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Free-induction decay after a pulse saturation for systems with noncorrelated frequency exchange

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The free-induction decay (FID) after saturation by a laser radiation pulse of finite duration is studied for systems with spectral diffusion. The exact solution of the FID signal shape has been obtained in the framework of a model of noncorrelated spectral exchange. The analysis accounts for the finite duration of the saturating field and is valid at an arbitrary value of the spectral exchange rate and amplitude of the coherent field. An exact expression for the FID signal is derived in the weak-external-field limit. A self-consistent explanation of the experimental field dependence of the FID rate [A. Szabo and T. Muramoto, Phys. Rev. A **39**, 3992 (1989)] is obtained using a model of slow noncorrelated spectral diffusion. [S1050-2947(97)00105-4]

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I. INTRODUCTION

Investigation of coherent nonlinear phenomena gives information about the microscopic mechanisms of relaxation processes in impurity-ion crystals. One of these phenomena is free-induction decay (FID) after steady-state saturation or after excitation by a laser pulse. Recently, a number of studies [1–10] of FID in ruby have been published that demonstrate the inapplicability of Bloch equations as a description of the field dependence of the FID rate. This results from the fact that the Bloch equations do not properly take into account the random modulation of impurity-ion frequencies in solids. The frequency modulation of the ruby R_1 transition is due to random reorientation of spins in the crystal lattice. The reorientation leads to a change in the local fields and correspondingly to a change in the impurity-ion frequencies. The random frequency modulation connected with the dephasing perturbations evokes a relaxation in the system. However, the coherent field influences the dephasing processes. As a result, the relaxation coefficients depend on the amplitude and frequency of the field in the master equation for the density matrix when averaged over the random perturbations. There is no such dependence in the Bloch scheme.

In study [11], the analysis of existing theories of FID, based on various models of random modulation of the transition frequency, was performed. In particular, the telegraph noise model was examined and the inability to explain the experimental data [2] was demonstrated. In this paper we restrict our theoretical discussions to the FID after pulse saturation in a system with noncorrelated spectral exchange. The purpose of this study is to present a general theory of a FID signal shape after saturation by a strong-field impulse of arbitrary duration, as well as to explain the experimental field dependence of the FID rate [2,3].

II. FID AFTER PULSE SATURATION

Let us consider an ensemble of impurity ions that interacts with a perturber reservoir and is driven by a resonant monochromatic radiation field $\mathcal{E}=\mathcal{E}_0\exp\{i\omega t\}$ within the time T. Polarization of the ions is induced by the radiation impulse and a FID signal is observed just after switching off the field. Each impurity ion is modeled by a two-level system (TLS) whose frequency $[E_2(t)-E_1(t)]/\hbar=\omega_0+\epsilon(t)$ is a stationary random process and whose mean value ω_0 and equilibrium distribution $\varphi(\epsilon)$ is conserved with respect to time [$E_{1,2}(t)$ are the energy levels of the TLS].

Since the value of ω_0 is distributed over the inhomogeneous line shape $\Phi(\omega_0)$ caused by the crystal-field dispersion, the FID signal shape is defined by

$$R(T+t)=\Phi_0\mathrm{Im}\int d\Delta\omega\overline{\sigma_{12}(T+t,\Delta\omega)}, \tag{1}$$

where $\Delta\omega=\omega_0-\omega$ is the detuning frequency, ω is the saturating field frequency, $\Phi_0=\Phi(\omega_0=\omega)=\mathrm{const}$, $\overline{\sigma_{12}(T+t,\Delta\omega)}$ is the off-diagonal element of a density matrix that determines the polarization at time t after switching off the field. The general expression for $\overline{\sigma_{12}(T+t,\Delta\omega)}$ has the following form:

$$\overline{\sigma_{12}(T+t,\Delta\omega)}=\left\langle \sigma_{12}(T,\Delta\omega)\exp\left\{ i\Delta\omega t + i\int_T^{T+t}\varepsilon(t')dt'-t/T_2\right\}\right\rangle, \tag{2}$$

where the angular brackets denote averaging over random realizations of the process $\varepsilon(t)$, $\sigma_{12}(T,\Delta\omega)$ is the initial polarization induced by the saturating field, and T_2 takes into account the spontaneous decay of the excited level (i.e., $T_2=2T_1$ where T_1 is the lifetime).

