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Publisher's version / Version de l'éditeur:
https://doi.org/10.4224/20378975
Building Practice Note, 1977-11

ISSN 0701-5216

BUILDING PRACTICE NOTE

ENERGY CONSERVATION AND ROOFS

by

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Division of Building Research
National Research Council of Canada

Ottawa
November 1977

PREFACE

The material in this paper was presented as part of the Technical Program of the Annual Meeting and National Convention of the Canadian Roofing Contractors Association held at the Queen Elizabeth Hotel, Montreal, 16 May 1977. It is now issued in this form by the Division to facilitate the provision of copies for use by roofers, and to make it available to others who may find it of value.

The author is an honorary member of the Canadian Roofing Contractors Association and has served on the Technical Committee of the Association for many years.

Ottawa
November 1977

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ENERGY CONSERVATION AND ROOFS

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M.C. Baker

Buildings are built to protect people, or their goods or possessions from some aspect of the natural weather. The walls, windows and roofs of a building act to separate an inside (usually controlled) environment from the outside natural (and usually variable) environments (Fig. 1).

The controlled indoor environment is created by using heating, ventilation and air-conditioning equipment. As it is usually expensive to provide heating and cooling, the building enclosure needs to be made thermally effective by using materials that have a high resistance to heat flow, to keep the heat in during the cold weather and out during the hot summer (Fig. 2).

The need for thermal insulation in buildings in Canada has been taken for granted for years. Most people accept the fact that a well insulated building costs less to heat or cool than one that is not insulated, and that such buildings where people live or work are more comfortable. So why all the fuss about energy conservation?

Firstly, despite a general knowledge about insulated buildings, some owners still largely ignore or intentionally bypass insulation in the interest of keeping initial building costs low. Even when insulation is used, the minimum amount required for reasonable comfort is often the criterion used without any real consideration of cost effectiveness. Even when cost effectiveness was considered in the past, the popular misconception that it only applied to the first one or two inches of insulation largely determined the amount used.

Secondly, only recently has the public been made aware that energy is indeed in short supply, and that energy costs are continuing to rise. It has been forecast that conventional oil and gas resources will probably be exhausted in the next 50 or 60 years, and thus other sources of energy for heating and cooling buildings will have to be developed. Most existing buildings use far more energy than necessary and many are extremely wasteful of energy. This is an urgent sociological problem; some believe energy conservation is the only hope of averting economic disaster.

Owners and designers are thus forced to take a new look at cost-effectiveness of insulation. The idea that maximum benefit to a building owner can be achieved by providing the desired indoor conditions at the least capital cost for the building and the heating and air-conditioning equipment, and at the least annual cost of operation and financing over the projected life of the

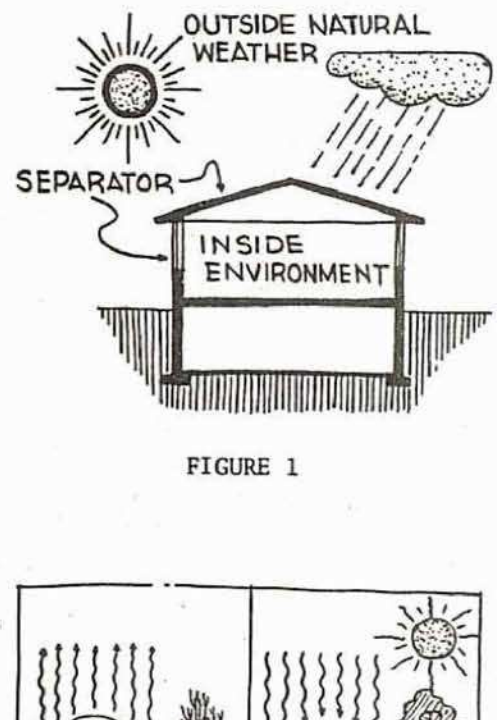


FIGURE 1

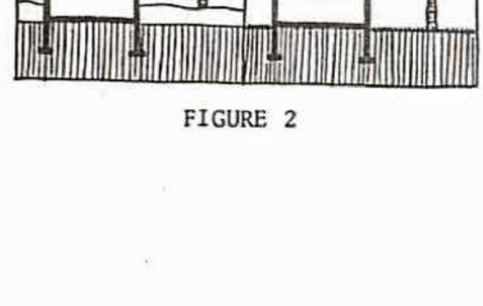


FIGURE 2

building, is not new. This "life-cycle" costing approach to building design has not in the past been taken seriously by most owners and their designers, and this has helped to precipitate the present energy crisis. Such a cost analysis frequently shows that the higher construction cost involved by making a building enclosure thermally efficient may result in net savings due to decreased fuel requirements for the heating and cooling over the projected life-span of the building. This is another way of saying that good thermal performance is more economical performance. Apart from the possibility of saving money in the long run by thermal upgrading, there will soon be energy conservation guidelines or standards enforced by law to help stretch out our dwindling resources. Although it now appears that the amount of new building construction is decreasing, it also appears that the use of renovated existing buildings is on the increase, and these almost certainly will require thermal upgrading.

The importance of the roof in relation to the rest of the building depends on the shape of the building. This can be seen by examining three buildings having equivalent volume, 40 storeys, 10 storeys and one storey high (Fig. 3). The exposed surface areas of the buildings expressed as a percentage of the gross floor areas are, respectively, 42.5, 30 and 118 per cent. This ratio has considerable importance in relation to energy conservation, because heat loss is a function of the amount of exposed surface; heat gains from occupants, lights and equipment are related to floor area. A building with a high ratio of exposed surface to floor area will have only a small part of its losses made up by heat produced from internal activities. It is appropriate, therefore, and very probable, that this type of building will be required by standards to have more thermal resistance in the exterior envelope than will buildings with low ratios of exposed surface to floor area. All single-storey buildings regardless of size, low-rise buildings of moderate size, and most residential buildings fall into this high ratio category. These generally are buildings with a large amount of roof area. In high-rise buildings, the roof represents only 6 per cent of the exposed area, in the medium rise it represents 33 per cent but in the single-storey building the roof is 85 per cent of the exposed area.

