

# NRC Publications Archive Archives des publications du CNRC

# Blown cellulose fiber thermal insulations: pt. I: density of cellulose fiber thermal insulation in horizontal applications

Bomberg, M. T.; Shirtliffe, C. J.

This publication could be one of several versions: author's original, accepted manuscript or the publisher's version. / La version de cette publication peut être l'une des suivantes : la version prépublication de l'auteur, la version acceptée du manuscrit ou la version de l'éditeur.

# Publisher's version / Version de l'éditeur:

ASTM Special Technical Publication, pp. 82-103, 1978-12

NRC Publications Archive Record / Notice des Archives des publications du CNRC : https://nrc-publications.canada.ca/eng/view/object/?id=258f5994-0667-4214-b91a-8c15a0c9a6cc https://publications-cnrc.canada.ca/fra/voir/objet/?id=258f5994-0667-4214-b91a-8c15a0c9a6cc

Access and use of this website and the material on it are subject to the Terms and Conditions set forth at <a href="https://nrc-publications.canada.ca/eng/copyright">https://nrc-publications.canada.ca/eng/copyright</a> READ THESE TERMS AND CONDITIONS CAREFULLY BEFORE USING THIS WEBSITE.

L'accès à ce site Web et l'utilisation de son contenu sont assujettis aux conditions présentées dans le site <u>https://publications-cnrc.canada.ca/fra/droits</u> LISEZ CES CONDITIONS ATTENTIVEMENT AVANT D'UTILISER CE SITE WEB.

**Questions?** Contact the NRC Publications Archive team at PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca. If you wish to email the authors directly, please see the first page of the publication for their contact information.

**Vous avez des questions?** Nous pouvons vous aider. Pour communiquer directement avec un auteur, consultez la première page de la revue dans laquelle son article a été publié afin de trouver ses coordonnées. Si vous n'arrivez pas à les repérer, communiquez avec nous à PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca.







National Research Council of Canada Conseil national de recherches du Canada

# BLOWN CELLULOSE FIBER THERMAL INSULATIONS

# Part 1

Part 2

Density of Cellulose Fiber Thermal Insulation in Horizontal Applications by M. Bomberg and C. J. Shirtliffe Thermal Resistance by C. J. Shirtliffe and M. Bomberg



69

Reprinted from American Society for Testing and Materials Special Technical Publication No. 660 December 1978, Part 1, 22 p. - Part 2, 26 p.

> DBR Paper No. 820 Division of Building Research

OTTAWA



#### SOMMAIRES Partie 1

Cet article présente les résultats d'une étude visant les objectifs suivants:

- déterminer les effets du transport et des conditions de mise en place sur la densité initiale de l'isolant,
- (ii) établir une méthode normalisée pour prélever des éprouvettes d'isolant en fibres cellulosiques injectées;
- (iii) faire des recherches sur les facteurs qui causent le tassement du matériau après la mise en place;
- (iv) établir une méthode normalisée qui provoque le tassement des éprouvettes tel qu'on l'observe lors des études sur-

Une methode re un qui simule le tar tassement par der	d deux procédés, tième qui cause le
La résistan souflées de d avec indicate la températur moins de prék teneur en pro de papier. Il a son épaisseu d'une marge	losiques en vrac utfante et d'essai e thermique avec mesures et avec bas les effets de la ire des particules proportionnelle à RC sous réserve

## M. Bomberg<sup>1</sup> and C. J. Shirtliffe<sup>1</sup>

# Blown Cellulose Fiber Thermal Insulations: Part 1—Density of Cellulose Fiber Thermal Insulation in Horizontal Applications

**REFERENCE:** Bomberg, M. and Shirtliffe, C. J., "Blown Cellulose Fiber Thermal Insulations: Part 1—Density of Cellulose Fiber Thermal Insulation in Horizontal Applications," *Thermal Transmission Measurements of Insulation, ASTM STP 660*, R. P. Tye, Ed., American Society for Testing and Materials, 1978, pp. 82–103.

ABSTRACT: This paper presents results of a study with the following objectives:

1. Determine the effects of transport and placement conditions on the initial density of the insulation.

2. Establish a standardized method for producing specimens of blown cellulose fiber insulations.

3. Investigate the factors that cause the material to settle after placement.

4. Establish a standardized method to produce settlement in the specimens comparable with those found in field studies.

A method recommended for producing settlement in the specimens consists of two procedures, one simulating settlement by impact produced on the standardized containers, and the other causing settlement under climatic cycling of the material.

**KEY WORDS:** cellulose fiber, thermal insulation, density, settlement, settled density, residual density, moisture effects, blown insulation, blowing, pneumatic transport of insulation, thermal conductivity

Cellulose fiber insulation consists of small tufts of fiber and minute pieces of paper mixed with fine particles of chemical additives. The thermal performance of the cellulose fiber insulation depends not only on the composition and structure of the material as produced during the milling operation, but also on the way the material is fluffed and configured while being blown into place. The blowing process produces a structural network of fibers. Both the density and the stability of the structure depend on the conditions of blowing.

<sup>1</sup>Research officers, National Research Council of Canada, Division of Building Research, Ottawa, Ont., Canada.

All blown fibrous insulations can be assumed to settle after being applied. The density may gradually increase until some equilibrium is reached. The density changes may be too small to be measured over the span of several months. Regardless of what actually occurs, the density at this stage is often called "settled density."

The settled density can be determined only by making measurements in the field. Results of field measurements, however, have shown a considerable scatter which can be explained only by studying the factors that have a significant effect on the density during blowing and on the subsequent settlement. Both sets of factors and the scope of the study on the relative significance of the factors are given in Tables 1 and 2.

The objective of this study was to find a method to produce specimens for testing that are representative of the material as it exists in the attics of buildings. Such a method must consist of two parts:

- 1. The technique for blowing specimens.
- 2. The method of obtaining settled density.

	Factors		Scope of	
Element	Variable	Effect	study in the research	
The material	degree of milling chemical content moisture content	<ul> <li>density variations:</li> <li>batch to batch, and</li> <li>bag to bag</li> </ul>	bag to bag variability	
The machine	design blower design air setting	- feeding to the blower - flow path changes - material to air ratio	3 machines with 3 blower designs studied	
The hose	size, design and length	- fluffing during transport	one size and design used air pressure in the hose studied - recycling of material performed	
The nozzle	geometry	- changes in air pressure and the flow of material	no nozzle used for horizontal applicațions	
Position of the nozzle	relative to the machine relative to the specimen	<ul> <li>density changes due</li> <li>to vertical transport</li> <li>impact on the material</li> <li>already blown</li> </ul>	0 and 91 cm height examined 8–30 cm and 91 cm examined	
The size of the container	shape and area depth	<ul> <li>flow pattern</li> <li>impact of material on walls and material already in the container</li> </ul>	2 shapes and 4 sizes examined 3 depths examined	

TABLE 1-Variables affecting density of blown cellulose fiber insulations.

Cause	Effect	Study	Conclusion of the study
barometric pressure temperature	non-reversible deformation	air pressure variation	not significant
climate	reversible thermal movements non-reversible deformation	thermal cycling	little significance
climate moisture accumulation in attics	adsorption, absorption and desorption, interparticle capillary forces causing movements	humidity cycling	significant
gravity field	time dependent displacement	observation in laboratory	little significance
environment	particle displacement	impact (drop test)	significant
	Cause barometric pressure temperature climate climate moisture accumulation in attics gravity field environment	CauseEffectbarometric pressure temperaturenon-reversible deformationclimatereversible thermal movements non-reversible deformationclimate moisture accumulation in atticsadsorption, absorption, and desorption, interparticle capillary forces causing movementsgravity fieldtime dependent displacementenvironmentparticle displacement	CauseEffectStudybarometric pressure temperaturenon-reversible deformationair pressure variationclimatereversible thermal movements non-reversible deformationthermal cyclingclimate moisture accumulation in atticsadsorption, absorption and desorption, interparticle causing movementshumidity cyclinggravity fieldtime dependent displacementobservation in laboratoryenvironmentparticle displacementimpact (drop test)

TABLE 2—Variables affecting settlement of blown cellulose fiber insulations.

#### Scope of the Research

#### Machines Used in the Tests

A number of different designs of blowing machines are available. Most of them break the compressed material from the bags into small lumps which pass through the blower, thus producing fine particles carried by the airstream.

Three blowing machines were used during the study.

Machine 1—A Shelter Shield blowing machine produced by Diversified Insulations Inc., Hamel, Minn. It was equipped with two 10-fingered agitators in the hopper. The air setting was continuously variable and the indicator was marked at 1/8-in. (3 mm) intervals from 0 to 2 in. (5.08 cm). A 1-hp (0.75 kW) Tornado blower (Model 8805) was used on the machine.

Machine 2—An Incel Corporation blowing machine produced by the Incel Corp., Bluffton, Ind., with one agitator in a hopper. The agitator had relatively long "fingers" which forced an ample supply of insulation into the blower. This machine used a 1.5-hp (1.1 kW) blower, Model RMI 8950, produced by Robbins and Myers, Springfield, Ohio. The air setting was continuously variable but was not graduated.

Machine 3—A Thermtron blowing machine produced by Thermtron Inc., Fort Wayne, Ind. Its three agitators, each having a different rate of rotation, provided a more than adequate supply of material to the blower. The unit had twin blowers, one 0.8 hp (0.6 kW) and one 1 hp (0.75 kW). In almost all applications the 0.8-hp (0.6 kW) blower (Model HP33WS), produced by Clement's Manufacturing Company, Chicago, Ill., was used. Unlike Machines 1 and 2, the air setting of Machine 3 was in discrete steps. There were five holes with diameters of about 1/8, 1/4, 1/2, 1, and  $1\frac{1}{4}$  in. (3, 6, 13, 25, and 32 mm). Adjustment of the air setting is shown in Fig. 1. The same value of air setting, that is, 3/8 in. (9.5 mm), does not represent the same rate of airflow in each of the three machines tested.

All three machines were supplied with standard 5-cm-inside diameter corrugated plastic hose. In the preliminary series, lengths of 15 and 23 m were used; in the main testing series, a 30-m length was used. The hose was used without a nozzle for blowing into horizontal space. (For simplicity, the end of the hose is termed the nozzle in this paper.)