Usually when calculating $R(t)$, the correlation of the TLS frequency fluctuations before and after switch off of the field is neglected. This makes possible the decoupling procedure in Eq. (2), where $\sigma_{12}(T,\Delta\omega)$ and the exponent are averaged separately. We obtain

$$\overline{\sigma_{12}(T+t,\Delta\omega)}=\overline{\sigma_{12}(T,\Delta\omega)}K(t)\exp\{i\Delta\omega t-t/T_2\}, \tag{3}$$

where

$$K(t)=\left\langle \exp\left\{ i\int_0^t \varepsilon(t')dt' \right\} \right\rangle \quad (4)$$

is the correlation function of the frequency modulation. It has been shown [9,12] that when exact expressions are used for the average $\overline{\sigma_{12}(T,\Delta\omega)}$ and $K(t)$, then results for the FID signal shape are obtained that are valid beyond the applicability range of perturbation theory (PT) for random detuning of the frequency. The condition for which PT is applicable (fast frequency modulation limit) is

$$q^2=\overline{\varepsilon^2}\tau_0^2\ll 1, \quad (5)$$

where $\overline{\varepsilon^2}$ is the dispersion of the frequency distribution and τ_0^{-1} is the spectral exchange rate. It is in the framework of approximation (5) that the majority of theoretical efforts [2–6] to explain the experiments [2] were undertaken. We note that the applicability range of Eq. (3) must be examined in each particular case.

Expressions (1) and (2) are exact and determine the FID signal shape in the general case. It has not yet been possible to carry out the averaging procedure defined in Eq. (2) in a generalized form. Therefore, we specifically model the random process $\varepsilon(t)$ and, in particular, consider the case where the TLS frequency is modulated by a Markovian noncorrelated random process. This specification of the random process $\varepsilon(t)$ allows the averaging in Eq. (2) to be performed and an exact expression for the FID signal shape to be obtained.

If the frequency of the TLS interacting with the laser radiation field is modulated by a purely discontinuous Markovian process, then in agreement with the sudden modulation theory [13], the averaging in Eq. (2) can be represented as

$$\overline{\sigma_{12}(T+t,\Delta\omega)}=\exp\{i\Delta\omega t-t/T_2\} \times\int d\varepsilon\, K(\varepsilon,t)\sigma_{12}(\varepsilon,T,\Delta\omega), \quad (6)$$

where $K(\varepsilon,t)$ and $\sigma_{12}(\varepsilon,T,\Delta\omega)$ are marginal or conditional averages [13] whose argument is equal to ε when the field is switched off.

Applying the Laplace transformation to Eq. (6), we get

$$\overline{\sigma_{12}(p,p_1,\Delta\omega)}=\int d\varepsilon\, K(\varepsilon,p_1)\sigma_{12}(\varepsilon,p,\Delta\omega), \quad (7)$$

where

$$K(\varepsilon,p_1)=\int_0^\infty dt\, K(\varepsilon,t)\exp\{-p_1t+i\Delta\omega t-t/T_2\}. \quad (8)$$

The frequency modulation function for Markovian spectral exchange has been thoroughly investigated [13–15]. To obtain a general solution for the FID signal, we have to find $\sigma_{12}(\varepsilon,p,\Delta\omega)$ (which describes the polarization induced by the radiation field) as a function of ε. For this purpose we will use the kinetic equations of the Markovian sudden modulation theory [13].

III. THE NONCORRELATED MARKOVIAN FREQUENCY MODULATION

In this section we consider noncorrelated Markovian frequency modulation. This model assumes that the frequency exchange takes place inside of packets whose centers are uniformly distributed over a wide spectral range. The frequency of the TLS changes instantly and then remains constant until the next jump. The distribution of the frequency value after the jump does not depend on the one before the jump [13].

