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Natural Convection Heat Transfer From Arrays of Isothermal Triangular Fins in Air

Measurements of the heat transfer to air by natural convection from arrays of isothermal triangular fins on a vertical base plate are reported for several array geometries, for a large range of Rayleigh number, and for two orientations (vertical fins and horizontal fins). The data are believed to be the first available for this important geometry. A single equation is provided that correlates the measured Nusselt numbers for the vertical orientation with an rms error of 4.8 percent. The horizontal fin orientation was shown to have inferior heat transfer performance.

Introduction

Heat transfer from fins is a topic of continuing interest in heat transfer. Although in practice the rectangle has been the most common fin shape, the triangular fin (Fig. 1) is known to have a higher rate of heat transfer per unit of material volume; indeed, by this measure, its performance approaches quite closely that of the optimal shape (Eckert and Drake, 1972). The common practice of using rectangular fins would seem to stem from the difficulty of manufacturing other shapes, particularly when the fins are fabricated from sheet metal. When the fins are extruded, however, they are more often triangular (or trapezoidal). This is often the case even when rectangular fins were intended, because of limitations in the extrusion process.

To design a fin properly one needs to know the convective coefficient to the surrounding fluid. Interestingly, it appears that no measurements have been reported in the literature of the natural convective heat transfer coefficients from triangular fins mounted in a vertical surface, which is the most common orientation. Fins can be mounted on a vertical surface with their midplanes vertical, as illustrated in Fig. 1(A), or horizontal, as illustrated in Fig. 1(B). The horizontal orientation seems to have gained recent popularity in electronic cooling applications.

Many workers (Elenbaas, 1942; Starner and McManus, 1963; Welling and Wooldridge, 1965; Schult, 1966; Aihara, 1970a; Chaddock, 1970) have measured heat transfer coefficients for vertical *rectangular* fins, and correlated their data into recommended equations for calculating the heat transfer in that situation. It seems likely that designers of triangular fins have used these equations to calculate heat transfer from the triangular fins assuming, for example, that the triangular fin will convect the same as a rectangular fin of the same perimeter facing a passage of the same cross-sectional area. Of course there are no data on which to judge the accuracy of such a practice, and the practice would certainly seem to be questionable for cases in which the fins interfere thermally; for a given fin spacing and height, a triangular fin would have less interference near its tips and more interference near its base. Also, heat transfer from the fin ends is going to be different in the two cases. Even if the rectangular fins and triangular fins did dissipate heat at the same rate, there would still be a problem: For vertical rectangular fins there are substantial differences between the recommended equations of different workers, and for *horizontal* fins there seem to be no data available at all.

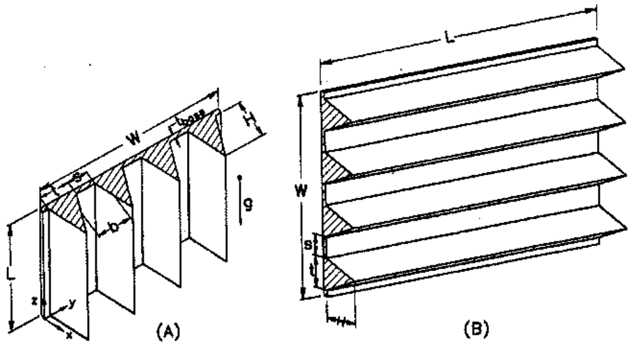

Fig. 1 Vertical fins on a vertical surface (A), and horizontal fins on a vertical surface (B)

Thus there would appear to be a real need for experimental measurements on natural convection heat transfer from both vertical and horizontal fins mounted in a vertical surface, and it is the purpose of this paper to address that need for triangular fins. In studying the literature reporting measurements on vertical rectangular fins, we have noted that the discrepancy between workers is most pronounced at low Rayleigh numbers. The reason is that the corrections for the radiant losses and for the back losses (i.e., the heat losses off the back or non-finned side of the base plate) become relatively large at low Rayleigh number, and there is usually a large uncertainty in both of these corrections. Thus in the present measurements, every effort was made to minimize or eliminate these losses.

