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BY

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REPARTITION ANGULAIRE DES MODES FONDAMENTAUX DE VIBRATION DANS UNE CHAMBRE D'ECHO

SOMMAIRE

L'auteur calcule la répartition angulaire des modes fondamentaux de vibration dans une enceinte rectangulaire de diverses dimensions. Il montre, en utilisant le résultat des mesures de réverbération dans l'enceinte, qu'une éprouvette dont la face est placée perpendiculairement à la longueur de l'enceinte reçoit des pressions acoustiques avec des angles d'incidence très divers. L'auteur propose différentes méthodes modifiées de mesure de l'absorption. Dans ces techniques, la face de l'éprouvette est généralement placée perpendiculairement à la largeur, de façon qu'elle reçoive les pressions acoustiques sous peu d'angles d'incidence.

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Angular Distribution of Lower Room Modes*

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The angular distribution of the lower-frequency modes in a rectangular room is calculated for various room dimensions. Applying the results to reverberation-room measurements, it is shown that a sample placed with its surface normal to a long dimension of the room experiences the most diffuse field. Various methods are proposed to modify the existing techniques of absorption measurement, where it usually happens that the sample is placed with its surface normal to the shortest axis, thus experiencing the least diffuse field.

INTRODUCTION

A REVERBERATION room is usually designed to be as small as possible, consistent with adequate simulation of a diffuse field at the lowest frequencies. Normally, it is required that there be a certain minimum number of room modes in the lowest frequency band. This, while fixing the number of discrete frequencies present in a given band, says nothing about how these modes are distributed with angle. Thus, it could happen that a specimen being tested in such an enclosure receives the bulk of its energy over a small range of angles. The ideal is a diffuse sound field, i.e., one in which all angles of incidence on the specimen are equally probable. At high frequencies, this situation is approached, and Bolt[1] shows that the calculated distribution more or less follows a $\sin\theta$ law, where θ is the angle measured from a perpendicular to a wall.

Other criteria exist for the design of an enclosure that might possibly be incompatible with one chosen for angular distribution. The room must have negligible redundancy of modes and the modes themselves should fall uniformly over the frequency band. Bolt[2] and Sepmeyer[3] have considered this problem for a rectangular room and have recommended that the room dimensions be related to one another in certain ways. Taking these ratios, we have investigated the angular distribution of modes relative to each of the room axes. We have performed the calculation for the lowest three third-octave bands. Beyond the third band, the calculations become intractable by hand, and in any case the number of modes then approaches that required for the condition of random distribution to be approximately satisfied.

The reverberation-room procedure for measuring absorption of materials has been established by long usage. Any modifications needed to make the sound field in a test room more diffuse should therefore not involve radical departure from well-tried practice. For instance, it is shown that a better distribution of angle would be obtained by mounting the test specimen perpendicular to a long dimension rather than on the floor (this assumes, as is generally the case, that the shortest room dimension is the height). To mount a specimen against the vertical wall with the necessary minimum distances from edges and corners might, in practice, be more difficult than simply placing the specimen in the floor. Thus, alternative methods are discussed in which the distribution of sound in the room may be altered so that the present positioning can be retained.

I. NUMBER AND ANGLE OF MODES

The number of modes in a rectangular room falling within a frequency band Δf is given by the formula

$$\Delta N = 4\pi f^2 \Delta f / c^3 (V + cA/8f + Lc^2/32\pi f^2), \qquad (1)$$

where c is the velocity of sound in air, f is the center frequency of the band, V is the volume of the room, A is the total wall area, and $L = 4(L_x + L_y + L_z)$; L_x, L_y, L_z being the room dimensions. Equation 1 may be re-

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[1] R. H. Bolt, J. Acoust. Soc. Am. 11, 74–79 (1940).
[2] R. H. Bolt, J. Acoust. Soc. Am. 19, 79–90 (1947).
[3] L. W. Sepmeyer, J. Acoust. Soc. Am. 37, 413–423 (1965).

written as

$$\Delta N=\frac{4\pi f^2\Delta f}{c^3}\left[V+\frac{c}{8f}\left(\frac{A}{V^{\frac{2}{3}}}\right)V^{\frac{2}{3}}+\frac{c^2}{32\pi f^2}\left(\frac{L}{V^{\frac{1}{3}}}\right)V^{\frac{1}{3}}\right]. \quad (2)$$