	40 STOREYS	10 STOREYS	1 STOREY
LENGTH, FT.	100	200	500
WIDTH, FT.	100	200	400
HEIGHT, FT.	400	100	20
VOLUME, CU. FT.	4 000 000	4 000 000	4 000 000
FLOOR AREA SQ. FT.	400 000	400 000	200 000
EXPOSED SURFACE, SQ. FT.	170 000	120 000	236 000
EXPOSED SURFACE TO FLOOR AREA	42.5%	30%	118%
ROOF AREA, SQ. FT.	10 000	40 000	200 000
ROOF AREA TO FLOOR AREA	2.5%	10%	100%
ROOF AREA TO EXPOSED SURFACE	5.9%	33%	84.7%

FIGURE 3

Heat Transfer

If one is to consider the energy savings that can be made through the use of insulation, one must understand some of the fundamentals of heat transfer. Heat transfer is a simple mechanical engineering expression related to where heat is going, how it flows, and how it can be stopped. A first fundamental fact is that heat energy always flows from warm to cold and nothing can stop it. All the heat added to a building eventually will flow out to the outdoors, but it can be slowed down by thermal insulation so that much less heat escapes each hour, each day or each winter. If the heat loss through the roof in winter is slowed down, much less heat will have to be replaced by the heating plant to maintain comfortable conditions. Likewise in summer if the heat coming into the building through the roof can be slowed down this will reduce the energy required for air-conditioning.

How does heat escape? All heat flow is by conduction, convection or radiation, although conduction is generally the dominant mode for solid building materials.

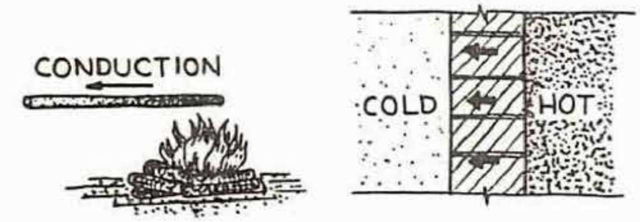


FIGURE 4

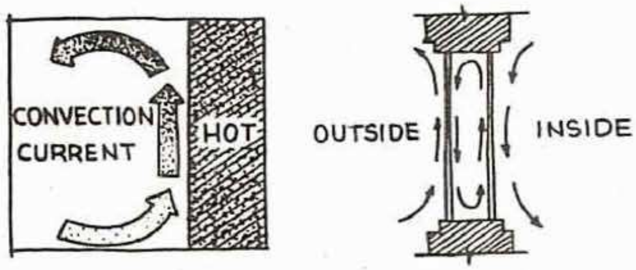


FIGURE 5

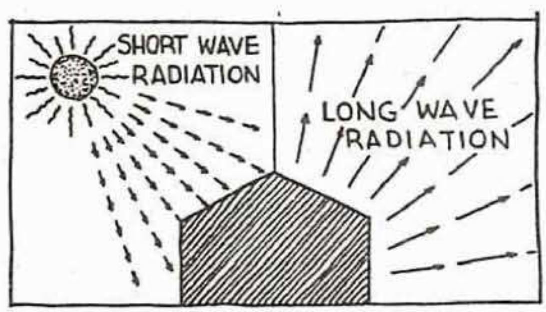


FIGURE 6

Conduction is the transfer or flow of heat through any type of matter by direct contact of particle to particle or molecule to molecule (Fig. 4). Activity of molecules is related to temperature. As heat is applied in one place the activity of the molecules is increased and is transferred from one molecule to the next. If a bar of good conduction material, such as metal, is heated in one place the heat is rapidly transferred throughout the bar by molecular activity. This is the only method by which heat can flow through an opaque solid. Heat can also pass by conduction from one material to another that is in intimate contact with it.

Convection is the carrying of heat by the movement of a liquid or gas (Fig. 5). If the liquid or gas is in contact with a heated surface it becomes heated and rises, thus creating a convection current, which may, in turn, transfer its heat to any colder surface it flows over. It it is cooled by contact with a colder surface, the convection current is downward. Convection currents set up in air spaces between the components of walls or roofs can transfer heat from the warm to the cold side. Air leaking through cracks and openings also transmits heat by convection.

Radiation is the transfer of heat energy by electro-magnetic waves through air or a vacuum (Fig. 6). All objects lose energy continuously by the emission of radiation; they gain energy by absorbing some of the radiation that comes to them from other objects. Short-wave radiation comes from the sun through empty space and some of it is absorbed by buildings to heat the materials. Building materials can also transfer heat energy by long-wave radiation through the air and across air spaces in the building construction.

No matter what the mode, heat flow means energy loss or gain, and usually this means greater expenditure for heating or cooling fuel dollars. Fortunately, however, all modes of heat flow can be easily slowed down by materials and methods readily available. The addition of insulation is the principal means of slowing down heat flow in roofs. To estimate the amount of heat loss or gain and the heating costs it is necessary to understand a few more specific terms that are used in calculations.

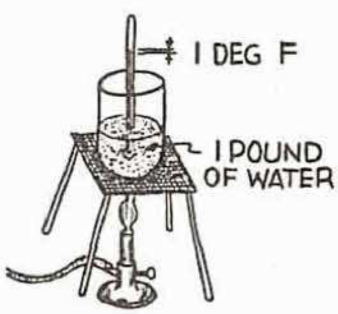
The British Thermal Unit, Btu, is the measuring unit of heat energy (Fig. 7). One Btu is the amount of heat required to raise the temperature of 1 pound of water by 1°F, or, conversely, the amount of heat given off by one pound of water cooling by 1°F. This is the unit used to describe the heat content of fuel and the heat flow through materials and building envelopes.

Thermal Resistance, (R), indicates the amount of resistance to heat transfer offered by one square foot of a material for each degree Fahrenheit difference of temperature between the faces of the material (Fig. 8). Most thermal insulation is now described by its R value. All materials in roofs have some resistance to heat flow as do air spaces, and air films at roof and ceiling surfaces. One inch thickness of a reasonably good insulation has an R value of about 3 to 4. This is an easy concept to understand but unfortunately it is not the only unit necessary in heat transfer analysis.

Thermal conductivity is actually the basic unit of heat flow, and this is a measure of the Btu's of heat that will be transmitted through one square foot of one-inch-thick material in one hour, when there is a temperature difference of 1°F between the opposite surfaces of the material. This is known as the k value of the material (Fig. 9).

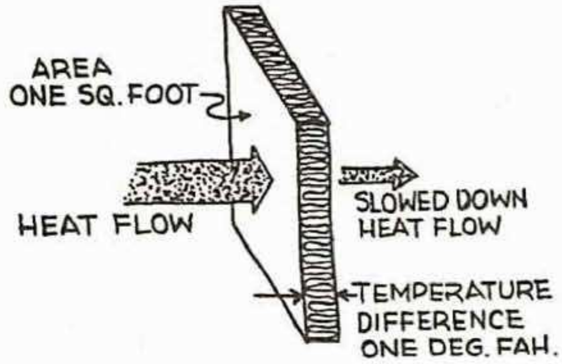
In analysing the heat flow for a total roof construction, the k or R value for each of the materials making up the construction must be considered . The two values are directly related; the R value for any material is simply the reciprocal of the k value ($R = \frac{1}{k}$ or $k = \frac{1}{R}$). To find the effectiveness of a roof or wall as a heat flow barrier the effectiveness of each component of the barrier must be added (Fig. 10). This can only be done by working with resistances, because k values cannot be added together. The sum of the resistances of the air film at the roof surface, the roofing membrane, the roof insulation, the roof deck, any dead air spaces, the ceiling and the inside air film is the total heat resistance of the construction. This allows calculation of the over-all heat flow for the roof system.