#### Materials Used in the Tests

All the 35 materials used for the tests were obtained from the regular production of manufacturers in the United States and in Canada. The fire retardant used was either aluminum sulphate or a combination of two or three of the following chemicals: aluminum sulphate, borax, boric acid,



FIG. 1—Adjustment of air setting in the three tested machines (1 in. = 25.4 mm).

ammonium sulphate, and calcium sulphate. The formula and quantity of fire retardant are not known exactly, but in general the amount varied from 16 to over 30 percent by weight. The source of the cellulose was newsprint except in one case. The moisture content of the paper varied due to the wetness of the cellulose stock and the variable hygroscopic properties of the fire retardants. The moisture content of the products varied between 5 and 10 percent by weight.

The materials were numbered randomly from 1 to 35.

#### Effect of Transport and Placement Conditions on the Applied Density

#### Effect of Nozzle Height and Hose Length

Changes in density caused by hose length and the height of the end of the hose above the machine were checked by blowing the same material in two different ways. In the first, the hose end was 3.3 m above the machine. In the second, the end of the hose was only 91 cm above the base of the machine. In each case a 10-mm (3/8 in.) air opening was used; the end of the hose was directed horizontally. Three containers, 91 by 35 by 15 cm, were filled. The densities obtained in these two tests were 31.6 and 31.7 kg/m<sup>3</sup>. It was judged that the height of the nozzle above the machine did not have a significant effect on the density of the material transported to the nozzle.

The effect of the hose length was checked by blowing the same material twice, that is, by recycling it. Several materials were recycled and the final density compared with the density after the first blowing. The densities of the specimens produced from the recycled materials were almost identical to the original densities. The variations were less than 1.5 percent or 0.5 kg/m<sup>3</sup>. In each case this is well within the standard deviation of 0.64 to 1.8 kg/m<sup>3</sup>. (It should be noted that standard deviations refer to a small specimen size.)

#### Effect of Specimen Size and Shape of the Container

The effect of shape and size of the container was studied to establish a controlled method for producing specimens of blown cellulose fiber insulations. Four different-size containers were used in a series of tests. Two, three; four, or six containers of each size were filled with each material tested. The material blown into the small (43 by 43 by 7.5 cm or 43 by 43 by 15 cm) containers showed greater variations in density than that blown into the larger (91 by 35 by 10 cm or 91 by 35 by 15 cm) containers. The variations were probably caused by the impact of the material on the walls of the container.

The importance of size and depth of the container is shown in Fig. 2.



FIG. 2-Density as affected by the thickness of the blown layer and container dimensions.

Two different techniques of placing the insulation were used: horizontal blowing from a height of 91 cm, and 10 deg upward blowing from a height of 28 cm.

The effect of container size and depth depends on the blowing technique and structure of the tested material. For the same material when blown with the nozzle 28 cm above the bottom of the container and pointed 10 deg upward, the effect of depth became negligible. For the same material and the aforementioned blowing technique, the effect of container size was less than 5 percent.

#### Effect of Air Setting

Products 10, 19, and 23 were blown using Machine 2 with the nozzle pointed horizontally at a height of 91 cm and various air settings. The results are plotted in Fig. 3.

Changes in the air setting significantly affect the density of the cellulose fiber insulations. With Machine 2, the minimum density was obtained at the 38 and 51 mm ( $1\frac{1}{2}$  and 2 in.) air settings. The density of Product 23 varied about 20 percent with air setting. The density of Product 10 varied about 15 percent but that of Product 19 varied only about 10 percent.

88



FIG. 3—Effect of air setting on the density of specimens produced with Machine 2 blowing horizontally from a 91-cm height.

It appears that a method specifying a selected air setting, position, and height of the nozzle will not produce minimum density for various cellulose fiber insulation products.

# Combined Effect of Hose Position and Air Setting

Figure 4 illustrates the different positions of the nozzle used in a series of tests. In each case the air settings were varied. The results of the tests can be obtained from the Division of Building Research/National Research Council of Canada (DBR/NRC).

The variation in density caused by changes in the air settings when blowing downward from a height of 15 cm is shown in Fig. 5. Products 3 and 12, when blown with Machine 1, gave a minimum density at the air setting between 6 and 3 mm ( $\frac{1}{4}$  and  $\frac{1}{8}$  in.). The minimum density was obtained with the same machine at an air setting between 38 and 51 mm ( $\frac{1}{2}$  and 2 in.) when the material was blown from a height of 91 cm. The air setting cannot be considered as an independent variable. The mass of material per volume of air or the rate of mass flow of the material and the



FIG. 4-Positions of the nozzle and path of material.



FIG. 5—Density versus air setting when blowing downward from a 15-cm height using Machines 1 and 2.

air velocity at the nozzle might be better indicators than the air setting, but these were not measured in the tests.

The effect of the air setting on the density can vary with the design of the machine. This is shown in Fig. 5, where it can be seen that Product 12 when blown with Machines 1 and 2 gave densities differing by several percentage points. There was a higher air velocity at the nozzle for the same air setting with Machine 2 than with Machine 1. Variations from bag to bag of the products were eliminated from most of the tested materials by recycling them two to four times. This produced a material that was more uniformly fluffed and allowed a better comparison of the machines.

The three techniques for blowing cellulose fiber, each using a range of air settings, were compared to see if the same density could be obtained on different machines. Figure 6 shows the density of three products as determined on three blowing machines and different blowing techniques. Differences of up to 20 percent occurred for the same material using different blowing machines. These differences can be significantly reduced if the optimum air setting is selected for the given machine. By a series of preliminary blowings, one can find the air setting that will give the minimum density and use it for the actual test. There are limitations to this approach, since the flow becomes nonuniform if the air setting is too low, and excessive dusting occurs if the air setting is too high. The air

#### PRODUCT - MACHINE NUMBERS AS SHOWN



FIG. 6—Density of Products 1, 2, and 19 determined with Machines 1, 2, and 3.

velocity at the nozzles could not be standardized because of the limited range of adjustment on the machines. The sensitivity of the density to this velocity made it impossible to standardize the blowing technique with the nozzle pointed downward. Figure 6 shows that the density was not seriously affected by this velocity when the hose was pointed 10 deg upward.

### Recommended Method for Producing Specimens of Blown Cellulose Fiber Insulations

The hose should be pointed 10 deg upward and the end of the hose kept 28 cm above the surface when blowing. This method is sufficiently reproducible to be accepted as the standard blowing technique. The air setting can be selected by conducting a series of tests with the given machine; a minimum of four settings should be used. Widely different air settings should be used first. The lowest setting should be that which will give a uniform flow of material and the highest that which will not produce excessive dust. Two intermediate air settings should then be used. The air setting which produces the minimum or near minimum density should then be chosen for the actual test. A minimum of four containers, 91 by 35 by 15 cm, should be used for the actual test.

Two products were blown with this technique, but applying three machines (the results are shown in Fig. 7). For Product 25 the greatest



FIG. 7—Density of Products 25 and 2 determined with recommended blowing technique on Machines 1, 2, and 3.

variation in density using the three machines was 4 percent, for Product 2 it was only 2.4 percent. This is much less than the 20 percent difference found for the same product using any of the other blowing techniques. The standard deviation was less than  $0.64 \text{ kg/m}^3$ . This method appears to be simple and effective even though it is not the usual method of installing insulation in attics by blowing.

### **Reproducibility of Density Determinations**

It is recommended that the proposed blowing technique have a reproducibility not worse than horizontal blowing at the 91-cm level. (Results of tests by blowing horizontal at this level are given in Table 3.) All products were manufactured from newsprint except Product 23, which was manufactured from cardboard. The standard deviation of the tests on Product 23 was about twice that determined when testing newsprint-based cellulose fiber insulations. The average standard deviation was 1.07 kg/m<sup>3</sup>. When testing six specimens, requesting a confidence level of 95 percent, and assuming a *t*-distribution function, the density should fall within the confidence interval:  $2 \times t \times s/\sqrt{n} = 2 \times 2.571 \times 1.07/\sqrt{6} = 2.24 \text{ kg/m<sup>3</sup>}$ . The densities of the tested materials were between 22.4 and 41.6 kg/m<sup>3</sup> with an average value of about 33.6 kg/m<sup>3</sup>. These figures show that the density determined using six specimens should, with a 95 percent confidence level, fall within 6.7 percent of a true average for 33.6-kg/m<sup>3</sup> specimens.

An estimate of the accuracy of the proposed blowing technique (28 cm, 10 deg upward) can be made using the mean standard deviation determined from tests performed according to the proposed method. The mean standard deviation for four density measurements in containers 91 by 35 by 15 cm was  $0.75 \text{ kg/m}^3$ . Using t = 3.182 for four specimens, the 95 percent confidence interval becomes  $2.40 \text{ kg/m}^3$ . That is, the density determination with four specimens tested according to the new method will be practically as accurate as the determination with six specimens and horizontal blowing from a height of 91 cm. These figures reflect, primarily, only the variability of the product from bag to bag, since two to three bags of material are used for density determinations. They do not show the differences that occur between batches from different production lots. Several production lots would have to be tested to examine the product variability, but this is beyond the scope of this research.