If the process is noncorrelated, then the kinetic equation for the density matrix may be written as

$$\dot r(\varepsilon)=-\hat L(\varepsilon)r(\varepsilon)+\varphi(\varepsilon)(\hat\Gamma\,\overline{r}+\hat\Lambda), \quad (9)$$

where

$$r(\varepsilon)=\begin{pmatrix}\sigma_{12}(\varepsilon)\\ \sigma_{21}(\varepsilon)\\ n(\varepsilon)\end{pmatrix},\quad \hat L(\varepsilon)=\hat L_0+i\varepsilon\hat L_1+\hat\Gamma,$$

$$\hat L_0=\begin{pmatrix}\frac{1}{T_2}-i\Delta\omega & 0 & -\frac{i\chi}{2}\\ 0 & \frac{1}{T_2}+i\Delta\omega & \frac{i\chi}{2}\\ -i\chi & i\chi & \frac{1}{T_1}\end{pmatrix},$$

$$\hat L_1=\begin{pmatrix}-1&0&0\\0&1&0\\0&0&0\end{pmatrix},$$

$$\hat\Gamma=\begin{pmatrix}1/\tau_2&0&0\\0&1/\tau_2&0\\0&0&1/\tau_1\end{pmatrix},\quad \hat\Lambda=\begin{pmatrix}0\\0\\ \frac{n_0}{T_1}\end{pmatrix},$$

$\sigma_{12}=\sigma_{21}^*=\varrho_{12}\exp\{i\omega t\}$, $n=\varrho_{22}-\varrho_{11}$ is the population difference, n_0 is the equilibrium population difference, $\chi=d_{12}E_0$ is the Rabi frequency, T_1 and T_2 are the longitudinal and transverse relaxation times due to spontaneous decay, and $\varphi(\varepsilon)$ is a static equilibrium frequency distribution.

We modified the terms that take into account the frequency exchange in the system by introducing two different parameters, $1/\tau_1$ and $1/\tau_2$. These parameters are the exchange rate for the populations and coherence, respectively. The same modifications have been used for another systems [16]. We will use following correlations $\tau_1=\tau_c$ and $\tau_2=u\tau_1$. In the case $u=1$, we have the model studied earlier for the FID signal after steady-state saturation [9].

Employing the Laplace transformation, we can easily obtain

$$r(\varepsilon,p)=\frac{\varphi(\varepsilon)}{p+\hat{L}(\varepsilon)}\hat{\Gamma}\frac{1}{1-\frac{\varphi(\varepsilon)}{p+\hat{L}(\varepsilon)}\hat{\Gamma}}\hat{\Gamma}^{-1}\left(r_0+\frac{1}{p}\hat{\Lambda}\right). \tag{10}$$

According to study [13–15], we have for the Laplace representation of the marginal function of the frequency modulation

$$K(\varepsilon,p_1)=\frac{1}{p_1+t_2-i(\Delta\omega+\varepsilon)}\times\frac{1}{1-\frac{1}{\tau_2}\int\frac{\varphi(\varepsilon)d\varepsilon}{p_1+t_2-i(\Delta\omega+\varepsilon)}}. \tag{11}$$

To obtain an analytical solution for the FID signal shape, we now make the distribution $\varphi(\epsilon)$ more concrete. From a physical point of view, the choice of a static contour shape is determined by the spatial packing of nuclei surrounding an ion. It is known that the dipole-dipole interaction with the nuclei yields a Gaussian inhomogeneous contour when the packing is regular, and a Lorentzian contour if it is random [17]. So, the Gaussian contour is more suitable as $\varphi(\epsilon)$ in the case of impurity-ion crystals. But it is difficult to get a simple analytical expression for the FID signal for a Gaussian contour. However, we know that the shape of distribution $\varphi(\epsilon)$ is not important for the case of the fast modulation; only the second moment of the distribution. The analysis shows that the situation is the same in the limit of slow noncorrelated exchange of frequency. In this case, we have to know just the rate of the spectral exchange. We consider the limits of fast and slow modulation for values of the spectral exchange parameters far from the threshold $\overline{\varepsilon^2}\tau_0^2=1$. The final result depends on the shape of the distribution $\varphi(\epsilon)$ only around the range of the parameters where $\overline{\varepsilon^2}\tau_0^2\approx 1$. Looking ahead, we have a case of slow frequency exchange and, in principle, we are free to choose the static equilibrium distribution.

