0.6705	Nm
Moment	Equipment Align	+/-	0.0005	Nm	*J*=0.7	+/-	0.1395		*J*=0.7	+/-	0.8173	Nm
	Static Zero Error	+/-	0.0037	Nm	*J*=0.8	+/-	0.1394		*J*=0.8	+/-	0.7102	Nm
	A/D Card Error	+/-	0.0042	Nm	*J*=0.9	+/-	0.1394		*J*=0.9	+/-	0.8720	Nm
					J=1.0	+/-	0.1393		*J*=1.0	+/-	0.9252	Nm
					J=1.1	+/-	0.1393		*J*=1.1	+/-	0.9596	Nm
					J=1.2	+/-	0.1392		*J*=1.2	+/-	0.9326	Nm

The biases and precision limits were combined using RSS to determine the overall uncertainty estimates for each of the variables of interests as shown in Table 2-3.

Table 2-3: Overall uncertainty estimates for podded propulsor variables.

J	U_p	U_D	U_{AA}	U_n	U_{VA}	U_{TProp}	U_Q	U_{TUnit}	U_{FY}	U_{FZ}	U_{MX}	U_{MY}	U_{MZ}
0.0	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940
0.1	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.2	0.0508	0.0525	0.0522	0.0528	0.0521	0.0530	0.0530	0.0521	0.0530	0.0522	0.0528	0.0521	0.0531
0.3	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154
0.4	2.2789	2.2760	2.2536	2.2733	2.2479	2.3497	2.2369	2.2485	2.3954	2.3153	2.3699	2.3569	2.4384
0.5	0.0662	0.0769	0.0749	0.0682	0.0664	0.0666	0.0803	0.0674	0.0794	0.0702	0.0852	0.0714	0.0662
0.6	1.0823	0.7920	0.8550	0.9309	1.0273	0.9319	1.0607	1.1925	3.7040	1.1706	0.8736	0.6411	-0.7897
0.7	1.2523	1.2528	1.4277	1.0634	1.3137	1.2702	1.1249	1.2815	1.2639	1.2136	1.0864	1.0182	1.1486
0.8	1.9170	1.9307	1.9085	1.8383	1.8736	1.8569	1.8593	1.8510	1.7746	1.7178	1.7100	1.6135	1.5827
0.9	1.9880	1.3465	1.1181	1.1284	0.9637	0.8465	0.9178	1.0239	0.7071	0.5921	-0.4653	-0.6530	-0.6875
1.0	2.5301	2.2517	2.2774	2.3626	2.3299	2.1401	2.1009	2.2418	2.2393	2.1941	2.1608	2.1106	-2.1477
1.1	1.3076	0.8556	0.9633	0.9827	1.2638	0.8930	0.6993	0.7820	0.6724	1.0635	0.9901	0.9568	0.9520
1.2	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940

Substitution of the uncertainty values from Table 2-3 into the appropriate uncertainty equations (equations 2-1 to 2-9) yielded the overall uncertainty levels for the propulsive performance coefficients of the podded propulsors as summarized in Table 2-4 and 2-5. The uncertainty estimates were based on the test and calibration data presented in the reports: Islam, 2006a, Islam 2006b and Islam 2006c).

Table 2-4: Overall uncertainties in advance coefficients, propeller thrust and torque coefficients and unit thrust coefficients.

Advance Coefficient Value	Advance Coefficient J (+/-)	Advance Coefficient Error (+/-)	Propeller Thrust Coefficient K_{TProp} (+/-)	Propeller Thrust Coefficient Error (+/-)	Propeller Torque Coefficient K_Q (+/-)	Propeller Torque Coefficient Error (+/-)	Unit Thrust Coefficient K_{TUnit} (+/-)	Unit Thrust Coefficient Error (+/-)
0.00	-	-	5.78E-03	1.21	7.61E-03	1.11	4.75E-03	1.01
0.10	5.20E-03	5.20	5.59E-03	1.24	7.57E-03	1.17	4.39E-03	1.00
0.20	5.27E-03	2.63	5.28E-03	1.28	7.25E-03	1.20	4.05E-03	1.02
0.30	5.37E-03	1.79	5.02E-03	1.34	6.68E-03	1.19	3.74E-03	1.05
0.40	5.52E-03	1.38	4.74E-03	1.42	6.27E-03	1.23	3.40E-03	1.08
0.50	5.70E-03	1.14	4.62E-03	1.60	5.90E-03	1.28	3.07E-03	1.13
0.60	5.90E-03	0.98	4.22E-03	1.71	5.92E-03	1.45	2.66E-03	1.17
0.70	6.15E-03	0.88	4.02E-03	1.96	5.18E-03	1.47	2.44E-03	1.33
0.80	6.40E-03	0.80	3.33E-03	2.03	5.30E-03	1.79	5.22E-03	3.71
0.90	6.71E-03	0.75	2.89E-03	2.40	4.66E-03	2.00	2.21E-03	2.30
1.00	7.02E-03	0.70	2.43E-03	3.21	4.94E-03	2.94	1.43E-03	2.86
1.10	7.32E-03	0.67	1.43E-03	4.85	4.15E-03	4.38	9.99E-04	2.80
1.20	7.67E-03	0.64	-2.26E-03	12.58	3.82E-03	45.91	-1.34E-03	2.70

Table 2-5: Overall uncertainties in global forces and moments in the three orthogonal directions for the podded propulsors.

Advance Coeff. Value	Axial Force Coeff. K_{FX} (+/-)	Axial Force Coeff. Error (+/-)	Side Force Coeff. K_{FY} (+/-)	Side Force Coeff. Error (+/-)	Vertical Force Coeff. K_{FZ} (+/-)	Vertical Force Coeff. Error (+/-)	Axial Moment Coeff. K_{MX} (+/-)	Axial Moment Coeff. Error (+/-)	Vertical Moment Coeff. K_{YX} (+/-)	Vertical Moment Coeff. Error (+/-)	Steering Moment Coeff. K_{ZX} (+/-)	Steering Moment Coeff. Error (+/-)
0.00	4.76E-03	1.01	1.37E-03	4.33	3.01E-03	17.02	1.83E-02	2.08	1.44E-02	5.23	1.89E-02	1.04
0.10	4.86E-03	1.11	1.47E-03	4.75	3.02E-03	15.71	1.12E-02	1.41	1.34E-02	4.93	1.96E-02	1.13
0.20	4.44E-03	1.11	1.46E-03	4.77	2.94E-03	15.14	9.88E-03	1.40	1.34E-02	4.87	1.82E-02	1.13
0.30	4.08E-03	1.15	1.35E-03	4.95	2.89E-03	14.92	9.91E-03	1.66	1.40E-02	4.89	1.72E-02	1.15
0.40	3.67E-03	1.16	1.49E-03	5.64	2.90E-03	15.98	8.06E-03	1.56	1.36E-02	5.75	1.60E-02	1.16
0.50	3.20E-03	1.18	1.44E-03	6.21	2.92E-03	15.59	6.39E-03	1.59	1.26E-02	4.79	1.44E-02	1.17
0.60	2.93E-03	1.29	1.45E-03	7.33	2.86E-03	16.63	6.51E-03	2.04	1.24E-02	5.09	1.26E-02	1.15
0.70	2.60E-03	1.42	1.37E-03	7.92	2.80E-03	16.51	6.41E-03	2.71	1.31E-02	5.98	1.16E-02	1.19
0.80	4.47E-03	3.18	1.39E-03	8.93	2.81E-03	17.53	4.23E-03	2.63	1.31E-02	7.25	1.04E-02	1.22
0.90	2.18E-03	2.27	1.39E-03	9.95	2.72E-03	16.70	3.75E-03	4.50	1.30E-02	9.81	9.88E-03	1.33
1.00	1.56E-03	3.13	1.40E-03	11.64	2.70E-03	16.68	-2.66E-03	4.76	1.25E-02	14.46	8.65E-03	1.36
1.10	9.99E-04	2.80	1.34E-03	11.90	2.56E-03	15.15	-3.98E-03	5.07	1.22E-02	69.82	7.47E-03	1.42
1.20	-1.36E-03	2.75	1.33E-03	16.78	2.45E-03	14.54	-3.77E-03	2.43	-1.24E-02	21.12	7.41E-03	1.82

From Table 2-4 and 2-5, it can be seen that the uncertainty levels of the propeller thrust coefficient are higher than those of the unit thrust coefficient. However, the uncertainty in torque coefficient is comparable with that of the bare propeller test uncertainty presented in Bose and Luznik (1998). Applying the uncertainty limits to the performance curves of average pod 01 in the form of error bars results in a plot as shown in Figure 2-2.

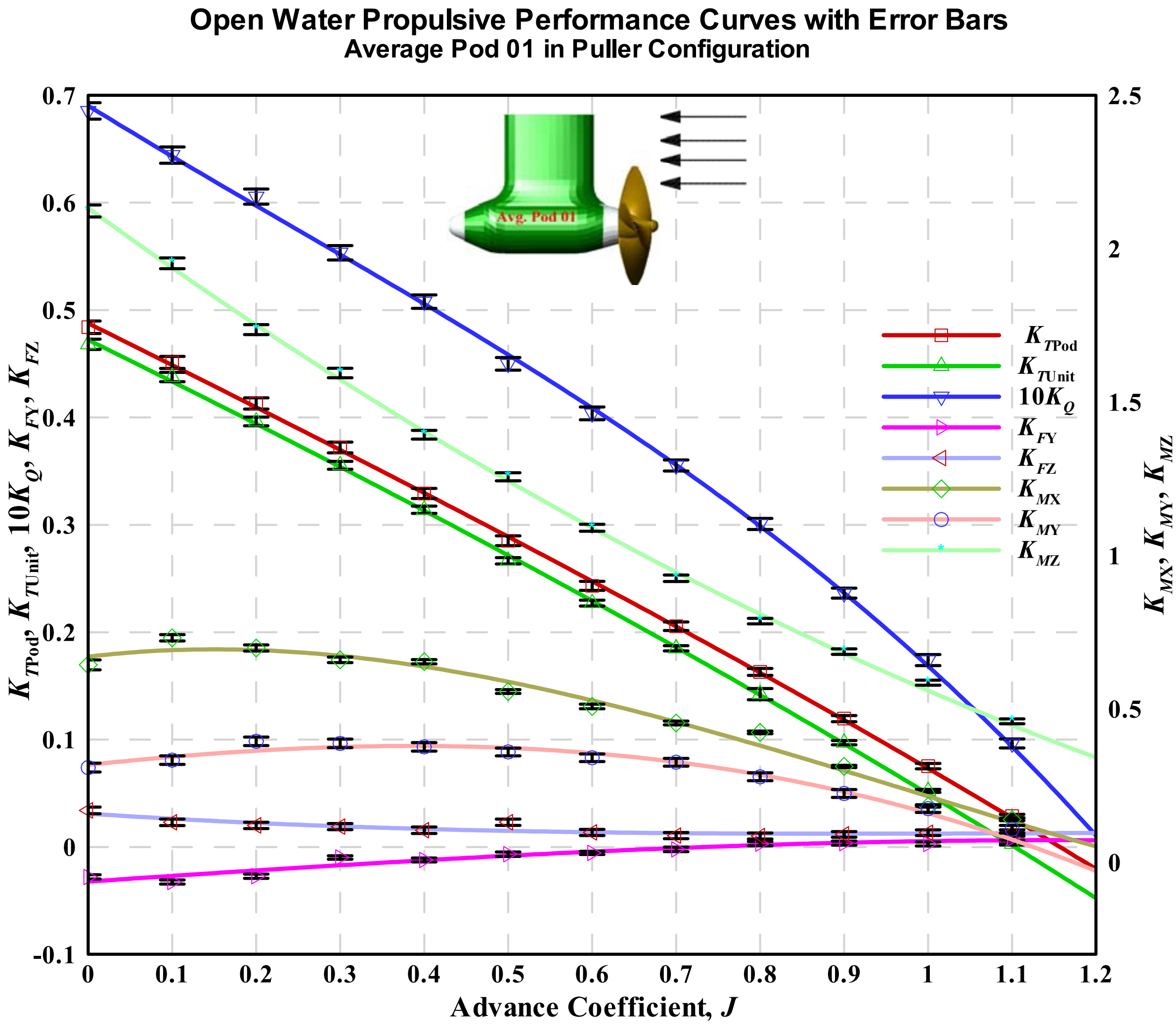


Figure 2-2: Performance curves for average pod 01 in puller configuration with uncertainty (error) bars.

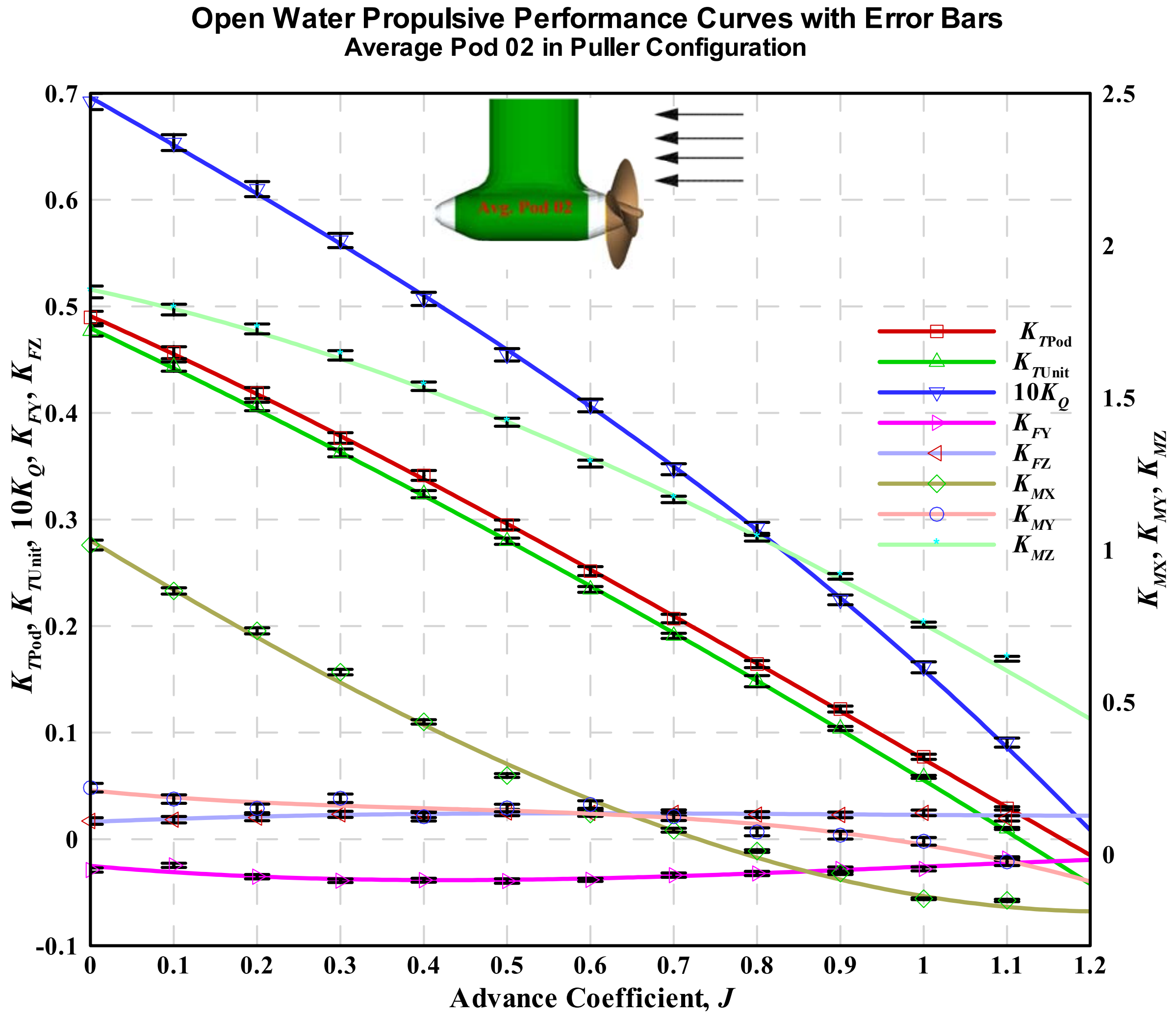


Figure 2-3: Performance curves for average pod 02 in puller configuration with uncertainty (error) bars.

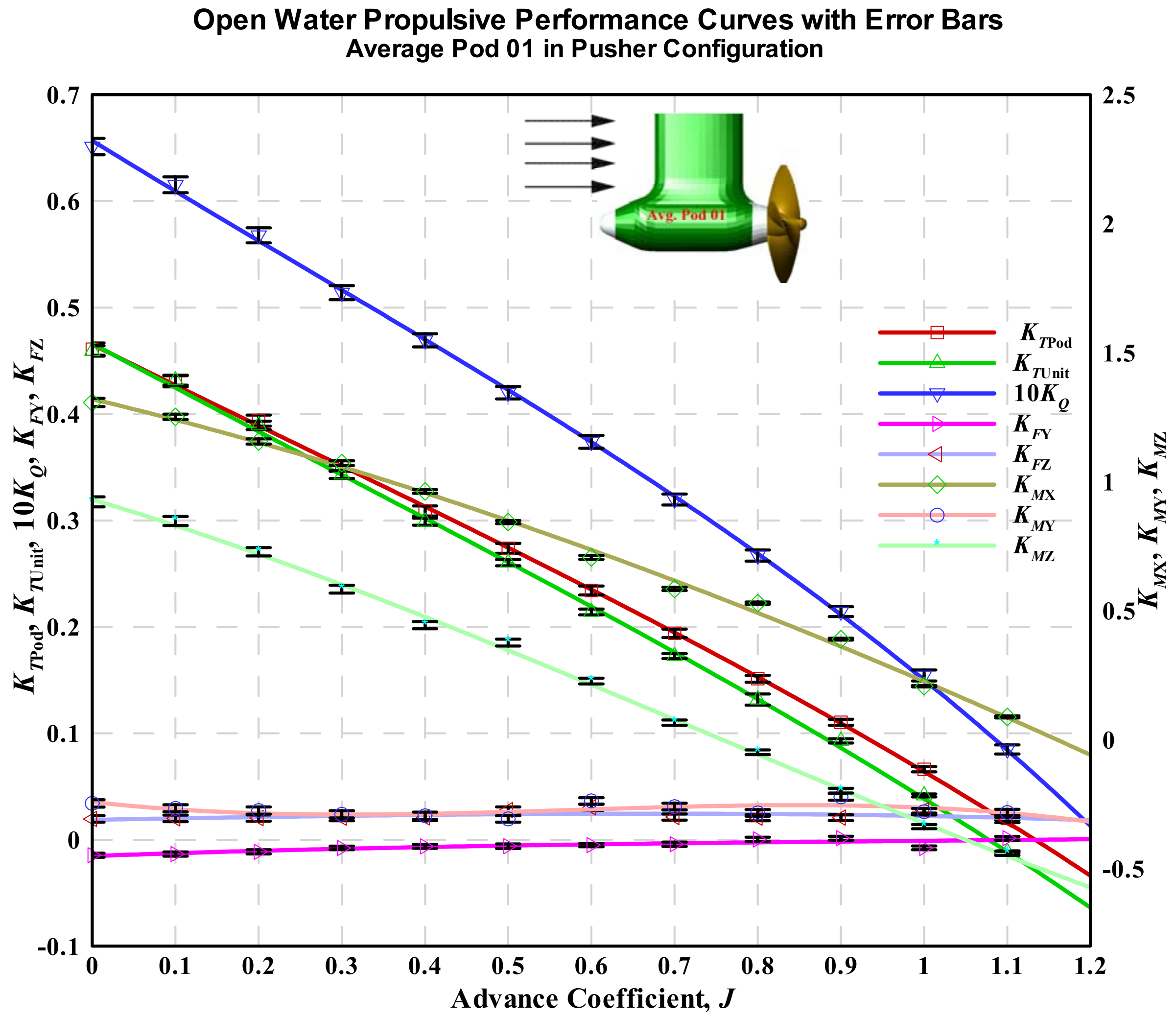


Figure 2-4: Performance curves for average pod 01 in pusher configuration with uncertainty (error) bars.

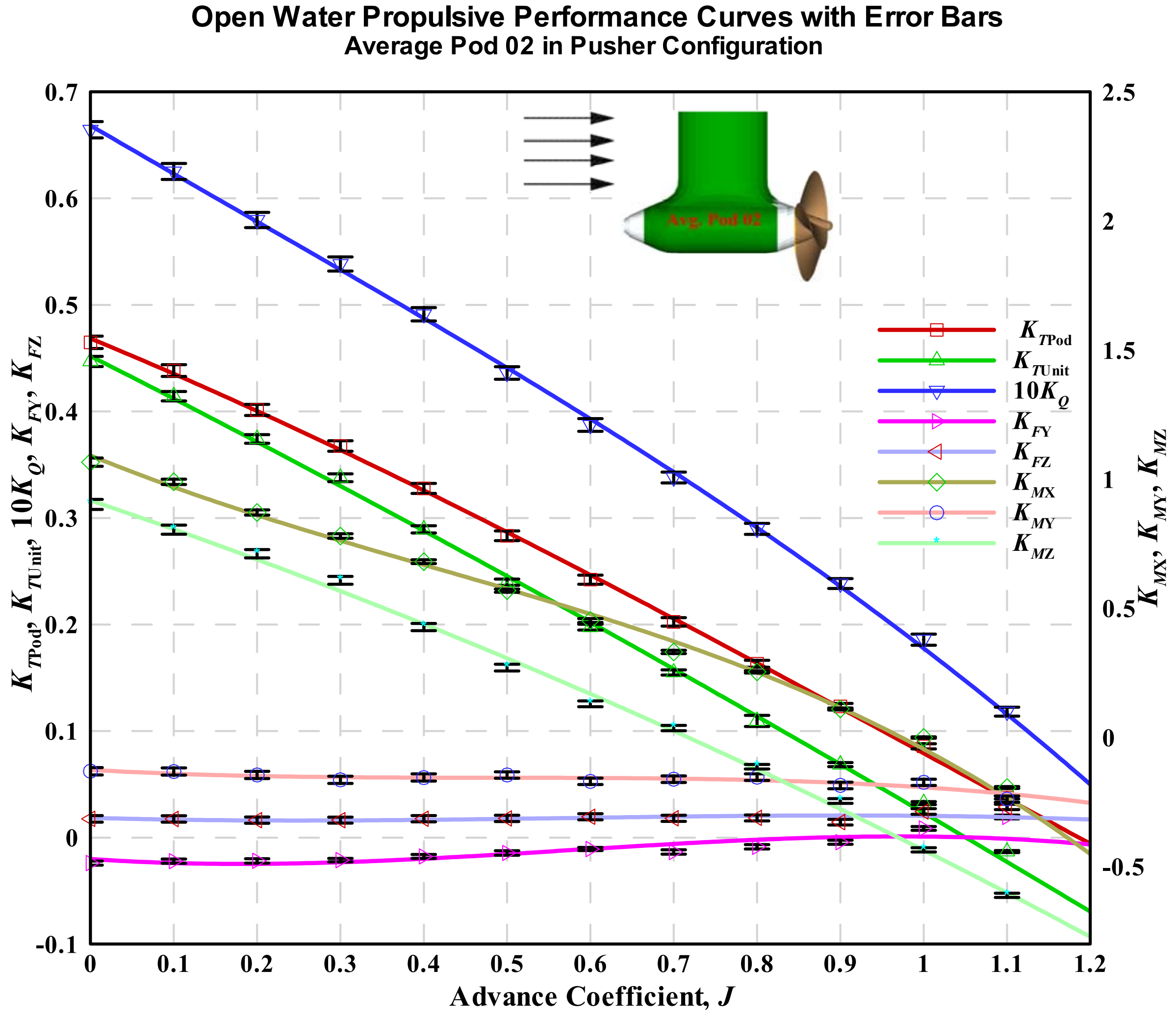


Figure 2-5: Performance curves for average pod 02 in pusher configuration with uncertainty (error) bars.