This over-all coefficient of thermal transmission, generally referred to as the U value, is the time rate of heat flow in Btu's per hour through an area one foot square,



BRITISH THERMAL UNIT (Btu)
AMOUNT OF HEAT TO RAISE TEMP. OF ONE POUND OF WATER BY 1° F.

FIGURE 7



THERMAL RESISTANCE

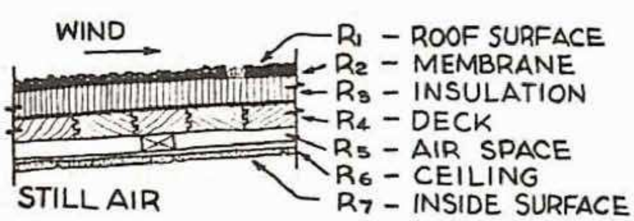
FIGURE 8

$\boxed{\text{THERMAL CONDUCTIVITY}} - \boxed{k}$

$\boxed{\text{THERMAL RESISTANCE}} - \boxed{R}$

$\boxed{R = 1/k}$ $\boxed{k = 1/R}$

FIGURE 9



$$\text{TOTAL } R = R_1+R_2+R_3+R_4+R_5+R_6+R_7$$

$$U = 1/R$$

FIGURE 10

under steady state conditions from the air on the inside of the roof to the air on the outside, for each 1°F of temperature difference between inside and outside air. The U value is $\frac{1}{R}$, where R represents the total heat resistance of the roof as already described. This is the value that must be known for the existing roof system for any proposed modifications, before the costs and possible savings can be calculated.

Heating Degree-Days

The discussion so far has related to the general situation, but for any specific building one needs to consider the location, the climate of which will influence the heat loss or gain of the building and consequently its energy requirements. Heating or cooling operating costs are influenced by the duration and severity of the heating or cooling season, by the cost of the fuel used, and by the efficiency of the heating or cooling plant. The aspect of climate that mainly affects this is the temperature and, for heating, the effect is measured in degree days. When this concept was originated heating facilities were generally put to use when the mean daily outdoor temperature fell below 65°F, and this is the figure generally used as the base for determining degree days. For any heating day there are as many degree days as there are Fahrenheit degrees difference between 65°F and the mean temperature when the mean temperature is below 65°F. If, on a given winter day the mean temperature is 35°F, for instance, that day represents 30 degree days. The degree days for the heating season are the sum of the daily values. Each weather station keeps track of this important figure, and a table of degree days below 65°F for most places in Canada is published as a supplement to the National Building Code of Canada. Values for representative cities in each province are given in Table I. The 65°F basis for degree days is no longer accurate enough for some calculations; tables are available that use a different temperature basis, and some formulae for energy calculations that use the degree days based on 65°F with a conversion factor to take this into account.

The Cost of Heat

Most buildings in Canada are heated by burning either oil or natural gas. The price of both of these fuels as well as electricity has been rising rapidly and it seems likely that the trend will continue. Forecasting the prices of these commodities is extremely difficult, but an estimate has to be made before any meaningful analysis of thermal upgrading for new or existing buildings can be done. It is predicted that oil prices will rise rapidly in the next few years -- perhaps at the rate of 20 per cent per year, until it catches up with the price of electricity, which is predicted to continue increasing in price at a fairly steady rate of 12 per cent per year (Fig. 11). The graph shows increases of 20 per cent and 15 per cent and indicates only that at the lower rate of increase it will take a little longer for the cost of oil to catch up with the cost of electricity. Natural gas is currently less expensive than fuel oil but it seems likely that it also will increase in price until it reaches parity with an alternate fuel, at which time it is assumed they will all increase at about 12 per cent per year.

Efficiency of the Heating Plant

The heating value of fuels will vary to some extent but Table II, based on information from the Guide of the American Society for Heating, Refrigeration and Air-Conditioning Engineers, gives reasonable values to use in calculations. If more accurate values are available from the owner in relation to the fuel being used, then they should be used. The Table indicates oil to have a heating value from 168 000 to 184 000 Btu per Canadian gallon, depending on the type, the 168 000 Btu per gallon being for No. 2 which is the common domestic fuel.