### Field Measurements on Cellulose Fiber Insulations

During March 1977 the density of Products 3, 10, and 12 was measured in situ after being exposed in Ottawa for two winters. The materials in two 2-story and three 1-story houses were tested. Insulation was added

				Densit	y in cont	ainers, k	g/m <sup>3</sup>		
Product	Machine	1	2	3	4	5	6	Mean	Standard Deviation
1	2	38.9	38.9	37.3	36.0	35.7	36.7	37.3	1.28
1	2	36.4	36.7	37.5	35.6	34.1	33.2	35.6	1,60
2	2	29.2	29.3	29.3	30.0	28.7	28.8	29.0	0.32
2	3	28.2	25.5	28,5	29.3	29.2	28.8	28.2	1.44
3	2	37.2	38.9	36,7	35.4	38.6	38.4	37.5	1.44
4	2	28.2	28.2	27.1	27.1	27.4	27.6	27.7	0.48
5	2	35.7	37.2	37.5	33.3	33.8	34.4	35.4	1.76
6	2	33.3	34.1	33.3	33.5	34.0	33.8	33.6	0.32
6	2	33.5	34,0	33.6	34.3	33.8	34.6	34.0	0.48
7	2	33.6	33.2	32.5	31.7	33.3	35.2	33.3	1.12
8	2	31.2	34.3	34.4	33.3	36.5	36.7	34.4	2.08
9	2	22.3	22.6	21.9	21.9	21.6	22.6	22.1	0.32
10	2	31.4	32.5	34.0	33.6	33.8	35.4	33.5	1.44
11	4	31.1	32.2	33.0	32.2	31.6	31.4	31.9	0.64
12	2	39.7	40.7	38.6	41.5	38.0	36.7	39.2	1.76
17	2	30.8	33.8	35.4	35,2	31.9	35.1	33.6	1.92
19	1	33.8	33,6	33.0	33.0		II	33.3	0.48
20	1	38.9	39.9	40.5	40.0	38.3	37.0	39.1	1.28
21	1	34.3	34.9	34.8	34,4	34.6	33.8	34.4	0.32
22	1	29,8	30.6	31.7	30.8	30.9	32.4	31.1	0.96
23	1	35.1	42.1	43.4	43.4	41.2	38.4	40.7	3.20
24	1	30.3	30.4	31.1	30.8	32.5	32.4	31.2	0.96

TABLE 3—Density of horizontal layer, in kilograms per cubic metre, determined for several products by horizontal blowing on 91-cm level into containers mainly 91 by 35 by 10 cm or sometimes 91 by 35 by 15 cm. Tests carried out in 1975 and 1976 at DBR/NRC.

83

to existing glass fiber batts or loose-fill material fibers in the fall of 1975. The thickness of the layer of cellulose fiber insulation varied between 10 and 25 cm.

The density of the material was determined in situ in the following way:

1. After removing an adjacent section of cellulose, a metal sheet was slowly inserted horizontally under the cellulose insulation.

2. A 25 by 25 cm area of material on the metal sheet was selected and five thickness measurements were made.

3. A 25 by 25 cm box with sides 25 cm high and open top and bottom was pushed through the insulation to the metal sheet.

4. The insulation within the metal box was removed and weighed. It was then dried in a  $50^{\circ}$ C oven and reweighed.

The results of these tests are given in Table 4.

Product 10 from House 1 was packed into plastic bags and taken to the laboratory. After selecting a proper air setting, five 91 by 35 by 25 cm containers were filled, and the density measured. The mean density was  $30.9 \text{ kg/m}^3$  and  $31.1 \text{ kg/m}^3$  for the 15- and 25-cm-deep containers, respectively. The density of  $31.1 \text{ kg/m}^3$  as blown in the laboratory and the density determined *in situ*, 46.9 kg/m<sup>3</sup>, can be compared directly because

Property Tested	House 1 product 10	House 2 product 10	House 3 product 3	House 4 product 12	House 5 product 12
mean layer thickness, cm	22.9	10.7	10.2	10.2	12.5
mean moisture content, % weight	7.0	9,9	10.2	9.8	9.3
density, kg/m <sup>3</sup>	49.5	45.3	35.1	42.1	40.5
wet material	47.9	40.2	35.2	38.4	45.3
	45.8	43.7	36.2	41.0	45.5
	45.8	42.0	34.8	40.8	42.8
	48.5	42.3	38,6		42.9
	46.0	43.6	40.8		51.6
	44.9	42.0	34.3		
	47.6	39.1			
mean wet density, kg/m <sup>3</sup>	46.9	42.3	36.4	40.5	44.9
mean density of a dry material, kg/m <sup>3</sup>	45.2	38.4	33.0	37.0	42.4
layer below the blown material	glass fiber batt	glass fiber batt	blown glas fiber	s glass fiber batt	blown rockwool

TABLE 4—Density determined in attics of five houses in Ottawa during March 1977.

1. the moisture content of material in the attics was almost the same as that of the material conditioned in the laboratory, and

2. it has been demonstrated that recycling has little effect on density.

The 51 percent apparent increase in density may not all be due to settlement, since the density at which the material was actually applied cannot be determined. Laboratory tests on the material removed from the attic showed a variation in density of 3 percent. The hose position would not cause a variation greater than 8 percent. It seems reasonable to assume that there was a settlement in the material of about 40 percent. In other houses the settlement seemed to be much lower. In House 2, Product 10 had an apparent increase in density of 36 percent. Probable settlement in House 2 was between 25 and 30 percent. The difference between the density determined in the house and the blown density obtained from the method used at DBR/NRC indicates a maximum possible settlement of the material. Settlement in the house will probably be smaller because the density at which the material was actually applied is likely to be higher than that obtained by the DBR method. In House 4 and 5, where Product 12 was used, probable settlements are in the range of 15 to 30 percent. In House 3, where Product 3 was used, there was no significant settlement. The variability in these estimations of settlement suggests a need for a study of the factors influencing the settlement of cellulose fiber insulations.

#### Laboratory Measurements of Moisture Content in Horizontal Layers

The ability of the material to absorb moisture was studied under laboratory conditions. The material was placed in containers located between two steady environments, one at 24°C and 50 percent relative humidity and the other at a temperature below the dew point so that internal condensation would occur close to the bottom of the container. The bottom surface of the containers was drilled with a few hundred small holes, allowing excessive moisture to pass to an underlying porous fiberboard layer.

Three series of tests were conducted with different temperature gradients. The gradients were chosen so that the zone of condensation varied in thickness. The resulting moisture contents in the condensation zones are given in Table 5.

Moisture content in the condensation zone appears to be in the range 150 to 200 percent by weight except for Product 23, which was made of cardboard. This specimen absorbed less moisture.

Water accumulated only in a very narrow layer adjacent to the lower surface of the material; the bulk of the material remained relatively dry. Moisture contents between 8 and 11 percent at the upper surface (Table 5) lie in the same range as average values determined *in situ*.

# Effect of Impact and Oscillation of the Climatic Conditions on the Settlement of the Material

From January 1974 to March 1977 the thermal resistance of cellulose fiber insulations tested at DBR were determined using a 45-cm vertical guarded hot plate (GHP) apparatus. Two matched specimens, 45 cm square and either 7.5 or 15 cm thick, were placed in polyethylene-covered frames and held in a vertical position on either side of the heater plate. Settlements occurred during the testing period, which usually lasted 2 to 5 days. The amount of settlement in this period is given in Table 6. Settlement occurred in every case even though it was different for the two thicknesses. The extent of the settlement was dependent on the amount of support from the surrounding surfaces. It was approximately 4 percent for 7.5-cm specimens and 10 percent for 15-cm specimens.

TABLE 5—Moisture contents, weight percent, in the layers adjacent to the upper and lower surfaces of the cellulose insulation exposed to the vapor condensation test.

Series 1		Prod.	Seri	Series 3	
Upper	Lower	No.	Upper	Lower	Lower
9.0	44.0	1	9.5	181	224
10.1	47.7	2	10.6	208	<b>a</b>
9.4	52.1	3	11.0	185	214
8.2	53.4	22	10.6	151	209
8.3	48.7	23	6.4	56	149
	Seri Upper 9.0 10.1 9.4 8.2 8.3	Series 1           Upper         Lower           9.0         44.0           10.1         47.7           9.4         52.1           8.2         53.4           8.3         48.7	Series 1         Prod.           Upper         Lower         No.           9.0         44.0         1           10.1         47.7         2           9.4         52.1         3           8.2         53.4         22           8.3         48.7         23	Series 1         Prod.         Series           Upper         Lower         No.         Upper           9.0         44.0         1         9.5           10.1         47.7         2         10.6           9.4         52.1         3         11.0           8.2         53.4         22         10.6           8.3         48.7         23         6.4	$\begin{array}{c c c c c c c c c c c c c c c c c c c $

TABLE 6—Density changes during thermal resistance testing in 45-cm GHP apparatus at DBR/NRC.

Test Number	Product Number	Density as blown kg/m <sup>3</sup>	Using for R before	7.5 cm -value after	frames test % change	Using for R before	l5 cm f -value after	rames test % change
1	3	37.6	39.7	42.0	5.6	42.6	48.7	14.3
2	4	26.6	27.4	28.5	4.1	29.5	32.7	10.9
3	5	35,2	38.4	39.7	3.3	36.7	42.3	15.3
4	6	34.0	34.6	36.2	4.6	36.0	41.6	15.6
5	7	33.3	33.4	36.2	6.8	39.4	42.3	7.3
6	8	34.4	33.8	36.0	6.6	35.0	40.7	16.0
7	9	22.1	23.4	23.9	2.1	24.0	24.7	2.7
8	10	33.5	34.9	35.6	1.8	34.4	38.0	10.2
				mean 4	1.4%		mean S	9.8%

#### Settlement During Air Pressure Changes

A cylindrical container with a diameter of 15 cm and a height of 30 cm was filled to the depth of 18.8 cm with a part of Product 10 that was taken from House 1. The density of the material in the container was 38.4 kg/m<sup>3</sup>.

A top was placed on the container and the pressure of the air in the cylinder was raised to about 200 Pa above atmospheric pressure. It was slowly lowered to about 200 Pa below atmospheric pressure. The pressure cycle lasted about 15 min. The cycling was continued for 2 days, then the container was opened.

The final thickness was 18.8 cm. No measurable settlement had occurred. The cycling of the air pressure is not a significant factor.

#### Settlement Due to Humidity Changes

Product 10 was blown into two open containers, 43 by 43 by 18 cm, at densities of 34.1 and 34.3 kg/m<sup>3</sup>. The containers were exposed alternately to 21°C and 50 percent relative humidity and 21°C and 98 percent relative humidity in 3- or 4-day intervals for a total of two weeks. Two cycles were completed. The final densities were 39.6 and 40.2 kg/m<sup>3</sup>—16 to 17 percent higher than at the beginning.

Product 3 was tested in the same way. The settlement was found to be 9.5 percent.

Product 2 was also blown into two frames 28 by 28 by 15 cm and two frames 28 by 28 by 30 cm. The frames were placed in a climatic chamber with a temperature of  $4^{\circ}$ C and 98 percent relative humidity. After two days the thickness was measured. The settlements were found to be 5.2 and 6.2 percent for the 15-cm-thick specimens and 7.4 and 8.6 percent for those 30 cm thick.

Humidity changes play a significant role in the settlement of the material. The thickness of the specimen appears to influence this effect. Tests should be performed on two sets of the specimens with different thicknesses.