From Figure 2-2 to 2-5, it is observed that the curves fitted to the data lie inside the error bars. Therefore, the fitted curves provide a good representation of the trends indicated by the results.

2-4 Discussions on Uncertainty:

The podded propulsor tests were conducted using a custom designed (NSERC-NRC) pod dynamometer system. The uncertainty analysis results of this system were compared to that of a very high quality, well-established equipment (tests done in the IOT towing tank) used to measure the performance of some bare podded propellers and using the NSERC-NRC dynamometer to conduce podded propulsor tests (Taylor 2006) similar to the current experiments. The uncertainties of the bare propeller tests results were used as a benchmark to compare the uncertainty levels of the new dynamometer system. When the results provided in Table 2-4 were compared with the corresponding results given in Taylor 2006 for the bare propellers and the pod test results, it was seen that the podded propulsor tests using the NSERC-NRC pod instrumentation in the current phase of tests provided the level of accuracy comparable with the established equipment. The uncertainty levels observed in the propeller thrust and unit thrust in the podded propulsor tests were found to be higher than the thrust uncertainty for the baseline tests, but less than the corresponding uncertainties in the pod tests as given in Taylor (2006).

It can be seen in Table B-1 in Appendix B that for majority of the cases, the primary element of the uncertainty of the performance coefficients was the bias error (90% or more on the total uncertainty). To reduce the overall uncertainty in the final results, the primary focuses should be to reduce the bias error in the equipment. Each individual variable in Table B-1 should be examined for possible ways to improve the bias errors.

Given the high degree of accuracy found in the temperature, density, propeller diameter, azimuthing angle, shaft speed and advance speed, these variables have not been given any further consideration for improvement. It can be seen from Table B-1 that the major component influencing the bias limits of propeller thrust, torque and unit thrust for the podded propulsor tests was the Curve-Fit Error. As suggested in Taylor (2006), the SEE analysis was incorporated into the calibration procedure and the error was reduced substantially (compare corresponding results in for the uncertainty of the pod tests as given in Taylor, 2006). The calibration of the propeller thrust and torque measurement gauges were repeated 5-8 times and SEE analysis to the results determined whether or not a curve-fit was acceptable and ascertained the functionality of the equipment. The uncertainty levels of the propeller torque were less than the baseline propellers or the pod tests as given in Taylor

2006. This is primarily because of less weight and curve-fit errors. The uncertainty level of the unit thrust was less than the pod test results as given in Taylor (2006). This is primarily because of the different calibration approaches.

One possible approach to improve accuracy of the uncertainties in propeller thrust and torque of the podded propulsor tests is to run experiments at higher shaft speeds. As identified in Table B-1, for the torque readings of the podded propeller experiments, there is no one dominant factor influencing the overall uncertainty. Despite low error levels in the variables in the torque uncertainty expression, the magnitudes of the actual test measurements were small which resulted in larger overall error. At higher shaft speeds, higher advance speeds will be required to achieve the desired advance coefficients. Under these conditions the magnitudes of the thrust and torque will be larger relative to the uncertainty levels. Correspondingly, the percent error for each of these measured variables would be reduced, which results in less overall uncertainty in the thrust and torque coefficients.

Chapter 3

Concluding Remarks

The calculated uncertainty levels in the podded propulsor experiments were found to be comparable with those determined by Taylor (2006) in similar pod tests. The custom-made podded propulsor dynamometer system demonstrated the capability of achieving uncertainty levels close to those of the established equipment.

For majority of the cases, the primary element of the uncertainty of the performance coefficients was the bias error (90% or more on the total uncertainty). To reduce the overall uncertainty in the final results, the primary focuses should be to reduce the bias error in the equipment.

Given the high degree of accuracy found in the temperature, density, propeller diameter, azimuthing angle, shaft speed and advance speed, these variables have not been given any further consideration for improvement. The major component influencing the bias limits of propeller thrust, torque and unit thrust for the podded propulsor tests was the Curve-Fit Error. The Standard Error Estimate (SEE) analysis was incorporated into the calibration procedure and the error was reduced substantially.

One possible approach to improve accuracy of the uncertainties in propeller thrust and torque of the podded propulsor tests is to run experiments at higher shaft speeds. At higher shaft speeds, higher advance speeds will be required to achieve the desired advance coefficients. Under these conditions the magnitudes of the thrust and torque will be larger relative to the uncertainty levels. Correspondingly, the percent error for each of these measured variables would be reduced, which results in less overall uncertainty in the thrust and torque coefficients.

The uncertainty analysis results provided strong evidence that the experimental data obtained using the NSERC-NRC dynamometer system presented the true performance characteristics of the model scale podded propulsors under consideration.

References

Bose N and Luznik, L, (1996), "Uncertainty Analysis in Propeller Open Water Tests", International Shipbuilding Progress, Vol. 43, no. 435, pp. 237-246.

Coleman, H W and Steele, W G, (1999), "Experimentation and Uncertainty Analysis for Engineers", Wiley Interscience.

Hess, D.E., Nigon, R.T., Bedel, J.W., (2000), "Dynamometer Calibration and Usage", Research and Development Report No. NSWCCD-50-TR-2000/040, Hydromechanics Directorate, Carderrock Division, Naval Surface Warfare Center, West Bethesda, Maryland, 31p.

Islam M. F. (2006a) "Experimental Results: Open Water Performance of Puller Podded Propulsors with Varied Geometry", Ocean Engineering Research Center (OERC) Report No. OERC-2006-02, St. John's, NL, Canada, 109p.

Islam M. F. (2006b) "Experimental Results: Open Water Performance of Pusher Podded Propulsors with Varied Geometry", Ocean Engineering Research Center (OERC) Report No. OERC-2006-03, St. John's, NL, Canada, 100p.

Islam M. F. (2006c) "Calibration and Usages of NSERC-NRC Pod Dynamometer System", Ocean Engineering Research Center (OERC) Report No. OERC-2006-04, St. John's, NL, Canada, 76p.

ITTC Quality Manual – Recommended Procedures, (2002), "Propulsion, Performance - Podded Propeller Tests and Extrapolation", 7.5-02-03-01.3, Revision 00.

ITTC Quality Manual – Recommended Procedures, (2002), "Propulsion, Propulsor Uncertainty Analysis, Example for Open Water Test", 7.5-02-03-02.2, Revision 00.

Macneill, A., Taylor, R., Molloy, S., Bose, N., Veitch, B., Randell, T. and Liu, P., (2004), "Design of Model Pod Test Unit", Proceedings of the 1st International Conference on Technological Advances in Podded Propulsion, Newcastle University, UK, April, pp. 447-458.

Taylor R (2006), "Experimental Investigation of the Influence of Hub Taper Angle on the Performance of Push and Pull Configuration Podded Propellers", Master in Engineering, Memorial University of Newfoundland, Canada, 150p.

Appendix A:
Particulars of the Pods

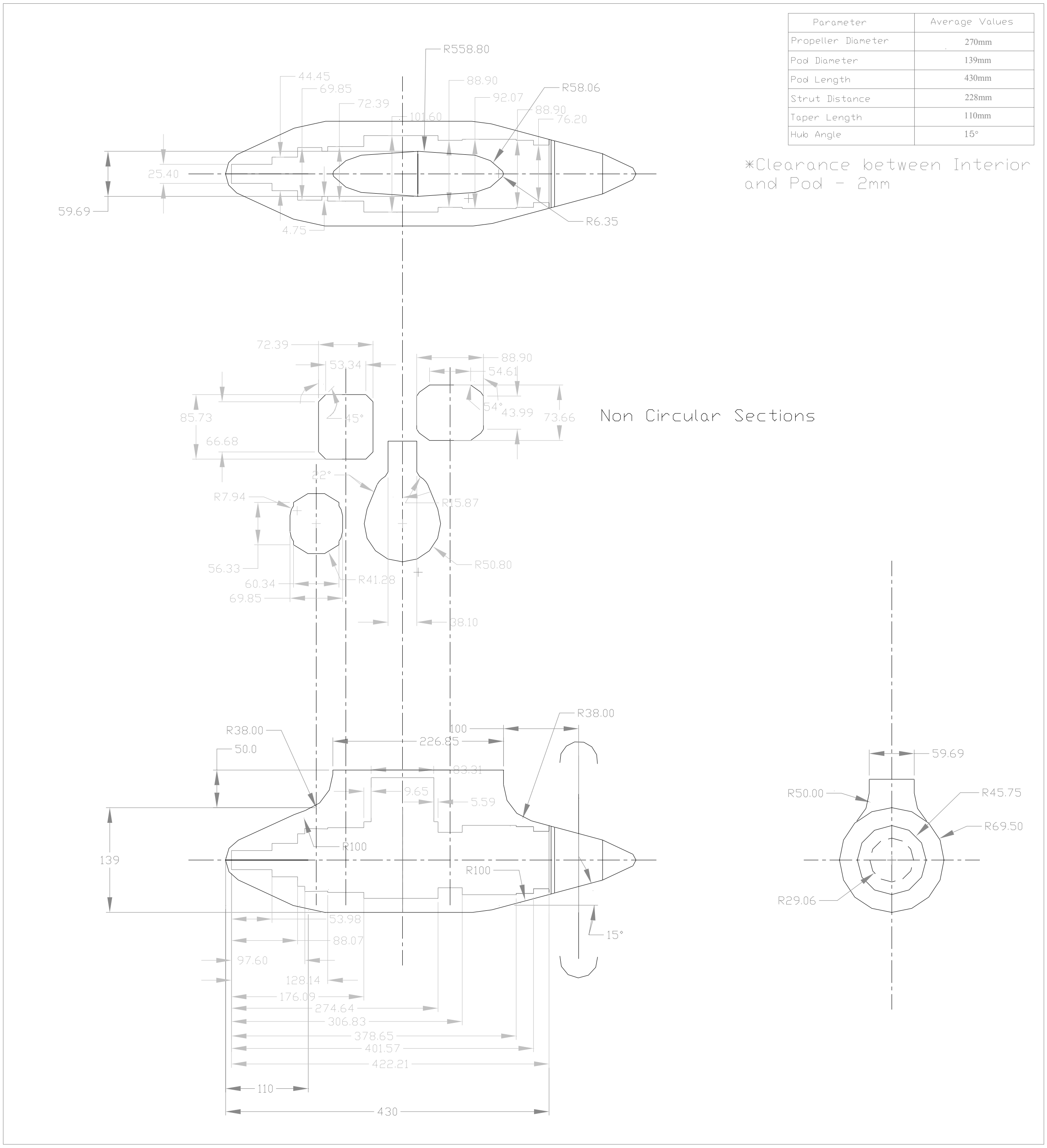
Particulars of Podded Propulsor Model Pods in Puller Configurations

Table A-1: Geometric particulars of Average Pod 01

Item Name	Symbol	Value	Unit
Propeller Diameter	D	0.270	m
Pod Diameter	D_{Pod}	0.139	m
Pod length	L_{Pod}	0.430	m
Pod Fore Taper Length	$L_{\text{Pod\_Fore}}$	0.085	m
Pod Aft Taper Length	$L_{\text{Pod\_Aft}}$	0.125	m
Pod Fore Taper Angle	$\theta_{\text{Pod\_Fore}}$	15	Deg.
Pod Aft Taper Angle	$\theta_{\text{Pod\_Aft}}$	25	Deg.
Pod-Strut Intersection Fillet Diameter	$R_{\text{P-S}}$	0.050	m
Strut Height	H_{Strut}	0.300	m
Strut Width	W_{Strut}	0.060	m
Strut Chord length	L_{Strut}	0.225	m
Strut Perimeter	P_{Strut}	0.484	m
Strut Distance	D_{Strut}	0.100	m

Table A-2: Average Performance characteristics of the pods in the pod series in puller configuration.

Advance Coeff. J	Carriage Speed V_A, m/s	Temp T deg.	Water Density ,ρ Kg/m^3	Shaft Speed n, rps	Average Propeller Thrust T_{Prop}, N	Average Propeller Torque Q, Nm	Average Unit Thrust T_{Unit}, N	Average Side Force F_Y, N	Average Vertical Force F_Z N	Average Axial Moment M_X Nm	Average Vertical Moment M_Y Nm	Average Steering Moment M_Z Nm
0.00	0.000	15.4	999.24	11.033	310.8529	11.8576	301.4786	20.3854	11.3666	158.5785	50.6204	331.539
0.10	0.297	15.5	999.24	11.036	290.0934	11.1791	281.4357	19.9165	12.3610	142.4916	49.9539	314.330
0.20	0.594	15.5	999.24	11.035	265.4980	10.5078	255.7397	18.8391	12.4948	126.4383	50.5798	292.554
0.30	0.891	15.8	999.24	11.037	240.1873	9.6000	229.0800	17.5256	12.4301	107.3857	52.8674	270.956
0.40	1.188	15.4	999.24	11.036	211.5920	8.8105	202.8701	16.9327	11.6735	94.3510	43.8099	250.175
0.50	1.485	15.4	999.24	11.039	183.3350	7.8058	174.1548	14.8987	12.0259	74.1375	48.3446	222.735
0.60	1.782	15.7	999.24	11.041	156.0190	7.0048	145.9534	12.6626	11.0506	59.8122	44.5884	198.180
0.65	1.931	15.7	999.24	11.040	144.6790	6.5060	131.0497	11.3449	10.5738	53.0111	41.6186	183.633
0.70	2.079	15.7	999.24	11.043	132.5470	6.1718	117.8805	11.0957	10.8779	45.3079	39.9235	176.561
0.75	2.228	15.8	999.24	11.046	117.1245	5.6068	103.6192	10.7944	10.6448	38.7896	35.3150	165.391
0.80	2.376	15.8	999.24	11.049	105.7160	5.2502	90.3858	9.9636	10.2967	31.8944	32.6700	154.156
0.85	2.525	15.8	999.24	11.054	91.2550	4.5673	76.3814	9.5771	10.2599	23.4676	28.4462	144.856
0.90	2.673	15.4	999.24	11.055	76.2420	4.0944	61.7347	8.9783	10.4554	17.8848	23.6824	134.354
0.95	2.822	15.4	999.24	11.052	62.0110	3.4328	47.7304	8.2458	10.4084	9.4350	20.9195	125.028
1.00	2.970	15.6	999.24	11.062	48.8660	2.9998	32.0558	7.7362	10.4095	0.4147	15.5071	114.338
1.10	3.267	15.6	999.24	11.055	18.5660	1.6719	2.1842	7.2081	10.8362	-11.9760	2.5564	95.281
1.20	3.564	15.5	999.24	11.065	-15.0680	0.0987	-31.7346	5.1084	10.8193	-26.7699	-11.6867	73.670

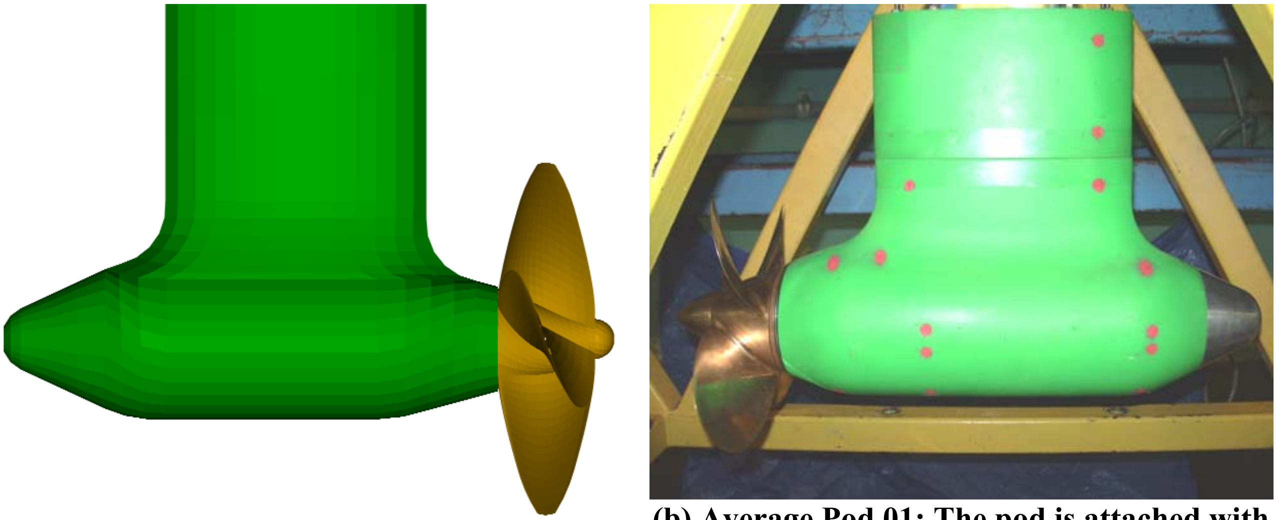


Parameter	Average Values
Propeller Diameter	270mm
Pod Diameter	139mm
Pod Length	430mm
Strut Distance	228mm
Taper Length	110mm
Hub Angle	15°

*Clearance between Interior and Pod - 2mm

Non Circular Sections

Figure A-1: The 2D drawing of the Average Pod 01.



(a) Average Pod 01: Computer generated rendered model.

(b) Average Pod 01: The pod is attached with the NSERC-NRC dynamometer before the test started (Side View).

(c) Average Pod 01: The pod is attached with the NSERC-NRC dynamometer before the test started (Front View).

(D) Average Pod 01: The pod model before it was attached to the measurement system.

Figure A-2: The Average Pod 01 physical model attached to the dynamometer system.

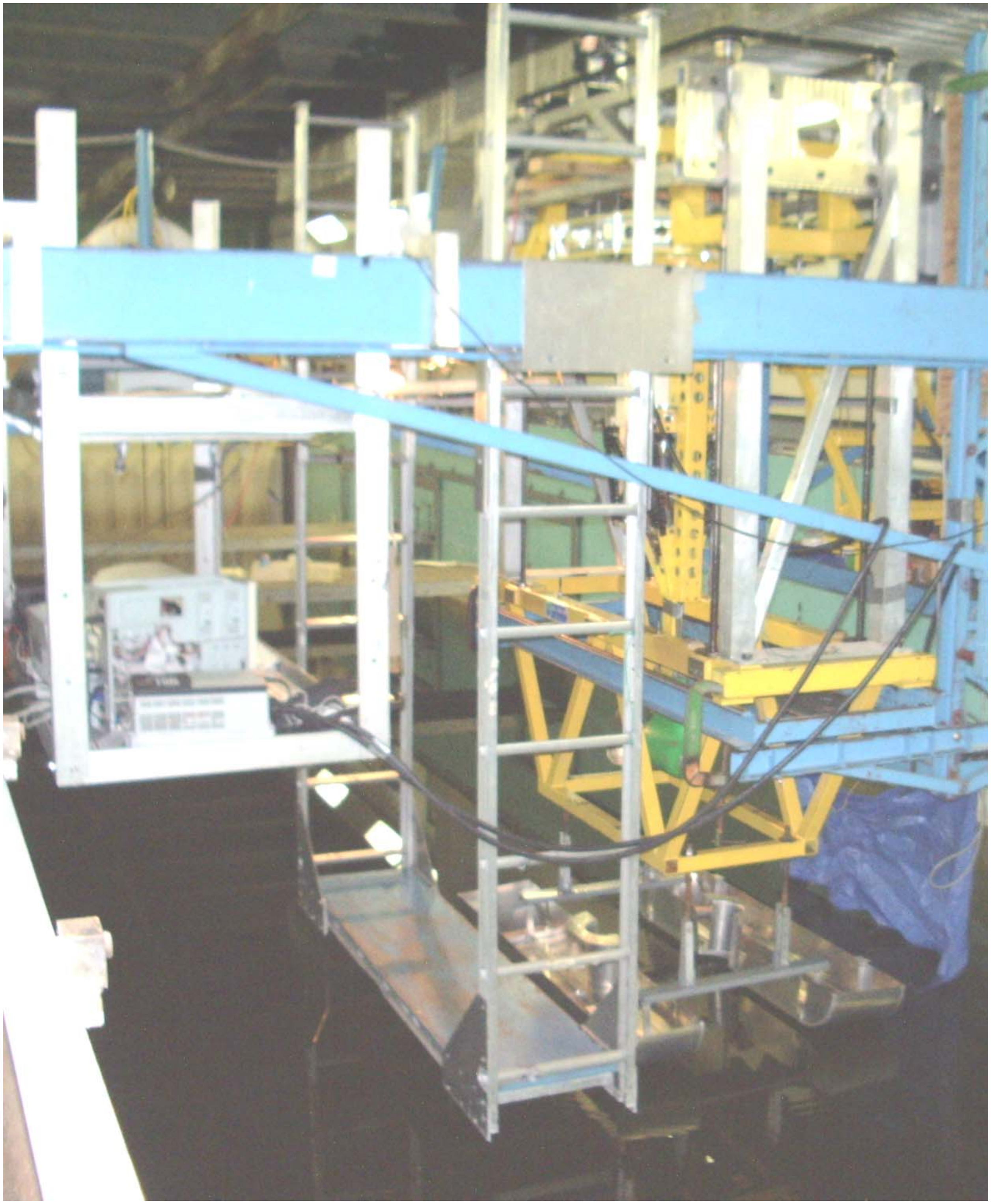


Figure A-3: Major component of the NSERC-NRC Pod Dynamometer system installed in the Towing Tank.