So we consider the model of the Lorentzian contour which allows development of an analytical solution. Using a Lorentzian for the equilibrium distribution $\varphi(\epsilon)$ in general expressions (7) and (1), we obtain for the FID signal shape

$$R(p,t)=\frac{\pi\Phi_0 n_0\chi}{2pF}\exp\left\{-\frac{t}{T_2}\right\}\left\{R_0\exp[-(a+F)t]-A\exp\left[-\left(\frac{1}{\tau_2}+\kappa\right)t\right]\right\}, \tag{12}$$

where a is the width of the Lorentzian distribution,

$$R_0=\frac{(p+t_2)a}{\kappa}-F+p+\frac{1}{T_2}+A,$$

$$A=\frac{a\left[p+\frac{1}{T_2}+\frac{(p+t_2)(a+F)}{\kappa}\right]\left(1-\frac{\kappa}{p+t_2}\right)}{1/\tau_2+\kappa-a-F},$$

$$\kappa^2=(p+t_2)^2+\chi^2\frac{p+t_2}{p+t_1},$$

$$F^2=\left[p+\frac{1}{T_2}+\frac{(p+t_2)a}{\kappa}\right]\left[p+\frac{1}{T_2}+\frac{a\kappa}{p+t_2}+\frac{\chi^2}{p+1/T_1}\right],$$

$$t_{1,2}=\frac{1}{T_{1,2}}+\frac{1}{\tau_{1,2}}.$$

Equation (12) is exact and determines the FID signal shape after saturation by a strong-field impulse assuming noncorrelated frequency modulation. The expression is valid for the arbitrary strength of the saturating field and for the arbitrary transition frequency modulation rate. In the limit

$$R^s(t)=\lim_{p\to 0}pR(p,t), \tag{13}$$

we obtain an expression for the FID signal shape in the case of steady-state saturation [9]. Naturally it is possible to compare this result to the one we obtained within the range of $T\gg T_1$ and, thus, observe the approach to steady-state saturation.

The approximate solution (3), neglecting correlation of the system motion before and after switch off of the field, can be represented as

$$\overline{\sigma_{12}(p,p_1,\Delta\omega)}=K(p_1)\overline{\sigma_{12}(p,\Delta\omega)}, \tag{14}$$

where according to Eq. (11)

$$K(p_1)=\int d\varepsilon\,\varphi(\varepsilon)K(\varepsilon,p_1)=\frac{1}{p_1+1/T_2+a-i\Delta\omega}, \tag{15}$$

and $\overline{\sigma_{12}(p,\Delta\omega)}$ is determined by Eq. (10) after the averaging by ε.

In this case, we finally find for the FID signal

$$R_d(p,t)=\frac{\pi\Phi_0 n_0\chi}{2pF}\left(p+\frac{1}{T_2}+\frac{(p+t_2)a}{\kappa}-F\right)\times\exp\left\{-\left(\frac{1}{T_2}+a+F\right)t\right\}. \tag{16}$$

The same result can be readily obtained from Eq. (12) in the limit $A=0$. It is evident that the appearance of the terms proportional to A in the exact solution (12) is related to the correlation of the TLS frequency fluctuations before and after switching off the field [9].

IV. DISCUSSION

A. Weak-external-field limit

The final expression for the FID signal shape contains two independent parameters a and τ_c. To determine these values for a theoretical explanation of experiments on the field dependence of the FID rate, the experimental data on the photon echo are usually employed. For the noncorrelated frequency modulation model, it is shown that the echo decay is exponential both for the fast [Eq. (5)] and slow spectral exchange ($a\tau_c \gg 1$) [14]. In these cases, the echo decay rate is equal to $\gamma_e = 1/T_2 + a$ (fast exchange) and $\gamma_e = 1/T_2 + 1/\tau_2$ (slow exchange). Thus, if the echo decay is exponential, we can determine either a assuming a fast spectral exchange, or τ_2 for a slow spectral exchange.