As will be clear from the dimensional analysis given in the next section, the dimensionless heat transfer, the Nusselt number, is a function of several dimensionless groups. These include the Rayleigh number, the Prandtl number, and other groups defining the fin geometry. Although it was clearly not possible experimentally to cover all combinations of values of all groups, a range fairly representative of common practice was covered. The Rayleigh number ranged over six decades (typically from 1 to 10^6) for any one fin geometry. The Prandtl number was constant at about 0.71. For the vertical fins, a single correlation equation, closely fitting the convective component of the Nusselt number's dependence on the Rayleigh, was developed, the equation being common to all fin geometries tested. Data for horizontal fins are also presented.

Dimensional Analysis

If Q_{CONV} is the convective heat transfer to the ambient fluid

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from the surface area A_s, where A_s is the entire surface area of the array except the "back" (see Fig. 2), the average heat transfer coefficient is embodied in the Nusselt number as follows:

$$\mathrm{Nu} = \frac{\bar{h}\, b}{k} = \frac{Q_{\mathrm{CONV}}\, b}{A_s \Delta T k} \quad (1)$$

The parameters on which Nu depends are obtained by a dimensional analysis. By retaining only the terms in the governing equations that are important for natural convection (see Raithby and Hollands, 1985), and neglecting property value variations, the nondimensional equations of continuity, momentum, and energy, for the vertical fin array in Fig. 1(A), can be expressed by

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \quad (2)$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = -\left(\frac{L}{b}\right)^2 \frac{\partial p^*}{\partial x^*} + \sqrt{\mathrm{Pr}\frac{(1+\mathrm{Pr})}{\mathrm{Ra}}}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{b^2}{L^2}\frac{\partial^2 u^*}{\partial z^{*2}}\right) \quad (3)$$

$$u^* \frac{\partial w^*}{\partial x^*} + v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{\partial p^*}{\partial z^*} + \sqrt{\mathrm{Pr}\frac{(1+\mathrm{Pr})}{\mathrm{Ra}}}\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}} + \frac{b^2}{L^2}\frac{\partial^2 w^*}{\partial z^{*2}}\right) + (1+\mathrm{Pr})\theta \quad (4)$$

$$u^* \frac{\partial \theta}{\partial x^*} + v^* \frac{\partial \theta}{\partial y^*} + w^* \frac{\partial \theta}{\partial z^*} = \sqrt{\frac{(1+\mathrm{Pr})}{\mathrm{Ra\,Pr}}}\left(\frac{\partial^2 \theta}{\partial x^{*2}} + \frac{\partial^2 \theta}{\partial y^{*2}} + \frac{b^2}{L^2}\frac{\partial^2 \theta}{\partial z^{*2}}\right) \quad (5)$$

$$\mathrm{Ra} = \frac{g\beta\Delta T(b^4/L)}{\nu\alpha} \quad (6)$$

(The v^* momentum equation has not been written since it is exactly the same as Eq. (3) with the dependent variable u^* replaced by v^*). The coordinates are shown in Fig. 1(A), and the dimensionless variables are defined in the nomenclature. It should be noted that in deriving these equations the length and velocity scales used in the vertical direction are different from those in the horizontal direction. The Rayleigh number based on the length scale in Eq. (6) is often referred to as the Elenbaas Rayleigh number.

The boundary conditions must still be recorded, and examined for additional dimensionless groups. On the fin surface the boundary conditions are

$$u^* = v^* = w^* = 0 \quad \theta = 1 \quad (7)$$

When the fin surface is plotted in (x^*, y^*, z^*) coordinates, all fin arrays will be coincident if the ratios H/b, t/b, W/b and t_{base}/b are all identical; it is important to note that L/b does not appear in this list because, by the definition $z^* = z/L$, all fin surfaces lie in the range $0 \le z^* \le 1$.