#### Settlement Due to Temperature Changes

Specimens of Products 3 and 10 were placed in a set of open containers, 12 and 30 cm deep. The containers were placed in a climatic chamber where the temperature was cycled, within a 24-h period, between 4 and 21°C. The relative humidity of the air was maintained at approximately 98 percent.

The thickness of the material was measured after 5 and 8 days of exposure (Table 7). The settlement for the 12-cm-thick specimens was not measurable; for the 30-cm thickness it was 6 to 8 percent. These data are insufficient to draw conclusions. It appears, however, that temperature

variations when applied together with changes in relative humidity and the elapse of time may contribute to settlement.

#### Settlement Due to Impact

Two containers, 91 by 35 by 15 cm deep and each weighing about 4 kg, were filled with Product 1 and then dropped three times from a height of 15 cm onto a concrete floor. The density was measured before and after dropping. This process was continued for a total of 42 drops; the density versus the number of drops was plotted (Fig. 8) for four tests performed on the same material. The scatter in the results becomes larger with increasing number of drops. The effect of each additional drop decreases continually, as would be expected.

The densities of a number of specimens of different products were measured after three, six, and twelve drops from a 15-cm height. Figure 9 shows on a semilogarithmic plot the dependence of the average of specimen densities on the number of drops. There is no visible limit of density increase during this test.

Figure 10 shows the effect of the initial density on the dependence of density on the number of impacts. The increase in density in the drop test does not appear to depend on the initial density. A product of light density does not settle more than denser, more compacted materials. Further testing has shown that the increase of density with impact (Fig. 9) appears to be representative of all the cellulose fiber insulations blown with this type of equipment.

Material taken from House 1 and reblown in the laboratory had a mean density before settlement of  $30.9 \text{ kg/m}^3$ . When dropped 18 times from a 15-cm height, the density reached  $36.8 \text{ kg/m}^3$ . This density was still far less than the in-place density of  $46.9 \text{ kg/m}^3$ . It appears that it is not practical to require the drop test alone to produce as much settlement as is found *in situ*.

Container depth, cm	Product	% change 5 days	after 8 days
10	3	0	0
	10	0	0
20	3	0	1
	10	1	. 4
30	3	5	6
	10	6	8

TABLE 7—Effect of temperature cycling between 4 and 21°C with constant relative humidity at 98 percent relative humidity on settlement of cellulose fiber blown insulation.



FIG. 8—Density of Product 1 versus number of drops; four containers of material tested.





FIG.10-Effect of initial density on settlement due to drop test.

### **Recommended Method for Producing Settlement in the Specimens**

Both temperature and humidity vary considerably in attics. These fluctuations can be assumed to play an important role in the settlement of the insulation material.

The following procedure is recommended for producing a settled density:

1. Blow the material into 90 by 35 by 15-cm and 45 by 35 by 30-cm containers and determine density as blown using the procedure already described.

2. Blow the material into two containers 28 by 28 by 15 cm and two containers 28 by 28 by 30 cm using the same blowing techniques.

3. Drop three 90 by 35 by 15-cm and three 45 by 35 by 30-cm containers six times from a 15-cm height onto a concrete floor.

100

4. Measure the thickness and calculate the average percent decrease in thickness during the drop test on six containers (designated as  $S_d$ ).

5. Place two 28 by 28 by 15-cm and two 28 by 28 by 30-cm containers in a climatic chamber at  $4 \pm 1^{\circ}$ C and  $98 \pm 1$  percent relative humidity for four days.

6. Remove the containers from the chamber and place in a conditioned room with climate  $23 \pm 2^{\circ}$ C and  $50 \pm 5$  percent relative humidity for at least three days.

7. Repeat steps (5) and (6) until four exposures in the 4°C-chamber have been completed.

8. Measure the thickness and calculate the average percent decrease for four containers (designated  $S_c$ ).

9. The settled density is determined by multiplying the density as blown into the 90 by 35 by 15-cm and 45 by 35 by 30-cm containers by the factor  $s = 100/(100 - S_d - S_c)$ .

Table 8 gives density as blown, percentage decreases during the drop test, and climatic cycling and settled density for several tested materials. The settlement percentages in cycling 15- and 30-cm-thick specimens do not show a significant difference. In a few cases, figures 2 to 3 percent higher are generated for the thick specimens. On average, however, the results are the same for both thicknesses tested. The dropping tests, reported in Table 8, were performed only on 15-cm-thick specimens. The scatter in the settlement determined on various containers is larger than the scatter in the climatic cycling.

To increase the reproducibility of the test and to account for the dependence of the settled density on the specimen thickness, an average of eight containers is recommended in the final version of the proposed method. The containers have the same volume but two thicknesses: 15 and 30 cm. Table 9 gives percentage of settlement during the drop test on containers 15 and 30 cm deep. The difference between 15- and 30-cmthick specimens is too small to analyze the effect of thickness on the settled density of cellulose fiber insulations. It justifies, however, testing both thicknesses and averaging the results.

#### **Comments on Settled Density Determination**

The goal of the proposed method of density determination is to ensure product quality assurance for the purpose of standardization, that is, to achieve an average material coverage and thickness for predicting the thermal resistance in material specifications. There is a wide variability in the settled density of cellulose fiber insulations. In about 40 products tested at NRC (until March 1978), settled densities varied between 35 and 58 kg/m<sup>3</sup>, with 50 percent of the materials falling in the range of settled

	Density	Settlement in Percent								Settled
Product a Number	as blown kg/m <sup>3</sup>		Dr	opping		Су 15 ст	cling	of Samp 30 cm	les	Density kg/m <sup>3</sup>
26	41.3	9.4	9.7	9.3	11.5	7.9	9.0	8.3	9.5	49.0
25	37.3	10.4	9.3	13.3	11.5	11.4	11.1	11.9	10.4	41.8
27	45.8	10.7	12.0	11.8	10.3	10.0	10.8	10.5	11.0	55.7
10*	30.9	14.6	9.8	12.7		20.9	20,1	21.8	20.0	41.2
28	34.6	9.4	9.7	8.4	9.7	13.5	12.5	12.8	13.1	42.3
29	30.6	10.5	7.7	10.0	9.7	10.4	10.6	13.0	13.4	37,2
30	35.7	11,6	7.0	9.3	9.1	10.0	6.0	9.2	8.6	42.1
31	37.5	11.7	10.7	9.9	8.6	6.9	8.0	9.2	7.9	44.4
32	38.1	9.0	9.2	8.7	9.9	6.8	7.1	9.7	9.9	44.9
6	33.2	8.6	9.0	9.3	9.4	7.9	8.8	8.5	9.6	39,1
33	47.1	13.5	11.1	11.4	9.6	13.0	10.9	11.3	10.7	57.8
5	34.3	9.8	10.5	12.1	9.3	11.0	9.3	11.7	10.8	41.4
34	30.8	9.6	9.8	10.9	10.3	14,3	15.5	14.0	15.0	38.4
35	30.1	10.3	9.2	9.7	10.0	10.3	10.6	13.8	12.8	36.7

TABLE 8—Density changes during settlement testing.

\* Material removed from the house

102

Material Code	container number										
		15 cm	deep		30 cm deep						
	1	2	3	4	5	6	7	8			
333-163	10.5	10.7	12.4	10.5	12.3	12.4	12.4	11.6			
335-190	10.3	8.6	11.4	9.4	11.5	11.7	13.4	10.4			
334-180	11.8	11.6	11.8	12.0	13.4	13.7	13.7	13.6			
339 - 37	9.9	9.3	11.8	11.4	14.9	14.6	14.8	15.5			

TABLE 9—Percent settlement during drop test.

densities between 40 and 45 kg/m<sup>3</sup>. It is therefore important for manufacturers to examine the several factors that influence the settled density, for example, size and length of fibers in the finished product, amount and type of added chemicals, their mixing, and sieve size. These factors were not studied in the reported work. Another aspect of settled density testing is product quality control, as the density of the finished product varies depending on the raw materials used in the actual production batch.

There is therefore a need for another, quicker method for settled density determinations. Comparison with one such method will be discussed in another paper.

#### Acknowledgment

This research was performed in the Thermal Properties Laboratory of the Energy and Services Section, Division of Building Research, National Research Council of Canada. The authors wish to express their gratitude to Gerry Theriault for his contribution to the development of the testing methods, to Roger Marchand for building the special instrumentation, and Nicole Normandin and Gareth Keatley for making many of the measurements.

This paper is a contribution from the Division of Building Research, National Research Council of Canada and is published with the approval of the director of the Division.

# Blown Cellulose Fiber Thermal Insulations: Part 2—Thermal Resistance

**REFERENCE:** Shirtliffe, C. J. and Bomberg, M., "Blown Cellulose Fiber Thermal Insulations: Part 2—Thermal Resistance," *Thermal Transmission Measurements of Insulation, ASTM STP 660, R. P. Tye, Ed., American Society for Testing and Materials, 1978, pp. 104–129.* 

**ABSTRACT:** The thermal resistance of a number of commercial blown loose-fill cellulose fiber thermal insulations has been measured using the guarded hot plate and heat flowmeter methods. An equation describing the variation of thermal resistance with temperature, temperature difference, density, and thickness has been derived from these measurements and, with lesser precision, from the data provided by other investigators. The equation does not include the effects of chemical content, moisture content, chemical composition, or structure of the paper particles. The thermal resistance of a layer of insulation was found not to be directly proportional to thickness. The equation for thermal resistance fits the National Research Council of Canada (NRC) data with a standard deviation of less than 3.5 percent for thickness of 50 to 305 mm.

**KEY WORDS:** cellulose fiber, cellulose fiber thermal insulation, blown insulation, cellulosic fiber insulation, thermal resistance, thermal conductivity, newsprint, paper, thermal properties, thickness effect

Cellulose (or cellulosic) fiber thermal insulation (CFI) is made primarily from ground newsprint. The ground newsprint is blended with finely powdered chemicals which impart a measure of resistance to fire, fungus, and vermin.