The expression for the FID signal after the steady-state excitation in the limit $\chi \to 0$ is obtained in Ref. [18] (in the framework of different spectral exchange models) for the fast and slow spectral exchange. In this regard, analysis of the FID signal shape for an arbitrary value of the spectral exchange rate will be of interest within the framework of our model.

Using Eq. (12), we obtain for the FID signal (for frequency modulation by a Markovian noncorrelated process) in the third-order contribution in χ

$$R(p,t) = -\chi^3 \frac{n_0 \pi \Phi_0}{4} \exp\{-(p+2/T_2)t\} F(p,t), \quad (17)$$

where

$$F(p,t) = \frac{1}{p(p+a+1/T_2)} \left\{ \left[\frac{a}{(p+t_1)(1/\tau_2 - a)} + \frac{1}{p+1/T_1} \right] \exp(-2at) - \frac{a(p+a+1/T_2)}{(p+t_1)(p+t_2)(1/\tau_2 - a)} \exp\left(-\frac{2t}{\tau_2}\right) \right\}.$$

Thus the FID signal shape is given by

$$R(T,t) = \begin{cases} -\chi^3 \dfrac{n_0 \pi \Phi_0}{4} \exp\{-2t/T_2\} f(T-t), & T > t \geq 0 \\ 0, \quad T < t. \end{cases}$$

where

$$f(T-t) = \frac{1}{2\pi i} \int_{b-i\infty}^{b+\infty} \exp\{p(T-t)\} F(p,t)\, dp.$$

Taking into account the form $F(p,t)$, we can write the expression for the FID signal at $\chi \to 0$ under $T > t \geq 0$ as

$$R(T,t) = -\chi^3 \frac{n_0 \pi \Phi_0}{4} \exp\left\{-\frac{2t}{T_2}\right\} \sum_{j=1}^{j=5} B_j(t) \exp\{p_j(T-t)\}, \quad (18)$$

where p_j and $B_j(t)$ are defined by

$$p_1 = 0,$$

$$B_1(t) = \frac{\exp(-2at)}{a+1/T_2} \left[\frac{a}{t_1(1/\tau_2 - a)} + T_1 \right] - \frac{a \exp\left(-\frac{2t}{\tau_2}\right)}{t_1 t_2 (1/\tau_2 - a)};$$

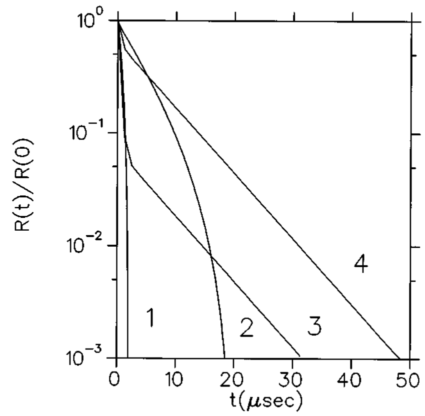

FIG. 1. The FID signals vs time for noncorrelated frequency modulation (plane-wave excitation) at $\chi \to 0$. $\tau_2 = \tau_1$, $a\tau_c = 20$, $T_1 = 4200$ μsec, $\gamma_e^{-1} = 15$ μsec. (1) $T = 2$ μsec, (2) $T = 20$ μsec, (3) $T = 200$ μsec, (4) steady-state regime.