Appendix B
Details of Uncertainty Analysis

Bias Limits for Temperature/Density/Viscosity

Temperature Error

Lowest measured temperature	15.4 °C
Highest measured temperature	15.8 °C
Average Temperature	15.6 °C

Calibration Error

The digital thermometer used in the experimentation has been calibrated by the manufacturer and has a guaranteed accuracy of +/- 0.2 °C and therefore the assumed calibration error is as follows:

B_{Temp_Cal} +/- 0.2 °C

Scale Error

The digital thermometer has scale accuracy up to 0.0010 °C. A reading error is assumed to be half of this error:

B_{Temp_Scale} +/- 0.0005 °C

Temperature Range Error

The range of test temperature observed during testing as different tank position was from 12.8-13.2 °C. As suggested by Bose and Luznik (1996), the precision error for temp range is half of the range:

B_{Temp_Range} +/- 0.2 °C

These error estimates are fossilized into a single temperature reading estimate using the methods of Coleman and Steele (1999):

$$\mathbf{B_{Temp}=(B_{Temp\_Cal}^{2}+B_{Temp\_Scale}^{2}+B_{Temp\_Range}^{2})^{1/2}}$$

B_{Temp} +/- 0.2828 °C

Density Error

Calibration error

The density-temperature expression given in ITTC 7.5-02-03-02.2 for the curve fit to the table given in ITTC Procedure 7.5-02-01-03 Rev 00 'Density and Viscosity of Water' for g=9.81 can be expressed as

$$\rho=1000.1+0.00552t-0.0077t^2+0.00004t^2$$

The derivative of the above expression gives the following

$$\left|\frac{\partial\rho}{\partial t}\right| = \left|0.0552 - 0.154t + 0.000120t^2\right|$$

Based on the above expression the calibration error of density as dependent on temperature is

B_{ρ_Cal}	0. 04407	Kg/m^3

Data Reduction Error

The error introduced when converting the temperature to a density can be calculated as two times the SEE of the curve fit to the density/temperature values for the whole temperature range. Comparing the tabulated value with the calculated value (expression) the bias error for data reduction can be calculated as follows:

B_{ρ_DataR}	0.070	Kg/m^3

Conceptual error

The nominal density according to the ITTC-1978 is ρ=1000 Kg/m^3. In the data analysis ρ =999 Kg/m^3 was assumed. Using this value introduces a bias limit as the difference between ρ (15.6 °C) =999.007 Kg/m^3 and ρ =999 Kg/m^3 such that

B_{ρ_Conc}	0.007	Kg/m^3

These components of the density expression error may be combined to give

$$\mathbf{B_{\rho\_Eqn}=(B_{\rho\_Cal}^2 + B_{\rho\_DataR}^2 + B_{\rho\_Conc}^2)^{1/2}}$$

$\mathbf{B_{\rho\_Eqn}}$	**0.0830**	$\mathbf{Kg/m^3}$

Temperature Related Density Error

Temperature related density errors may be calculated by considering the density at the lower and upper limits of difference between the average temperature and the fossilized temperature errors:

	Temp	Density
at T=(15.6+0.28284)	15.88284	999.1946 Kg/m3
at T=(15.6-0.28284)	15.31716	999.2827 Kg/m3
Density Difference	0.088153	Kg/m3

The bias limit for this is taken as half of the difference of the density over this temperature range:

B_{ρ_Temp}	0.0441	Kg/m^3

The two contributing elements of error may be combined to give the overall limit using RSS:

$$\mathbf{B_\rho=(B_{\rho\_Eqn}^2+B_{\rho\_Temp}^2)^{1/2}}$$

B_ρ	**0.0939**	**Kg/m^3**

Viscosity Error

Calibration Error

Viscosity is a temperature dependent parameter. For the viscosity, the assumed viscosity-temperature relationship adopted by ITTC Procedure 7.5-02-01-03 has been used. This is given as

$$\nu=(1.72256-0.0476t+0.000585t^2)10^{-6}$$

Taking the partial derivative of this expression with respect to the temperature give;

$$\left|\frac{\partial \nu}{\partial t}\right|=\left|-0.0476-0.00117t\right|$$

Based on the test temperature, the bias in viscosity due to error in temperature calibration is:

B_{ν_Cal}	1.36278E-08	m^2/s

Data Reduction Error

Since the above expression is representative of a curve fit to a table of data, there is an inherent error that may be estimated by taking the difference of the viscosity value calculated using the expression and the viscosity value given in the tables of the ITTC procedure 7.5-02-01-03. This bias error may be estimated as follows:

B_{ν_DataR}	4.15E-10	m^2/s

These components of the viscosity expression error may be combined to give

$$\mathbf{B_\nu=(B_{\nu\_Cal}^2+B_{\nu\_DataR}^2)^{1/2}}$$

B_ν	**1.36341E-08**	**m^2/s**

Propeller Diameter Error

Bias Error

The errors in the propeller geometry due to error in the CNC machining process is estimated as

B_{D_CNC}	+/-	0.0001 m

The propeller blades were hand finished to get the desired surface finish. This error is estimated as:

B_{D_Polish}	+/-	0.0001 m

Based on the above findings the bias limit for the propeller diameter is estimated using RSS of the two components:

$$B_D = (B_{D\_CNC}^2 + B_{D\_Polish}^2)^{1/2}$$

B_D 0.000141 m

Azimuthing Angle Error

Bias Error

Radius of the Azimuthing angle-defining Disk 0.494225 m

The errors in the circular disk geometry due to error in the CNC machining process is estimated as

B_{AA_CNC}	+/- 0.0001 m	0.000202 rad

The disk plate were hand finished to get the desired surface finish. This error is estimated as:

B_{AA_Polish}	+/- 0.001 m	0.002023 rad

The allowance at each azimuthing angle defining holes were:

$B_{AA\_allowance}$	+/- 0.004 m	0.008093 rad

Test Condition Errors

The errors in the Azimuthing angle during testing was calculated as follows:

Equipment Positioning Error

Errro in Equipment Angle B_{AA_EPA}	+/- 0.5 deg	0.00873 rad

Based on the above findings the bias limit for the Azimuthing Angle is estimated using RSS of the two components:

$$B_{AA} = (B_{AA\_CNC}^2 + B_{AA\_Polish}^2 + B_{AA\_Allowance}^2 + B_{AA\_EPA}^2)^{1/2}$$

B_{AA} 0.012077 **rad**

B_{AA} 0.691964 **Deg**

Shaft Speed Error

Bias Error in Shaft Speed

Calibration Error in the Tachometer

When considering the data associated with the calibration of the equipment, the first step is to map out the process and identify sources of error in that process. During calibration of the shaft speed (Tech Generator) sensor, the increase of shaft speed is accomplished by increasing motor speed through a computer operated motor controller. The rps was measured using the tachometer corresponding to each motor speed in Hz

The tachometer used in the experimentation has been calibrated by the manufacturer and has a guaranteed accuracy of +/- 0.5 rpm and therefore an assumed bias limit for calibrating the shaft speed sensor is as follows:

$B_{n\_techcalib}$ +/- 0.008 rps

The error in this measured rps using the tachometer may be estimated ad follows

Techo, rps	0.000	0.750	1.717	3.183	4.200	5.700	7.217	7.717	8.983	10.217	12.700
error, rps	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008

Total rps	12.7	rps
Error in rps	0.026533	rps
B_{n_techo}	0.026533	rps

A/D Card Error

Since the calibration data is also acquired using an AD card, an error due to this conversion also occurs and is calculated as follows:

AD Card Resolution	12	bits
Ad Card Voltage Range	10	Volts
AD Card Error	1	bits

B_{n_ADConv} 1/2(10v/4096) V x slope of the calibration curve

B_{n_ADConv} 0.0012207 V

0.0078734 rps

Curve Fit Error

The calibration data is used to establish the propeller rotational speed-voltage relationship from a curve fit of the data which introduces an additional error, estimated using SEE as follows:

Data #	Load set Motor Hz	Load Set Techo, rpm	Load Set Techo, rps	rps Prop Volt Avg	Predict rps	Y-Y'	(Y-Y')^2
1	0.00	0.00	0.000000	0.003734	-0.00398	0.0040	0.0000
2	3.00	45.00	0.750000	-0.109563	0.726771	0.0232	0.0005
3	7.00	103.00	1.716667	-0.263933	1.722439	-0.0058	0.0000
4	13.00	191.00	3.183333	-0.495708	3.21737	-0.0340	0.0012

5	17.00	252.00	4.200000	-0.650237	4.214065	-0.0141	0.0002
6	23.00	342.00	5.700000	-0.882365	5.711268	-0.0113	0.0001
7	29.00	433.00	7.216667	-1.114659	7.209541	0.0071	0.0001
8	31.00	463.00	7.716667	-1.192140	7.709283	0.0074	0.0001
9	36.00	539.00	8.983333	-1.385691	8.95767	0.0257	0.0007
10	41.00	613.00	10.216667	-1.579685	10.20891	0.0078	0.0001
11	51.00	762.00	12.700000	-1.967076	12.70754	-0.0075	0.0001
12	29.00	433.00	7.216667	-1.116802	7.223361	-0.0067	0.0000
13	17.00	253.00	4.216667	-0.652536	4.228893	-0.0122	0.0001
14	3.00	45.00	0.750000	-0.111126	0.736849	0.0132	0.0002
15	0.00	0.00	0.000000	0.003730	-0.00396	0.0040	0.0000
Sum						0.0006	0.0033
SEE	rps						0.0160
			B_{n_CF}	0.032038 rps			

Test Conditions Errors

Since the shaft speed test data is also acquired using an AD card, an error due to this conversion also occurred and is calculated as follows:

AD Card Resolution	12	bits
Ad Card Voltage Range	10	Volts
AD Card Error	1	bits
Bn_ADConv 1/2(10v/4096)	V	times the slope of the rps vs volt cal curve

B_{n_ADConv} 0.0012207 V

0.0078734 rps

Static Zero Error

Errors in the static zero of the shaft speed affect the test condition reading and is included in the following way:

$B_{n\_StaticZero}$ 0.0037341 V

0.0240847 rps

Total Shaft Speed Bias Error

$$B_n = (B_{n\_techcalib}^2 + B_{n\_techo}^2 + B_{n\_ADConv}^2 + B_{n\_CF}^2 + B_{n\_ADConv}^2 + B_{n\_StaticZero}^2)^{1/2}$$

B_n 0.0499848 rps

Precision Error in Shaft Speed

The precision limit for the shaft speed was estimated by collecting at least 10 sets of data at each of the advance coefficient tested and then applying the standard deviation theory as suggested in Bose and Luznik (1996):

Precision Limit Table

	J=0.00	**J=0.1**	**J=0.2**	**J=0.3**	**J=0.4**	**J=0.5**	**J=0.6**	**J=0.7**	**J=0.8**	**J=0.9**	**J=1.0**	**J=1.1**	**J=1.2**
Read	***n***	***n***	***n***	***n***	***n***	***n***	***n***	***n***	***n***	***n***	***n***	***n***	***n***
#	rps	rps	rps	rps	rps	rps	rps	rps	rps	rps	rps	rps	rps
1	11.0655	11.0055	11.0055	11.0055	11.0055	11.0055	11.0055	11.0055	11.0055	11.0055	11.0055	11.0055	11.0055
2	11.0136	11.0136	11.0136	11.0136	11.0136	11.0136	11.0136	11.0136	11.0136	11.0136	11.0136	11.0136	11.0136
3	11.0406	11.0132	11.0073	11.0132	11.0083	11.0134	11.0088	11.0113	11.0070	11.0082	11.0097	11.0099	11.0102
4	11.0346	11.0133	11.0103	11.0135	11.0103	11.0135	11.0124	11.0120	11.0099	11.0102	11.0106	11.0102	11.0120
5	11.0387	11.0133	11.0081	11.0134	11.0089	11.0135	11.0099	11.0116	11.0082	11.0086	11.0100	11.0101	11.0102
6	11.0364	11.0133	11.0084	11.0134	11.0092	11.0135	11.0105	11.0118	11.0086	11.0100	11.0103	11.0102	11.0106
7	11.0377	11.0133	11.0082	11.0134	11.0090	11.0135	11.0104	11.0117	11.0084	11.0088	11.0101	11.0101	11.0105
8	11.0372	11.0133	11.0082	11.0134	11.0092	11.0135	11.0105	11.0117	11.0085	11.0091	11.0101	11.0102	11.0106
9	11.0375	11.0133	11.0082	11.0134	11.0091	11.0135	11.0104	11.0117	11.0084	11.0089	11.0101	11.0101	11.0105
10	11.0372	11.0133	11.0082	11.0134	11.0091	11.0135	11.0105	11.0117	11.0084	11.0089	11.0101	11.0102	11.0106
Mean:	11.0379	11.0125	11.0086	11.0126	11.0092	11.0127	11.0103	11.0113	11.0086	11.0092	11.0100	11.0100	11.0104
StDev	0.0124	0.0025	0.0021	0.0025	0.0020	0.0025	0.0021	0.0021	0.0021	0.0020	0.0019	0.0019	0.0020
Mean, ri=(1/10)*sum(r1;;r10)													
	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2
1	0.0008	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
2	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006
3	0.0000	0.0006	0.0009	0.0006	0.0009	0.0006	0.0008	0.0007	0.0010	0.0009	0.0008	0.0008	0.0008
4	0.0000	0.0006	0.0008	0.0006	0.0008	0.0006	0.0006	0.0007	0.0008	0.0008	0.0007	0.0008	0.0007
5	0.0000	0.0006	0.0009	0.0006	0.0008	0.0006	0.0008	0.0007	0.0009	0.0009	0.0008	0.0008	0.0008
6	0.0000	0.0006	0.0009	0.0006	0.0008	0.0006	0.0007	0.0007	0.0009	0.0008	0.0008	0.0008	0.0007
7	0.0000	0.0006	0.0009	0.0006	0.0008	0.0006	0.0008	0.0007	0.0009	0.0008	0.0008	0.0008	0.0008
8	0.0000	0.0006	0.0009	0.0006	0.0008	0.0006	0.0008	0.0007	0.0009	0.0008	0.0008	0.0008	0.0007
9	0.0000	0.0006	0.0009	0.0006	0.0008	0.0006	0.0008	0.0007	0.0009	0.0008	0.0008	0.0008	0.0007
10	0.0000	0.0006	0.0009	0.0006	0.0008	0.0006	0.0008	0.0007	0.0009	0.0008	0.0008	0.0008	0.0007
Sum	0.0014	0.0065	0.0086	0.0064	0.0083	0.0064	0.0077	0.0071	0.0086	0.0083	0.0078	0.0078	0.0076
Precision Index of Population: [(1/9)*sum(r-ri)]^0.5													
Sr	0.0124	0.0269	0.0310	0.0267	0.0303	0.0267	0.0292	0.0281	0.0309	0.0303	0.0295	0.0295	0.0290
Precision Index of Mean: Sr/(10)^0.5													
Srm	0.0039	0.0085	0.0098	0.0085	0.0096	0.0084	0.0092	0.0089	0.0098	0.0096	0.0093	0.0093	0.0092
Precision Limit of Average Results: nt*Srm													
P_n	0.0086	0.0187	0.0216	0.0186	0.0211	0.0186	0.0203	0.0196	0.0215	0.0211	0.0205	0.0205	0.0202
Where t=2.262 (value taken from t distribution for 95% confidence coverage for degree of freedom 10-1=10													

Another approach is to use the standard deviation approach for each set of data calculated according to the expression given in ITTC QM 7.5-02-01-01. As outlined in this method, the precision limit is then estimated using:

$$P_i=t*S_i$$

This results in

P_n	0.0344	0.0043	0.0046	0.0061	0.0056	0.0045	0.0049	0.0056	0.0046	0.005	0.0046	0.0052	0.0054

The total bias error and the total precision error were then combined using RSS to get the total uncertainty in shaft speed as follows:

$$U_n^2 = B_n^2 + P_n^2$$

	J=0.00	J=0.1	J=0.2	J=0.3	J=0.4	J=0.5	J=0.6	J=0.7	J=0.8	J=0.9	J=1.0	J=1.1	J=1.2
B_n	0.04998	0.04998	0.04998	0.04998	0.04998	0.04998	0.04998	0.04998	0.04998	0.04998	0.04998	0.04998	0.04998
P_n	0.0086	0.0187	0.0216	0.0186	0.0211	0.0186	0.0203	0.0196	0.0215	0.0211	0.0205	0.0205	0.0202
U_n	0.051	0.0513	0.0514	0.0512	0.0511	0.0512	0.0512	0.052	0.0512	0.0515	0.0512	0.0516	0.0512

Advance Speed

The specifications of the carriage speed measuring equipment, as taken from the carriage user manuals and from the work of Bose and Luznik (1996) at the MUN towing tank facilities are:

Resolution	10000	Pulse/m
Diameter of Wheel	0.5	m
Max. Pulse Duration	2.00E-07	sec
Min pulse Duration	1.20E-07	sec
Max Output Signal	5	volts
Circuit Speed	100	ms
AD/DA Card	12	bits

Using methods based on an adaptation of ITTC recommended procedure 7.5-02-02-02 the following carriage speed error estimate has been founded by dividing the measurement system into components and estimating the elemental errors associated with each components:

Bias Error in Advance Speed

Information provided in the user manual and from the Manufacturer

B_{V_Cal} +/- 0.005 m/s

A/D Card Error

The 12 bit card with a full range from -5 to 5 volts the bias error is estimated as:

B_{V_AD} +/- 0.001221 volts/unit

0.000959 m/s

The bias associated with the diameter of the wheel is specified by the manufacturer is

B_{V_Wheel} +/- 0.0001 m

Tide Error

During testing, the pod unit stirs up the tank and a small amount of remaining circulation is estimated based on the experience of Bose and Luznik (1996) to produce an error of

B_{V_Tide} +/- 0.001 m/s

Curve Fit Error

A data reduction error is introduced when the voltage signal is converted back to a frequency that can be used to calculate speed. This bias limit is introduced by the linear curve fit obtained from a set of calibration data that is applied to that measured data to allow this conversion. This is estimated using the standard error estimate and based on the ITTC guidelines as shown below:

Data Point	Load set m/s, Y	Load Set Volt Avg.	Predict m/s, Y'	Y-Y'	(Y-Y')^2
1	0.00	0.00	-0.00491	0.0049	0.0000
2	0.79	1.02	0.797382	-0.0074	0.0001
3	1.20	1.54	1.201143	-0.0011	0.0000
4	1.60	2.05	1.603732	-0.0037	0.0000
5	1.80	2.29	1.792352	0.0076	0.0001
Sum					0.0002
SEE	m/s				0.0071

B_{V_CF} 0.014253312 m/s

Test Conditions Errors

Since the carriage speed test data is also acquired using an AD card, an error due to this conversion also occurred and is calculated as follows:

AD Card Resolution	12	bits
Ad Card Voltage Range	10	Volts
AD Card Error	1	bits

BV_ADConv 1/2(10v/4096) V times the slope of the rps vs volt cal curve

B_{V_ADConv} 0.001221 V

0.000959 m/s

Static Zero Error

Errors in the static zero of the shaft speed affect the test condition reading and is included in the following way

$B_{V\_StaticZero}$ 0.003038 V

0.002387 m/s

Total Shaft Speed Bias Error

$$B_V=(B_{V\_Cal}^{2} + B_{V\_AD}^{2}+ B_{V\_Wheel}^{2}+ B_{V\_Tide}^{2}+ B_{V\_CF}^{2}+ B_{V\_ADConv}^{2}+B_{V\_StaticZero}^{2})^{1/2}$$

B_V 0.015385 m/s

Precision Error in Advance Speed

The precision limit for the Advance speed was estimated by collecting at least 10 sets of data at each of the advance coefficient tested and then applying the standard deviation theory as suggested in Bose and Luznik (1996);

Read #	J=0.00 V_A m/s	J=0.1 V_A m/s	J=0.2 V_A m/s	J=0.3 V_A m/s	J=0.4 V_A m/s	J=0.5 V_A m/s	J=0.6 V_A m/s	J=0.7 V_A m/s	J=0.8 V_A m/s	J=0.9 V_A m/s	J=1.0 V_A m/s	J=1.1 V_A m/s	J=1.2 V_A m/s
1	0.0003	0.2951	0.5943	0.8934	1.1939	1.4940	1.7939	2.0932	2.3932	2.6943	2.9917	3.2620	3.4976
2	-0.0001	0.2951	0.5948	0.8924	1.1942	1.4946	1.7927	2.0898	2.3907	2.6906	2.9951	3.2679	3.4914
3	0.0002	0.2951	0.5946	0.8926	1.1941	1.4944	1.7936	2.0923	2.3916	2.6918	2.9944	3.2677	3.4962
4	0.0002	0.2951	0.5947	0.8924	1.1941	1.4944	1.7930	2.0913	2.3914	2.6906	2.9950	3.2679	3.4939
5	0.0002	0.2951	0.5947	0.8926	1.1941	1.4944	1.7933	2.0916	2.3916	2.6916	2.9947	3.2678	3.4961
6	0.0002	0.2951	0.5947	0.8925	1.1941	1.4944	1.7930	2.0915	2.3914	2.6913	2.9950	3.2679	3.4954
7	0.0002	0.2951	0.5947	0.8926	1.1941	1.4944	1.7932	2.0915	2.3915	2.6914	2.9950	3.2678	3.4956
8	0.0002	0.2951	0.5947	0.8925	1.1941	1.4944	1.7931	2.0915	2.3915	2.6913	2.9950	3.2678	3.4954
9	0.0002	0.2951	0.5947	0.8925	1.1941	1.4944	1.7932	2.0915	2.3915	2.6913	2.9950	3.2678	3.4954
10	0.0002	0.2951	0.5947	0.8925	1.1941	1.4944	1.7931	2.0915	2.3915	2.6913	2.9950	3.2678	3.4954
Mean:	0.0002	0.2951	0.5947	0.8926	1.1941	1.4944	1.7932	2.0916	2.3916	2.6915	2.9946	3.2672	3.4952
StDev	0.0001	0.0000	0.0001	0.0003	0.0001	0.0001	0.0003	0.0008	0.0006	0.0010	0.0010	0.0018	0.0016

	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2
1	2.53E-08	3.42E-11	1.59E-07	6.93E-07	3.21E-08	1.64E-07	4.52E-07	2.65E-06	2.55E-06	7.44E-06	8.12E-06	2.74E-05	5.46E-06
2	9.94E-08	1.85E-10	6.76E-09	4.99E-08	1.27E-09	2.91E-08	2.39E-07	3.07E-06	7.12E-07	9.10E-07	2.76E-07	3.80E-07	1.50E-05
3	1.79E-09	2.96E-11	7.70E-09	1.36E-10	3.20E-10	7.26E-10	1.62E-07	5.63E-07	9.63E-10	5.74E-08	2.98E-08	2.44E-07	1.01E-06
4	7.51E-11	3.99E-12	5.83E-09	3.85E-08	3.23E-10	1.01E-09	5.85E-08	9.44E-08	3.73E-08	8.78E-07	1.94E-07	3.78E-07	1.77E-06
5	3.68E-10	3.52E-12	2.37E-09	7.01E-10	3.22E-10	8.40E-10	1.26E-08	3.96E-11	4.30E-10	4.41E-09	1.31E-08	2.87E-07	7.34E-07
6	2.61E-10	5.29E-14	3.19E-09	1.54E-08	3.23E-10	8.59E-10	4.09E-08	4.49E-09	2.84E-08	8.55E-08	1.60E-07	3.77E-07	1.92E-08
7	3.27E-10	1.58E-12	2.94E-09	2.30E-09	3.22E-10	8.54E-10	8.39E-12	3.86E-09	9.32E-09	2.14E-08	1.43E-07	3.42E-07	1.09E-07
8	2.72E-10	7.42E-15	3.16E-09	7.49E-09	3.23E-10	8.55E-10	1.50E-08	4.31E-09	1.03E-08	6.72E-08	1.56E-07	3.60E-07	2.61E-08
9	3.10E-10	8.13E-13	3.15E-09	4.41E-09	3.23E-10	8.54E-10	2.66E-09	4.31E-09	1.02E-08	4.91E-08	1.46E-07	3.43E-07	3.96E-08
10	3.10E-10	3.39E-13	3.16E-09	5.30E-09	3.23E-10	8.55E-10	5.87E-09	4.31E-09	1.03E-08	5.00E-08	1.50E-07	3.43E-07	2.75E-08
Sum	1.28E-07	2.59E-10	1.97E-07	8.17E-07	3.60E-08	2.00E-07	9.88E-07	6.40E-06	3.36E-06	9.56E-06	9.39E-06	3.04E-05	2.41E-05

Precision Index of Population: [(1/9)*sum(r-ri)]^0.5

Sr	1.19E-04	5.36E-06	1.48E-04	3.01E-04	6.32E-05	1.49E-04	3.31E-04	8.43E-04	6.11E-04	1.03E-03	1.02E-03	1.84E-03	1.64E-03

Precision Index of Mean: Sr/(10)^0.5

Srm	3.78E-05	1.70E-06	4.68E-05	9.53E-05	2.00E-05	4.71E-05	1.05E-04	2.67E-04	1.93E-04	3.26E-04	3.23E-04	5.81E-04	5.18E-04

Precision Limit of Average Results: nt*Srm

Pn	8.54E-05	3.84E-06	1.06E-04	2.15E-04	4.52E-05	1.07E-04	2.37E-04	6.03E-04	4.37E-04	7.37E-04	7.31E-04	1.32E-03	1.17E-03

Where t=2.262 (value taken from t distribution for 95% confidence coverage for degree of freedom 10-1=9

The total bias error and the total precision error were then combined using RSS to get the total uncertainty in advance speed as follows:

$$U_{V_A}^2 = B_{V_A}^2 + P_{V_A}^2$$

	J=0.00	**J=0.1**	**J=0.2**	**J=0.3**	**J=0.4**	**J=0.5**	**J=0.6**	**J=0.7**	**J=0.8**	**J=0.9**	**J=1.0**	**J=1.1**	**J=1.2**
B_{V_A}	0.1538	0.1538	0.1538	0.1538	0.1538	0.1538	0.1538	0.1538	0.1538	0.1538	0.1538	0.1538	0.1538
P_{V_A}	0.0001	0.0000	0.0001	0.0002	0.0000	0.0001	0.0002	0.0006	0.0004	0.0007	0.0007	0.0013	0.0012
U_{V_A}	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154

Propeller Thrust

The errors associated with the propeller thrust measurements was conveniently divided into three primary categories: Calibration errors, curve fit errors, and testing errors. The data acquisition errors influence the uncertainty levels of the calibration data, in the way that the data is used to generate the curves to convert the test data voltage signals into loading values. The data acquisition error actually influences the overall uncertainty in two ways, in that it first introduces uncertainty into the calibration data used to get the curve fits, and secondly the actual test data collected is subject to these uncertainties. Therefore, the errors associated with the data acquisition would be best treated as contributors to the error of both to the calibration and test data sets.