Commercial cellulose fiber insulations may contain between 1 and 38 percent of chemicals by weight. Most of those meeting the standards contain between 18 and 25 percent. The additives are usually a blend of borax, boric acid, and aluminum sulphate. Aluminum sulphate alone has been used in insulations that are not intended to meet rigorous standards on corrosion and fungus growth. Other chemicals, such as soda ash,

<sup>1</sup>Research officers, National Research Council of Canada, Division of Building Research, Ottawa, Ont., Canada.

ammonium sulphate, oxides, phosphates, silicates, clay, portland cement, and garden fertilizer, have been used but have not been satisfactory.

After being blown into attics or walls, the material has a final density of about 40 to 50 and 60 to 100 kg/m<sup>3</sup> (2.6 to 3.5 and 4.0 to 7.5 lb/ft<sup>3</sup>), respectively. The material provides more thermal resistance per unit thickness at a competitive price than low-density mineral fiber insulation. As its manufacture entails a recycling of a material that is normally wasted, the material may play a major part in the retrofitting of residential buildings.

The first large-scale commercial production of cellulose fiber insulations in North America began between 1925 and 1935. Reports of research studies on this material are scarce. The earliest documented work known to the authors was that carried out at the University of Saskatchewan in the early 1950's and for the National Cellulose Manufacturers Association by Dynatech Inc. in the 1960's. Little information was published, however, until after 1970. Since then, a few technical and semitechnical papers have appeared and there are now four materials standards for the material three in the United States:

1. ASTM Standard Specification for Cellulosic Fiber (Wood-Base) Loose-Fill Thermal Insulation (C 739-73);

2. General Services Administration Specification for "Thermal Insulation Blanket; and Insulation Thermal (Loose Fill for Pneumatic or Poured Application): Cellulose Vegetable and Wood Fiber (GSA HH-I-515c);"

3. National Cellulose Insulation Manufacturer's Association Standard Specification for "Cellulosic Fiber (Wood Base) Loose-Fill Thermal Insulation (NCIMA N101-73);" and one in Canada:

Canadian Government Specifications Board Provisional Standard for "Thermal Insulation, Cellulose Fiber, Loose Fill (CGSB 51-GP-60P)."

The adoption of these standards has reduced the variability of the composition of cellulose fiber insulation and enabled the properties to be predicted with more accuracy than formerly.

This paper presents the results of measurements of the thermal resistance of a layer of cellulose fiber insulation. An equation was developed that approximates the results for thicknesses between 50 and 305 mm and densities between 30 and 100 kg/m<sup>3</sup>. The reason for restricting the range of the study is evident from the simplicity of the dependence of thermal resistance on density and thickness shown in Fig. 1. Most of the cellulose fiber insulations used in practice have properties that fall in this region.

A complete equation would include the effects of thickness of the layer, density, mean temperature, temperature difference, moisture content, degree of milling, amount and formulation of the chemical treatment, and the integrity of the cellulose fibers in the basic newsprint or paper stock. The last three factors have the least effect on thermal resistance. Few manufacturers control any of the variables except the chemical formulation



FIG. 1—Approximate relationship between thermal resistance of cellulose fiber insulation and density and thickness of layer showing complexity at thickness below 50 mm and densities below 40 kg/m<sup>3</sup>.

and quantity of the chemical in the product. Results of another study have shown these to be of little significance. Any deviation of individual measurements from the average curve reflects the consequences of ignoring these and other variables.

The moisture content of the material affects its density. At standard conditions the moisture content will depend on the paper stock, chemical formulation, and the amount of chemical used. As the variation of thermal resistance with density is included in the equation, the effect of the chemical formulation on the thermal resistance is at least partially included.

Moisture distribution in the material will be nonuniform due to the temperature gradient imposed across the material during the test. The degree of dependence of the measured thermal resistance on this temperature gradient indicates the importance of the moisture distribution.

The equation that has been derived for thermal resistance does not

describe the performance of any specific material for a given application; it represents a good estimate of the expected performance for current commercial materials. During the initial study, over 75 measurements were made on 30 commercial materials. Results of 39 measurements on 20 materials made by Tye  $[1]^2$  as well as 14 on three materials made by Anderson and Wilkes [2] and for the Public Service Co. of Colorado were also analyzed. Additional tests on 29 materials measured at 76-mm thickness, three high-density specimens, and 12 measurements on two materials differing only in chemical content, communicated to the authors by Anderson [3], are included as a secondary check on the analysis.

#### **Previous Measurements**

Tye [1] showed that the apparent thermal conductivity, and therefore the thermal resistance, of 25- and 35-mm layers of cellulose insulation is dependent on the density, the mean temperature, and the moisture content. Tye's measurements, on 29 materials, covered a temperature range of -20 to  $+40^{\circ}$ C, a density range of 24 to 123 kg/m<sup>3</sup>, moisture contents of 0 to 12.5 percent, and thicknesses from 6 to 52 mm. A special procedure was used by Tye to produce the specimens.

The 14 measurements collected from other sources such as Anderson and Wilkes [2] and a private communication from the Public Service Co. of Colorado included those on materials with thicknesses from 127 to 203 mm and densities from 29 to 64 kg/m<sup>3</sup>. Large commercial blowers were used to produce the specimens and it was reported that there were problems with fire-retardant separation during specimen preparation. The specimens may also have contained some high-density clumps due to the characteristics of the large blowers. Some settlement could also have occurred during the initial stages of the measurements. These data are listed in Table 1b.

The measurements in all the studies were made by experienced laboratory personnel using either ASTM Tests for Steady-State Thermal Transmission Properties by Means of the Guarded Hot Plate (C 177-76) or Steady-State Thermal Transmission Properties by Means of the Heat Flow Meter (C 518-76). The test conditions were essentially identical in all three studies.

#### Preparation of Specimens in Current Study

The materials used in the present study were obtained directly from manufacturers in standard commercial packages and stored in an air-con-

<sup>2</sup>The italic numbers in brackets refer to the list of references appended to this paper.

ditioned space with humidity ranging from 30 to 60 percent until the specimens were prepared.

The materials were blown through a small commercial cellulose fiber blower into frames of appropriate thickness, using, in most cases, a machine made by Diversified Insulation, Inc. of Minneapolis, Minn. A 0.75kW (1 hp) Model 8806 Toronado blower on the machine fluffed the material, then the air and material were transported through a hose 15 m long and 5 cm in diameter. The end of the hose was held either horizontally, 91 cm above the bottom of the specimen frames, or at a 10-deg upward slope, 28 cm above the bottom of the frames [4]. The specimens of the same material blown by the two methods appeared to have a consistent texture and structure and on average had the same thermal resistance.

Before testing, the specimens were conditioned for 1 to 4 weeks in a room at 22°C and 50 percent relative humidity. Moisture content of the conditioned specimens varied between 8 and 12 percent by weight.

The frames that contained the material during the measurement of the thermal resistance were made of Plexiglas, plywood, or extruded polystyrene foam and were covered on one or both sides before testing with 0.05 to 0.10 mm clear or black polyethylene.

Equipment used in the testing was either a 30- or 40-cm-square horizontal heat flowmeter apparatus of the Armstrong Cork design which conforms to ASTM C 518-76 or a vertical guarded hot-plate apparatus, 46 or 60 cm square, conforming to ASTM C 177-76. Earlier calibration had shown that the apparatus would give results agreeing to within approximately  $\pm \frac{1}{2}$  percent. Table 1*a* lists the test conditions, apparatus, and specimen size.

Tests were performed generally in accordance with the requirements of the test methods. The amount of edge insulation required by the test methods for conditions where the ambient did not equal the mean temperature of the specimens could not always be achieved because of space limitations. The effects of edge losses were calculated and the results corrected.

#### **Precision of Measurements**

The electronic measuring equipment used to measure the heat flux, temperature differences, and heat meter thermopile outputs contributed errors of less than 0.1 percent to the results. The principal errors were those due to edge losses and uncertainties in the calibration of the heat flowmeters. From a number of checks, the errors in the calibration of the heat flowmeters were found to be less than 1 percent; errors due to edge losses were more difficult to estimate.

#### **Estimation of Edge Losses**

A two-dimensional finite-element computer program was used to solve the steady-state temperature field at the center of the edge of each apparatus for various thicknesses of specimens, temperature differences, ambient temperatures, and amounts of edge insulation. The calculated temperatures were compared with edge temperatures, measured with thermocouples. Additional measurements were made with ambient temperatures other than the mean of the hot and cold surfaces to introduce known additional errors. The calculations agreed well with the measurements. It is considered that edge losses were estimated with sufficient accuracy to allow their use for making corrections where necessary.

In a few cases the ambient temperature was 4°C below the mean temperature. The edge-loss corrections for this condition for the 229- and 305mm-thick specimens in the heat flowmeter apparatus with 2.7 K  $\cdot$  m<sup>2</sup>/W edge insulation ranged from 8 to 22 percent. After the uniformity of the ambient temperature was improved and the level was brought to the mean temperature of the specimens, estimates showed this loss to be less than 1 percent. This is less than the uncertainty in the calculation, and so corrections were not made on tests performed with these conditions.

In testing thinner specimens, edge losses introduced errors of less than 1 percent.

#### Results

#### General

The experimental results are given in Table 1a and illustrated in Figs. 2-7.

Specimens for testing were generally taken from different samples of material. Variability between samples was not known but observations in another study [4] showed that the density variations from batch to batch of the same material could be considerable. Measurements on specimens of the same material from three samples with the same density gave identical thermal resistances.

All calculations were made in the imperial system of units and then converted to the SI system.