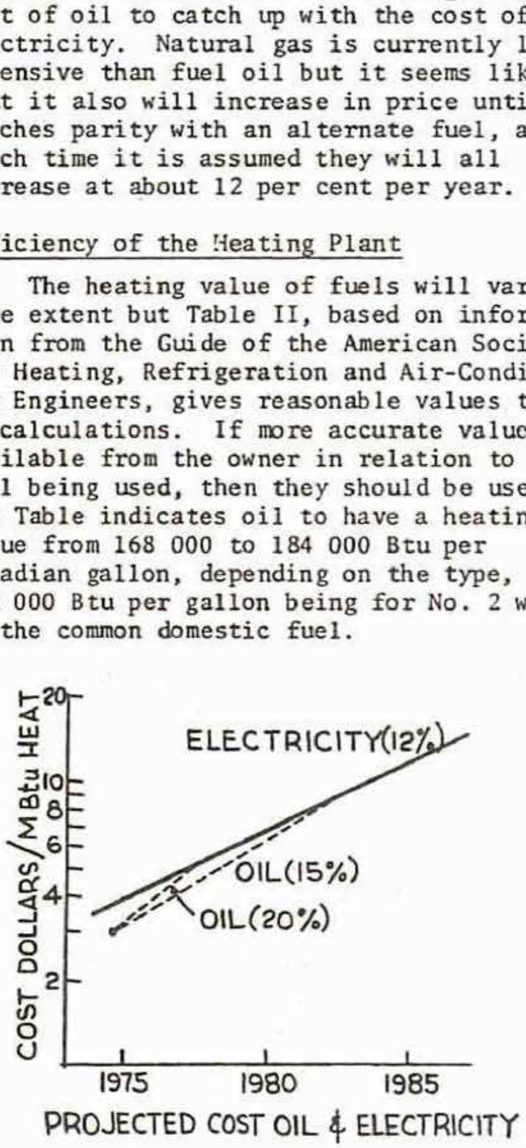


FIGURE 11

TABLE I

DEGREE DAYS BELOW 65°F - SELECTED LOCATIONS CANADA

Place	F Degree Days	Place	F Degree Days
Northwest Territories		Ontario	
Alert	23 488	Kenora	10 796
Frobisher Bay	17 876	Ottawa	8 693
Yukon		Timmins	11 400
Dawson	15 067	Toronto	6 827
Whitehorse	12 475	Welland	6 691
Newfoundland		Manitoba	
Grand Falls	9 352	Churchill	16 728
Labrador City	14 200	Morden	10 068
St. John's	8 991	Thompson	13 900
Prince Edward Island		Winnipeg	10 679
Charlottetown	8 486	Saskatchewan	
Nova Scotia		Maple Creek	9 500
Halifax	7 361	North Battleford	11 082
Sydney	8 049	Regina	10 806
Yarmouth	7 340	Saskatoon	10 856
New Brunswick		Alberta	
Edmundston	9 796	Calgary	9 703
Fredericton	8 671	Edmonton	10 268
Saint John	8 453	Fort Vermilion	13 113
Québec		Lethbridge	8 644
Drummondville	8 700	British Columbia	
Gaspé	9 800	Fort Nelson	12 777
Montréal	8 200	Kamloops	6 799
Québec	8 937	Prince George	9 755
Seven Islands	11 327	Vancouver	5 515

TABLE II

HEATING VALUE AND COST FACTORS OF FUELS

Fuel	Btu's	Cost per Btu
Anthracite coal	12 910 per lb.	0.000 077 4 x cost/lb.
Bituminous coal	9 150 per lb.	0.000 109 2 x cost/lb.
#2 Fuel oil (domestic)	168 000 per gal.	0.000 005 9 x cost/gal.
#5 Fuel oil (bunker C)	180 000 per gal.	0.000 005 5 x cost/gal.
#6 Fuel oil (bunker C)	184 000 per gal.	0.000 005 4 x cost/gal.
Natural gas	1 000 per cu. ft.	0.001 x cost/cu. ft.
LPG	91 690 per gal.	0.000 010 9 x cost/gal.
Steam	1 000 per lb.	0.001 x cost/lb.
Electricity	3 413 per kwh	0.000 293 x cost/kwh

Apart from the heat content of the fuel it is necessary to consider the efficiency of utilization. Heating systems, even when well designed and maintained, are not 100 per cent efficient. The rated efficiencies will probably seldom be much higher than 80 per cent and they relate to continuous operation at full capacity. The actual over-all seasonal efficiency may drop to three-quarters or less of the rated capacity. Typical efficiency ranges of heating systems are included in Table III based on figures from the ASHRAE Guide. The low values probably best represent seasonal efficiency.

Quantity of Oil Consumed

A formula to determine the quantity of oil consumed per year is:

$$Q = \frac{D \times 24}{R \times H \times E}$$

where:

- Q = oil consumption, gallons per square foot
- D = degree days below 65°F per year
- 24 = conversion factor (For residential and some other buildings this may be given a lower value)
- R = total resistance of the roof to heat flow
- H = heat content of the oil Btu per gallon
- E = seasonal heating efficiency

TABLE III

EFFICIENCES OF HEATING SYSTEMS

Hand-fired Anthracite furnace	60-75%
Hand-fired bituminous	50-65
Stoker-fired coal	60-75
Oil- or gas-fired	70-80
Gas designed unit	75-80
Gas converted unit	60-80
Oil designed unit	65-80
Oil converted unit	60-80
Direct electric	Close to 100%

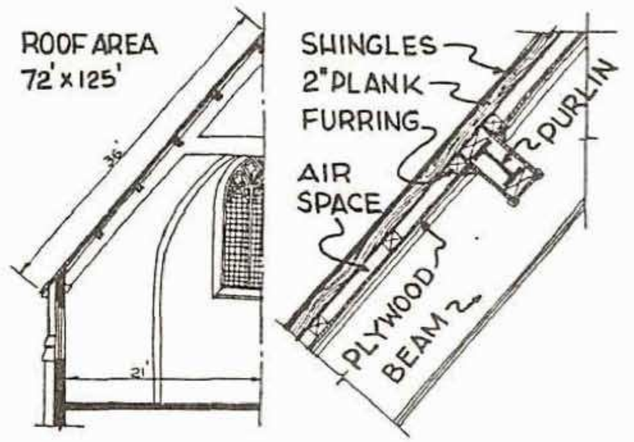


FIGURE 12

$$\text{OIL CONSUMPTION} = \frac{\text{DEGREE DAYS} \times 24}{\text{RESISTANCE} \times \text{FUEL HEAT CONTENT} \times \text{EFFICIENCY}} \text{ GAL/YEAR/SQ. FT.}$$

An Example of Thermal Upgrading

It may be useful to consider an actual example. A church building has an asphalt shingle roof, twenty years old and needing replacement (Fig. 12). The construction, from exterior to interior, consists of asphalt shingles, 2-in. thick tongue-and-groove wood plank, 2 in. by 3 in. furring strips creating an air space, and 3/8 in. thick fir plywood interior finish. The roof area is 72 by 125 ft = 9000 sq. ft. The congregation would like to know if they should insulate at the same time as the roof is replaced.

The first step is to determine the resistance to heat flow for the existing roof, which can be designated as R_E. Resistance values for air films, air spaces and typical roofing materials can be obtained from Tables IV, V and VI.

Outside air film (7½ mph wind)	0.25
Asphalt shingles	0.44
2-in. wood plank	2.50
Air space	0.90
Plywood	0.47
Inside air film	0.62
TOTAL R_E	5.18

This is not a very thermally efficient roof considering that an inch of glass fibre insulation has an R value of about 3 and extruded polystyrene an R of about 5.

The quantity of fuel consumed in relation to Ottawa, where the building is located, can now be estimated by using the formula described earlier:

$$Q = \frac{D \times 24}{R \times H \times E} \text{ gal/year/sq ft}$$

$$= \frac{8690 \times 24}{168000 \times 0.6} \times \frac{1}{R_E}$$

$$= \frac{2.069}{R_E} = \frac{2.069}{5.18} = 0.399 \text{ gal/year/sq ft}$$

TABLE IV

AIR FILM RESISTANCE AT SURFACES FOR ROOFS

Surface	Conditions	Flat	45° Slope
Outside at roofing surface	15 mph wind for winter	0.17	0.17
	7½ mph wind for summer	0.25	0.25
Inside at ceiling surface	Heat flow up-winter	0.61	0.62
	Heat flow down-summer	0.92	0.76

Note: Values are for nonreflective building materials with surface emissivity of 0.90

TABLE V

RESISTANCE OF AIR SPACES

Roof	Conditions	Resistance	
		¾ in. space	4 in. space
Flat	Heat flow up-winter	0.87	0.94
	Heat flow down-summer	0.84	0.99
45° Slope	Heat flow up-winter	0.94	0.96
	Heat flow down-summer	0.84	0.90

Note: 1. Assumes both surfaces of the air space to be non-reflective building materials.