Calibration Error

When considering the data associated with the calibration of the equipment, the first step is to map out the process and identify sources of error in that process. Loading during calibration is accomplished by adding weights to a calibration fixture suspended from the pod unit. These weights were weighted on a scale accurate to +/- 0.1g. This error in these weights may be estimated as follows:

load cond	Load set kg	Error in Weight, Kg
1	0.0000	0.00001
2	3.1175	0.00001
3	2.5100	0.00001
4	5.0100	0.00001
5	7.5140	0.00001
6	7.5168	0.00001
7	7.0543	0.00001
8	4.5480	0.00001
Total Weight		37.2706 Kg
Error in Weight		2.82843E-05 Kg
B_{TPod_Cal}		0.000277469 N

Loading Angle Error

The weight pan used for calibrating the propeller thrust is a vertical fixture. There may be slight alignment errors due to misalignment of this fixture. The error may be estimated as follows:

Error in loading Angle +/- 0.5 deg	0.008730159	rad
Error in loading Angle Tpod(1-cosO)		
B_{TPod_LA}	0.011432	N

Load cell Alignment Error

For this instrumentation, it may be assumed that a small error may be attributed to the misalignment of the load cell inside the pod. This would have a slight influence on the thrust voltage signal generated and based on a adaptation of ITTC recommended procedure 7.5-02-02-02 the error is estimated as:

Error in Load cell alignment +/- 0.1 deg	0.001746 rad
Error in Loadcell alignment	Tpod(1-CosO)
BTPod_LCA	0.000457 N

In should be noted that the Tpod was assumed to be 300 N for convenience.

A/D Card Error

Since the calibration data is also acquired using an AD card, an error due to this conversion also occurs and is calculated as follows

AD Card Resolution	12	bits
Ad Card Voltage Range	10	Volts
AD Card Error	1	bits
$B_{TPod\_ADConvC}$	1/2(10v/4096)	V times the slope of the thrust calibration curve
$B_{TPod\_ADConvC}$	0.001220703	V
	0.604101563	N Page 133. Coleman and Steele (1999)

Curve Fit Error

The calibration data (Islam 2006c) is used to establish the force-voltage relationship from a curve fit of the data, which introduces an additional error, estimated using SEE as follows

Reading #	Add Weight kg	Total Load kg	App Thrust N, Y	Output Voltage	Predicted Load N, Y’	(Y-Y')^2
1	0.0000	0.4000	3.9240	-0.0894	3.4133	0.2608
2	0.7209	1.1209	10.9960	-0.1022	9.7134	1.6452
3	0.9963	2.1172	20.7697	-0.1273	22.1647	1.9458
4	1.0003	3.1175	30.5827	-0.1445	30.6596	0.0059
5	2.5100	5.6275	55.2058	-0.1956	55.9084	0.4937
6	5.0100	10.6375	104.3539	-0.2956	105.3448	0.9818
7	7.5140	18.1515	178.0662	-0.4453	179.3315	1.6008

8		16.6028	34.7543	340.9397	-0.7727	341.1044	0.0271
9		-9.5480	25.2063	247.2738	-0.5840	247.8618	0.3457
10		-9.5628	15.6435	153.4627	-0.3950	154.4498	0.9743
11		-5.0060	10.6375	104.3539	-0.2952	105.1022	0.5600
12		7.5140	18.1515	178.0662	-0.4453	179.3315	1.6008
13		16.6028	34.7543	340.9397	-0.7737	341.5975	0.4327
14		-9.5480	25.2063	247.2738	-0.5858	248.7439	2.1611
15		-9.5628	15.6435	153.4627	-0.3951	154.5200	1.1179
16		-10.0160	5.6275	55.2058	-0.1951	55.6517	0.1989
17		-3.5103	2.1172	20.7697	-0.1257	21.3686	0.3586
18		-1.7172	0.4000	3.9240	-0.0876	2.5100	1.9994
Total							16.7107
SEE	N						1.0220
			B_{TPod_CF}	2.0439 N			

Test Condition Errors

The errors in the propeller thrust during testing is broken down into elemental components as follows:

Equipment Positioning Error

Errro in Equipment Angle +/-	0.5 deg	0.00873	rad
B_{TPod_EPA}		0.011432	N

Again similar to the calibration error components, a small error occurs due to the misalignment of the load cell while acquidign data using the test equipment. This error is estimated as

Error in Loadcell alignment +/- 0.1 deg	0.001746	rad
Error in Loadcell alignment Tpod(1-CosO)		
B_{TPod_LCA}	0.000457	N

Static Zero Error

The static zero load error was observed and estimated as

B_{TPod_SZT}	0.00007195	V
B_{TPod_SZT}	0.03560532	N

Note: the zero load in thrust measurement was done by taking at least 40 sec of data with the shaft not turning. Between each measurements for calibration the shaft was moved in and out to get rid of any static error.

A/D Card Error

Since the test data is also acquired using an AD card, an error due to this conversion also occurs and is calculated as follows:

AD Card Resolution		12	bits
Ad Card Voltage Range		10	Volts
AD Card Error		1	bits
$B_{TPod\_ADConvT}$	1/2(10v/4096) V	times the slope of the thrust vs volt calibration curve	
$B_{TPod\_ADConvT}$		0.001220703	V
		0.604101563	N Page 133. Coleman and Steele 1999

Total Propeller Thrust Bias Error

$$\mathbf{B_{TPod}}=\left(B_{TPod\_Cal}{}^{2} + B_{TPod\_LA}{}^{2} + B_{TPod\_ADConvC}{}^{2} + B_{TPod\_CF}{}^{2} + B_{TPod\_EPA}{}^{2} + B_{TPod\_LCA}{}^{2} + B_{TPod\_SZT}{}^{2} + B_{TPod\_ADConvT}{}^{2}\right)^{1/2}$$

B_{TPod} 2.215933341 N

Precision Error in Propeller Thrust

The precision limit for the Propeller thrust were estimated for several advance speeds and the data from one run is subdivided into 10 segments of sufficient length (at least 10 sec of data acquisition time) and then applying the standard deviation theory as suggested in Bose and Luznik (1996).

Read #	J=0.00 T_{Pod} N	J=0.1 T_{Pod} N	J=0.2 T_{Pod} N	J=0.3 T_{Pod} N	J=0.4 T_{Pod} N	J=0.5 T_{Pod} N	J=0.6 T_{Pod} N	J=0.7 T_{Pod} N	J=0.8 T_{Pod} N	J=0.9 T_{Pod} N	J=1.0 T_{Pod} N	J=1.1 T_{Pod} N	J=1.2 T_{Pod} N
1	299.361	282.019	258.836	233.288	214.182	184.841	159.181	135.041	106.650	79.897	56.747	28.262	-1.310
2	300.813	281.383	257.089	231.338	212.970	182.788	157.125	133.503	107.119	79.806	56.107	30.450	-1.704
3	300.425	281.597	258.151	231.424	213.316	183.972	158.231	134.572	106.655	79.860	56.506	30.352	-1.654
4	300.152	281.772	258.453	232.727	213.886	184.177	157.968	133.783	107.096	79.846	56.308	30.197	-1.669
5	300.429	281.831	258.180	231.607	213.856	184.052	157.823	133.896	107.035	79.868	56.351	29.561	-1.426
6	300.125	281.640	258.196	231.440	213.314	182.992	158.073	134.180	107.116	79.867	56.307	28.280	-1.532
7	300.359	281.508	258.739	232.271	213.087	184.817	158.087	134.862	106.696	79.847	56.738	28.708	-1.575
8	300.178	281.672	257.435	232.433	213.583	183.525	158.706	133.661	107.028	79.887	56.496	30.203	-1.412
9	300.666	281.613	258.814	232.942	213.254	184.776	158.015	133.564	106.903	79.831	56.464	29.371	-1.691
10	300.673	281.989	258.015	231.925	214.168	183.333	157.546	134.312	107.103	79.816	56.687	29.628	-1.510
Mean:	300.318	281.702	258.191	232.139	213.562	183.927	158.076	134.137	106.940	79.853	56.471	29.501	-1.548
StDev	0.411	0.202	0.577	0.698	0.439	0.754	0.568	0.548	0.199	0.029	0.210	0.836	0.135
	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2
1	0.916	0.100	0.416	1.320	0.385	0.835	1.222	0.817	0.084	0.002	0.076	1.536	0.057
2	0.245	0.102	1.215	0.642	0.350	1.299	0.904	0.402	0.032	0.002	0.133	0.900	0.024
3	0.012	0.011	0.002	0.513	0.060	0.002	0.024	0.188	0.081	0.000	0.001	0.724	0.011
4	0.028	0.005	0.069	0.345	0.105	0.063	0.011	0.125	0.024	0.000	0.026	0.484	0.015
5	0.012	0.016	0.000	0.284	0.087	0.016	0.064	0.058	0.009	0.000	0.014	0.004	0.015
6	0.037	0.004	0.000	0.489	0.061	0.875	0.000	0.002	0.031	0.000	0.027	1.491	0.000

7	0.002	0.038	0.300	0.017	0.226	0.791	0.000	0.526	0.060	0.000	0.071	0.629	0.001
8	0.020	0.001	0.571	0.086	0.000	0.162	0.398	0.227	0.008	0.001	0.001	0.493	0.019
9	0.121	0.008	0.388	0.644	0.095	0.721	0.004	0.329	0.001	0.000	0.000	0.017	0.020
10	0.126	0.082	0.031	0.046	0.367	0.353	0.281	0.030	0.027	0.001	0.046	0.016	0.001
sum	1.517	0.367	2.991	4.386	1.736	5.116	2.907	2.706	0.358	0.008	0.397	6.294	0.163
Sr, Precision Index of Population: [(1/9)*sum(r-ri)]^0.5													
Sr	0.411	0.202	0.577	0.698	0.439	0.754	0.568	0.548	0.199	0.029	0.210	0.836	0.135
Srm, Precision Index of Mean: Sr/(10)^0.5													
Srm	0.130	0.064	0.182	0.221	0.139	0.238	0.180	0.173	0.063	0.009	0.066	0.264	0.043
Pn, Precision Limit of Average Results: nt*Srm													
Pn	0.331	0.151	0.481	0.560	0.329	0.487	0.514	0.363	0.114	0.026	0.150	0.507	0.091

Where t=2.262 (value taken from t distribution for 95% confidence coverage for degree of freedom 10-1=9

The total bias error and the total precision error were then combined using RSS to get the total uncertainty in propeller thrust as follows:

$$U_{T_{\text{Pod}}}^2 = B_{T_{\text{Pod}}}^2 + P_{T_{\text{Pod}}}^2$$

	J=0.00	**J=0.1**	**J=0.2**	**J=0.3**	**J=0.4**	**J=0.5**	**J=0.6**	**J=0.7**	**J=0.8**	**J=0.9**	**J=1.0**	**J=1.1**	**J=1.2**
$\mathbf{B}_{\text{TPod}}$	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159
$\mathbf{P}_{\text{TPod}}$	0.331	0.151	0.481	0.560	0.329	0.487	0.514	0.363	0.114	0.026	0.150	0.507	0.091
$\mathbf{U}_{\text{TPod}}$	2.2406	2.2211	2.2674	2.2856	2.2402	2.2688	2.2747	2.2455	2.2189	2.2161	2.2210	2.2733	2.2178

Propeller Torque

As described in ITTC 7.5-02-03-02.2, the errors in torque should consider both moment arm errors and weight errors

B_{Q_Cal} =[(Ma*Bw)^2+(F*Bma)^2]

To reduce the error associated with the calibration weights, each individual weight was measured on a high quality digital scale accurate =/- 0.01 g the weight errors are calculated as follows.

load cond	Load set kg	Error in Weight, Kg
1	0.0000	0.00001
2	1.1209	0.00001
3	0.4999	0.00001
4	0.4575	0.00001
5	1.0003	0.00001
6	0.9963	0.00001

Total Weight 4.0749 Kg
Error in Weight 2.44949E-05 Kg
$B_{Q\_Calload}$ 0.000240295 N

It should be noted that the lever arm angle error was neglected, since the moment arm used designed such that it had a radius equal to the moment arm length, and therefore, regardless of the angle, the effective moment arm distance was constant over the range of angles considered. In addition, the static zero estimate and frictional effects are not present in this design as with other equipment, since the strain gauge was located such that there were no seals, bearings or other mechanical components between the gauge and the propeller.

The moment arm used to calibrate the torque was assumed to be accurate to 0.2mm and had a length as specified below. The bias contribution due to the error in this length is estimated as follows:

Ma 0.5 m
Bma 0.0002 m
B_{Q_Cal} 0.000823785 Nm

Static Zero Error

As with propeller thrust, errors in the static zero of the torque reading were included basked on the observed differences in the zero torque value.

B_{Q_SZC} 0.00108360 V

B_{Q_SZC} 0.021669821 Nm

A/D Card Error

Since the calibration data is also acquired using an A/D card, an error due to this conversion also occurs and is calculated as follows:

AD Card Resolution	12	bits
Ad Card Voltage Range	10	Volts
AD Card Error	1	bits

$B_{Q\_ADConvC}$ 1/2(10v/4096) V times the slope of the rps vs volt cal curve

$B_{Q\_ADConvC}$ 0.001220703 V

0.017087402 Nm Page 133. Coleman and Steele 1999

Curve Fit Error

The calibration data is used to establish the torque-voltage relationship from a curve fit of the data which introduces an additional error, estimated using SSE as follows:

Counter Clockwise Applied Torque

load cond	Add Load kg	Load Total kg	App Torque Nm	Torque Volt Avg	Predicted Load N	(Y-Y')^2
1	0.0000	0	0	0.733636895	0.010515	0.000111
2	1.1209	1.1209	5.498004	1.126466517	5.507772	9.54E-05
3	0.4999	1.62078	7.94991	1.30025954	7.939832	0.000102
4	0.4575	2.07828	10.19394	1.460832091	10.18688	4.98E-05
5	1.0003	3.07858	15.1004	1.810080218	15.07426	0.000683
6	0.9963	4.07488	19.98725	2.162683737	20.0086	0.000456
7	-0.4999	3.575	17.53534	1.987249262	17.55357	0.000332
8	-1.0003	2.5747	12.62888	1.634627564	12.61898	9.8E-05
9	-0.9963	1.5784	7.742037	1.284566453	7.720223	0.000476
10	-0.4575	1.1209	5.498004	1.124537981	5.480785	0.000296
11	-1.1209	0	0	0.734720495	0.025679	0.000659
12	0.0000	0	0	0.734128449	0.017394	0.000303
13	1.1209	1.1209	5.498004	1.127896496	5.527784	0.000887
14	0.4999	1.62078	7.94991	1.30047071	7.942787	5.07E-05
15	0.4575	2.07828	10.19394	1.460627672	10.18402	9.84E-05
16	1.0003	3.07858	15.1004	1.810731906	15.08338	0.00029
17	0.9963	4.07488	19.98725	2.160754705	19.9816	3.19E-05
18	2.5100	6.58488	32.29877	3.040501682	32.29278	3.59E-05
19	-0.4999	6.085	29.84687	2.865956428	29.85019	1.11E-05
20	-1.0003	5.0847	24.9404	2.516280687	24.95683	0.00027
21	-0.9963	4.0884	20.05356	2.165420133	20.04689	4.45E-05
22	-0.4575	3.6309	17.80953	2.0059999	17.81596	4.14E-05
23	-2.5100	1.1209	5.498004	1.125173472	5.489678	6.93E-05
24	-1.1209	0	0	0.737965824	0.071094	0.005054
Total						0.010545

SEE 0.021893

B_{Q_CF} 0.043787 Nm

Clockwise Applied Torque

load cond	Load set kg	Load Total kg	App Torque Nm	Torque Volt Avg	Predicted Load N	(Y-Y')^2
1	0.0000	0	0	0.725924273	-0.00878	7.7E-05
2	1.1209	1.1209	5.498004	0.330222437	5.525509	0.000757
3	0.4999	1.62078	7.94991	0.153776224	7.993286	0.001881
4	0.4575	2.07828	10.19394	-0.00407754	10.20103	5.02E-05
5	1.0003	3.07858	15.1004	-0.35608696	15.12423	0.000568
6	0.9963	4.07488	19.98725	-0.70144378	19.95439	0.001079
7	-0.4999	3.575	17.53534	-0.53069686	17.56633	0.00096
8	-1.0003	2.5747	12.62888	-0.18408717	12.71864	0.008058
9	-0.9963	1.5784	7.742037	0.165614198	7.82772	0.007342
10	-0.4575	1.1209	5.498004	0.327626947	5.56181	0.004071
11	-1.1209	0	0	0.724191814	0.015453	0.000239
12	0.0000	0	0	0.720823459	0.062563	0.003914
13	1.1209	1.1209	5.498004	0.334221661	5.469576	0.000808
14	0.4999	1.62078	7.94991	0.160827078	7.894672	0.003051
15	0.4575	2.07828	10.19394	0.003300004	10.09785	0.009235
16	1.0003	3.07858	15.1004	-0.34730558	15.00142	0.009799
17	0.9963	4.07488	19.98725	-0.70116792	19.95053	0.001348
18	2.5100	6.58488	32.29877	-1.58195732	32.26926	0.000871
19	-0.4999	6.085	29.84687	-1.40776396	29.83299	0.000193
20	-1.0003	5.0847	24.9404	-1.06051285	24.97633	0.001291
21	-0.9963	4.0884	20.05356	-0.71000748	20.07416	0.000424
22	-0.4575	3.6309	17.80953	-0.55239733	17.86983	0.003636
23	-2.5100	1.1209	5.498004	0.327674871	5.561139	0.003986
24	-1.1209	0	0	0.723997281	0.018174	0.00033
Total						0.025082
SEE	Nm					0.033765

B_{Q_CF} 0.06753 Nm

The curve fit error for the torque calibration was assumed to be the average of the two erroes

B_{Q_CF} 0.0556585 Nm

Test Condition Errors

The errors in the propeller torque during testing is broken down into elemental components as follows

Equipment Positioning Error

Errro in Equipment Angle +/- 0.5 deg	0.00873	rad
B_{Q_EPA}	0.000457	Nm

Static Zero Error

As with propeller thrust, errors in the static zero of the torque reading was included basked on the observed defferences in the zero torque value.

B_{Q_SZT}	0.00108360 V
B_{Q_SZT}	0.01516822 Nm

A/D Card Error

Since the calibration data is also acquired using an AD card, an error due to this conversion also occurs and is calculated as follows

AD Card Resolution	12	bits
Ad Card Voltage Range	10	Volts
AD Card Error	1	bits

$B_{Q\_ADConvT}$ 1/2(10v/4096) V times the slope of the rps vs volt cal curve

$B_{Q\_ADConvT}$ 0.001220703 V

0.017087402 Nm Page 133. Coleman and Steele 1999

Total Propeller Torque Bias Error

$$B_Q=\left(B_{Q\_Cal}^{2} + B_{Q\_SZC}^{2} + B_{Q\_ADConvC}^{2} + B_{Q\_CF}^{2} + B_{Q\_EPA}^{2} + B_{Q\_SZT}^{2} + B_{Q\_ADConvT}^{2}\right)^{1/2}$$

B_Q 0.0662 Nm

Precision Error in Propeller Torque

The precision limit for the Propeller torque were estimated for several advance speeds and the data from one run is subdivided into 10 segments of sufficient length (at least 10 sec of data acquisition time) and then applying the standard deviation theory as suggested in Bose and Luznik (1996).

Read #	J=0.00 T_Pod N	J=0.1 T_Pod N	J=0.2 T_Pod N	J=0.3 T_Pod N	J=0.4 T_Pod N	J=0.5 T_Pod N	J=0.6 T_Pod N	J=0.7 T_Pod N	J=0.8 T_Pod N	J=0.9 T_Pod N	J=1.0 T_Pod N	J=1.1 T_Pod N	J=1.2 T_Pod N
1	11.6907	10.9548	10.3732	9.5886	8.8051	7.9972	7.0605	6.3314	5.3760	4.5545	3.6139	2.7157	1.3640
2	11.6844	11.0876	10.5002	9.5210	8.8228	8.0243	7.2265	6.2819	5.5361	4.6409	3.8463	2.8212	1.4029
3	11.6902	10.9896	10.4530	9.5588	8.8224	8.0102	7.0624	6.2865	5.4474	4.6155	3.7149	2.7586	1.3902
4	11.6896	11.0608	10.4981	9.5704	8.8153	8.0026	7.2142	6.2864	5.4856	4.6305	3.7567	2.7488	1.3919
5	11.6902	11.0708	10.4513	9.5748	8.8075	8.0184	7.1967	6.2846	5.4571	4.6132	3.7870	2.7675	1.4020
6	11.6887	10.9832	10.4058	9.5225	8.8224	8.0195	7.2081	6.3255	5.4452	4.5946	3.6769	2.7342	1.3745
7	11.6859	10.9733	10.4804	9.5318	8.8160	7.9981	7.1166	6.3070	5.4599	4.5925	3.7762	2.7231	1.3788
8	11.6879	11.0374	10.4260	9.5739	8.8179	8.0098	7.0680	6.3120	5.4278	4.6039	3.8438	2.7159	1.3851
9	11.6855	11.0272	10.4521	9.5680	8.8220	8.0011	7.2011	6.2895	5.5252	4.5928	3.7704	2.8161	1.3806
10	11.6886	10.9738	10.4740	9.5705	8.8095	8.0069	7.0988	6.3261	5.4384	4.5914	3.6362	2.8125	1.3952

Mean:	11.6882	11.0158	10.4514	9.5580	8.8161	8.0088	7.1453	6.3031	5.4599	4.6030	3.7422	2.7614	1.3865
StDev	0.0022	0.0470	0.0405	0.0240	0.0067	0.0094	0.0700	0.0196	0.0467	0.0241	0.0803	0.0418	0.0124

	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2
1	0.0000	0.0037	0.0061	0.0009	0.0001	0.0001	0.0072	0.0008	0.0070	0.0023	0.0165	0.0021	0.0005
2	0.0000	0.0052	0.0024	0.0014	0.0000	0.0002	0.0066	0.0005	0.0058	0.0014	0.0108	0.0036	0.0003
3	0.0000	0.0007	0.0000	0.0000	0.0000	0.0000	0.0069	0.0003	0.0002	0.0002	0.0007	0.0000	0.0000
4	0.0000	0.0020	0.0022	0.0002	0.0000	0.0000	0.0048	0.0003	0.0007	0.0008	0.0002	0.0002	0.0000
5	0.0000	0.0030	0.0000	0.0003	0.0001	0.0001	0.0026	0.0003	0.0000	0.0001	0.0020	0.0000	0.0002
6	0.0000	0.0011	0.0021	0.0013	0.0000	0.0001	0.0039	0.0005	0.0002	0.0001	0.0043	0.0007	0.0001
7	0.0000	0.0018	0.0008	0.0007	0.0000	0.0001	0.0008	0.0000	0.0000	0.0001	0.0012	0.0015	0.0001
8	0.0000	0.0005	0.0006	0.0003	0.0000	0.0000	0.0060	0.0001	0.0010	0.0000	0.0103	0.0021	0.0000
9	0.0000	0.0001	0.0000	0.0001	0.0000	0.0001	0.0031	0.0002	0.0043	0.0001	0.0008	0.0030	0.0000
10	0.0000	0.0018	0.0005	0.0002	0.0000	0.0000	0.0022	0.0005	0.0005	0.0001	0.0112	0.0026	0.0001
sum	0.0000	0.0199	0.0147	0.0052	0.0004	0.0008	0.0441	0.0035	0.0197	0.0052	0.0580	0.0157	0.0014

Sr, Precision Index of Population: [(1/9)*sum(r-ri)]^0.5

Sr	0.0022	0.0470	0.0405	0.0240	0.0067	0.0094	0.0700	0.0196	0.0467	0.0241	0.0803	0.0418	0.0124

Srm, Precision Index of Mean: Sr/(10)^0.5

Srm	0.0007	0.0149	0.0128	0.0076	0.0021	0.0030	0.0221	0.0062	0.0148	0.0076	0.0254	0.0132	0.0039

Pn, Precision Limit of Average Results: nt*Srm

Pn	0.0016	0.0336	0.0290	0.0172	0.0048	0.0067	0.0501	0.0140	0.0334	0.0172	0.0574	0.0299	0.0088

Where t=2.262 (value taken from t distribution for 95% confidence coverage for degree of freedom 10-1=9

The total bias error and the total precision error were then combined using RSS to get the total uncertainty in propeller torque in Nm as follows:

$$U_Q^2 = B_Q^2 + P_Q^2$$

	J=0.00	J=0.1	J=0.2	J=0.3	J=0.4	J=0.5	J=0.6	J=0.7	J=0.8	J=0.9	J=1.0	J=1.1	J=1.2
B_Q	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662
P_Q	0.0016	0.0336	0.0290	0.0172	0.0048	0.0067	0.0501	0.0140	0.0334	0.0172	0.0574	0.0299	0.0088
U_Q	0.0662	0.0742	0.0723	0.0684	0.0664	0.0665	0.0830	0.0677	0.0742	0.0684	0.0876	0.0726	0.0668

Global Forces and Moments In the Three Coordinate Directions (F_X, F_Y and F_Z)

Bias Error

All of the components of the bias errors of the global forces (F_X, F_Y and F_Z) in the three coordinate directions were calculated in a similar fashion as done for the propeller thrust. The only component of the bias error that was calculated differently is the Least Square Curve Fit Error. All of the components of the bias errors of the global moments (M_X, M_Y and M_Z) in the three coordinate directions are calculated in a similar fashion as done for the propeller torque. The only component of the bias error that was calculated differently is the Least Square Curve fit error. The theoretical background to calculate the curve fit error for the six-component global dynamometer is described in section 2-2. The following section details out the procedure for the case of global forces applied to the average pod 01 (Islam 2006).