### Fitting Curves to Data

The National Research Council of Canada (NRC) data and that obtained from Tye were divided into sets according to the thickness of the specimens. Sets of data for thicknesses of approximately 25, 35, 76, 150, and

Test No.	Product No.	Apparatus <sup>2</sup>	Thickness, mm	Density, kg/m <sup>3</sup>	Moisture Content, % weight	Thermal Resistance, $R, K \cdot m^2/W$
1	10	2	50.8	56.2	8.8	1.28
1	10	2	50.8	48 1	6.0	1.30
2	10	2	50.8	48 1	1.7	1.32
3	15	1	50.8	48 1	7.5	1.28
4	13	1	76.2	32.2	87	1.87
5	14	2	25.4	74 3	3.9	0.64
7	13	I	63.5	73.4	11.4	1.46
12	15	î	76.4	35.6	7.0	2.06
14	10	1	76.4	36.4	15.3	2.06
16	9	1	76.5	23.4	8.8	1.94
19	8	i	76.4	35.6	8.1	2.02
20	7	i	76.7	36.2	10.2	2.00
20	6	1	76.7	36.3	7.2	2.04
24	3	i	76.7	42.0	9.2	2.04
24	3	i	63.8	44.7		1.68
27	3	i	89.2	79.5		2.19
30	3	3	305.7	47.8		7.48
31	3	3	305.7	47.8		7.20
37	4	ĩ	76.8	28.5	8.4	2.00
34	5	i	76.8	39.7	10.1	2.03
36	1	i	76.7	44.7	5.4	2.00
37	18	1	76.8	55.0	8.6	1.96
38	17	î	76.8	40.7	12.1	1.94
30	2	1	76.1	34.0	5.0	2.04
40	19	í.	76.8	54.8	5.1	1.99
40	20	i	76.6	43.1	5.8	1.97
43	21	i	76.9	36.5	7.2	2.00
44	22	1	76.8	34.8	5.5	2.03
45	16	1	76.9	45.2	8.0	1.94
46	23	1	76.7	48.1	5.9	1.90
47	28	1	76.7	43.4		1.96
48	32	4	75.4	47.6	2	1.99
49	30	1	76.7	44.9		2.00
50	31	1	76.7	48.2		1.94
51	31	1	76.7	48.2		1.94
52	31	1	76.7	48.2		1.94
53	26	1	76.9	48.1		1.98
54	35	1	76.7	37.8		2.04
55	36	1	76.9	37.3		2.09
56	34	1	76.9	38.6		2.02
57	33	1	76.9	68.6		1.98
81	3	1	76.0	40.7	8.5	1.99
82	3	1	228.2	40.5	8.5	5.69
83	3	3	230.0	49.0	9.2	5.53
84	3	2	12.8	40.7	8.5	0.37
85	3	2	39.4	39.7	8.5	1.12
86	3	2	65.9	40.5	8.5	1.88

TABLE 1a—NRC data on cellulose fiber insulation.

a Apparatus:

3: 60-cm guarded hot plate.4: 45-cm heat flowmeter.

45-cm guarded hot plate.
 30-cm guarded hot plate.

Test No.	Product No.	<b>App</b> aratus <sup>a</sup>	Thickness, mm	Density, kg/m <sup>3</sup>	Moisture content, % weight	Thermal Resistance, R, K·m²/W
87	32	1	75.4	47.8		1.99
88	32	1	299.8	47.8		7.25
89	28	1	76.7	43.2		1.96
90	28	1	299.7	41.8		7.40
91	3	3	305.5	53.0	14.8	7.48
92	37	1	76.6	42.0		1.97
93	37	1	152.6	39.4		3.72
94	30	1	76.7	44.9		2.00
96	25	1	76.9	46.0		1.97
97	25	1	299.9	46.0	•••	7.52
98	25	1	299.9	46.0		7.26
99	37	1	76.0	78.8	•••	1.83
33	4	1	152.7	32.7	8.4	3.68 <sup>b</sup>
35	5	1	152.4	42.3	10.1	3.92
13	11	1	152.4	38.0	7.0	3.99
15	10	1	152.2	37.8	15.3	3.83
17	9	I	152.4	24.8	8.8	3.71
19	8	1	152.2	40.7	8.1	3.84 <sup>b</sup>
21	7	1	152.8	42.3	10.2	3.79
23	6	1	152.7	41.7	7.2	3.85
25	3	1	152.8	48,7	9.2	3,69
28	3	1	152.3	80,1	***	3.51 <sup>b</sup>
33	4	1	152.7	32.7	8.4	3.67 <sup>b</sup>
35	5	1	152.4	42.3	10.1	3.92 <sup>b</sup>

TABLE la—Continued.

<sup>a</sup>Apparatus:

1: 45-cm guarded hot plate

3: 60-cm guarded hot plate.

2: 30-cm guarded hot plate 4: 45-cm heat flowmeter.

<sup>b</sup>Values of thermal resistance corrected with regard to side heat losses.

TABLE1b—Thermal resistance tests on cellulose fiber insulation: data from Public Service Co. of Colorado (measurements according to ASTM C 518-76).

Product No.	Thickness, mm	Density, kg/m <sup>3</sup>	Thermal Resistance, $R, K \cdot m^2/W$
40	152.4	43.5	3.77
40	139.7	47.7	3.47
40	127.0	52.3	3.16
12	203.2	39.7	5.14
12	190.5	42.3	4.94
12	177.8	45.4	4.62
12	165.7	48.9	4.28
12	152.4	52.9	3.86
12	139.7	57.7	3,58
12	127.0	63.6	3.22
12	152.4	38.5	4.10
12	139.7	42.0	3.66
12	127.0	46.3	3.27
12	76.0	78.8	1.82

#### 112 THERMAL TRANSMISSION MEASUREMENTS OF INSULATION

Product No.	Thickness, mm	Density, kg/m <sup>3</sup>	Thermal Resistance, $R, K \cdot m^2/W$	
30a	101.6	24.1	2.58	
30a	101.6	32.1	2.61	
30a	101.6	40.1	2.69	
30a	101.6	48.1	2.59	
30a	101.6	56.1	2.54	
30a	101.6	64.2	2.44	
30b	101.6	24.1	2.61	
30b	101.6	32.1	2.63	
30b	101.6	40.1	2.70	
30b	101.6	48.1	2.61	
30b	101.6	56.1	2.60	
30b	101.6	64.2	2.45	
30b	25.4	43.3	0.66	
30b	50.8	43.3	1.29	
30b	101.6	43.3	2.53	
30b	152.4	43.3	3.81	
30b	203.2	43.3	5.03	
30b	25.4	64.2	0.61	
30b	50.8	64.2	1.22	
30b	101.6	64.2	2.39	
30b	152.4	64.2	3.56	
30b	203.2	64.2	4.76	

 TABLE 1c—Thermal resistance tests on cellulose fiber insulation: data from Anderson
 [3]

 [3] (Product 30b has a 4 percent higher chemical addition than Product 30a; moisture contents are approximately 10 percent.)

 TABLE 1d—NRC data on cellulose fiber insulation obtained after analysis of data in

 Table 1.

Test No.	Product No.	Apparatus	Thickness, mm	Density, kg/m <sup>3</sup>	Moisture Content, % Weight	Thermal Resistance, R, K·m <sup>2</sup> /W
100	41	4	75.1	38.3		1.946
101	42	4	74.9	51.9		1.923
102	43	4	74.9	54.3		1.912
103	44	4	75.1	42.7		1.925
104	45	4	74.9	39.9		1.945
105	46a	4	75.2	38.3		2.015
106	46b	4	75.2	37.3		
107	47	4	74.8	42.5		1.954
108	48	4	75.0	36.0		1.931
109	49	4	74.9	41.2	1224	1.963
110	50	4	75.4	44.8		1.994
111	51	4	76.7	44.3		1.937
112	52	4	76.9	45.2		1.973
113	53	4	75.0	42.1		
114	54	4	75.0	37.6		
115	55	4	75.0	37.8		2.018
116	56	4	76.7	36.9		1.983

Test No.	Product No.	Apparatus	Thickness, mm	Density, kg/m <sup>3</sup>	Moisture Content, % Weight	Thermal Resistance, $R$ , K $\cdot$ m <sup>2</sup> /W
117	57	4	75.0	41.5		
118	58	4	76.7	36.6		2.045
119	59	4	76.9	48.7		1.982
120	60	4	76.0	57.9		
121	61	4	76.6	39.5		1.972
122	62	4	76.9	38.5		2.021
123	63	4	76.9	55.6		1.912
124	64	4	75.1	48.8		1.959
125	65	4	76.7	42.3		1.959
126	66	4	76.9	32.2		2.091
127	67	4	75.0	32.7		
128	68	4	75.2	42.0		1.943
129	69	4	75.1	42.7		1.983
130	70	4	75.0	36.1		1.979
131	71	4	75.0	45.9		1.916
132	72	4	75.0	36.6	1992	1.959
133	73	4	75.2	34.8		1.943
134	74	4	76.8	49.1		1.967
135	75	4	75.1	46.3		1.946
136	3	4	153.1	114.4		3,416
137	3	1	153.1	64.1		3.792

TABLE 1d—Continued





FIG. 2—Thermal resistance versus density for 25-mm thickness of cellulose fiber insulation.



FIG. 3—Thermal resistance versus density for 35-mm thickness of cellulose fiber insulation.

305 mm (1, 1.37, 3, 6, and 12 in.) were selected. A variation up to 10 percent in thickness was allowed in any set.

The dependence of thermal resistance (R) on the density was examined first. Equations of the form  $R = A + B\rho$  were fitted to the data by linear regression, where R is thermal resistance  $(K \cdot m^2/W)$  and  $\rho$  the density (kg/m<sup>3</sup>). Coefficients A and B were assumed to have the form A = C + DL and B = E + FL, where L is the specimen thickness (mm); coefficients C, D, E, and F were determined by linear regression. The equations for A and B were substituted back into the equation for R, giving an equation of the form  $R = (C + DL) + (E + FL)\rho$ .

The equation was then used to adjust the measured R so as to eliminate the variation due to thickness within the sets. The corrected data sets are shown in Figs. 2-5 and 7; the scatter within each corrected set was still too large to justify using higher-order equations in the fitting of the curves to the data.

Coefficients A and B were recalculated from the corrected data by linear regression (Figs. 8 and 9). Values of A and B and the correlation coeffi-



FIG. 4—Thermal resistance versus density for 76-mm thickness of cellulose fiber insulation.

cients are listed in Table 2*a*. Equations were again fitted to the *A* and *B* coefficients using linear regression (Table 2*b*). The correlation coefficients indicate that the fit of the data for the four greatest thicknesses is adequate to describe the relationship and does not mask any special effects near zero thickness. Equations were also fitted through all points; the equations, correlation coefficients, and standard deviation are listed in Table 2*b*. The data for the 102-mm specimens provided by Anderson [3] and the 30 extra measurements are plotted in the same figures to indicate the type of agreement obtained.

Substituting the third set of coefficients into  $A + B\rho$  gives the following equation, which describes the dependence of thermal resistance on density and thickness

$$R = (0.205 + 0.0247L) - (0.00201 + 0.0000143L)\rho$$
(1)

or, in imperial units, °F/(Btu/h·ft<sup>2</sup>), in., and lb/ft<sup>3</sup>

$$R = (1.165 + 3.56L) - (0.183 + 0.0331L)\rho$$
<sup>(2)</sup>



FIG. 5—Thermal resistance versus density for 152-mm thickness of cellulose fiber insulation.