$$p_2 = -a - \frac{1}{T_2},$$

$$B_2(t) = -\frac{\exp(-2at)}{a+1/T_2} \left[\frac{a}{\left(t_1 - a - \frac{1}{T_2}\right)(1/\tau_2 - a)} + \frac{1}{\frac{1}{T_1} - a - \frac{1}{T_2}} \right];$$

$$p_3 = -t_1,$$

$$B_3(t) = \frac{a}{t_1(1/\tau_2 - a)} \left[\frac{\exp(-2at)}{t_1 - a - \frac{1}{T_2}} + \frac{\exp\left(-\frac{2t}{\tau_2}\right)}{t_2 - t_1} \right];$$

$$p_4 = -t_2,$$

$$B_4(t) = -\frac{a \exp\left(-\frac{2t}{\tau_2}\right)}{t_2(1/\tau_2 - a)(t_2 - t_1)};$$

$$p_5 = -\frac{1}{T_1},$$

$$B_5(t) = \frac{T_1 \exp(-2at)}{\frac{1}{T_1} - a - \frac{1}{T_2}}.$$

Figure 1 shows FID signal shapes [Eq. (18)] in the weak-external-field limit for different coherent pulse durations. In general, the FID signal after steady-state excitation or after excitation by a pulse of finite duration is biexponential. The FID signal is mainly determined by $\exp\{-2at\}$ for the fast modulation and $\exp\{-2t/\tau_2\}$ for the slow modulation of the frequency. The FID signal behavior at long times is the same as that for the standard Bloch equations with a finite pulse

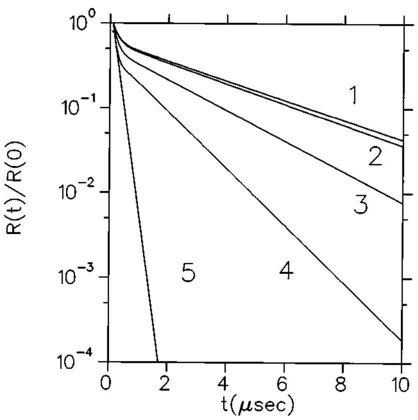

FIG. 2. The FID signals vs time for noncorrelated frequency modulation (plane wave excitation). $\tau_2=\tau_1, a\tau_c=20, T_1=4200$ μsec, $\gamma_e^{-1}=15$ μsec, $T=200$ μsec. (1) $\chi/2\pi=0.5$ kHz, (2) $\chi/2\pi=5$ kHz, (3) $\chi/2\pi=20$ kHz, (4) $\chi/2\pi=50$ kHz, (5) the approximate solution at $\chi/2\pi=5$ kHz [Eq. (16)].

duration [11]. It is seen that the FID signal suddenly disappears after a time equal to the pulse duration. This type of behavior has been discussed theoretically [19,20] and observed experimentally [21]. Qualitatively, the effect is related to the fact that a pulse of duration T excites a frequency band of $\approx\pi/T$. The dipoles then dephase completely in a time T which is the Fourier transform of the bandwidth.

It should be noted that for $\chi\rightarrow 0$, the theorem on coherent transients [20] for the Bloch model is also valid both for the noncorrelated frequency modulation and anticorrelated frequency modulation models [11].

B. The FID signal after saturation

To analyze the FID signal shape after saturation by a finite duration impulse, we employed an algorithm for the Laplace numerical inverse transformation [22]. The correctness of performing the numerical transformation of $R(p,t)$ was tested by comparing the results in the case of $T\geqslant T_1$, where the exact analytical solution for the FID signal is

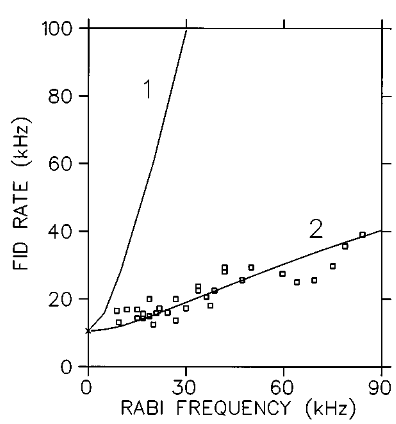

FIG. 3. Field dependence of the FID rate for a Gaussian shaped beam. (1) Bloch theory, (2) exact theory for noncorrelated spectral exchange at $a\tau_c=100, \tau_2=2\tau_c, \tau_c=7.5$ μsec, $T=200$ μsec, □, experimental data (Ref. [2]), the point designated by × at zero Rabi frequency is obtained from photon-echo data.