2. The air spaces must be dead air spaces, that is with no venting to inside or outside, and not used as a plenum.

TABLE VI

THERMAL RESISTANCE OF BUILDING MATERIALS

Material	R
Steel deck	Negligible
4-in. concrete deck	0.32
Wood plank (per inch)	1.25
Gypsum concrete (per inch)	0.60
Insulating concrete zonolite or perlite at 30#/cu ft (per inch)	1.41
Plywood (per 1/8 inch)	0.16
Gypsum board (per half-inch)	0.45
Gypsum plaster (3/4 inch)	0.47
Vapour barriers	Negligible
Wood fibreboard (per inch)	2.78
Perlite board (per inch)	2.78
Glass fibre (per inch)	3.00
Rock wool (per inch)	3.00
Foam glass (per inch)	2.63
Urethane (per inch)	6.25
Polystyrene bead board (per inch)	3.50
Polystyrene extruded (per inch)	5.00
Built-up roofing (per ply)	0.08
Asphalt shingles	0.44
Wood shingles	0.94

This has been expressed in relation to R because everything else will remain the same in consideration of the thermal upgrading.

The most logical way of upgrading this roof would be to leave the existing system as is, place 2-in. by 2-in. or 2-in. by 4-in. wood members on the roof nailed through to the wood deck; fill the spaces between the wood framing with insulation and apply plywood sheathing and new roofing of shingles or other material as desired (Fig. 13). To isolate the new from the old and to provide a good air vapour barrier it would be good practice to apply heavy polyethylene over the old shingles before the new system is applied. 2 by 2's will allow 1½ in. of insulation; 2 by 4's will allow 3½ in. The new total resistance will depend on the type of insulation; the cost effectiveness will depend on the cost per resistance unit for the insulation. Typical R values and costs per resistance unit are indicated for several types of insulation in Table VII.

The easiest material to use would be glass fibre which can be fitted between the wood members at 2 ft on centres to take the plywood sheathing. It also is the cheapest per unit of resistance. Insulation in board form would have to be cut to fit between the wood members with additional labour and some wastage. If friction-fit glass fibre is used the R contributed by the added materials can be computed as well as the fuel consumption for the upgraded roof.

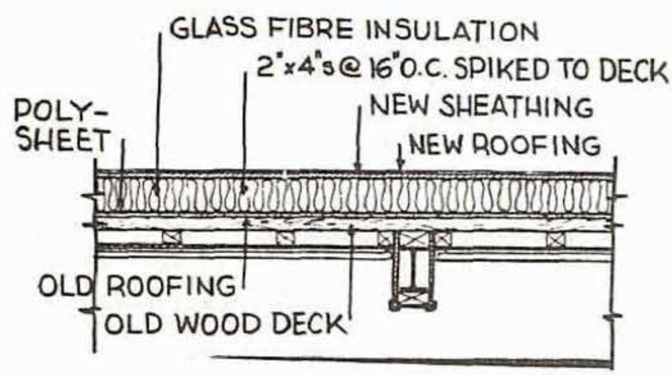


FIGURE 13

1½ in. insulation	4.50
Plywood sheathing	0.47
New shingles	0.44
Added R	5.41
R_E	5.18
Total R_N	10.59
Fuel $\frac{2.069}{R_N}$	0.195 gal/year/sq ft

3½ in. insulation	10.50
Plywood sheathing	0.47
New shingles	0.44
Added R	11.41
R_E	5.18
Total R_N	16.59
Fuel $\frac{2.069}{R_N}$	0.125 gal/year/sq ft

TABLE VII

COST OF INSULATION

Insulation Type	R per inch	Cost/sq ft per R unit	Total R 1½ in.	Total R 3½ in.	Cost $/sq ft 1½ in.	Cost $/sq ft 3½ in.
Bead polystyrene	3.5	2.3	5.25	12.25	0.12	0.28
Extruded polystyrene	5	5.2	7.50	17.50	0.39	0.91
Urethane	5 *	9	7.50	17.50	0.68	1.58
Glass fibre	3.0	1.3	4.50	10.50	0.06	0.14

* May be rated higher; this figure is for non-foil-covered on a long-term basis

Fuel savings can now be calculated for each case by subtracting the fuel consumed with the upgraded roofs from the fuel consumed with the existing roof. This amounts to 0.399 - 0.195 = 0.204 gallon per year per square foot for 1½-in. insulation and 0.399 - 0.125 = 0.274 gallon per year per square foot for 3½-in. insulation. Fuel savings for the roof of 9000 sq ft would be 1836 gallons and 2466 gallons, and with fuel at, say, 60¢ per gallon, this represents \$1102 for 1½-in. insulation and \$1480 per year for 3½-in insulation. For ten years the savings are \$11 020 and \$14 800. This of course, does not include any analysis of the cost of doing the work, the cost of borrowing money, the rise in cost of fuel, or whether and how long it would take to pay off the investment from the savings. It is possible to take all these things into account, but some additional knowledge is required.

Optimal Thermal Resistance

Another approach to thermal upgrading is to determine the optimum amount of insulation. This is somewhat more difficult. The optimum amount of insulation is the amount at which the savings resulting from adding any more insulation just equals the cost of adding the insulation (Fig. 14). Assuming insulation can be added without structural changes, the cost of insulation increases directly as the thickness is increased. Also the amount of heat supplied (and therefore the cost) decreases as insulation is added. The total annual cost of heat loss from the building through the roof is made up of the amortized cost of the insulation plus the cost of the fuel. This reduces to a minimum and then increases again as insulation is added. The low point on the total cost curve represents the optimum thickness. The formula for optimum resistance is:

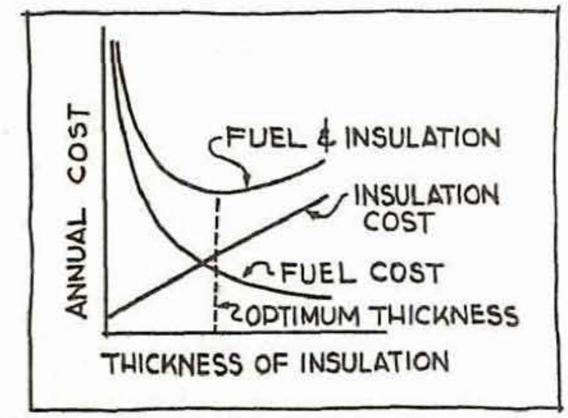


FIGURE 14

This introduces some factors that were not used in the previously described calculations. The cost of heat in cents per Btu can be obtained by multiplying the cost per Btu (found in Table II) by the cost per unit of the fuel. For No. 2 oil at 60¢ per gallon it is 0.0000059 x 60 = 0.00035 cents per Btu. The cost of insulation in this formula is the cost per square foot per resistance unit and is given in Table VII for some insulations.