Table 3 gives a comparison of the equations obtained by evaluating Eq 1 at given thicknesses with the equations obtained by linear regression, that is, the least-squares fit to the data of R versus  $\rho$  at these thicknesses. The agreement is reasonable for thicknesses over 50 mm.

Equation 1 does not give a zero resistance at zero thickness. Higherorder equations for the coefficients A and B would be required to describe the thermal resistance of thin layers of materials. A second-order fit for A and B is included in Table 2b for comparison. The thermal resistance for specimens with thicknesses over 50 mm can be represented adequately by the linear equation. Again the equations for Anderson's data and the 30 additional measurements are included as a check.

Equation 1 was evaluated at each thickness and for a number of densities. The resulting curves are plotted in Figs. 2-7 for comparison with the data and curves found by linear regression. The density term in Eq 1 is related to the conduction through the cellulose fiber.

Equation 1 should also contain a term containing the variable 1/density to describe the reduction of the radiation component of heat transfer in the material with increasing density. This would justify a variable in the equation of the form  $\rho + C\rho^{-1}$ , where C is a constant that can be determined from the density at which the maximum resistance occurs. The results of both Tye [1] and Anderson [3] can be used to show that at densities below about 24 kg/m<sup>3</sup> (1.5 lb/ft<sup>3</sup>) the radiation component must increase at a rapid rate with density [5]. The result of such a fit to Tye's data for 35-mm specimens and Anderson's results for 102-mm specimens is shown in Figs. 3 and 10, respectively. Above 40 kg/m<sup>3</sup> with the 35-mm specimens, the curve falls about 0.5 percent higher than the line shown, within about the width of the line. This more complex form of the equation was not used since there were insufficient data to establish the curve below densities of 32 kg/m<sup>3</sup> and it did not affect the results for density greater than 40 kg/m<sup>3</sup>. In Fig. 10, the linear equation and the equation using the  $\rho$  +  $1302\rho^{-1}$  term do not agree. Further data would be necessary to distinguish which equation is more representative at higher densities or if the Anderson data contained errors.



FIG. 6—Thermal resistance versus density for 229-mm thickness of cellulose fiber insulation.



FIG. 7—Thermal resistance versus density for 305-mm thickness of cellulose fiber insulation.

The equation for R in terms of  $\rho$  and L was checked against the remainder of the data, including that by Tye, at thicknesses of 13, 25, 39, 51, 64, 89, and 228 mm (0.5, 1.0, 1.55, 2.0, 2.5, 3.5, and 9 in.). The results are shown in Fig. 6 and in Table 4. The agreement is good, except at the 13- and 25mm thicknesses, but this region was of little interest.

#### Thermal Resistance Versus Thickness

A check of the relationship between thermal resistance and thickness was made by using a carefully selected set of data in which the thickness varied from 13 to 305 mm and the density between 40 and 48 kg/m<sup>3</sup>. Adjustments to the thermal resistance were made to correct the data to a density of 40.4 kg/m<sup>3</sup>. One curve of the form R = G + HL was fitted to the corrected data for the 50-mm thickness and above using linear regression. Other equations were obtained by including the data for thicknesses below 50 mm. The resulting equations are listed in Table 5. The linear equations obtained using 15 and 13 points, respectively, are

$$R = 0.143 + 0.0240L \tag{3}$$

$$R = 0.192 + 0.0238L \tag{3a}$$

Equation 1 at the same density gives

$$R = 0.124 + 0.0241L \tag{4}$$

Equations 3 and 4 are plotted with the data in Fig. 11. The two techniques for curve fitting yield approximately the same equation at this density; thus the form of Eq 1 and the value of the constants in it appear to be adequate to describe the dependence of thermal resistance in the range of variables considered. Anderson's data agree well with both curves.

Anderson's data and Eq 1 evaluated at a density of  $64.2 \text{ kg/m}^3$  are plotted in Fig. 12. The agreement is somewhat poorer than at the lower density; this may be due to error in the data caused by transient heat flows.

#### Thickness and Density Changes

A further check of the adequacy of the form of the equation was made by comparing Eq 1 for resistance in terms of density and thickness with



FIG. 8—Three-point linear fit for coefficient A in the equation  $R = A + B\rho$  (Table 2a).

THICKNESS, in.



FIG. 9—Three-point linear and second-order fit for coefficient B in equation  $R = A + B\rho$  (Table 2a).

Thickness, mm	$A, K \cdot m^2/W$	$B, K \cdot m^5/W^* kg \times 10^{-3}$	Correlation Coefficient
25	0.642	-0.772	-0.285
35	0.942	-1.487	-0.846
75	2.132	-3.913	-0.563
76	2.086	-2.565	-0.501
102	2.744	-3.606	-0.654
152	3.966	-4.776	-0.520
228	5.845*	$-5.710^{b}$	
305	7.726	-5.896	-0.427

TABLE 2a—Values of coefficients A and B in the equation  $R = A + B\rho$ .

<sup>a</sup>Interpolated

<sup>b</sup>Using value of A and data in Fig. 6.

Number of		Correlation	Standard Deviation of $A + B\rho$ : 83 Points	
Points	Coefficients	Coefficient	%	Value
5 6	A = 0.0731 + 0.02561L -B = 0.957 × 10 <sup>-3</sup> + 1.881 × 10 <sup>-5</sup> L	r = 0.99971 r = -0.954	3.7	0.109
4 5	A = 0.1310 + 0.02498L -B = 1.383 × 10 <sup>-3</sup> + 1.696 × 10 <sup>-5</sup> L	r = 0.99987 r = -0.949	3.7	0.110
3 4	${}^{a}A = 0.2052 + 0.02468L$ ${}^{b}-B = 2.015 \times 10^{-3} + 1.430 \times 10^{-5}L$	r = 1.0000 r = -0.923	3.3	0.099
6 6	A = -0.007934 + 0.02704L - 5.663 × 10 <sup>-6</sup> L <sup>2</sup> -B = -1.652 × 10 <sup>-4</sup> + 4.382 × 10 <sup>-5</sup> L	r = 0.99990 r = -0.998	5.1	0.135
6 6	+ $7.856 \times 10^{-8}L^2$ As above with $A' = A + 0.9294$ B' = B		2.8	0.075

TABLE 2b—Equations in coefficients A and B in terms of the thickness, L, mm, of the layer.

<sup>a</sup>These coefficients were used in deriving Eq. 8.

Thickness, mm	Identification	Equation for Thermal Resistance, $K \cdot m^2/W$	Correlation Coefficient
25	least squares Eq 1	$\begin{array}{c} 0.624\!-\!0.00077\rho\\ 0.822\!-\!0.00237\rho\end{array}$	-0.285
35	least squares Eq 1	$\begin{array}{c} 0.942 \!-\! 0.00149 \rho \\ 1.069 \!-\! 0.00251 \rho \end{array}$	0.846
75ª	least squares Eq 1	2.132-0.00391p 2.058-0.00308p	-0.563
76	least squares Eq 1	$2.086 - 0.00256\rho$ $2.081 - 0.00310\rho$	-0.501
102 <sup>b</sup>	least squares Eq 1	$\begin{array}{c} 2.744 - 0.00355 \rho \\ 2.724 - 0.00347 \rho \end{array}$	-0.638
152	least squares Eq 1	$3.966 - 0.00477 \rho$ $3.956 - 0.00418 \rho$	-0.520
305	least squares Eq 1	7.727 - 0.00589  ho 7.732 - 0.00637  ho	-0.427

TABLE 3—Comparison of Eq 1 with least-squares fit of thermal resistance versus density,  $\rho \ kg/m^3$ .

<sup>a</sup>Additional 30 data points.

<sup>b</sup>Data by Anderson [3].



FIG. 10—Thermal resistance versus density for 102-mm-thickness cellulose fiber insulation (data from Anderson [3]).

	No. of Points	Pc Pm	% Diffe	% Difference	
Thickness, mm		$K \cdot m^2/W$	Average	Max	
13		0.06	16	16	
25	7	0.09	14	27	
35	31	0.06	7	16	
39		0.05	4	4	
51		0.02	1	2	
64	2	-0.03	-2	2	
75	30	+0.02	$^{-2}$	4	
76	33	-0.01	-0.6	6	
89		-0.04	$^{-2}$	2	
150	12	0.04	1.2	4	
228	4	+0.05	+1	9	
305	8	+0.03	+0.4	3	
Average		0.01			
50 to 305			-0.4	4	

 TABLE 4—Difference between eq 8 for calculated thermal resistance, Rc, and the measured thermal resistance, Rm.

Range of Thickness, mm	No. of Measurements Used	Equation for $R$ , $K \cdot m^2/W$	Correlation Coefficient
(1) 40 to 305	13	0.192 + 0.0238L	0.9991
(2) $25$ to $305$	14	0.162 + 0.239L	0.9991
(3) 13 to 305	15	0.143 + 0.240L	0.9992
(4) 13 to 305	15	$0.092 + 0.0250L - 2.98 \times 10^{-6}L^2$	0.9992
Notes:			

TABLE 5—Equations derived from data on selected materials with varying thicknesses and a corrected density of 40.4 kg/m<sup>3</sup>. Density range: 40.4 to 47.6 kg/m<sup>3</sup>; thicknesses measured: 13, 25, 40, 75, 152, 228, and 305 mm.

1. Differences in R given by (1), (2), and (3) at 76 = 1.7 percent and at 305 mm = 0.2 percent.

2. Equation 1, when evaluated at 40.4 kg/m<sup>3</sup>, gives R = 0.124 + 0.0241L.

3. Differences between all four equations at both 76 and 305 mm thickness were less than 2.5 percent.





FIG. 11—Thermal resistance versus thickness for cellulose fiber thermal insulation, density 40 kg/m<sup>3</sup> (2.52 lb/ft<sup>3</sup>).