For a given volume, the smallest number of modes occurs when $A/V^{\frac{2}{3}}$ and $L/V^{\frac{1}{3}}$ have minimum values. If $L_y=pL_x$, $L_z=qL_x$, then the minima occur when $p=q=1$; thus, the worst shape is a perfect cube. If we require, after Sepmeyer, that there be a minimum of nine modes in the lowest third-octave band, then

$$Vf^3=2.55\times10^9 \text{ ft}^3 \text{ sec}^{-3}. \quad (3)$$

This differs slightly from the value 2.46×10^9 quoted by Sepmeyer. His value is presumed to be the more accurate one because of the approximate nature of Eq. 1 in expressing a discrete by a continuous function; hence it is used in our subsequent calculations.

The conditions for the existence of a mode is

$$f=(c/2L_x)(l^2+m^2/p^2+n^2/q^2)^{\frac{1}{2}},$$

when l, m, n are integers (including zero). For convenience, we define a dimensionless parameter μ given by

$$\mu=2fLx/c=(l^2+m^2/p^2+n^2/q^2)^{\frac{1}{2}}. \quad (4)$$

Then, if θ_i is an angle of incidence measured from axis ($i=x$, y, or z), we have $\cos\theta_i=\mu_i/\mu$, where the value μ_i is obtained from Eq. 3 by putting the integers for axes different from i equal to zero. Thus

$$\cos\theta_z=(n/q)(l^2+m^2/p^2+n^2/q^2)^{-\frac{1}{2}}.$$

II. CALCULATIONS AND RESULTS

When applying the rule for minimum V we have chosen $f=50$ Hz as the center frequency of the lowest third-octave band, but the results are easily adapted to any other limiting frequency merely by scaling down the room dimensions; for example, if the lowest band is to be centered on 125 Hz, the room dimensions would be divided by 125/50. The analysis proceeds for two Sepmeyer room ratios and the Bolt ratio for the first three one-third-octave bands. We merely fit maximum and minimum values for μ corresponding to the limits of the frequency band and find the sets of values of l, m, n that satisfy Eq. 3. Finally, Eq. 4 is used to calculate values for the angles.

Table I shows the minimum room dimensions, assuming nine modes to exist at the lowest $\frac{1}{3}$ oct centered on 50 Hz. It can be noted that by actual count the number of modes in this band ranges from 7 to 10 for the rooms considered, even though the volumes are the same.

TABLE I. Dimensions (in feet) for a room satisfying minimum conditions.

Dimension	Recommended room proportions (Sepmeyer I) 1:0.82:0.72	(Bolt) 1:0.79:0.63	(Sepmeyer II) 1:0.69:0.43
L_x	32.2	34.1	40.4
L_y	26.4	26.9	27.8
L_z	23.2	21.4	17.4

TABLE II. Deviations and proportional deviations for three room-dimension ratios.

Room proportions	Band No.	No. of modes	δ_z^2	δ_x^2	$p(\delta_z^2)$	$p(\delta_x^2)$
1:0.69:0.43	1	7	10.7	3.5	0.22	0.07
1:0.79:0.63	1	8	11.8	3.6	0.19	0.06
1:0.82:0.72	1	10	13.4	8.7	0.13	0.09
1:0.69:0.43	2	19	39.5	6.7	0.11	0.02
1:0.79:0.63	2	18	31.3	3.7	0.09	0.01
1:0.82:0.72	2	15	13.6	14.2	0.06	0.06
1:0.69:0.43	3	30	84.4	21.9	0.09	0.02
1:0.79:0.63	3	31	27.7	7.2	0.03	0.01
1:0.82:0.72	3	32	40.8	9.1	0.04	0.01