P in the formula represents Present Worth Factor. When buying a piece of equipment or making an improvement in a building it is necessary to compare the initial cash investment with the saving it will produce during its projected lifetime. If the projected life is 10 years on a \$1000 investment, and the saving is \$100 per year it might be thought that the savings have paid for the investment. In actual fact at an interest rate of 10 per cent the \$100 per year would merely pay off the interest. What must be known is how much money can be

$$R(opt) = \sqrt{\frac{24 \times \text{Degree Days} \times \text{Cost of Heat (Cents/Btu)} \times P}{\text{Cost of insulation} \times \text{efficiency of fuel utilization}}}$$

justified in investing now in order to get the benefit of \$100 per year for a period of 10 years. At an interest rate of 10 per cent this is \$614.45, the present value of a series of 10 annual payments of \$100 required to pay the interest and repay the \$614.45 borrowed. The present worth factor is $\frac{614.45}{100}$ or 6.1445. The values of P can be determined in relation to the rate of interest and the length of term, and can be found in financial tables for present value of an annuity of 1 at compound interest.

The formula is

$$P = \frac{1 - (1 + i)^{-N}}{i}$$

where i = the rate of interest

and N = the length of term

But that's not the whole story. In the case of fuel savings, the cost of fuel is going up and as it rises the savings due to the use of extra insulation also increase. This can be taken into account by using an effective rate of interest that allows for the percentage increase in fuel costs in place of the nominal rate of interest. The formula is:

$$Y = \frac{i - x}{1 + x}$$

where i = the rate of interest

and x = the rate of increase in fuel cost

Example:

Consider an oil-heated building in Montreal with a steel deck to be insulated with glass fibre insulation or extruded polystyrene foam. Assume 10 per cent as the rate of interest and 12 per cent as the increase per year in the cost of fuel. The effective interest rate is -1.79 (Table VIII) and the value of P for a projected life of 30 years is about 40 (Table IX). The F degree days below 65°F for Montreal are 8200. The cost per square foot of glass fibre per resistance unit is 1.3; for extruded polystyrene it is 5.2. The efficiency is taken to be 75 per cent

$$R\ (\text{opt}) = \sqrt{\frac{24 \times 8200 \times 0.000354 \times 40}{1.3 \times 0.75}}$$

$$\doteq \sqrt{\frac{2787}{0.975}}$$

$$= 53.5 \text{ for glass fibre}$$

The thickness of insulation, therefore, will be $\frac{53.5}{3}$ = 17.8 in. Obviously this thickness cannot be placed on top of the

TABLE VIII

EFFECTIVE INTEREST RATE

Increase in cost of fuel % per year	INTEREST RATE %										
	10	11	12	13	14	15	16	17	18	19	20
15	-4.35	-3.48	-2.61	-1.74	-0.87	0	0.87	1.74	2.61	3.48	4.35
14	-3.51	-2.63	-1.75	-0.88	0	0.88	1.75	2.63	3.51	4.39	5.26
13	-2.65	-1.77	-0.88	0	0.88	1.77	2.65	3.54	4.42	5.31	6.19
12	-1.79	-0.89	0	0.89	1.79	2.68	3.57	4.46	5.36	6.25	7.14
11	-0.90	0	0.90	1.80	2.70	3.60	4.50	5.41	6.31	7.21	8.11
10	0	0.91	1.82	2.73	3.64	4.55	5.45	6.36	7.27	8.18	9.09

TABLE IX

PRESENT WORTH FACTORS

Year	EFFECTIVE INTEREST										
	-4%	-3%	-2%	-1%	0%	1%	2%	3%	4%	5%	10%
5	5.7	5.5	5.3	5.2	5.0	4.9	4.7	4.6	4.5	4.3	3.8
10	12.6	11.9	11.2	10.6	10.0	9.5	9.0	8.5	8.1	7.7	6.1
15	21.1	19.3	17.7	16.3	15.0	13.9	12.8	11.9	11.1	10.4	7.6
20	31.6	28.0	24.9	22.2	20.0	18.0	16.4	14.9	13.6	12.5	8.5
25	44.4	38.1	32.9	28.6	25.0	22.0	19.5	17.4	15.6	14.1	9.1
30	60.1	49.8	41.7	35.2	30.0	25.8	22.4	19.6	17.3	15.4	9.4

TABLE X

PAYBACK PERIOD OF THERMAL UPGRADING INVESTMENT IN YEARS

Ratio $\left(\frac{\text{Annual Savings}}{\text{Investment}}\right)$	EFFECTIVE INTEREST								
	-3%	-2%	-1%	0%	1%	2%	3%	4%	10%
0.01	45.5	54.5	69.0	100.0					
0.02	30.1	34.3	40.0	50.0	69.7				
0.03	22.8	25.3	28.6	33.3	40.7	55.5			
0.04	18.4	20.1	22.2	25.0	28.9	35.0	46.9		
0.05	15.4	16.7	18.1	20.0	22.4	25.8	31.0	41.0	
0.06	13.3	14.2	15.3	16.7	18.3	20.5	23.4	28.0	
0.07	11.7	12.4	13.3	14.3	15.5	17.0	18.9	21.6	
0.08	10.5	11.0	11.7	12.5	13.4	14.5	15.9	17.7	
0.09	9.4	9.9	10.5	11.1	11.8	12.7	13.7	15.0	
0.10	8.6	9.0	9.5	10.0	10.6	11.3	12.1	13.0	
0.11	7.9	8.3	8.7	9.1	9.6	10.1	10.8	11.5	25.2
0.12	7.3	7.6	8.0	8.3	8.7	9.2	9.7	10.3	18.8
0.13	6.8	7.1	7.4	7.7	8.0	8.4	8.9	9.4	15.4
0.14	6.4	6.6	6.9	7.1	7.4	7.8	8.2	8.6	13.1
0.15	6.0	6.2	6.4	6.7	6.9	7.2	7.6	7.9	11.5
0.16	5.6	5.8	6.0	6.3	6.5	6.7	7.0	7.3	10.3
0.17	5.3	5.5	5.7	5.9	6.1	6.3	6.6	6.8	9.3
0.18	5.1	5.2	5.4	5.6	5.7	5.9	6.2	6.4	8.5
0.19	4.8	5.0	5.1	5.3	5.4	5.6	5.8	6.0	7.8
0.20	4.6	4.7	4.9	5.0	5.2	5.3	5.5	5.7	7.3
0.21	4.4	4.5	4.6	4.8	4.9	5.1	5.2	5.4	6.8
0.22	4.2	4.3	4.4	4.5	4.7	4.8	5.0	5.1	6.4
0.23	4.0	4.1	4.2	4.3	4.5	4.6	4.7	4.9	6.0
0.24	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	5.7
0.25	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	5.4

steel deck in the conventional roofing system.