The F and V matrices are given as follows:

F=[0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
-10.626100	1.221400	-1.221400	11.857000	-23.713900	11.857000
-81.864600	9.409700	-9.409700	91.347500	-182.695000	91.347500
-167.961100	19.305900	-19.305900	187.417000	-374.834000	187.417000
-239.240400	27.498900	-27.498900	266.953000	-533.906000	266.953000
-333.997800	38.390500	-38.390500	372.686600	-745.373200	372.686600
-191.804900	22.046500	-22.046500	214.022800	-428.045500	214.022800
-105.708400	12.150400	-12.150400	117.953200	-235.906400	117.953200
-34.469900	3.962100	-3.962100	38.462700	-76.925400	38.462700
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
-10.626100	0.000000	0.000000	11.857000	-23.713900	11.857000
-58.099500	0.000000	0.000000	64.829500	-129.659000	64.829500
-124.865500	0.000000	0.000000	139.329400	-278.658800	139.329400
-215.396700	0.000000	0.000000	240.347200	-480.694500	240.347200
-310.154000	0.000000	0.000000	346.080800	-692.161700	346.080800
-167.961100	0.000000	0.000000	187.417000	-374.834000	187.417000
-81.864600	0.000000	0.000000	91.347500	-182.695000	91.347500
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
-10.626100	-1.221400	1.221400	11.857000	-23.713900	11.857000
-58.099500	-6.678100	6.678100	64.829500	-129.659000	64.829500
-124.865500	-14.352400	14.352400	139.329400	-278.658800	139.329400
-215.396700	-24.758200	24.758200	240.347200	-480.694500	240.347200
-310.154000	-35.649900	35.649900	346.080800	-692.161700	346.080800
-167.961100	-19.305900	19.305900	187.417000	-374.834000	187.417000
-81.864600	-9.409700	9.409700	91.347500	-182.695000	91.347500
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
58.061600	-6.673700	6.673700	-64.787200	129.574400	-64.787200
144.158100	-16.569900	16.569900	-160.856700	321.713500	-160.856700
262.680600	-30.193200	30.193200	-293.108300	586.216600	-293.108300
310.154000	-35.649900	35.649900	-346.080800	692.161700	-346.080800
167.923200	-19.301500	19.301500	-187.374700	374.749400	-187.374700
101.157200	-11.627300	11.627300	-112.874800	225.749700	-112.874800
10.626100	-1.221400	1.221400	-11.857000	23.713900	-11.857000

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
58.061600	0.000000	0.000000	-64.787200	129.574400	-64.787200
144.158100	0.000000	0.000000	-160.856700	321.713500	-160.856700
262.680600	0.000000	0.000000	-293.108300	586.216600	-293.108300
310.154000	0.000000	0.000000	-346.080800	692.161700	-346.080800
167.923200	0.000000	0.000000	-187.374700	374.749400	-187.374700
101.157200	0.000000	0.000000	-112.874800	225.749700	-112.874800
10.626100	0.000000	0.000000	-11.857000	23.713900	-11.857000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
58.099500	6.678100	-6.678100	-64.829500	129.659000	-64.829500
144.196000	16.574300	-16.574300	-160.899000	321.798100	-160.899000
262.718500	30.197500	-30.197500	-293.150600	586.301200	-293.150600
310.154000	35.649900	-35.649900	-346.080800	692.161700	-346.080800
167.961100	19.305900	-19.305900	-187.417000	374.834000	-187.417000
101.100400	11.620700	-11.620700	-112.811400	225.622800	-112.811400
10.626100	1.221400	-1.221400	-11.857000	23.713900	-11.857000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	-11.266100	-23.125100	63.772700	0.000000	-63.772700
0.000000	-26.817700	-55.046900	151.804100	0.000000	-151.804100
0.000000	-55.021800	-112.939400	311.455500	0.000000	-311.455500
0.000000	-86.062900	-176.655500	487.166900	0.000000	-487.166900
0.000000	-47.236600	-96.959400	267.387100	0.000000	-267.387100
0.000000	-17.598500	-36.123200	99.617800	0.000000	-99.617800
0.000000	-3.481000	-7.145100	19.704300	0.000000	-19.704300
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	-29.049800	-29.049800	107.735700	0.000000	-107.735700
0.000000	-72.098000	-72.098000	267.387100	0.000000	-267.387100
0.000000	-131.359200	-131.359200	487.166900	0.000000	-487.166900
0.000000	-155.077000	-155.077000	575.128100	0.000000	-575.128100
0.000000	-83.980600	-83.980600	311.455500	0.000000	-311.455500
0.000000	-50.550200	-50.550200	187.473500	0.000000	-187.473500
0.000000	-5.313000	-5.313000	19.704300	0.000000	-19.704300
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	-39.066900	-19.032600	107.735700	0.000000	-107.735700
0.000000	-96.959400	-47.236600	267.387100	0.000000	-267.387100
0.000000	-176.655500	-86.062900	487.166900	0.000000	-487.166900
0.000000	-208.551800	-101.602200	575.128100	0.000000	-575.128100
0.000000	-112.939400	-55.021800	311.455500	0.000000	-311.455500
0.000000	-67.981300	-33.119100	187.473500	0.000000	-187.473500
0.000000	-7.145100	-3.481000	19.704300	0.000000	-19.704300
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	3.481000	7.145100	-19.704300	0.000000	19.704300
0.000000	31.685000	65.037600	-179.355700	0.000000	179.355700
0.000000	70.511300	144.733700	-399.135500	0.000000	399.135500
0.000000	101.602200	208.551800	-575.128100	0.000000	575.128100
0.000000	55.009300	112.913900	-311.385200	0.000000	311.385200
0.000000	33.137700	68.019500	-187.579000	0.000000	187.579000
0.000000	3.481000	7.145100	-19.704300	0.000000	19.704300
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	26.860900	26.860900	-99.617800	0.000000	99.617800
0.000000	95.740000	95.740000	-355.067100	0.000000	355.067100
0.000000	131.359200	131.359200	-487.166900	0.000000	487.166900
0.000000	155.077000	155.077000	-575.128100	0.000000	575.128100
0.000000	107.622500	107.622500	-399.135500	0.000000	399.135500
0.000000	48.361300	48.361300	-179.355700	0.000000	179.355700
0.000000	5.313000	5.313000	-19.704300	0.000000	19.704300
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	36.123200	17.598500	-99.617800	0.000000	99.617800
0.000000	128.753700	62.726200	-355.067100	0.000000	355.067100
0.000000	176.655500	86.062900	-487.166900	0.000000	487.166900
0.000000	208.551800	101.602200	-575.128100	0.000000	575.128100
0.000000	144.733700	70.511300	-399.135500	0.000000	399.135500
0.000000	65.037600	31.685000	-179.355700	0.000000	179.355700
0.000000	7.145100	3.481000	-19.704300	0.000000	19.704300

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	25.837700	3.941300	25.837700
0.000000	0.000000	0.000000	92.093200	14.048100	92.093200
0.000000	0.000000	0.000000	126.355800	19.274600	126.355800
0.000000	0.000000	0.000000	149.170100	22.754800	149.170100
0.000000	0.000000	0.000000	103.523200	15.791700	103.523200
0.000000	0.000000	0.000000	46.519200	7.096200	46.519200
0.000000	0.000000	0.000000	5.110700	0.779600	5.110700
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	18.538900	18.538900	18.538900
0.000000	0.000000	0.000000	66.078200	66.078200	66.078200
0.000000	0.000000	0.000000	90.662100	90.662100	90.662100
0.000000	0.000000	0.000000	107.031700	107.031700	107.031700
0.000000	0.000000	0.000000	74.279400	74.279400	74.279400
0.000000	0.000000	0.000000	33.378200	33.378200	33.378200
0.000000	0.000000	0.000000	3.667000	3.667000	3.667000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	11.240100	33.136500	11.240100
0.000000	0.000000	0.000000	40.063200	118.108300	40.063200
0.000000	0.000000	0.000000	54.968300	162.049500	54.968300
0.000000	0.000000	0.000000	64.893200	191.308600	64.893200
0.000000	0.000000	0.000000	45.035500	132.767000	45.035500
0.000000	0.000000	0.000000	20.237200	59.660200	20.237200
0.000000	0.000000	0.000000	2.223300	6.554400	2.223300
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	20.578900	-2.384300	37.422200
0.000000	0.000000	0.000000	73.349100	-8.498200	133.383800
0.000000	0.000000	0.000000	100.638000	-11.659900	183.008100
0.000000	0.000000	0.000000	118.808800	-13.765200	216.051400
0.000000	0.000000	0.000000	82.452600	-9.553000	149.938400
0.000000	0.000000	0.000000	37.050900	-4.292700	67.376400
0.000000	0.000000	0.000000	4.070500	-0.471600	7.402100
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	37.422200	-2.384300	20.578900
0.000000	0.000000	0.000000	133.383800	-8.498200	73.349100
0.000000	0.000000	0.000000	183.008100	-11.659900	100.638000
0.000000	0.000000	0.000000	216.051400	-13.765200	118.808800
0.000000	0.000000	0.000000	149.938400	-9.553000	82.452600
0.000000	0.000000	0.000000	67.376400	-4.292700	37.050900
0.000000	0.000000	0.000000	7.402100	-0.471600	4.070500
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	-0.344400	39.462100	16.499000
0.000000	0.000000	0.000000	-1.227400	140.654600	58.807300
0.000000	0.000000	0.000000	-1.684000	192.984100	80.686100
0.000000	0.000000	0.000000	-1.988100	227.828600	95.254500
0.000000	0.000000	0.000000	-1.379700	158.111700	66.106100
0.000000	0.000000	0.000000	-0.620000	71.049100	29.705500
0.000000	0.000000	0.000000	-0.068100	7.805600	3.263500
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	16.499000	39.462100	-0.344400
0.000000	0.000000	0.000000	58.807300	140.654600	-1.227400
0.000000	0.000000	0.000000	80.686100	192.984100	-1.684000
0.000000	0.000000	0.000000	95.254500	227.828600	-1.988100
0.000000	0.000000	0.000000	66.106100	158.111700	-1.379700
0.000000	0.000000	0.000000	29.705500	71.049100	-0.620000
0.000000	0.000000	0.000000	3.263500	7.805600	-0.068100
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
10.626100	-0.447900	1.598200	-13.990100	23.713900	-9.723800
34.410200	12.798300	12.117200	-84.597700	76.792200	7.805500

```
77.411000       36.833800       31.177300       -212.493100     172.756000      39.737100
148.687500      76.530100       62.700600       -424.090600     331.821600      92.269000
58.251100       26.025300       22.636400       -155.233500     129.997300      25.236200
10.626100       -0.447900       1.598200        -13.990100      23.713900       -9.723800
0.000000        0.000000        0.000000        0.000000        0.000000        0.000000
105.383400      -11.736200      12.886500       -119.723700     235.181100      -115.457400
129.167500      -6.684900       31.600400       -190.331200     288.259400      -97.928100
172.168400      2.490000        65.521100       -318.226700     384.223200      -65.996500
243.444800      17.627900       121.602800      -529.824200     543.288800      -13.464600
153.008400      -1.646300       50.308000       -260.967100     341.464500      -80.497400
105.383400      -11.736200      12.886500       -119.723700     235.181100      -115.457400
0.000000        0.000000        0.000000        0.000000        0.000000        0.000000
0.000000        0.000000        0.000000        0.000000        0.000000        0.000000
105.383400      0.575200        0.575200        -119.723700     235.181100      -115.457400
129.167500      12.457700       12.457700       -190.331200     288.259400      -97.928100
172.168400      34.005600       34.005600       -318.226700     384.223200      -65.996500
243.444800      69.615400       69.615400       -529.824200     543.288800      -13.464600
153.008400      24.330800       24.330800       -260.967100     341.464500      -80.497400
10.626100       0.575200        0.575200        -13.990100      23.713900       -9.723800
0.000000        0.000000        0.000000        0.000000        0.000000        0.000000
0.000000        0.000000        0.000000        0.000000        0.000000        0.000000
10.607100       1.219200        -1.219200       20.529500       101.082700      -12.511300
81.826700       9.405400        -9.405400       -37.061000      312.350900      -92.437300
124.827600      14.348000       -14.348000      -71.807400      439.971300      -140.695500
77.392100       8.895600        -8.895600       -33.457200      299.238400      -87.460900
34.391200       3.953000        -3.953000       1.289200        171.618000      -39.202800
10.607100       1.219200        -1.219200       20.529500       101.082700      -12.5113];


V=[0.100573094  0.033651395     -0.028351541    0.456497536     0.635478946     0.442100244
0.119063327     0.035450101     -0.026246207    0.464947882     0.618818701     0.45021955
0.273361609     0.04866858      -0.00957585     0.527647159     0.497089956     0.509072786
0.415671209     0.065570735     0.010003195     0.603375125     0.35157019      0.579304106
0.54880381      0.078106236     0.055186581     0.665116002     0.233400317     0.63603759
0.720400493     0.094775768     0.045645163     0.744689427     0.082654141     0.710002745
0.486962537     0.072773585     0.017392616     0.635757902     0.286181429     0.609325694
0.327634803     0.055431478     -0.002159695    0.562134257     0.427708708     0.540021999
0.179587262     0.040369877     -0.020231107    0.493425809     0.56187972      0.476317109
0.100690163     0.032552632     -0.029084444    0.456600349     0.635374559     0.442147471
0.10035822      0.033468248     -0.026563189    0.456708964     0.635253433     0.44223975
0.11966535      0.033333357     -0.027011829    0.465425535     0.617958088     0.450955806
0.205620539     0.034620174     -0.027882768    0.502917385     0.5411898       0.490045848
0.327255136     0.035971133     -0.028274136    0.556740371     0.432090071     0.54628331
0.491454481     0.036299762     -0.029350391    0.629380469     0.285481307     0.621496933
0.656478282     0.038657992     -0.029738094    0.701158002     0.141750699     0.696644106
0.422625684     0.036782093     -0.028689779    0.598224775     0.344813636     0.590589281
0.269386533     0.036074796     -0.027501842    0.530218832     0.48175123      0.520004497
0.101327399     0.034601652     -0.026748001    0.456555796     0.635157816     0.442479222
0.100559785     0.033220946     -0.028659638    0.45664799      0.635538427     0.442205443
0.118028657     0.030389441     -0.031485871    0.464983192     0.61961529      0.449735484
0.203234266     0.01769809      -0.04148131     0.504117403     0.543262567     0.486809011
0.323559772     0.000644179     -0.056739749    0.559670021     0.4352903       0.539929393
0.48955573      -0.023043135    -0.077529069    0.635803583     0.288021348     0.611971673
0.661736963     -0.047460795    -0.099022482    0.715173646     0.136294436     0.68677041
0.429080499     -0.014225839    -0.070382101    0.607208052     0.338841985     0.586415456
0.271505277     0.00835497      -0.050721427    0.535083043     0.478155086     0.517940209
0.100007619     0.03237973      -0.028957481    0.456533378     0.635177787     0.441934809
0.101323926     0.029904769     -0.027223823    0.45840356      0.636423342     0.442798607
-0.00422297     0.016739723     -0.040834859    0.410982152     0.730443531     0.395384684
-0.156862869    -0.002306125    -0.060945679    0.343504918     0.866078461     0.32835098
-0.377999467    -0.031058614    -0.0868926      0.246360903     1.062491769     0.231246631
-0.466476935    -0.042414746    -0.097774659    0.207983651     1.140944437     0.192904164
-0.235327951    -0.011877361    -0.069730743    0.307130585     0.935454832     0.289807088
-0.114508565    0.003253783     -0.054583577    0.360237489     0.827976913     0.342658553
0.074487891     0.028179573     -0.030932674    0.445790824     0.660061724     0.429711257
0.101894187     0.030261428     -0.028874565    0.458907202     0.636410625     0.442860256
0.001054458     0.030318744     -0.028561363    0.413958934     0.726413324     0.398123298
-0.156940505    0.030243475     -0.029658966    0.343893823     0.86694938      0.32780946
-0.378527941    0.029323435     -0.030401711    0.246198476     1.064131684     0.229643618
```

-0.465203678	0.028589518	-0.030522059	0.208975903	1.140702598	0.192382918
-0.231390926	0.028151773	-0.029603976	0.308532012	0.93313469	0.291951586
-0.112368654	0.029793874	-0.028981516	0.360104057	0.826861266	0.344185143
0.075211587	0.030242565	-0.028756536	0.445731867	0.660351408	0.430330204
0.100167113	0.031185937	-0.02849164	0.457815611	0.637347494	0.441919385
-0.010986449	0.044165314	-0.015362751	0.406270573	0.737233451	0.390953171
-0.165618909	0.062180884	0.002679476	0.338168984	0.874463372	0.322376961
-0.38352153	0.087590008	0.026632144	0.241853789	1.068872713	0.226157287
-0.464957749	0.096785347	0.035325002	0.208402887	1.140864014	0.191394802
-0.233394378	0.069871415	0.009198156	0.306106757	0.934126094	0.291316975
-0.1129204	0.055906827	-0.003777601	0.358643411	0.82689463	0.344237162
0.073229533	0.034354317	-0.055037046	0.444584274	0.660597463	0.429279154
0.107518958	0.030312984	-0.031751059	0.46232362	0.633610175	0.448079288
0.108936719	0.084763752	-0.045043141	0.412252057	0.630191074	0.498028052
0.11028921	0.156852089	-0.061755173	0.347142852	0.625925068	0.564002234
0.112591199	0.28883642	-0.092844955	0.227357978	0.61863876	0.684898912
0.115283744	0.441321352	-0.129086109	0.089532935	0.610958861	0.822246176
0.112516211	0.270293893	-0.088848209	0.244044274	0.61902803	0.665364717
0.109822243	0.129424811	-0.055494111	0.371623797	0.626919936	0.536272625
0.107863596	0.053042798	-0.038047262	0.441022072	0.632471448	0.466127528
0.107981211	0.031167366	-0.031591996	0.460703421	0.634567054	0.445052372
0.107699134	0.102404114	-0.071640432	0.377404584	0.631983785	0.526622775
0.107048853	0.20562081	-0.129586172	0.276152139	0.629175952	0.64496975
0.105489136	0.349314509	-0.270487854	0.089104024	0.62620679	0.808323128
0.104979184	0.402069331	-0.239921745	0.028230562	0.626077048	0.867863382
0.106148486	0.270781349	-0.15470965	0.203198616	0.62734924	0.69498022
0.106703631	0.172005547	-0.110701314	0.294938579	0.62913729	0.604900383
0.10709218	0.049130181	-0.041069486	0.439426769	0.634027662	0.464173669
0.106353576	0.032111448	-0.030483598	0.459712887	0.63512638	0.444232998
0.104749994	0.084508829	-0.091234999	0.373462455	0.63445802	0.526446077
0.102859719	0.163146732	-0.180782564	0.245355342	0.633622034	0.648448801
0.09946605	0.271653071	-0.304901758	0.067652985	0.633090114	0.817770554
0.098062155	0.313631231	-0.352593517	-0.000642631	0.633283189	0.883053158
0.101110633	0.198902411	-0.220908295	0.185497133	0.63296371	0.703813173
0.102984831	0.139848048	-0.152449048	0.282960053	0.633060994	0.610439771
0.105190054	0.045099268	-0.044547135	0.437902144	0.634589101	0.46397396
0.098292091	0.027652328	-0.028369521	0.45883153	0.636332936	0.444323382
0.097588946	0.002834735	-0.022834091	0.48039831	0.63675826	0.42198582
0.095589697	-0.178411828	0.018729949	0.641402098	0.6416997	0.275724645
0.091540055	-0.392785217	0.06775366	0.831587935	0.652479867	0.060024399
0.088634136	-0.545272923	0.101946893	0.966390158	0.662862898	-0.077070082
0.092342005	-0.303633319	0.046403999	0.752735113	0.64882161	0.143423104
0.095095689	-0.180613314	0.018271607	0.644060358	0.643041284	0.274693046
0.098611586	0.003802612	-0.023450931	0.48060651	0.636967123	0.422296574
0.098887715	0.028073124	-0.028233836	0.458993951	0.636432453	0.44445242
0.098155026	0.028977011	-0.029080947	0.458657916	0.636657206	0.443829395
0.090805477	-0.059792909	0.019881305	0.555985507	0.644422864	0.338578141
0.0744426	-0.278841161	0.12810049	0.775394779	0.664531785	0.104162799
0.066933877	-0.349736071	0.179206727	0.875420856	0.675360406	-0.001920624
0.062602399	-0.407179237	0.270542015	0.93824151	0.682681184	-0.067461991
0.072219709	-0.28586381	0.143397331	0.804632757	0.66850057	0.07510525
0.086172017	-0.122491094	0.053237235	0.624796749	0.650860347	0.267467659
0.097454213	0.009786714	-0.018802779	0.478301399	0.638068955	0.423613144
0.099112519	0.027702734	-0.028446746	0.458272486	0.636593618	0.444632875
0.098322349	0.027594208	-0.02827392	0.458503715	0.636166261	0.444554979
0.099546442	-0.033051659	0.040967246	0.55543662	0.635823616	0.349233974
0.102538648	-0.171422388	0.198197749	0.77602629	0.637247486	0.134360922
0.104164533	-0.235709862	0.270926872	0.878752644	0.638804089	0.035850665
0.105173451	-0.276367137	0.317111681	0.943962028	0.639698083	-0.026118911
0.10308688	-0.190169281	0.279792078	0.808043576	0.638059451	0.105173906
0.100608354	-0.076741027	0.090442705	0.625601898	0.636674133	0.281860817
0.098222681	0.014958337	-0.015116063	0.478211663	0.636553933	0.425389022
0.098140322	0.026752265	-0.029861075	0.458257702	0.636397763	0.445007633
0.09895839	0.028943295	-0.029066207	0.456703098	0.636528055	0.445879953
0.098795051	0.029429756	-0.028566636	0.461300094	0.675097826	0.450996771
0.098322811	0.029900026	-0.029003662	0.473410531	0.77445091	0.463123025
0.097188285	0.029381156	-0.030120451	0.479808375	0.825990537	0.469675718
0.097198265	0.029322696	-0.029753113	0.484257209	0.859888068	0.474185643
0.097229096	0.028917384	-0.029345048	0.475935012	0.791740558	0.465563414
0.097769976	0.028976736	-0.029161665	0.465467923	0.706190685	0.454065082
0.098128983	0.029246615	-0.029124178	0.45789515	0.644187292	0.446190899
0.098060557	0.029009978	-0.029264267	0.457074471	0.636427166	0.44559022