Table II contains a measure of how the actual angular distribution departs from the ideal one of $dN/d\theta=K\sin\theta$. The parameter shown is defined by

$$\delta^2=\sum_1^9 |N_{\text{calc}}-N_{\text{ideal}}|^2,$$

where the angular range 0°–90° is split into 10° zones. The parameter δ^2 represents a true deviation, but may only be used for comparison between bands when the number of modes is constant. The suffixes are used to denote the directions of the perpendicular. As an alternative, we can express deviations by $p(\delta^2)=\delta^2/N^2$, where N is the number of modes being considered. This is a more satisfactory form of presentation because it involves proportionalities, and this now permits us to compare the deviations in different bands. This fraction may also be defined by

$$p(\delta^2)=\sum_1^9 |p_{\text{calc}}-p_{\text{ideal}}|^2.$$

The calculated values are given in the last two columns of Tables II. Both these parameters differ from the one used by Sepmeyer[3] in which the ratio of N_{calc} to N_{ideal} is formed for each zone, squared, and then averaged. This parameter tends to overemphasize the contributions made by smaller zones, corresponding to small angles of incidence. From Table II, we see that δ^2 is usually smaller for a sample placed normal to the longest axis of the room (the x axis); in this position the sample experiences a more diffuse field than if it were perpendicular to the z axis. The evidence from $p(\delta^2)$ values confirms that of the δ^2 values. Values for conditions normal to the intermediate y axis fall between those shown in the Tables, but are not given. The Table also indicates that the Bolt ratio of 1:0.79:0.63 is probably the best one to use. In Fig. 1, which summarizes the distributions for the first three third-octave bands lumped together,

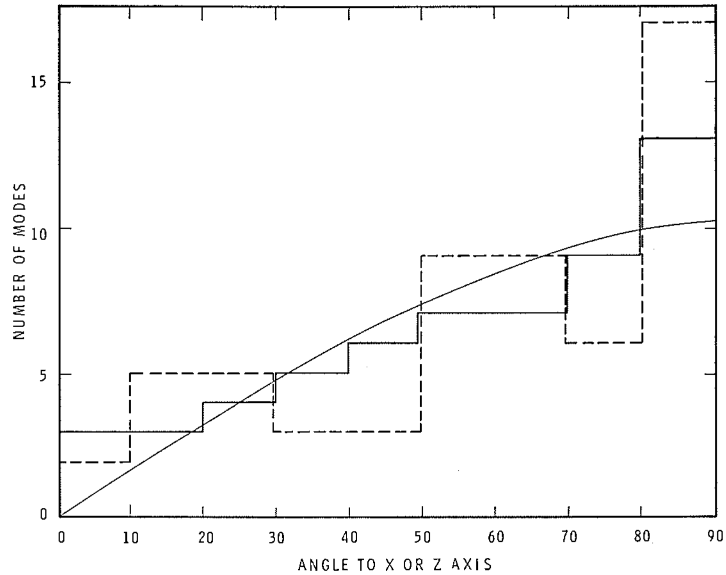

FIG. 1. Angular distribution of modes relative to x and z axes in first three frequency bands. ——: distribution perpendicular to x axis; – – –: distribution perpendicular to z axis.

the advantage of using the test sample perpendicular to the longest axis is apparent

III. MODIFICATIONS TO EXISTING PROCEDURES

If, as is usual, the shortest dimension of the room is the vertical, the conclusions of the previous analysis suggest that absorption test specimens should be mounted on an end wall. This is not always convenient, and we consider alternative measures that would improve the distribution—especially when specimens are placed on the floor, perpendicular to the short dimension. The most obvious method is to average the curve in some way so that energies in some angular ranges are diverted into others. This is not as easy as it may seem, for it is never possible to manipulate one range without modifying all the others.

One way to present a more uniformly diffuse field to a specimen would be to fragment the original specimen and arrange the new elements at different angles with respect to the sound field. One procedure, formerly used, is to split the total specimen into three widely separated areas mounted on three mutually perpendicular surfaces. This arrangement has the disadvantage that, for a given total area of material, the effect of edge diffraction is increased as compared to a single large specimen. Diffraction effects could be reduced by increasing the size of the individual areas, but increasing the total absorbing area results in a still greater departure from diffuse field conditions.