If polystyrene is to be used, the only item in the formula that changes is the cost per square foot per unit of resistance. Thus:

$$R\ (\text{opt}) = \sqrt{\frac{2787}{5.2 \times .75}} = 26.73$$

The thickness of insulation will be $\frac{26.73}{5} = 5.34$ inches. It might be possible to place this amount of insulation in a conventional roof, but it should be in one thickness as multiple layers do not readily adhere one to the other.

There is also a formula to determine the payback period for any thermal upgrading investment related to the saving of heat energy. The annual saving of heat energy converted to dollar savings, the effective interest, and the cost of construction improvement are required in this formula. The formula is:

$$N = \frac{\text{Ln}\ (\frac{R}{R - Y})}{\text{Ln}\ (1 + Y)}$$

where N = the payback period

R = the ratio of savings expected to investment required

Y = the effective interest rate and,

Ln = the natural logarithm

Table X gives the payback periods worked out from this formula related to effective interest and the annual savings to investment ratio.

The church example can now be further analyzed, assuming the cost of installation for the roofing to be $10,000; the cost of the shingles, which have to be replaced anyway, can be neglected. The ratio of savings per year to the total investment for the 3½ in. of insulation would be $\frac{1480}{10\ 000} = 0.15$. Using a 10 per cent interest on money and a 12 per cent increase in fuel cost the effective interest is -1.79. The payback period would be about 6.3 years (Table X).

If extruded polystyrene were used, the extra cost for the insulation would be around $7,000 making the total cost closer to $17 000. The total resistance would be 23.59 producing a savings of 2799 gallons of oil or $1 679 annually. Ratio of annual savings to total cost is 0.098. At the same effective interest it would take 9.3 years payback period (Table X).

Effects of Increased Insulation

The optimum thicknesses of insulation for roofs are substantially higher than the thicknesses currently being used. One concern arising from this is whether this will result in life shortening effects on the roofing. When the insulation is installed directly below the roofing membrane which is usual for conventional roofing systems, it has been suggested that there will be accelerated weathering, impact resistance and lateral stability will be reduced, and the splitting hazard will be increased. All but the impact resistance are related to the greater temperature range to which the membrane will be subjected in comparison with a heavily insulated roof.

The main difference in temperature range will result from radiation effects both day and night, with a slightly warmer temperature for the more heavily insulated roof on a sunny day, and a slightly cooler temperature on a clear cool night. The rate of chemical degradation of materials accelerates as temperatures rise, so any increase in temperature over the normal hot summer temperatures can increase the degradation of materials, such as bitumens.

In addition to any increase in temperature ranges from increased insulation, the membrane is located further away from the structural deck. The importance of resistance within the insulation to lateral movement of the membrane certainly assumes added importance. It has also been suggested that there might be an increase in differential contraction of the membrane caused by some areas being snow covered and others bare. In actual fact, increased insulation reduces this effect.

Calculations by the Division of Building Research, by the National Bureau of Standards in Washington, and by several individual consultants in the U.S.A. all show that the fears about damage to the roofing membranes from increased thicknesses of insulation are not justified. The increase in membrane temperature for heavily insulated roofs as compared with lightly insulated roofs is marginal, and no one suggests omitting all insulation to increase membrane life. The NBS report indicated that the difference between the high membrane temperature in summer for a black surface built-up roof system with no insulation and one having one half-inch of insulation, is greater than the difference between a system going from ½ in. to 5 in. of insulation. A ten-fold increase from 1 in. probably will not produce a membrane temperature increase in excess of 5 F deg. It should also be noted that the colour of the roof has a significantly greater effect on the membrane temperature. For roofs of equal insulation thickness it has been calculated and substantiated by measurement that the temperature difference between a black and white surface can be close to 30°F (17°C). On the basis of temperature, therefore, it can be said that insulation thickness is a minor factor in promoting chemical degradation of built-up roofing compared with surface colour.

Resistance to membrane movement in a membrane over insulation system is transferred to the structural deck through the insulation. The fastening of the membrane to the insulation, the shear resistance of the insulation, and the fastening of the insulation to the deck are all involved in this transfer. Thinner insulation may be achieved in a single thickness, but to obtain greater thicknesses multiple layers may have to be used. In order to obtain adequate resistance to movement it may be necessary to use some form of mechanical fastening for the insulation. The movement of the membrane can be reduced by applying white gravel or other reflective coatings to the membrane to reduce surface temperatures. The impact resistance of a roof is related to the compressive shortening under loading which will be greater as the insulation thickness increases. Presumably, this might increase the vulnerability of the roof membrane to damage from falling objects during construction operations, roof traffic and in some geographical areas to hailstones.

Methods of Thermal Upgrading

How does one achieve the thermal upgrading necessary to meet the energy crisis? For the purposes of this discussion all roof systems are divided into three types: the vented loft or attic, the compact double membrane low slope roof, and the protected membrane low slope roof.

The vented loft or attic system was common for commercial buildings in Canada some 50 years ago and is still common in parts of Europe (Fig. 15). The system consists of a secondary roof deck usually of wood, above the main structural deck to carry the roofing membrane. When insulation was used with this system, it was placed on top of the main structural deck; when it was realized that air vapour control was necessary, a membrane was placed on the deck below the insulation. This is a very good system. Tolerance to air leakage is good because the air vapour barrier retards the moisture flow from inside the building. Small amounts that penetrate through any imperfections to the attic or loft space have a large volume of air to mix with and then can pass harmlessly to the outside if ventilation is provided. Membrane problems are likely to be fewer, because the roofing is attached directly to the secondary deck. The roofing membrane is subjected to a wide range of temperatures but not as much as when over insulation, and the deck and membrane go through approximately the same temperature range, thus reducing the likelihood of differential movement and splitting.