0.098304356	0.028663321	-0.02955755	0.459673804	0.643547522	0.448301639
0.09806793	0.028832637	-0.029936063	0.467844097	0.662753139	0.456788343
0.09825595	0.028735546	-0.029981695	0.46772931	0.662793974	0.456807075
0.098103064	0.02830549	-0.032006723	0.472337938	0.67226293	0.460441825
0.097833296	0.02890983	-0.03228954	0.495769637	0.72969575	0.485464129
0.097417833	0.02867909	-0.031903457	0.501508841	0.742766123	0.491150673
0.097091526	0.028766564	-0.031099667	0.510619709	0.764398734	0.50084373
0.096930455	0.028627558	-0.030239843	0.520270978	0.787333005	0.510950314
0.098475276	0.028385085	-0.030638356	0.4575593	0.637530113	0.444380033
0.098246534	0.028884017	-0.030029829	0.473727408	0.650059219	0.461453517
0.098245857	0.029397312	-0.030908939	0.518282904	0.684691199	0.507437278
0.097569989	0.029881231	-0.030615976	0.541453492	0.702464091	0.531199038
0.096731868	0.030153903	-0.030461985	0.556631126	0.714512897	0.547015849
0.097626075	0.029689769	-0.030615838	0.526246224	0.690683606	0.515352867
0.098431681	0.029502026	-0.031222884	0.487782562	0.660685153	0.475993448
0.098641512	0.029754676	-0.030442454	0.46015636	0.639188169	0.44698384
0.099013637	0.029444293	-0.0310798	0.456449721	0.636531597	0.443331577
0.099396752	0.029473142	-0.029810044	0.456471039	0.63633435	0.443457868
0.099222713	0.02899112	-0.030479974	0.451117995	0.680633063	0.453268907
0.098447345	0.029273908	-0.031010852	0.437000933	0.793755541	0.477982854
0.098252554	0.029625497	-0.030375997	0.430332163	0.852021981	0.490683151
0.098021361	0.029971707	-0.029629117	0.426120479	0.888390652	0.498742656
0.097822094	0.029689065	-0.029087562	0.435075668	0.81320941	0.482474805
0.098387492	0.029781831	-0.029013812	0.446950042	0.716124163	0.461030916
0.099249648	0.029382297	-0.029569839	0.455965919	0.643981883	0.445334105
0.099305019	0.029132878	-0.028564044	0.456936694	0.636519528	0.443720194
0.098715055	0.02927578	-0.029237255	0.456680236	0.637592325	0.443575613
0.098562801	0.029225518	-0.028586412	0.466001057	0.680590531	0.438557675
0.098362936	0.02968926	-0.028560646	0.490552159	0.79312944	0.425732069
0.09803275	0.030068615	-0.029379708	0.503469307	0.850616259	0.419339176
0.097625855	0.031082717	-0.029927867	0.511769607	0.889257888	0.415129627
0.097651148	0.030364555	-0.029857866	0.494876748	0.812469909	0.423270163
0.098658998	0.02979466	-0.028902475	0.473625311	0.715608563	0.434314335
0.09845982	0.029274547	-0.02981172	0.458564905	0.645428192	0.442542008
0.098038251	0.029165583	-0.030455452	0.456772372	0.636759363	0.443551186
0.099049076	0.02948506	-0.030519252	0.457087475	0.636394523	0.44355119
0.098616849	0.029677968	-0.030480689	0.469027756	0.644842898	0.471955104
0.097629386	0.029330697	-0.030985919	0.500887286	0.66635055	0.543863888
0.096309787	0.029051566	-0.032457051	0.517321526	0.677730566	0.580597235
0.096462438	0.029456926	-0.031851012	0.528548208	0.6851393	0.604726767
0.097087086	0.029116625	-0.031632755	0.506146717	0.669740749	0.556212347
0.098248534	0.029243447	-0.031687478	0.479047333	0.651328222	0.493917433
0.098547654	0.028944052	-0.030696622	0.459203358	0.638141673	0.448688177
0.098781373	0.028701329	-0.030037546	0.456769118	0.636580067	0.443110782
0.099167245	0.028598479	-0.029863779	0.456776492	0.636353533	0.44317789
0.098251909	0.029198274	-0.030084725	0.484059631	0.644184327	0.456669316
0.097081931	0.029498758	-0.031649222	0.553920119	0.66533634	0.491291911
0.096629809	0.029825915	-0.031836662	0.589569301	0.676696495	0.509233114
0.09716243	0.029808594	-0.032798357	0.613321962	0.684260305	0.521226586
0.098086523	0.029266613	-0.032002262	0.565482697	0.669392168	0.497116474
0.098098138	0.029264065	-0.030504507	0.505628217	0.651076226	0.467354912
0.098591984	0.028801971	-0.030808523	0.462015504	0.638268991	0.445667053
0.098156764	0.028117864	-0.030249255	0.456869621	0.636707265	0.442583772
0.110153508	0.055077277	-0.032534393	0.463194769	0.63103201	0.447867041
0.107750068	0.012011099	-0.016642968	0.484292703	0.633107734	0.425027086
0.062308397	-0.009909782	0.020655613	0.507332809	0.673853303	0.360155872
-0.012364989	-0.047724849	0.082869628	0.548832127	0.741538851	0.272254597
-0.142408616	-0.102312212	0.179831165	0.603314418	0.860293203	0.082179661
0.012178685	-0.028469693	0.056708614	0.52392426	0.719684564	0.297450158
0.104408204	0.01399492	-0.015349483	0.482139251	0.635410959	0.424232617
0.106210668	0.027117336	-0.028532954	0.464949388	0.634737253	0.445580874
0.088842397	-0.202787902	0.02009725	0.661176118	0.651813474	0.227633005
0.041698813	-0.233629969	0.035365158	0.672214747	0.69567035	0.172207295
-0.041824394	-0.301960017	0.061569973	0.703145995	0.773253072	0.064229356
-0.147846159	-0.365314698	0.10163684	0.722708396	0.869985692	-0.047144562
-0.175621688	-0.405731978	0.049560681	0.748778851	0.896326773	-0.096263455
-0.057893335	-0.273571683	0.018950329	0.673004166	0.785912283	0.087112522
-0.009877743	-0.268007203	-0.030599271	0.683816364	0.742248352	0.11903046
0.105840477	0.028442846	-0.029840912	0.460516629	0.632910799	0.447683833
0.097235584	-0.145399559	0.066663998	0.656415398	0.642853948	0.242615443
0.048111903	-0.180399019	0.086173972	0.67391955	0.687857695	0.17992569
-0.032756895	-0.241353337	0.118601937	0.707237277	0.762396525	0.074715083

```
-0.1683407      -0.33583112     0.169705302     0.754300195     0.887081068     -0.090857938
-0.003143883    -0.271876062    0.103260037     0.686710725     0.735949135     0.123354317
0.104216613     0.02863518      -0.02997424     0.461785527     0.635050746     0.446941622
0.100981402     0.028238627     -0.031440729    0.455946542     0.628600075     0.444544007
0.120352688     0.026827392     -0.033491351    0.51761455      0.626762071     0.479504151
0.249261632     0.009443392     -0.050768392    0.611268218     0.51677224      0.555439167
0.327564193     -0.000852755    -0.061160029    0.667483803     0.450681571     0.601480662
0.247600008     0.009426987     -0.050759632    0.60896159      0.51600587      0.554081857
0.17081028      0.020164697     -0.041146091    0.552668621     0.581952827     0.508445235
0.124156663     0.055760703     -0.034882572    0.519798162     0.622275205     0.48131992
0.10103432      0.029143672     -0.030326129    0.456129176     0.628414245     0.444068514];
```

The uncertainties in the interaction matrix was obtained using the equations 2-10 and 2-11. It is to be noted that the uncertainties in each elements in the **F** and **V** matrix is assumed to be equal for simplicity, as described in Appendix B. A small program in Matlab is written to calculate the uncertainties in each of the 36 elements of the interaction matrix using the procedure described in section 2-2.

The interaction matrix and the uncertainties in the interaction matrix was obtained as:

Interaction/Calibration Matrix

-718.30	107.90	-195.60	249.00	-79.80	11.90
-23.70	42.20	519.20	48.80	-15.50	1.70
7.30	-491.10	246.90	-131.60	12.10	156.60
1380.60	978.00	-1421.40	-248.20	414.60	-755.60
-2552.90	-80.00	-165.50	866.70	-714.20	773.90
1333.10	-1666.20	2303.90	-1178.70	455.10	514.90

Uncertainties in the Interaction/Calibration Matrix

0.1043	0.1508	0.4143	0.1109	0.1171	0.0882
0.0537	0.0767	0.207	0.057	0.0601	0.0458
0.0513	0.0732	0.1969	0.0544	0.0573	0.0438
0.2223	0.3224	0.8899	0.2363	0.2498	0.1876
0.2364	0.3428	0.9464	0.2513	0.2656	0.1995
0.205	0.2972	0.8201	0.2179	0.2303	0.1730

Now to calculate the uncertainties in the forces and moments in the three orthogonal directions for the case of average forces and moments of the podded propulsors, the procedure as described in section 2-2 was used.

The voltage output for the test conditions for the six-components were:

J	V1	V2	V3	V4	V5	V6
0.0000	0.7513	0.0311	-0.0199	0.8893	0.0173	0.9804
0.1000	0.7091	0.0286	-0.0125	0.8821	0.0960	0.9047
0.2000	0.6574	0.0325	-0.0174	0.8506	0.1448	0.8851
0.3000	0.5874	0.0295	-0.0318	0.7624	0.2099	0.9329
0.4000	0.5437	0.0406	-0.0300	0.7659	0.2772	0.8357
0.5000	0.4777	0.0380	-0.0330	0.7181	0.3405	0.8213
0.6000	0.4255	0.0399	-0.0324	0.6956	0.4154	0.7649
0.6500	0.3970	0.0390	-0.0317	0.6764	0.4485	0.7498
0.7000	0.3680	0.0440	-0.0339	0.6580	0.4816	0.7339
0.7500	0.3348	0.0402	-0.0347	0.6398	0.5258	0.7093
0.8000	0.3087	0.0421	-0.0371	0.6211	0.5572	0.6973
0.8500	0.2766	0.0377	-0.0345	0.6037	0.5991	0.6724
0.9000	0.2454	0.0367	-0.0345	0.5881	0.6384	0.6514
0.9500	0.2061	0.0298	-0.0365	0.5505	0.6835	0.6594
1.0000	0.1840	0.0300	-0.0307	0.5598	0.7161	0.6039
1.1000	0.1126	0.0291	-0.0331	0.5107	0.8078	0.5609
1.2000	0.0389	0.0171	-0.0269	0.4828	0.9121	0.4830

The bias uncertainties in each components of the voltage signal were assumed to be equal for simplicity as described in the section 2-2.

J	U_{V1}	U_{V2}	U_{V3}	U_{V4}	U_{V5}	U_{V6}
0.00	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.10	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.20	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.30	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.40	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.50	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.60	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.65	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.70	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.75	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.80	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.85	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.90	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
0.95	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
1.00	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
1.10	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04
1.20	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04	4.07E-04

The bias uncertainties in the forces and moments at the advance coefficient tested were calculated as:

J	U_{FX}	U_{FY}	U_{FZ}	U_{MX}	U_{MY}	U_{MZ}
0.0000	0.0076	0.4162	1.7620	-0.5235	0.4192	0.1394
0.1000	0.0077	0.4160	1.7624	-0.5233	0.4191	0.1394
0.2000	0.0013	0.4097	1.7220	-0.5122	0.4303	0.1393
0.3000	-0.0103	0.3982	1.6477	-0.4923	0.4508	0.1392
0.4000	-0.0128	0.3955	1.6323	-0.4877	0.4552	0.1392
0.5000	-0.0204	0.3878	1.5835	-0.4744	0.4687	0.1391
0.6000	-0.0240	0.3841	1.5607	-0.4679	0.4750	0.1391
0.6500	-0.0265	0.3816	1.5453	-0.4636	0.4793	0.1390
0.7000	-0.0292	0.3789	1.5277	-0.4588	0.4842	0.1390
0.7500	-0.0326	0.3754	1.5063	-0.4529	0.4901	0.1390
0.8000	-0.0355	0.3725	1.4879	-0.4478	0.4952	0.1389
0.8500	-0.0376	0.3703	1.4743	-0.4440	0.4990	0.1389
0.9000	-0.0400	0.3679	1.4591	-0.4398	0.5032	0.1389
0.9500	-0.0441	0.3638	1.4329	-0.4326	0.5105	0.1389
1.0000	-0.0441	0.3637	1.4335	-0.4325	0.5104	0.1389
1.1000	-0.0511	0.3565	1.3886	-0.4201	0.5228	0.1388
1.2000	-0.0559	0.3516	1.3589	-0.4116	0.5311	0.1388

The uncertainties in the forces and moments were then expressed in terms of the forces and moments in the corresponding directions in shown in the following table.

J	***Fx***	***Fy***	***Fz***	***Mx***	***My***	***Mz***	***%Fx***	***%Fy***	***%Fz***	***%Mx***	***%My***	***%Mz***
0.00	301.4931	20.3854	11.3666	158.5785	50.6204	331.5387	0.0025	2.0415	15.5012	-0.3301	0.8281	0.0420
0.10	281.4178	19.9165	12.3610	142.4916	49.9539	314.3295	0.0027	2.0888	14.2580	-0.3672	0.8390	0.0443
0.20	255.7239	18.8391	12.4948	126.4383	50.5798	292.5544	0.0005	2.1745	13.7817	-0.4051	0.8508	0.0476
0.30	229.0597	17.5256	12.4301	107.3857	52.8674	270.9564	-0.0045	2.2719	13.2557	-0.4584	0.8528	0.0514
0.40	202.8949	16.9327	11.6735	94.3510	43.8099	250.1751	-0.0063	2.3359	13.9827	-0.5169	1.0390	0.0556
0.50	174.1263	14.8987	12.0259	74.1375	48.3446	222.7351	-0.0117	2.6031	13.1673	-0.6398	0.9695	0.0624
0.60	145.9614	12.6626	11.0506	59.8122	44.5884	198.1799	-0.0165	3.0336	14.1235	-0.7824	1.0653	0.0702
0.65	131.0192	11.3449	10.5738	53.0111	41.6186	183.6329	-0.0202	3.3640	14.6142	-0.8746	1.1517	0.0757
0.70	117.8528	11.0957	10.8779	45.3079	39.9235	176.5613	-0.0248	3.4145	14.0445	-1.0127	1.2128	0.0787
0.75	103.6280	10.7944	10.6448	38.7896	35.3150	165.3906	-0.0315	3.4781	14.1502	-1.1675	1.3879	0.0840
0.80	90.3610	9.9636	10.2967	31.8944	32.6700	154.1560	-0.0393	3.7390	14.4498	-1.4042	1.5159	0.0901
0.85	76.3839	9.5771	10.2599	23.4676	28.4462	144.8560	-0.0493	3.8667	14.3698	-1.8920	1.7542	0.0959
0.90	61.7356	8.9783	10.4554	17.8848	23.6824	134.3539	-0.0648	4.0975	13.9559	-2.4590	2.1249	0.1034
0.95	47.7069	8.2458	10.4084	9.4350	20.9195	125.0279	-0.0925	4.4115	13.7666	-4.5854	2.4403	0.1111
1.00	32.0455	7.7362	10.4095	0.4147	15.5071	114.3377	-0.1375	4.7012	13.7714	-4.3054	3.2912	0.1215
1.10	2.1979	7.2081	10.8362	-11.9760	2.5564	95.2807	-2.3270	4.9464	12.8143	3.5080	4.4520	0.1457
1.20	-31.7039	5.1084	10.8193	-26.7699	-11.6867	73.6704	0.1762	6.8835	12.5596	1.5377	-4.5448	0.1884

Precision Error

The precision limit for the global axial force component was estimated for several advance speeds and the data from one run is subdivided into 10 segments of sufficient length (at least 10 sec of data acquisition time) and then applying the standard deviation theory as suggested in Bose and Luznik (1996).

Read #	J=0.00 T_{Pod}/F_X N	J=0.1 T_{Pod}/F_X N	J=0.2 T_{Pod}/F_X N	J=0.3 T_{Pod}/F_X N	J=0.4 T_{Pod}/F_X N	J=0.5 T_{Pod}/F_X N	J=0.6 T_{Pod}/F_X N	J=0.7 T_{Pod}/F_X N	J=0.8 T_{Pod}/F_X N	J=0.9 T_{Pod}/F_X N	J=1.0 T_{Pod}/F_X N	J=1.1 T_{Pod}/F_X N	J=1.2 T_{Pod}/F_X N
1	299.36	282.02	258.84	233.29	214.18	184.84	159.18	135.04	106.65	79.90	56.75	28.26	-1.31
2	302.81	284.38	261.09	236.34	217.07	187.79	156.12	131.50	119.12	83.81	59.11	28.26	-3.27
3	301.19	282.36	259.83	235.84	215.98	186.22	159.09	133.59	116.78	82.43	58.50	28.26	-2.77
4	301.97	282.11	261.01	234.28	216.65	185.10	156.74	132.43	113.04	81.81	58.67	28.26	-1.50
5	299.59	283.06	259.25	233.45	214.27	185.76	159.14	134.47	117.14	81.53	58.93	28.26	-2.01
6	300.07	283.78	260.99	234.47	215.43	186.20	157.62	133.85	109.48	81.47	57.66	28.26	-2.37
7	301.59	283.83	259.12	235.30	215.46	187.23	158.41	134.26	115.10	82.05	58.69	28.26	-1.34
8	301.86	282.29	259.92	235.84	216.89	187.15	158.46	131.75	117.93	80.79	57.35	28.26	-2.68
9	299.78	282.91	260.72	234.48	214.55	185.94	157.44	134.51	118.30	81.15	57.45	28.26	-2.80
10	301.85	283.14	260.91	235.28	214.52	185.20	156.84	134.80	117.21	83.38	58.91	28.26	-1.49
Mean:	301.01	282.99	260.17	234.86	215.50	186.14	157.90	133.62	115.08	81.83	58.20	28.26	-2.15
StDev	1.21	0.81	0.88	1.03	1.11	0.99	1.11	1.28	4.12	1.16	0.82	0.00	0.72
	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2	(r-ri)^2
1	2.7079	0.9431	1.7718	2.4588	1.7371	1.6957	1.6311	2.0192	70.9904	3.7373	2.1151	0.0000	0.7123
2	3.2616	1.9408	0.8501	2.1958	2.4647	2.7035	3.1641	4.4820	16.3544	3.9004	0.8186	0.0000	1.2472
3	0.0331	0.3938	0.1165	0.9754	0.2326	0.0064	1.4065	0.0007	2.9111	0.3599	0.0910	0.0000	0.3813
4	0.9314	0.7710	0.7032	0.3355	1.3281	1.0850	1.3607	1.4150	4.1470	0.0007	0.2198	0.0000	0.4249
5	2.0099	0.0054	0.8337	1.9850	1.5177	0.1472	1.5273	0.7289	4.2754	0.0925	0.5302	0.0000	0.0217
6	0.8775	0.6216	0.6735	0.1489	0.0054	0.0034	0.0821	0.0512	31.2963	0.1306	0.2959	0.0000	0.0461
7	0.3398	0.7048	1.0922	0.1957	0.0015	1.1707	0.2572	0.4113	0.0006	0.0469	0.2399	0.0000	0.6637
8	0.7210	0.4855	0.0599	0.9601	1.9266	1.0235	0.3084	3.5133	8.1484	1.0784	0.7317	0.0000	0.2811
9	1.4970	0.0057	0.3039	0.1420	0.8975	0.0432	0.2189	0.7887	10.4264	0.4614	0.5647	0.0000	0.4118
10	0.7030	0.0239	0.5518	0.1825	0.9672	0.8853	1.1273	1.3855	4.5388	2.4048	0.5070	0.0000	0.4395
sum	13.0822	5.8956	6.9565	9.5797	11.0783	8.7640	11.0837	14.7958	153.0889	12.2130	6.1138	0.0000	4.6296
Sr, Precision Index of Population: [(1/9)*sum(r-ri)]^0.5													
Sr	1.2056	0.8094	0.8792	1.0317	1.1095	0.9868	1.1097	1.2822	4.1243	1.1649	0.8242	0.0006	0.7172
Srm, Precision Index of Mean: Sr/(10)^0.5													
Srm	0.3813	0.2559	0.2780	0.3263	0.3508	0.3121	0.3509	0.4055	1.3042	0.3684	0.2606	0.0002	0.2268
Pn, Precision Limit of Average Results: nt*Srm													
Pn	0.8624	0.5789	0.6289	0.7380	0.7936	0.7059	0.7938	0.9172	2.9501	0.8333	0.5896	0.0004	0.5130

Where t=2.262 (value taken from t distribution for 95% confidence coverage for degree of freedom 10-1=9

The total bias error and the total precision error were then combined using RSS to get the total uncertainty in propeller thrust as follows:

$$U_{T_{\text{Pod}}}^2 = B_{T_{\text{Pod}}}^2 + P_{T_{\text{Pod}}}^2$$

	J=0.00	**J=0.1**	**J=0.2**	**J=0.3**	**J=0.4**	**J=0.5**	**J=0.6**	**J=0.7**	**J=0.8**	**J=0.9**	**J=1.0**	**J=1.1**	**J=1.2**
B_{FX}	0.6391	0.6391	0.6391	0.6392	0.6392	0.6394	0.6395	0.6397	0.6401	0.6403	0.6406	0.6411	0.6415
P_{FX}	0.6963	0.6624	0.4704	0.7059	0.6585	0.5856	0.7842	0.7171	3.0795	1.1557	0.6913	0.0004	0.5189
U_{FX}	9.8E-06	1.07E-05	9.63E-06	1.73E-05	2.05E-05	2.48E-05	4.81E-05	6.65E-05	1.21E-03	4.58E-04	8.64E-04	8.62E-02	6.7E-04

In a similar fashion, other force components and moment components in the three orthogonal directions were calculated and the results are summarized in Table B-1 and B-2

Propeller Thrust Coefficient-Total Bias Error

The total bias limit for propeller thrust coefficient is calculated based on ITTC recommended procedure 7.5-02-03-02.2 as follows:

Item	Value
Water Density, ρ	999.00
Shaft Rotational Speed, n	11.00
Propeller Diameter, D	0.27 m

Advance Coefficient, *J*	**0.00**
Thrust Coefficient, $K_{T\text{Pod}} = \dfrac{T_{\text{Pod}}}{\rho n^2 D^4}$	0.4791