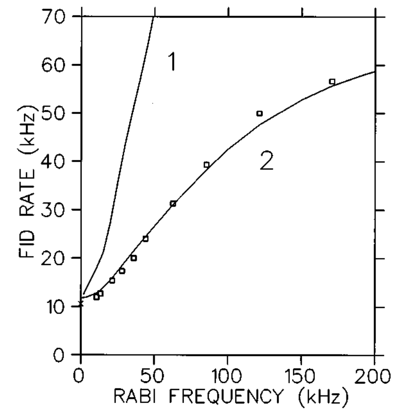

FIG. 4. Field dependence of the FID rate for a Gaussian shaped beam. (1) Bloch theory, (2) exact theory for noncorrelated spectral exchange at $a\tau_c=100, \tau_2=2\tau_c, \tau_c=7.5$ μsec, $T=50$ μsec, □, experimental data (Ref. [3]), the point designated by × at zero Rabi frequency is obtained from photon-echo data.

known [9]. Our study showed that when the saturating impulse duration increases, i.e., with $T\rightarrow T_1$, the FID signal shape approaches the steady-state saturation shape. We note that this result is true both for the case of a fast and slow exchange. Moreover, we have shown that the condition $T\gg\gamma_e^{-1}$, with $T_1>T$ is far from sufficient to employ the expression for the FID signal shape under steady-state saturation.

Figure 2 shows the FID signal shape calculated for different values of the Rabi frequency. It demonstrates the influence of the field strength on the FID kinetics. As the Rabi frequency increases, the decay rate of the induced polarization is accelerated. The increase of the FID rate with increasing Rabi frequency is related to the power broadening of the hole burned into the inhomogeneous profile during the preparation pulse. A stronger field burns a wider hole and the FID rate increases after the field is switched off.

In general, the kinetics of the FID signal is described by two exponents at a noncorrelated exchange. For fast modulation, the main term is the first one in the expression for $F(p,t)$ while the second term is dominant for slow exchange. The correction (second) term in the equation for $F(p,t)$ accounts for the correlation of the frequency fluctuations before and after turn off of the field. The calculated FID signal, using the approximate expression (16), is also shown in the Fig. 2. It is emphasized that we have to take into consideration the correlation of the system motion before and after switch off of the field.

We used a plane-wave model and infinitely wide inhomogeneous line in the discussion of the FID signal above. To allow comparison of the theory with experimental data, we have to account for the Gaussian shape of the laser beam used in the experiments [2,3]. As is easily shown, the beam shape can be included in the theory as follows:

$$S(T,t)=\text{const}\int_0^{\chi}d\chi' R(\chi',T,t), \qquad (19)$$

where $R(\chi',T,t)$ is determined by Eq. (12).

Experimental studies [2,3] showed that the shape of the FID signal is nearly exponential. However, as has been discussed earlier [11], FID decay is not an exponential function of the time in the telegraph noise model. To explain the experimental results [2,3] of the FID rate dependence on the field strength in ruby, we used a noncorrelated frequency modulation model. Figures 3 and 4 compare theory and experiment for $\tau_2 = 2\tau_c, \tau_c = 7.5$ μsec and $a\tau_c = 100$. The good agreement supports our conclusion is that the spectral exchange in ruby is slow and the noncorrelated frequency modulation model is more suitable than the telegraph noise model for this system.

V. CONCLUSION

The exact solution of the FID signal shape after saturation by a radiation pulse of finite duration under noncorrelated spectral exchange has been obtained. Analysis of the FID signal for an arbitrary spectral exchange rate and Rabi frequency showed that the correlation of the fluctuations before and after the turn off of the field must be taken into consideration.

The exact analytic expressions of the FID signal were derived in the weak-external-field limit. These expressions are valid for an arbitrary frequency modulation rate. It was shown that a decrease of the saturation pulse duration leads to nonexponential decay of induced polarization for the limits $a\tau_c \ll 1$ and $a\tau_c \gg 1$, while the FID signal after steady-state saturation is exponential. It was demonstrated that the field dependence of the FID rate found by Szabo and coauthors [2,3] have a self-consistent explanation in the limit of the slow noncorrelated spectral diffusion.

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