FIG. 12—Thermal resistance versus thickness for cellulose fiber thermal insulation, density  $64.2 \text{ kg/m}^3$  (4 lb/ft<sup>3</sup>).

data on three materials obtained from The Public Service Co. of Colorado. In these measurements the thermal resistance of a given specimen was measured first at full thickness, then at a number of progressively reduced thicknesses. Five measurements were made on one specimen; three measurements were made on each of the other two (Fig. 13). Equation 1, which has the same slope as the data in every case, accounts for simultaneous density and thickness changes.

The thermal resistances of the specimens used in these tests were higher than those predicted by Eq 1. This may be partly due to the degree of milling of the paper in the materials and differences in chemical formulation. This behavior was similar to that observed in NRCC measurements on the same materials.

#### Effect of Temperature Difference

Measurements were made at 25, 76, and 305 mm with temperature differences of 11 and 44°C rather than the usual 22°C. The percentage variation in R was found to be 0, +0.0013, and +0.12 percent/deg C, respectively.

An average value of 0.04 percent/deg C could have been used. Because the measurements at 305 mm are least reliable, a value of 0 percent/deg C is recommended until further information becomes available.

#### Effect of Mean Temperature on Thermal Resistance

Cellulose fiber insulation is an air-filled material operating well above the region where the mean free path length of air molecules approaches that of the spacing between fibers and controls the thermal conductance. It was assumed that variation of conductance, C, with mean temperature, T, is given by

$$C = C_{\rho} \left[ 1 + H(T - 24) \right]$$
(5)

where  $C_o$  is the thermal conductance at 24°C and H is a constant. The dependence of thermal resistance on temperature will therefore be

$$R = R_0 / \left[ 1 + H(T - 24) \right] \tag{6}$$

The term  $\{1/[1 + H(T - 24)]\}$  cannot be readily inverted to give a convergent series, so this form must be retained.

The data obtained by Tye [1] were used to establish the value of H. His data covered a range of -18 to  $+50^{\circ}$ C. The final value as determined by least squares in SI units was

$$R = R_0 / [1 + 0.00289 (T - 24)]$$
<sup>(7)</sup>

Equation for Thermal Resistance

The equation for thermal resistance becomes

$$R = \frac{(0.205 + 0.0247L) - (0.00201 + 0.0000143L)\rho}{[1 + 0.00289(T - 24)]}$$
(8)

The equation fits the NRC data with a standard deviation of less than 3 percent (approximately 0.1). Almost all the observations fall within 10 percent of Eq 8; most of the data fall within 4 percent of the equation. The remaining difference between the observations and the equation are more likely due to the form of the paper particles and to chemical composition than to errors in the measurements.

#### Adjusting the Constant Term in Eq 8

Equation 8 describes the dependence of thermal resistance on density and with thickness of the specimen, for thicknesses greater than 50 mm. The equation appears to have the correct form to fit the data. There does appear to be a problem, however, in determining the correct constant or zero-thickness intercept. A check was made to see if the constant in Eq 8 could be adjusted to yield better results. The difference between the thermal resistance calculated from Eq 8, Rc, and the measured thermal resistance, Rm, is summarized in Table 4 as Rc - Rm. The increase of the 0.01 units to the constant in Eq 8 indicated by the average from 50 to 305 mm is hardly justified by precision in the measurements.

#### Adding Higher-Order Terms in Thickness and Adjustment of Slope

The fit of Eq 8 to the observations is shown in Figs. 2-7 and 13. Figure 2, for the 25-mm thickness, shows a large discrepancy between the curve from Eq 8 and that from the linear regression. This discrepancy indicates that either A or both A and B in the equation  $R = A + B\rho$  should have higher-order terms in L. The equation for R obtained from a second-order fit to both A and B gives better agreement. The standard deviation from the data, however, is large and, to improve the agreement, the equation must be shifted as indicated by the last set of coefficients in Table 2b.

As already mentioned, most cellulose fiber insulation is generally applied at densities over 30 kg/m<sup>3</sup> and at thicknesses greater than 50 mm. There



FIG. 13—Comparison of Eq 8 with data from Table 1b when thickness decreases and density increases.

appears to be little reason to complicate the equations with terms that become negligible in the normal range of use. The uncorrected linear equation is sufficiently accurate for material in thicknesses greater than 50 mm.

#### Discussion

#### Thickness Effects

The thermal resistance of low-density insulation materials is not necessarily directly proportional to thickness. A theoretical model was proposed by Poltz [6] in 1962 to explain this phenomenon in cellular plastics. His equations can be rearranged to produce an equation in the form

$$R(L) = R(0) + r(\infty)L$$
<sup>(9)</sup>

The thermal resistance at thickness L is equal to an apparent thermal resistance for zero thicknesses of material R(0) plus the thermal resistance per unit thickness of material measured at large thicknesses,  $r(\infty)$ , multiplied by the thickness of the specimen. The equation will hold only at thicknesses greater than the mean free path length [7] for radiation in the insulation. The equation for the thermal resistance of the layer of material can be interpreted as the sum of the thermal resistances of two layers for which there are widely different heat-transfer mechanisms. The same form of equation can be deduced by examining the radiation heat transfer through absorbing or scattering gases.

Examples of such dependence were given by Shirtliffe [8], Cammerer [9], and more recently by Lao and Skochdopole [7]. At least two manufacturers of mineral fiber insulation investigated this dependence and have been using similar equations in the design of such insulations since the mid-1960's. The existence of a thickness dependence for cellulose fiber insulation was measured in the early 1970's at the Division of Building Research. The effect in this material may be due to radiation as in plastic foams and also to moisture gradients.

It is not simple to measure the thickness effect. Marechal [10], in a discussion of Cammerer's paper, attempted to show that there was no such effect. He claimed it was due to errors in measurement of surface temperatures in the test apparatus. Edge losses can also vary with the thickness of the specimen and can either mask the thickness effect or make it appear more pronounced. This is especially true if a small amount of edge insulation is used on the apparatus. When the ambient temperature is lower than the mean temperature of the specimen, the error will produce results that indicate a thickness effect greater than actually exists. Other errors in measurement may also tend to have the same effect. The con-

tribution of such errors is recognized in ASTM Specifications C 177-76 and C 518-76.

The thickness effect measured in this study is not due to surface contact or errors in surface temperature measurements. The errors in such measurements are at least two orders of magnitude smaller than the measured thickness effect. The computer calculations of edge-loss errors, although not exact, showed that the edge losses were too small to cause the observed thickness effect. Unless some as yet unexplained or unrecognized effect is found, the thickness effect determined in these measurements must be assumed to be real.

#### Second Analysis of the Data

The observations were analyzed to see if the resulting equation was highly dependent on the data selected. Additional data were used, equal weight was placed on the data, and less precise corrections were made to individual measurements.

The equation in SI units obtained was

$$R = \frac{(0.253 + 0.0241L) - (0.00178 + 0.0000075L)\rho}{[1 + 0.00289(T - 24)]}$$
(10)

The standard deviation on 77 measurements made at NRCC was about 3 percent (approximately 0.1).

Equation 10 gives values for a 40-kg/m<sup>3</sup> material very close to those given by Eq 8. The difference in R at 50 mm was +0.032 (1.6 percent) and at 305 mm +0.043 (0.6 percent). These are small differences. The slopes between 76 and 305 mm differ by 1.3 percent.

#### Conclusions

The equation for thermal resistance (in SI units) (Eq 8)

$$R = \frac{(0.205 + 0.0247L) - (0.00201 + 0.0000143L)\rho}{[1 + 0.00289(T - 24)]}$$

will, on average, describe the thermal resistance of layers of newsprintbased cellulose fiber insulation with thicknesses between 50 mm and 305 mm, with densities of 32 to about 100 kg/m<sup>3</sup>, for applied temperature differences of 5 to 45°C, and for mean temperature from -20 to +50°C. The thermal resistance of individual materials may differ from the value obtained by the equation by as much as 10 percent. This difference is probably due to the shape of the particles of paper and the formulation and amount of powdered fire retardant in the product. The standard deviation of the fit was about 3 percent (approximately 0.1) for the NRC data and 6 percent (approximately 0.2) when data from the other sources were included. Anderson's data and 30 additional data points agree well with the equations, and inclusion of this in the analysis would improve the equations slightly.

Terms were suggested that would extend the equation to densities between 15 and 32 kg/m<sup>3</sup> and thicknesses of 10 to 50 mm. Additional observations would be required, however, to establish these terms with certainty.

The equation developed in this study contains terms to account for the so-called "thickness effect" on thermal resistance for low-density insulations over 50 mm thick.

#### Acknowledgments

The authors would like to acknowledge the excellent cooperation received from R.P. Tye, A. Déjarlais, R.W. Anderson, The Public Service Co. of Colorado, and a number of members of ASTM Committee C-16 in the gathering of data and discussion of the results. We would like also to acknowledge the many suggestions for improving the specimen preparation made by J.G. Theriault and C. St. Jacques and the meticulous work of J.G. Theriault, R. Marchand, G. Keatley and N. Normandin in the specimen preparation and measurements. This paper is a contribution from the Division of Building Research, National Research Council of Canada, and is published with the approval of the director of the Division.

#### References

- [1] Tye, R.P., Journal of Testing and Evaluation, Vol. 2, No. 3, May 1974, pp. 176-179
- [2] Anderson, R.W. and Wilkes, P., "Survey of Cellulosic Insulation Materials," ERDA 77-23, UC-95d, Energy Research and Development Administration, Jan. 1977.
- [3] Anderson, R.W., private communication.
- [4] Bomberg, M. and Shirtliffe, C.J. this publication, pp. 82-103.
- [5] Pelanne, C.M. in *Proceedings*, Eighth Conference on Thermal Conductivity, Plenum Press, New York, 1969, pp. 897-911.
- [6] Poltz, H. Allgemeine Wärmetechnik, Vol. 4, 1962, pp. 64-71.
- [7] Lao, B.Y. and Skochdopole, R.E. in *Proceedings*, Society of the Plastics Industry, International Cellular Plastics Meeting, Montreal, Canada, Nov. 1976, pp. 175-182.
- [8] Shirtliffe, C.J. Canadian Building Digest, National Research Council of Canada, Division of Building Research, No. 149, Ottawa, Ont., Canada, 1972.
- [9] Cammerer, W.F. in *Proceedings*, International Institute of Refrigeration (IIR) Commission B, Zurich, 1973-1974, pp. 189-195.
- [10] Marechal, D., Discussion of Ref 9, pp. 196-200.