$(B_{KT\text{Pod}})^2 = \left(\dfrac{\partial K_{T\text{Pod}}}{\partial T_{\text{Pod}}} \times B_{T\text{Pod}}\right)^2 + \left(\dfrac{\partial K_{T\text{Pod}}}{\partial n} \times B_n\right)^2 + \left(\dfrac{\partial K_{T\text{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\dfrac{\partial K_{T\text{Pod}}}{\partial D} \times B_D\right)^2$	
$\dfrac{\partial K_{T\text{Pod}}}{\partial T_{\text{Pod}}} = \dfrac{1}{\rho n^2 D^4}$	0.001557
$\dfrac{\partial K_{T\text{Pod}}}{\partial n} = \dfrac{-2T_{\text{Pod}}}{\rho n^3 D^4}$	-0.08711
$\dfrac{\partial K_{T\text{Pod}}}{\partial \rho} = \dfrac{-T_{\text{Pod}}}{\rho^2 n^2 D^4}$	-0.00048
$\dfrac{\partial K_{T\text{Pod}}}{\partial D} = \dfrac{-4T_{\text{Pod}}}{\rho n^2 D^5}$	-7.09777
$B_{K\,T\text{Pod}}\big\|_{J=0.00}$	0.005645
$100 * B_{K\,T\text{Pod}}\big\|_{J=0.00} / K_{T\text{Pod}}$	1.178266%

Advance Coefficient, *J*	**0.10**
Thrust Coefficient, $K_{T\text{Pod}} = \dfrac{T_{\text{Pod}}}{\rho n^2 D^4}$	0.4512

$(B_{KT\text{Pod}})^2 = \left(\dfrac{\partial K_{T\text{Pod}}}{\partial T_{\text{Pod}}} \times B_{T\text{Pod}}\right)^2 + \left(\dfrac{\partial K_{T\text{Pod}}}{\partial n} \times B_n\right)^2 + \left(\dfrac{\partial K_{T\text{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\dfrac{\partial K_{T\text{Pod}}}{\partial D} \times B_D\right)^2$	
$\dfrac{\partial K_{T\text{Pod}}}{\partial T_{\text{Pod}}} = \dfrac{1}{\rho n^2 D^4}$	0.001557
$\dfrac{\partial K_{T\text{Pod}}}{\partial n} = \dfrac{-2T_{\text{Pod}}}{\rho n^3 D^4}$	-0.08204
$\dfrac{\partial K_{T\text{Pod}}}{\partial \rho} = \dfrac{-T_{\text{Pod}}}{\rho^2 n^2 D^4}$	-0.00045

$\frac{\partial K_{T\mathrm{Pod}}}{\partial D} = \frac{-4T_{\mathrm{Pod}}}{\rho n^2 D^5}$	-6.68467
$B_{K\,T\mathrm{Pod}}\big\|_{J=0.10}$	0.005442
$100 * B_{K\,T\mathrm{Pod}}\big\|_{J=0.10} / K_{T\mathrm{Pod}}$	1.205969%

Advance Coefficient, *J*	**0.20**
Thrust Coefficient, $K_{T\mathrm{Pod}} = \frac{T_{\mathrm{Pod}}}{\rho n^2 D^4}$	0.4131

$(B_{KT\mathrm{Pod}})^2 = \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} \times B_{T\mathrm{Pod}}\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} = \frac{1}{\rho n^2 D^4}$	0.001557
$\frac{\partial K_{T\mathrm{Pod}}}{\partial n} = \frac{-2T_{\mathrm{Pod}}}{\rho n^3 D^4}$	-0.07511
$\frac{\partial K_{T\mathrm{Pod}}}{\partial \rho} = \frac{-T_{\mathrm{Pod}}}{\rho^2 n^2 D^4}$	-0.00041
$\frac{\partial K_{T\mathrm{Pod}}}{\partial D} = \frac{-4T_{\mathrm{Pod}}}{\rho n^2 D^5}$	-6.12036
$B_{K\,T\mathrm{Pod}}\big\|_{J=0.20}$	0.005172
$100 * B_{K\,T\mathrm{Pod}}\big\|_{J=0.20} / K_{T\mathrm{Pod}}$	1.251839%

Advance Coefficient, *J*	**0.30**
Thrust Coefficient, $K_{T\mathrm{Pod}} = \frac{T_{\mathrm{Pod}}}{\rho n^2 D^4}$	0.3742

$(B_{KT\mathrm{Pod}})^2 = \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} \times B_{T\mathrm{Pod}}\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} = \frac{1}{\rho n^2 D^4}$	0.001557
$\frac{\partial K_{T\mathrm{Pod}}}{\partial n} = \frac{-2T_{\mathrm{Pod}}}{\rho n^3 D^4}$	-0.06804
$\frac{\partial K_{T\mathrm{Pod}}}{\partial \rho} = \frac{-T_{\mathrm{Pod}}}{\rho^2 n^2 D^4}$	-0.00037
$\frac{\partial K_{T\mathrm{Pod}}}{\partial D} = \frac{-4T_{\mathrm{Pod}}}{\rho n^2 D^5}$	-5.54437

$B_{K\,T\mathrm{Pod}}\Big\vert_{J=0.30}$	0.004907
$100 * B_{K\,T\mathrm{Pod}}\Big\vert_{J=0.30} / K_{T\mathrm{Pod}}$	1.311289%

Advance Coefficient, *J*	**0.40**
Thrust Coefficient, $K_{T\mathrm{Pod}} = \dfrac{T_{\mathrm{Pod}}}{\rho n^2 D^4}$	0.3337

$\left(B_{KT\mathrm{Pod}}\right)^2 = \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} \times B_{T\mathrm{Pod}}\right)^2 + \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial n} \times B_n\right)^2 + \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial D} \times B_D\right)^2$	
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} = \dfrac{1}{\rho n^2 D^4}$	0.001557
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial n} = \dfrac{-2T_{\mathrm{Pod}}}{\rho n^3 D^4}$	-0.06067
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial \rho} = \dfrac{-T_{\mathrm{Pod}}}{\rho^2 n^2 D^4}$	-0.00033
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial D} = \dfrac{-4T_{\mathrm{Pod}}}{\rho n^2 D^5}$	-4.94342
$B_{K\,T\mathrm{Pod}}\Big\vert_{J=0.40}$	0.004646
$100 * B_{K\,T\mathrm{Pod}}\Big\vert_{J=0.40} / K_{T\mathrm{Pod}}$	1.392332

Advance Coefficient, *J*	**0.50**
Thrust Coefficient, $K_{T\mathrm{Pod}} = \dfrac{T_{\mathrm{Pod}}}{\rho n^2 D^4}$	0.2895

$\left(B_{KT\mathrm{Pod}}\right)^2 = \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} \times B_{T\mathrm{Pod}}\right)^2 + \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial n} \times B_n\right)^2 + \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial D} \times B_D\right)^2$	
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} = \dfrac{1}{\rho n^2 D^4}$	0.001557
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial n} = \dfrac{-2T_{\mathrm{Pod}}}{\rho n^3 D^4}$	-0.05263
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial \rho} = \dfrac{-T_{\mathrm{Pod}}}{\rho^2 n^2 D^4}$	-0.00029
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial D} = \dfrac{-4T_{\mathrm{Pod}}}{\rho n^2 D^5}$	-4.28823
$B_{K\,T\mathrm{Pod}}\Big\vert_{J=0.50}$	0.00438
$100 * B_{K\,T\mathrm{Pod}}\Big\vert_{J=0.50} / K_{T\mathrm{Pod}}$	1.513306

Advance Coefficient, *J*	**0.60**
Thrust Coefficient, $K_{T\mathrm{Pod}} = \dfrac{T_{\mathrm{Pod}}}{\rho n^2 D^4}$	0.2468

$(B_{KT\mathrm{Pod}})^2 = \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} \times B_{T\mathrm{Pod}}\right)^2 + \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial n} \times B_n\right)^2 + \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial D} \times B_D\right)^2$	
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} = \dfrac{1}{\rho n^2 D^4}$	0.001557
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial n} = \dfrac{-2T_{\mathrm{Pod}}}{\rho n^3 D^4}$	-0.04487
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial \rho} = \dfrac{-T_{\mathrm{Pod}}}{\rho^2 n^2 D^4}$	-0.00025
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial D} = \dfrac{-4T_{\mathrm{Pod}}}{\rho n^2 D^5}$	-3.65595
$B_{K\,T\mathrm{Pod}}\big\|_{J=0.60}$	0.004147
$100^* B_{K\,T\mathrm{Pod}}\big\|_{J=0.60} / K_{T\mathrm{Pod}}$	1.680414

Advance Coefficient, *J*	**0.70**
Thrust Coefficient, $K_{T\mathrm{Pod}} = \dfrac{T_{\mathrm{Pod}}}{\rho n^2 D^4}$	0.2054

$(B_{KT\mathrm{Pod}})^2 = \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} \times B_{T\mathrm{Pod}}\right)^2 + \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial n} \times B_n\right)^2 + \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\dfrac{\partial K_{T\mathrm{Pod}}}{\partial D} \times B_D\right)^2$	
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} = \dfrac{1}{\rho n^2 D^4}$	0.001557
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial n} = \dfrac{-2T_{\mathrm{Pod}}}{\rho n^3 D^4}$	-0.03734
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial \rho} = \dfrac{-T_{\mathrm{Pod}}}{\rho^2 n^2 D^4}$	-0.00021
$\dfrac{\partial K_{T\mathrm{Pod}}}{\partial D} = \dfrac{-4T_{\mathrm{Pod}}}{\rho n^2 D^5}$	-3.0423
$B_{K\,T\mathrm{Pod}}\big\|_{J=0.70}$	0.003946
$100^* B_{K\,T\mathrm{Pod}}\big\|_{J=0.70} / K_{T\mathrm{Pod}}$	1.921326

Advance Coefficient, *J*	**0.80**
Thrust Coefficient, $K_{T\mathrm{Pod}} = \dfrac{T_{\mathrm{Pod}}}{\rho n^2 D^4}$	0.1639

$(B_{KT\text{Pod}})^2 = \left(\frac{\partial K_{T\text{Pod}}}{\partial T_{\text{Pod}}} \times B_{T\text{Pod}}\right)^2 + \left(\frac{\partial K_{T\text{Pod}}}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_{T\text{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_{T\text{Pod}}}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_{T\text{Pod}}}{\partial T_{\text{Pod}}} = \frac{1}{\rho n^2 D^4}$	0.001557
$\frac{\partial K_{T\text{Pod}}}{\partial n} = \frac{-2T_{\text{Pod}}}{\rho n^3 D^4}$	-0.0298
$\frac{\partial K_{T\text{Pod}}}{\partial \rho} = \frac{-T_{\text{Pod}}}{\rho^2 n^2 D^4}$	-0.00016
$\frac{\partial K_{T\text{Pod}}}{\partial D} = \frac{-4T_{\text{Pod}}}{\rho n^2 D^5}$	-2.4278
$B_{K\,T\text{Pod}}\big\|_{J=0.80}$	0.003773
$100 * B_{K\,T\text{Pod}}\big\|_{J=0.80} / K_{T\text{Pod}}$	2.302303

Advance Coefficient, *J*	**0.90**
Thrust Coefficient, $K_{T\text{Pod}} = \frac{T_{\text{Pod}}}{\rho n^2 D^4}$	0.1206

$(B_{KT\text{Pod}})^2 = \left(\frac{\partial K_{T\text{Pod}}}{\partial T_{\text{Pod}}} \times B_{T\text{Pod}}\right)^2 + \left(\frac{\partial K_{T\text{Pod}}}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_{T\text{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_{T\text{Pod}}}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_{T\text{Pod}}}{\partial T_{\text{Pod}}} = \frac{1}{\rho n^2 D^4}$	0.001557
$\frac{\partial K_{T\text{Pod}}}{\partial n} = \frac{-2T_{\text{Pod}}}{\rho n^3 D^4}$	-0.02192
$\frac{\partial K_{T\text{Pod}}}{\partial \rho} = \frac{-T_{\text{Pod}}}{\rho^2 n^2 D^4}$	-0.00012
$\frac{\partial K_{T\text{Pod}}}{\partial D} = \frac{-4T_{\text{Pod}}}{\rho n^2 D^5}$	-1.78608
$B_{K\,T\text{Pod}}\big\|_{J=0.90}$	0.003628
$100 * B_{K\,T\text{Pod}}\big\|_{J=0.90} / K_{T\text{Pod}}$	3.009375

Advance Coefficient, *J*	**1.00**
Thrust Coefficient, $K_{T\text{Pod}} = \frac{T_{\text{Pod}}}{\rho n^2 D^4}$	0.0757

$\left(B_{KT\mathrm{Pod}}\right)^2 = \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} \times B_{T\mathrm{Pod}}\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} = \frac{1}{\rho n^2 D^4}$	0.001557
$\frac{\partial K_{T\mathrm{Pod}}}{\partial n} = \frac{-2T_{\mathrm{Pod}}}{\rho n^3 D^4}$	-0.01377
$\frac{\partial K_{T\mathrm{Pod}}}{\partial \rho} = \frac{-T_{\mathrm{Pod}}}{\rho^2 n^2 D^4}$	-7.6E-05
$\frac{\partial K_{T\mathrm{Pod}}}{\partial D} = \frac{-4T_{\mathrm{Pod}}}{\rho n^2 D^5}$	-1.12204
$B_{K\,T\mathrm{Pod}}\big\rvert_{J=1.00}$	0.003521
$100{*}\,B_{K\,T\mathrm{Pod}}\big\rvert_{J=1.00} / K_{T\mathrm{Pod}}$	4.648986

Advance Coefficient, *J*	**1.10**
Thrust Coefficient, $K_{T\mathrm{Pod}} = \frac{T_{\mathrm{Pod}}}{\rho n^2 D^4}$	0.0295

$\left(B_{KT\mathrm{Pod}}\right)^2 = \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} \times B_{T\mathrm{Pod}}\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} = \frac{1}{\rho n^2 D^4}$	0.001557
$\frac{\partial K_{T\mathrm{Pod}}}{\partial n} = \frac{-2T_{\mathrm{Pod}}}{\rho n^3 D^4}$	-0.00537
$\frac{\partial K_{T\mathrm{Pod}}}{\partial \rho} = \frac{-T_{\mathrm{Pod}}}{\rho^2 n^2 D^4}$	-3E-05
$\frac{\partial K_{T\mathrm{Pod}}}{\partial D} = \frac{-4T_{\mathrm{Pod}}}{\rho n^2 D^5}$	-0.43719
$B_{K\,T\mathrm{Pod}}\big\rvert_{J=1.10}$	0.00346
$100{*}\,B_{K\,T\mathrm{Pod}}\big\rvert_{J=1.10} / K_{T\mathrm{Pod}}$	11.72619

Advance Coefficient, *J*	**1.20**
Thrust Coefficient, $K_{T\mathrm{Pod}} = \frac{T_{\mathrm{Pod}}}{\rho n^2 D^4}$	-0.0194

$\left(B_{KT\mathrm{Pod}}\right)^2 = \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} \times B_{T\mathrm{Pod}}\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_{T\mathrm{Pod}}}{\partial D} \times B_D\right)^2$

$\frac{\partial K_{T\mathrm{Pod}}}{\partial T_{\mathrm{Pod}}} = \frac{1}{\rho n^2 D^4}$	0.001557
$\frac{\partial K_{T\mathrm{Pod}}}{\partial n} = \frac{-2T_{\mathrm{Pod}}}{\rho n^3 D^4}$	0.003264
$\frac{\partial K_{T\mathrm{Pod}}}{\partial \rho} = \frac{-T_{\mathrm{Pod}}}{\rho^2 n^2 D^4}$	1.8E-05
$\frac{\partial K_{T\mathrm{Pod}}}{\partial D} = \frac{-4T_{\mathrm{Pod}}}{\rho n^2 D^5}$	0.265916
$B_{K\,T\mathrm{Pod}}\Big\vert_{J=1.20}$	0.003454
$100 * B_{K\,T\mathrm{Pod}}\Big\vert_{J=1.20} / K_{T\mathrm{Pod}}$	-19.2404

Propeller Torque Coefficient-Total Bias Error

The total bias limit for propeller torque coefficient is calculated based on ITTC recommended procedure 7.5-02-03-02.2 as follows:

Item	Value
Water Density, ρ	999.00
Shaft Rotational Speed, n	11.00
Propeller Diameter, D	0.27 m

Advance Coefficient, *J*	**0.00**
Torque Coefficient, $K_Q = \dfrac{Q}{\rho n^2 D^5}$	0.06832

$(B_{KQ})^2 = \left(\dfrac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\dfrac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\dfrac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\dfrac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\dfrac{\partial K_Q}{\partial Q} = \dfrac{1}{\rho n^2 D^5}$	0.005765
$\dfrac{\partial K_Q}{\partial n} = \dfrac{-2Q}{\rho n^3 D^5}$	-0.01242
$\dfrac{\partial K_Q}{\partial \rho} = \dfrac{-Q}{\rho^2 n^2 D^5}$	-6.8E-05
$\dfrac{\partial K_Q}{\partial D} = \dfrac{-5Q}{\rho n^2 D^6}$	-1.26527
$B_{KQ}\big\|_{J=0.00}$	0.000751
$100 * B_Q\big\|_{J=0.00} / K_Q$	1.098484

Advance Coefficient, *J*	**0.10**
Torque Coefficient, $K_Q = \dfrac{Q}{\rho n^2 D^5}$	0.06463

$(B_{KQ})^2 = \left(\dfrac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\dfrac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\dfrac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\dfrac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\dfrac{\partial K_Q}{\partial Q} = \dfrac{1}{\rho n^2 D^5}$	0.005765
$\dfrac{\partial K_Q}{\partial n} = \dfrac{-2Q}{\rho n^3 D^5}$	-0.01175

$\frac{\partial K_Q}{\partial \rho} = \frac{-Q}{\rho^2 n^2 D^5}$	-6.5E-05
$\frac{\partial K_Q}{\partial D} = \frac{-5Q}{\rho n^2 D^6}$	-1.19696
$B_{K\,Q}\big\vert_{J=0.10}$	0.000721
$100 * B_Q\big\vert_{J=0.10} / K_Q$	1.115033

Advance Coefficient, *J*	**0.20**
Torque Coefficient, $K_Q = \frac{Q}{\rho n^2 D^5}$	0.06041

$(B_{KQ})^2 = \left(\frac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\frac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho n^2 D^5}$	0.005765
$\frac{\partial K_Q}{\partial n} = \frac{-2Q}{\rho n^3 D^5}$	-0.01098
$\frac{\partial K_Q}{\partial \rho} = \frac{-Q}{\rho^2 n^2 D^5}$	-6E-05
$\frac{\partial K_Q}{\partial D} = \frac{-5Q}{\rho n^2 D^6}$	-1.1186
$B_{K\,Q}\big\vert_{J=0.20}$	0.000687
$100 * B_Q\big\vert_{J=0.20} / K_Q$	1.137479

Advance Coefficient, *J*	**0.30**
Torque Coefficient, $K_Q = \frac{Q}{\rho n^2 D^5}$	0.05611

$(B_{KQ})^2 = \left(\frac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\frac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho n^2 D^5}$	0.005765
$\frac{\partial K_Q}{\partial n} = \frac{-2Q}{\rho n^3 D^5}$	-0.0102

$\frac{\partial K_Q}{\partial \rho} = \frac{-Q}{\rho^2 n^2 D^5}$	-5.6E-05
$\frac{\partial K_Q}{\partial D} = \frac{-5Q}{\rho n^2 D^6}$	-1.03902
$B_{KQ}\big\rvert_{J=0.30}$	0.000654
$100 * B_Q\big\rvert_{J=0.30} / K_Q$	1.165059

Advance Coefficient, *J*	**0.40**
Torque Coefficient, $K_Q = \frac{Q}{\rho n^2 D^5}$	0.05086

$(B_{KQ})^2 = \left(\frac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\frac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho n^2 D^5}$	0.005765
$\frac{\partial K_Q}{\partial n} = \frac{-2Q}{\rho n^3 D^5}$	-0.00925
$\frac{\partial K_Q}{\partial \rho} = \frac{-Q}{\rho^2 n^2 D^5}$	-5.1E-05
$\frac{\partial K_Q}{\partial D} = \frac{-5Q}{\rho n^2 D^6}$	-0.9418
$B_{KQ}\big\rvert_{J=0.40}$	0.000614
$100 * B_Q\big\rvert_{J=0.40} / K_Q$	1.207405

Advance Coefficient, *J*	**0.50**
Torque Coefficient, $K_Q = \frac{Q}{\rho n^2 D^5}$	0.04591

$(B_{KQ})^2 = \left(\frac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\frac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho n^2 D^5}$	0.005765
$\frac{\partial K_Q}{\partial n} = \frac{-2Q}{\rho n^3 D^5}$	-0.00835

$\frac{\partial K_Q}{\partial \rho} = \frac{-Q}{\rho^2 n^2 D^5}$	-4.6E-05
$\frac{\partial K_Q}{\partial D} = \frac{-5Q}{\rho n^2 D^6}$	-0.85024
$B_{KQ}\big\|_{J=0.50}$	0.000578
$100 * B_Q\big\|_{J=0.50} / K_Q$	1.259228

Advance Coefficient, *J*	**0.60**
Torque Coefficient, $K_Q = \frac{Q}{\rho n^2 D^5}$	0.04073

$(B_{KQ})^2 = \left(\frac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\frac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho n^2 D^5}$	0.005765
$\frac{\partial K_Q}{\partial n} = \frac{-2Q}{\rho n^3 D^5}$	-0.00741
$\frac{\partial K_Q}{\partial \rho} = \frac{-Q}{\rho^2 n^2 D^5}$	-4.1E-05
$\frac{\partial K_Q}{\partial D} = \frac{-5Q}{\rho n^2 D^6}$	-0.75436
$B_{KQ}\big\|_{J=0.60}$	0.000542
$100 * B_Q\big\|_{J=0.60} / K_Q$	1.331344

Advance Coefficient, *J*	**0.70**
Torque Coefficient, $K_Q = \frac{Q}{\rho n^2 D^5}$	0.03522

$(B_{KQ})^2 = \left(\frac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\frac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho n^2 D^5}$	0.005765
$\frac{\partial K_Q}{\partial n} = \frac{-2Q}{\rho n^3 D^5}$	-0.0064

$\frac{\partial K_Q}{\partial \rho} = \frac{-Q}{\rho^2 n^2 D^5}$	-3.5E-05
$\frac{\partial K_Q}{\partial D} = \frac{-5Q}{\rho n^2 D^6}$	-0.65231
$B_{KQ}\big\|_{J=0.70}$	0.000507
$100 * B_Q\big\|_{J=0.70} / K_Q$	1.438278

Advance Coefficient, *J*	**0.80**
Torque Coefficient, $K_Q = \frac{Q}{\rho n^2 D^5}$	0.02963

$(B_{KQ})^2 = \left(\frac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\frac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho n^2 D^5}$	0.005765
$\frac{\partial K_Q}{\partial n} = \frac{-2Q}{\rho n^3 D^5}$	-0.00539
$\frac{\partial K_Q}{\partial \rho} = \frac{-Q}{\rho^2 n^2 D^5}$	-3E-05
$\frac{\partial K_Q}{\partial D} = \frac{-5Q}{\rho n^2 D^6}$	-0.54882
$B_{KQ}\big\|_{J=0.80}$	0.000474
$100 * B_Q\big\|_{J=0.80} / K_Q$	1.597849

Advance Coefficient, *J*	**0.90**
Torque Coefficient, $K_Q = \frac{Q}{\rho n^2 D^5}$	0.02329

$(B_{KQ})^2 = \left(\frac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\frac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho n^2 D^5}$	0.005765
$\frac{\partial K_Q}{\partial n} = \frac{-2Q}{\rho n^3 D^5}$	-0.00424