In this analysis, we evade the diffraction problem by considering essentially one absorbing area, and investigate the effect of tilting one portion slightly with respect

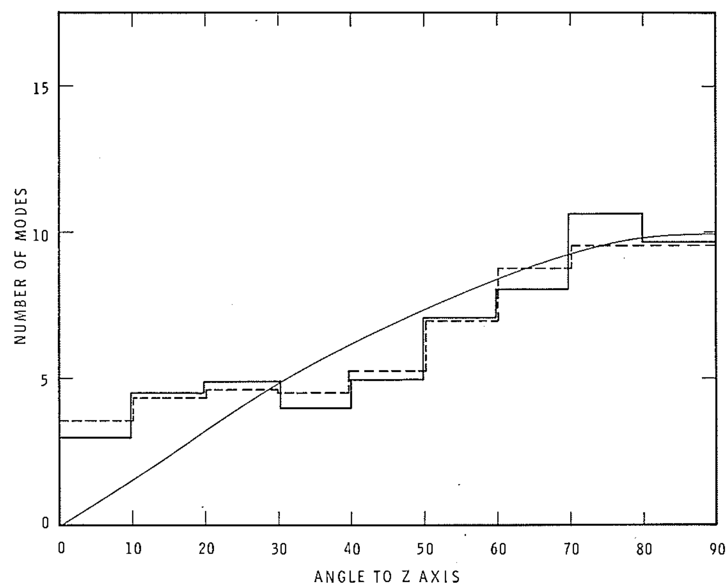

FIG. 2. Angular distribution of modes in first three frequency bands for slightly tilted absorbers. ——: two absorbers $A\,(0°)+0.7A\,(10°)$; – – –: three absorbers $A\,(0°)+0.6A\,(10°)+0.2A\,(20°)$.

to the remainder. Assuming an initial position in the plane of the floor, we first tilt one section by 10°. Energy hitting the original area in the interval 10°–20° will then strike the tilted section in the range 0°–30° (the range for a given contribution, depending on orientation relative to the direction of tilt). By varying the ratio of the two areas we can find, by statistical analysis, the ratio that gives the best approximation to the ideal distribution. This has been done for the room proportion 1.0:0.79:0.63. We consider the first three third-octaves, and for convenience in calculation use as a criterion δ_0^2 calculated for the whole octave band.

The optimum ratio of tilted to untilted portions is found to be 0.7, and this reduces δ_0^2 from 95 to 20. For the individual third-octave bands δ_z^2 becomes 3.1, 2.5, and 5.2, respectively, as compared with 11.8, 31.3, and 27.7 for the untilted case. Table II shows that these compare favorably also with the values of δ_x^2, corresponding to the end wall position for the specimen.

Obviously, the same procedure might be extended further. By adding a third segment, tilted at 20°, δ_{oct}^2 may be reduced to 16. The optimum ratio of areas is then found to be 1.0:0.6:0.2 for sections tilted at angles of 0°, 10° and 20°, respectively. For the first three third-octaves, δ_z^2 now becomes 2.9, 1.2, and 4.8. Clearly, the first tilting operation is the most important, and further elaboration beyond the second is unwarranted.

IV. DISCUSSION

The calculations of Sec. III were based on the modal distribution for a bare rectangular room. It is of interest to consider, qualitatively at least, the behavior of actual rooms and the effects of various special techniques that have been devised for reverberation rooms.

We can expect that the actual shape of a room, flatness of its surfaces, etc., will depart from the nominal condition, even when great care is taken in construction. As a result, modes beyond the first few will probably not conform precisely to the predicted distribution, either in angle or in frequency. In the lowest bands, the alterations to the angular distribution may be small enough merely to shift individual modes slightly, and the theorectical analysis might apply reasonably well.

In absorption measurements, another distorting effect is introduced by the presence of the absorptive specimen itself, which will alter the relative importance of modes, depending on their degree of involvement with the specimens. A similar effect would result from the presence of absorptive patches introduced as part of the "empty" room, as has sometimes been done. This, in effect, could be another method of altering and perhaps improving the angular distribution. It would have the undesirable effect, however, of increasing the total room absorption.

A procedure in current vogue is to use a series of reflecting panels, tilted with respect to each other and to the room boundaries. The effect of such an arrangement will be similar to that of tilting portions of the specimen: part of the energy incident on a boundary from a given direction will be diverted to a different direction and, thus, the angular distribution at the specimen will be altered. A few panels strategically located and oriented might suffice. Alternatively, a large number of randomly arranged panels may be expected to provide an approach to the desired uniform distribution. The rotating vane frequently used is a special case of a diffusing panel, which over a period of time is equivalent to a series of different tilting arrangements.