The wooden structure and secondary deck is no longer looked on with favour by building codes, although the potential fire hazard

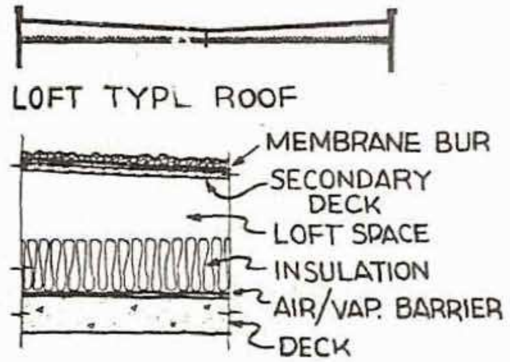


FIGURE 15

would not seem to be high. The upgrading of such buildings from an energy conservation standpoint is relatively easy. For existing buildings it simply means adding additional thicknesses of the cheapest insulation available that meets any other requirements necessary such as fire and rot resistance. For new buildings allowance can be made for the optimal thickness without worrying about adhesion and shear resistance of the insulation. The principle of the system is so sound that it is hoped someone can design a superstructure and secondary deck that meet the fire code requirements. Most residential roofs in Canada have been and still are basically this type of roof, and they have proved eminently successful when a good air vapour barrier at the ceiling and adequate attic ventilation have been provided.

The compact double membrane low slope roof, made less effective by trying to build it flat, is the usual system for most public, commercial and industrial buildings today (Fig. 16). It has an air vapour barrier membrane applied to the structural deck, partially or fully adhered or otherwise fastened. Insulation is adhered over this membrane and the principal roofing membrane is applied over the top of the insulation, usually intended to be fully adhered. The system can only be successful if no moisture or air is trapped between the two membranes, none gets in during service, and adequate shear resistance is provided in the system so that resistance to membrane movement can be provided from the deck through the insulation to the membrane. It is a credit to roofing contractors that so many successful roofs of even the "flat" version of this type have been built. Ways have been suggested by DBR to give this system a much greater chance of success by draining and venting. How can such systems be thermally upgraded?

As insulation is added between the air vapour barrier at the deck and the principal membrane, the distance between the membrane and the deck becomes greater, and adhesion between the layers to achieve shear transfer becomes more important. There are more pieces of material, more joints, and more air and moisture that can be trapped. One might think of providing all the insulation required in one thickness but the warping potential for the board becomes greater with the greater thickness, and little is known of what may happen at the joints of such systems. It appears that the limit for single thickness material should not be greater than 3 to 4 in. It may be possible in some instances to add insulation to the inside, and one could consider a sprayed-on type that would wick moisture to the surface and breathe, such as asbestos or cellulose insulation (Fig. 17). The deck and air vapour barrier membrane would, of course, still need to be at a temperature below the dew point and, as a rule of thumb only, the inside insulation should not constitute more than one-third of the total thickness.

The third type of roof system, the protected membrane low slope roof (Fig. 18),

DOUBLE MEMBRANE FLAT ROOF

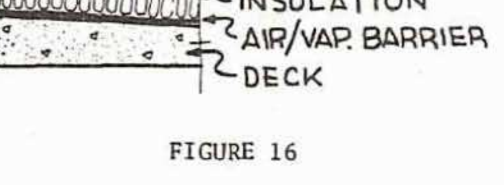


FIGURE 16

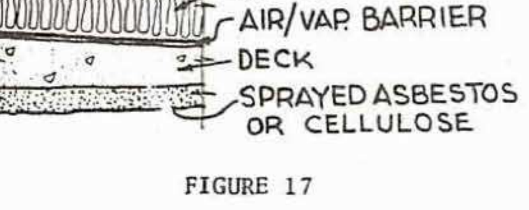


FIGURE 17

PROTECTED MEMBRANE ROOF

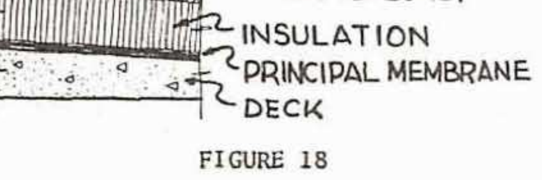


FIGURE 18

has been finding considerable favour in Canada and Europe and to a slight extent in the United States. In this system, the upper membrane is eliminated, the principle membrane and the air vapour barrier at the structural deck are combined and, the insulation is exposed or partially exposed to the weather.

This system or slight variations of it would seem to offer the best potential for thermal upgrading with board-type insulation on new and existing roofs. The only insulation material now suitable for use with this system is an extruded type of polystyrene, and it is deteriorated by ultraviolet light. The material is somewhat difficult to stick down because of its sensitivity to solvents and heat and if used on ponded roofs (whether due to lack of slope or controlled flow drains) it will float readily. If the membrane is a non-curving type such as rubberized asphalt, a separation sheet must be used to avoid bonding. These factors necessitate the use of protection and ballast; a heavy layer of gravel is usually used for this purpose. Increasing the thickness of the insulation increases the buoyant force, and consequently the weight of the gravel has to be increased. If, however, the roof is properly sloped and drained and the insulation adhered to the membrane, flotation will not be a problem, and a reasonable amount of gravel can be used to give protection against ultraviolet light and insurance against uplift during occasional unintentional flooding.

One possible approach to reduce the uplift force, and consequently the weight of gravel, is to consider putting part of the insulation below the membrane and part above (Fig. 19). The insulation above the membrane will still protect it from weather and traffic; the insulation below will make the membrane colder. Two factors need to be considered: drainage at the top of the membrane, which can take place only if the membrane temperature is above freezing, and condensation below the membrane, which will occur when the temperature of the membrane is below the dewpoint of the inside air. Again as a rule of thumb, if the thermal resistance of the insulation below the membrane is kept to about one-third of the total no difficulties would normally be anticipated. The insulation under the membrane should be a type that could be well and properly adhered.

This is the situation encountered in retrofitting or thermal upgrading an existing roof if the existing system is in good condition. In this case it would be reasonable to remove the existing gravel, apply a new flood coat and in some cases additional plies of felt, and then apply new extruded polystyrene insulation and ballast and protection as discussed. The physical and loading design limitations of the structure and the ease of application would have to be considered. An amount of insulation with a resistance value at least twice the existing should be used. The effect of this on flashings, cants and drains would have to be taken into account when considering ease of application.

Cost-effectiveness of Upgrading

Another question of considerable importance in relation to a building as a whole is where the greatest value for money spent can be achieved in relation to energy conservation. Indications are that for many types of buildings there are several areas that need to be considered before roofs unless the roofing has to be replaced anyway, in which case it is obviously an area for upgrading.

It has been shown that modifications to equipment and operating procedures, modification of windows and doors, and general air tightening can produce quite large savings in energy conservation for many existing buildings, and undoubtedly these will be the first areas to receive attention. Only after that will thermal upgrading of walls and roofs be considered.

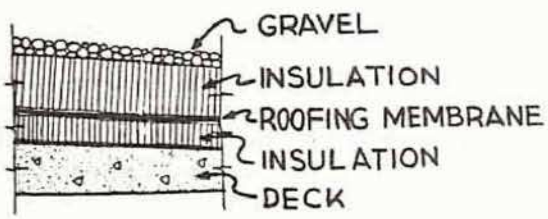


FIGURE 19