$\frac{\partial K_Q}{\partial \rho} = \frac{-Q}{\rho^2 n^2 D^5}$	-2.3E-05
$\frac{\partial K_Q}{\partial D} = \frac{-5Q}{\rho n^2 D^6}$	-0.43134
$B_{KQ}\big\vert_{J=0.90}$	0.000441
$100 * B_Q\big\vert_{J=0.90} / K_Q$	1.891976

Advance Coefficient, *J*	**1.00**
Torque Coefficient, $K_Q = \frac{Q}{\rho n^2 D^5}$	0.01681

$(B_{KQ})^2 = \left(\frac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\frac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho n^2 D^5}$	0.005765
$\frac{\partial K_Q}{\partial n} = \frac{-2Q}{\rho n^3 D^5}$	-0.00306
$\frac{\partial K_Q}{\partial \rho} = \frac{-Q}{\rho^2 n^2 D^5}$	-1.7E-05
$\frac{\partial K_Q}{\partial D} = \frac{-5Q}{\rho n^2 D^6}$	-0.31127
$B_{KQ}\big\vert_{J=1.00}$	0.000413
$100 * B_Q\big\vert_{J=1.00} / K_Q$	2.459809

Advance Coefficient, *J*	**1.10**
Torque Coefficient, $K_Q = \frac{Q}{\rho n^2 D^5}$	0.00946

$(B_{KQ})^2 = \left(\frac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\frac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho n^2 D^5}$	0.005765
$\frac{\partial K_Q}{\partial n} = \frac{-2Q}{\rho n^3 D^5}$	-0.00172

$\frac{\partial K_Q}{\partial \rho} = \frac{-Q}{\rho^2 n^2 D^5}$	-9.5E-06
$\frac{\partial K_Q}{\partial D} = \frac{-5Q}{\rho n^2 D^6}$	-0.17528
$B_{KQ}\big\rvert_{J=1.10}$	0.000392
$100 * B_Q\big\rvert_{J=1.10} / K_Q$	4.14176

Advance Coefficient, *J*	**1.20**
Torque Coefficient, $K_Q = \frac{Q}{\rho n^2 D^5}$	0.00083

$(B_{KQ})^2 = \left(\frac{\partial K_Q}{\partial Q} \times B_Q\right)^2 + \left(\frac{\partial K_Q}{\partial n} \times B_n\right)^2 + \left(\frac{\partial K_Q}{\partial \rho} \times B_\rho\right)^2 + \left(\frac{\partial K_Q}{\partial D} \times B_D\right)^2$	
$\frac{\partial K_Q}{\partial Q} = \frac{1}{\rho n^2 D^5}$	0.005765
$\frac{\partial K_Q}{\partial n} = \frac{-2Q}{\rho n^3 D^5}$	-0.00015
$\frac{\partial K_Q}{\partial \rho} = \frac{-Q}{\rho^2 n^2 D^5}$	-8.3E-07
$\frac{\partial K_Q}{\partial D} = \frac{-5Q}{\rho n^2 D^6}$	-0.0154
$B_{KQ}\big\rvert_{J=1.20}$	0.000382
$100 * B_Q\big\rvert_{J=1.20} / K_Q$	45.90882

Propeller Advance Coefficient-Total Bias Error

The total bias limit for propeller advance coefficient is calculated based on ITTC recommended procedure 7.5-02-03-02.2 as follows:

Item	Value
Water Density, ρ	999.00
Shaft Rotational Speed, n	11.00
Propeller Diameter, D	0.27 m

Advance Coefficient, *J*	**0.00**
Advance Coefficient, $J = \frac{V_A}{nD}$	0.00

$(B_J)^2 = \left(\frac{\partial J}{\partial V_A} \times B_{V_A} \right)^2 + \left(\frac{\partial J}{\partial n} \times B_n \right)^2 + \left(\frac{\partial J}{\partial D} \times B_D \right)^2$	
$\frac{\partial J}{\partial V_A} = \frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-6.8E-05
$B_J \Big\rvert_{J=0.00}$	0.000751
$100^* B_J \Big\rvert_{J=0.00} / K_J$	1.098484

Advance Coefficient, *J*	**0.10**
Advance Coefficient, $J = \frac{V_A}{nD}$	0.10

$(B_J)^2 = \left(\frac{\partial J}{\partial V_A} \times B_{V_A} \right)^2 + \left(\frac{\partial J}{\partial n} \times B_n \right)^2 + \left(\frac{\partial J}{\partial D} \times B_D \right)^2$	
$\frac{\partial J}{\partial V_A} = \frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-6.8E-05
$B_J \Big\rvert_{J=0.10}$	0.000751
$100^* B_J \Big\rvert_{J=0.10} / J$	1.098484

Advance Coefficient, *J*	**0.20**
Advance Coefficient, $J = \frac{V_A}{nD}$	0.20

$(B_J)^2 = \left(\frac{\partial J}{\partial V_A} \times B_{V_A}\right)^2 + \left(\frac{\partial J}{\partial n} \times B_n\right)^2 + \left(\frac{\partial J}{\partial D} \times B_D\right)^2$	
$\frac{\partial J}{\partial V_A} = \frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-6.8E-05
$B_J \big\rvert_{J=0.20}$	0.000751
$100 * B_J \big\rvert_{J=0.20} / J$	1.098484

Advance Coefficient, *J*	**0.30**
Advance Coefficient, $J = \frac{V_A}{nD}$	0.30

$(B_J)^2 = \left(\frac{\partial J}{\partial V_A} \times B_{V_A}\right)^2 + \left(\frac{\partial J}{\partial n} \times B_n\right)^2 + \left(\frac{\partial J}{\partial D} \times B_D\right)^2$	
$\frac{\partial J}{\partial V_A} = \frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-6.8E-05
$B_J \big\rvert_{J=0.30}$	0.000751
$100 * B_J \big\rvert_{J=0.30} / J$	1.098484

Advance Coefficient, *J*	**0.40**
Advance Coefficient, $J = \frac{V_A}{nD}$	0.40

$(B_J)^2 = \left(\frac{\partial J}{\partial V_A} \times B_{V_A}\right)^2 + \left(\frac{\partial J}{\partial n} \times B_n\right)^2 + \left(\frac{\partial J}{\partial D} \times B_D\right)^2$

$\frac{\partial J}{\partial V_A}=\frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n}=-\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n}=-\frac{V_A}{n^2 D}$	-6.8E-05
$B_J\big\|_{J=0.40}$	0.000751
$100* B_J\big\|_{J=0.40}/J$	1.098484

Advance Coefficient, *J*	**0.50**
Advance Coefficient, $J=\frac{V_A}{nD}$	0.50

$(B_J)^2=\left(\frac{\partial J}{\partial V_A}\times B_{V_A}\right)^2+\left(\frac{\partial J}{\partial n}\times B_n\right)^2+\left(\frac{\partial J}{\partial D}\times B_D\right)^2$	
$\frac{\partial J}{\partial V_A}=\frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n}=-\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n}=-\frac{V_A}{n^2 D}$	-6.8E-05
$B_J\big\|_{J=0.50}$	0.000751
$100* B_J\big\|_{J=0.50}/J$	1.098484

Advance Coefficient, *J*	**0.60**
Advance Coefficient, $J=\frac{V_A}{nD}$	0.60

$(B_J)^2=\left(\frac{\partial J}{\partial V_A}\times B_{V_A}\right)^2+\left(\frac{\partial J}{\partial n}\times B_n\right)^2+\left(\frac{\partial J}{\partial D}\times B_D\right)^2$	
$\frac{\partial J}{\partial V_A}=\frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n}=-\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n}=-\frac{V_A}{n^2 D}$	-6.8E-05
$B_J\big\|_{J=0.60}$	0.000751
$100* B_J\big\|_{J=0.60}/J$	1.098484

Advance Coefficient, *J*	**0.70**
Advance Coefficient, $J = \frac{V_A}{nD}$	0.70

$(B_J)^2 = \left(\frac{\partial J}{\partial V_A} \times B_{V_A}\right)^2 + \left(\frac{\partial J}{\partial n} \times B_n\right)^2 + \left(\frac{\partial J}{\partial D} \times B_D\right)^2$	
$\frac{\partial J}{\partial V_A} = \frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-6.8E-05
$B_J\big\|_{J=0.70}$	0.000751
$100^* B_J\big\|_{J=0.70} / J$	1.098484

Advance Coefficient, *J*	**0.80**
Advance Coefficient, $J = \frac{V_A}{nD}$	0.80

$(B_J)^2 = \left(\frac{\partial J}{\partial V_A} \times B_{V_A}\right)^2 + \left(\frac{\partial J}{\partial n} \times B_n\right)^2 + \left(\frac{\partial J}{\partial D} \times B_D\right)^2$	
$\frac{\partial J}{\partial V_A} = \frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-6.8E-05
$B_J\big\|_{J=0.80}$	0.000751
$100^* B_J\big\|_{J=0.80} / J$	1.098484

Advance Coefficient, *J*	**0.90**
Advance Coefficient, $J = \frac{V_A}{nD}$	0.90

$(B_J)^2 = \left(\frac{\partial J}{\partial V_A} \times B_{V_A}\right)^2 + \left(\frac{\partial J}{\partial n} \times B_n\right)^2 + \left(\frac{\partial J}{\partial D} \times B_D\right)^2$

$\frac{\partial J}{\partial V_A} = \frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-6.8E-05
$B_J \big\rvert_{J=0.90}$	0.000751
$100{*}\, B_J \big\rvert_{J=0.90} / J$	1.098484

Advance Coefficient, *J*	**1.00**
Advance Coefficient, $J = \frac{V_A}{nD}$	1.00

$(B_J)^2 = \left(\frac{\partial J}{\partial V_A} \times B_{V_A}\right)^2 + \left(\frac{\partial J}{\partial n} \times B_n\right)^2 + \left(\frac{\partial J}{\partial D} \times B_D\right)^2$	
$\frac{\partial J}{\partial V_A} = \frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-6.8E-05
$B_J \big\rvert_{J=1.00}$	0.000751
$100{*}\, B_J \big\rvert_{J=1.00} / J$	1.098484

Advance Coefficient, *J*	**1.10**
Advance Coefficient, $J = \frac{V_A}{nD}$	1.10

$(B_J)^2 = \left(\frac{\partial J}{\partial V_A} \times B_{V_A}\right)^2 + \left(\frac{\partial J}{\partial n} \times B_n\right)^2 + \left(\frac{\partial J}{\partial D} \times B_D\right)^2$	
$\frac{\partial J}{\partial V_A} = \frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-6.8E-05
$B_J \big\rvert_{J=0.00}$	0.000751
$100{*}\, B_J \big\rvert_{J=0.00} / J$	1.098484

Advance Coefficient, *J*	**1.20**
Advance Coefficient, $J = \frac{V_A}{nD}$	1.20

$(B_J)^2 = \left(\frac{\partial J}{\partial V_A} \times B_{V_A}\right)^2 + \left(\frac{\partial J}{\partial n} \times B_n\right)^2 + \left(\frac{\partial J}{\partial D} \times B_D\right)^2$	
$\frac{\partial J}{\partial V_A} = \frac{1}{nD}$	0.005765
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-0.01242
$\frac{\partial J}{\partial n} = -\frac{V_A}{n^2 D}$	-6.8E-05
$B_J\big\rvert_{J=0.00}$	0.000751
$100 * B_J\big\rvert_{J=0.00} / J$	1.098484

Overall Uncertainty

Table B-1: Bias and Precision uncertainties of the components of podded propulsor variables.

	J=0.0	J=0.1	J=0.2	J=0.3	J=0.4	J=0.5	J=0.6	J=0.7	J=0.8	J=0.9	J=1.0	J=1.1	J=1.2
P (Kg/m^3)	999	999	999	999	999	999	999	999	999	999	999	999	999
B_ρ	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940
P_ρ	-	-	-	-	-	-	-	-	-	-	-	-	-
$(U_\rho/\rho)^2$	8.85E-09	8.85E-09	8.85E-09	8.85E-09	8.85E-09	8.85E-09	8.85E-09	8.85E-09	8.85E-09	8.85E-09	8.85E-09	8.85E-09	8.85E-09
AA (deg)	30	30	30	30	30	30	30	30	30	30	30	30	30
B_{AA}	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920
P_{AA}	-	-	-	-	-	-	-	-	-	-	-	-	-
$(U_{AA}/AA)^2$	5.32E-04	5.32E-04	5.32E-04	5.32E-04	5.32E-04	5.32E-04	5.32E-04	5.32E-04	5.32E-04	5.32E-04	5.32E-04	5.32E-04	5.32E-04
D (m)	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
B_D	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
P_D	-	-	-	-	-	-	-	-	-	-	-	-	-
$(U_D/D)^2$	2.74E-07	2.74E-07	2.74E-07	2.74E-07	2.74E-07	2.74E-07	2.74E-07	2.74E-07	2.74E-07	2.74E-07	2.74E-07	2.74E-07	2.74E-07
N (rps)	11	11	11	11	11	11	11	11	11	11	11	11	11
B_n	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
P_n	0.0096	0.0284	0.0269	0.0253	0.0282	0.0278	0.0274	0.0267	0.0275	0.0256	0.0273	0.0268	0.0279
$(U_n/n)^2$	2.14E-05	2.73E-05	2.66E-05	2.59E-05	2.72E-05	2.70E-05	2.69E-05	2.66E-05	2.69E-05	2.61E-05	2.68E-05	2.66E-05	2.71E-05
V_A (m/s)	0.0000	0.2970	0.5940	0.8910	1.1880	1.4850	1.7820	2.0790	2.3760	2.6730	2.9700	3.2670	3.5640
J	0.0000	0.1000	0.2000	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	1.0000	1.1000	1.2000
B_{VA}	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154
P_{VA}	0.0001	0.0000	0.0001	0.0002	0.0000	0.0001	0.0002	0.0006	0.0005	0.0007	0.0007	0.0011	0.0012
$(U_{DA}/V_A)^2$	#DIV/0!	2.68E-03	6.71E-04	2.98E-04	1.68E-04	1.07E-04	7.46E-05	5.48E-05	4.20E-05	3.32E-05	2.69E-05	2.23E-05	1.88E-05

	J=0.0	J=0.1	J=0.2	J=0.3	J=0.4	J=0.5	J=0.6	J=0.7	J=0.8	J=0.9	J=1.0	J=1.1	J=1.2
T_{Prop} (N)	307.77	289.86	265.39	240.42	214.36	185.95	158.53	131.92	131.92	105.27	77.45	48.65	18.96
K_{TProp}	0.4791	0.4512	0.4131	0.3742	0.3337	0.2895	0.2468	0.2054	0.1639	0.1206	0.0757	0.0295	-0.0179
B_{TProp}	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159	2.2159
P_{TProp}	0.4651	0.7181	0.4167	0.5090	0.3925	0.7459	0.3420	0.3826	0.8855	0.6028	0.7953	0.8763	1.0194
$(U_{TProp}/T_{Prop})^2$	5.41E-05	6.46E-05	7.22E-05	8.94E-05	1.10E-04	1.58E-04	2.00E-04	2.91E-04	3.27E-04	4.76E-04	9.24E-04	2.40E-03	1.66E-02
Q (Nm)	11.85	11.21	10.48	9.73	8.82	7.96	7.07	6.11	5.14	4.04	2.92	1.64	0.14
K_Q	0.0683	0.0646	0.0604	0.0561	0.0509	0.0459	0.0407	0.0352	0.0296	0.0233	0.0168	0.0095	0.0008
B_Q	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662	0.0662
P_Q	0.0016	0.0356	0.0272	0.0177	0.0054	0.0065	0.0390	0.0122	0.0458	0.0195	0.0570	0.0305	0.0000
$(U_Q/Q)^2$	3.12E-05	4.49E-05	4.66E-05	4.96E-05	5.67E-05	6.98E-05	1.18E-04	1.21E-04	2.45E-04	2.92E-04	8.97E-04	1.97E-03	2.11E-01
F_X or T_{Unit} (N)	301.47	281.47	255.73	229.08	202.87	174.15	145.95	117.88	90.38	61.73	32.05	2.18	-31.73
K_{TUnit}	0.4693	0.4381	0.3981	0.356600	0.315800	0.271100	0.227200	0.183500	0.140700	0.096100	0.049900	0.003400	-0.0494
B_{Tunit}	0.6391	0.6391	0.6391	0.6392	0.6392	0.6394	0.6395	0.6397	0.6401	0.6403	0.6406	0.6411	0.6415
P_{TProp}	0.6963	0.6624	0.4704	0.7059	0.6585	0.5856	0.7842	0.7171	3.0795	1.1557	0.6913	0.0004	0.5189
$(U_{TUnit}/T_{Unit})^2$	9.83E-06	1.07E-05	9.63E-06	1.73E-05	2.05E-05	2.48E-05	4.81E-05	6.65E-05	1.21E-03	4.58E-04	8.64E-04	8.62E-02	6.76E-04
F_Y (N)	20.39	19.92	19.67	17.53	16.93	14.90	12.66	11.10	9.96	8.98	7.74	7.21	5.11
K_{FY}	0.0317	0.0310	0.0306	0.027281	0.026358	0.023192	0.019711	0.017272	0.015510	0.013976	0.012043	0.011221	0.0080
B_{FY}	0.7626	0.7626	0.7591	0.752966	0.751577	0.747551	0.745634	0.742936	0.739731	0.737397	0.735318	0.731805	0.7294
P_{FY}	0.9865	1.1183	1.0995	0.846057	1.045572	1.000031	0.699036	0.903499	0.983355	1.130782	1.057046	0.712972	0.9811
$(U_{FY}/F_Y)^2$	3.74E-03	4.62E-03	4.61E-03	4.18E-03	5.78E-03	7.02E-03	6.51E-03	1.11E-02	1.53E-02	2.26E-02	2.77E-02	2.01E-02	5.73E-02

	J=0.0	**J=0.1**	**J=0.2**	**J=0.3**	**J=0.4**	**J=0.5**	**J=0.6**	**J=0.7**	**J=0.8**	**J=0.9**	**J=1.0**	**J=1.1**	**J=1.2**
F_Z (N)	11.37	12.36	12.49	12.43	11.67	12.03	11.05	10.88	10.30	10.46	10.41	10.84	10.82
K_{FZ}	0.0177	0.0192	0.0195	0.019350	0.018172	0.018720	0.017202	0.016933	0.016029	0.016276	0.016204	0.016868	0.0168
B_{FZ}	1.8743	1.8747	1.8368	1.767300	1.752919	1.707577	1.686506	1.656024	1.619302	1.592962	1.569538	1.528580	1.5016
P_{FZ}	0.4793	0.5142	0.4522	0.416251	0.848629	0.835418	0.690880	0.715301	0.854036	0.765266	0.935634	0.501086	0.4873
$(U_{FZ}/F_Z)^2$	2.90E-02	2.47E-02	2.29E-02	2.13E-02	2.78E-02	2.50E-02	2.72E-02	2.75E-02	3.16E-02	2.86E-02	3.08E-02	2.20E-02	2.13E-02
M_X (Nm)	153.00	137.92	122.30	103.45	89.56	69.69	55.27	40.99	27.83	14.45	-2.62	-13.60	-26.95
K_{MX}	0.8821	0.7952	0.7051	0.596456	0.516368	0.401775	0.318653	0.236332	0.160471	0.083331	-0.015084	-0.078431	-0.1554
B_{MX}	0.5243	0.5240	0.5130	0.493094	0.488548	0.475208	0.468798	0.459689	0.448742	0.440690	0.433457	0.421076	0.4126
P_{MX}	2.1139	1.1792	1.0653	0.952723	0.738724	0.726402	0.990374	1.237505	0.432988	0.485318	0.146373	0.417215	0.4174
$(U_{MX}/M_X)^2$	2.03E-04	8.75E-05	9.35E-05	1.08E-04	9.78E-05	1.55E-04	3.93E-04	1.04E-03	5.02E-04	2.06E-03	3.06E-02	1.90E-03	4.74E-04
M_Y (Nm)	47.75	47.21	47.59	49.70	41.20	45.56	42.27	37.94	31.34	23.02	14.97	3.03	-10.21
K_{MY}	0.2753	0.2722	0.2744	0.286552	0.237531	0.262658	0.243688	0.218752	0.180665	0.132699	0.086301	0.017467	-0.0589
B_{MY}	2.0743	2.0743	2.0765	2.080896	2.081839	2.084836	2.086270	2.088374	2.090965	2.092872	2.094600	2.097675	2.0998
P_{MY}	1.3015	1.0308	1.1672	0.979184	1.038114	0.562152	0.209076	0.636427	0.767946	0.670870	0.514977	0.267345	0.4604
$(U_{MY}/M_Y)^2$	2.63E-03	2.41E-03	2.51E-03	2.14E-03	3.19E-03	2.25E-03	2.46E-03	3.31E-03	5.05E-03	9.12E-03	2.08E-02	4.87E-01	4.43E-02
M_Z (Nm)	316.23	300.06	279.43	259.31	239.67	213.44	190.06	169.28	147.75	128.68	110.02	91.57	70.74
K_{MZ}	1.8232	1.7299	1.6110	1.495039	1.381777	1.230556	1.095784	0.975947	0.851867	0.741903	0.634286	0.527946	0.4078
B_{MZ}	0.1398	0.1398	0.1398	0.139648	0.139628	0.139553	0.139521	0.139471	0.139411	0.139370	0.139334	0.139267	0.1392
P_{MZ}	1.3497	0.9229	1.0803	0.931339	0.910767	1.030657	0.631036	0.756710	0.830758	0.891493	1.101379	1.065461	0.8242
$(U_{MZ}/M_Z)^2$	1.84E-05	9.68E-06	1.52E-05	1.32E-05	1.48E-05	2.37E-05	1.16E-05	2.07E-05	3.25E-05	4.92E-05	1.02E-04	1.38E-04	1.40E-04

The overall uncertainties of the measured parameters were then obtained by combining the precision limit and bias limit estimates of each of the parameters as shown in Table B-1. Using the uncertainty expressions (equations 2-2 to 2-9) the estimates for the data reduction equations as given in Table B-2.

Table B-2: Overall Uncertainties of the components of podded propulsor variables.

	J=0.0	J=0.1	J=0.2	J=0.3	J=0.4	J=0.5	J=0.6	J=0.7	J=0.8	J=0.9	J=1.0	J=1.1	J=1.2
U_{ρ}	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940	0.0940
U_{D}	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
U_{n}	0.0508	0.0573	0.0573	0.0577	0.0575	0.0582	0.0576	0.0577	0.0578	0.0575	0.0569	0.0571	0.0580
U_{AA}	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920	0.6920
U_{VA}	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154
U_{TProp}	2.2736	2.3043	2.2581	2.2536	2.2391	2.3060	2.2367	2.2578	2.3221	2.2883	2.3935	2.3617	2.4437
U_{Q}	0.0662	0.0756	0.0724	0.0679	0.0663	0.0665	0.0753	0.0675	0.0798	0.0693	0.0811	0.0713	0.0662
U_{TUnit}	1.1456	0.8682	0.7844	1.0566	0.9297	0.9883	0.9991	1.1576	2.6752	1.0806	0.8700	0.6411	-0.8398
U_{FY}	1.3186	1.2530	1.1501	1.2148	1.2475	1.3140	1.0316	1.3158	1.2627	1.2739	1.2731	1.0507	1.0882
U_{FZ}	1.9412	1.9468	1.9282	1.8426	1.9038	1.8947	1.8222	1.8121	1.8258	1.7497	1.7813	1.5990	1.5688
U_{MX}	2.2553	1.6666	1.0842	1.3142	0.9236	0.8928	1.1391	1.0914	0.7193	0.5921	-0.4611	-0.6303	-0.6857
U_{MY}	2.4117	2.3751	2.2809	2.2985	2.2461	2.1271	2.1037	2.2397	2.2379	2.2265	2.1475	2.1154	-2.1465
U_{MZ}	1.1873	1.0638	1.0031	1.0536	0.9611	0.9577	0.9345	0.7207	0.7315	1.0925	0.9346	1.0044	1.0622