The outer surface, far removed from the fin array, can be assumed to be a sphere, of radius R_o, centered at the origin. Hence for

$$\sqrt{x^{*2} + y^{*2} + z^{*2}} = R_o^* = R_o/b: \quad u^* = v^* = w^* = \theta = 0 \quad (8)$$

If R_o^* is not large, there will be hydrodynamic and thermal interference between the fin array and the outer surface (i.e., enclosure effects).

Equation (1) for Nusselt number can be rewritten as

$$\mathrm{Nu} = \int_{A_s^*} [\nabla^*\theta]_s \cdot \hat{n}\, dA_s^*; \quad \nabla^*\theta = \frac{\partial\theta}{\partial x^*}\hat{\imath} + \frac{\partial\theta}{\partial y^*}\hat{\jmath} + \frac{b}{L}\frac{\partial\theta}{\partial z^*}\hat{k} \quad (9)$$

where $[\nabla^*\theta]_s$ is the nondimensional gradient of θ evaluated on the fin surface, $\hat{n}$ is the unit surface normal, and A_s^* is the

Nomenclature

- A_s = surface area of fin array, excluding the "back" (see Fig. 2), m^2
- b = mean fin spacing, Fig. 1, mm
- C = thermal capacitance of array, J/K
- g = gravitational acceleration, m/s^2
- $\bar{h}$ = convective heat transfer coefficient = $Q_{\mathrm{CONV}}/A_s\Delta T$, W/mK
- $\hat{\imath}, \hat{\jmath}, \hat{k}$ = unit vectors in the x, y, and z directions
- H = fin height, Fig. 1, mm
- k = thermal conductivity of air at T_f, W/mK
- L = fin length, Fig. 1, mm
- Nu = Nusselt number for convection = $\bar{h}\, b/k$
- Nu_L = Nusselt number for convection = $\bar{h}\, L/k$
- $\mathrm{Nu}_{\mathrm{COND}}$ = Nusselt in the conduction limit ($\mathrm{Ra} \to 0$)
- $\mathrm{Nu}_{\mathrm{RAD}}$ = $Q_{\mathrm{RAD}}\, b/(A_s\, \Delta T\, k)$
- p = pressure, relative to hydrostatic, Pa
- p^* = $p/\rho w_o^2$
- Pr = Prandtl number = ν/α
- Q_{CONV} = total convective heat transfer from A_s, W
- Q_L = heat loss from fin array via lead wires, W
- Q_{RAD} = heat loss from A_s by radiation, W
- Q_{TOT} = defined in Eq. (13), W
- Ra = Rayleigh number = $g\beta_\infty\Delta T b^4/\nu\alpha L$
- Ra_L = Rayleigh number = $g\beta_\infty\Delta T L^3/\nu\alpha$
- s = spacing between fins at base plate, Fig. 1, m
- t = width of fin at its base, Fig. 1, m
- T_f = film temperature = $(T_s + T_\infty)/2$
- T_l = temperature of tank liner, K
- T_s = surface temperature of fin array, K
- T_∞ = ambient fluid temperature, K
- ΔT = $T_s - T_\infty$
- u^*, v^* = nondimensional velocities = u/v_o, u/v_o
- u, v, w = velocity components in x, y, z directions
- v_o = $\sqrt{g\beta\Delta TL/(1+\mathrm{Pr})}\ (b/L)$
- w^* = nondimensional velocity = w/w_o
- w_o = $\sqrt{g\beta\Delta TL/(1+\mathrm{Pr})}$
- W = width of base plate, Fig. 1, m
- x^*, y^*, z^* = nondimensional coordinates = x/b, y/b, z/L, respectively
- x, y, z = Cartesian coordinates, see Fig. 1, m
- α = thermal diffusivity at T_f, m^2/s
- β = thermal expansion coefficient at T_∞, 1/K
- ΔT = $T_s - T_\infty$, K
- ϵ = surface emissivity of fin array
- θ = $(T - T_\infty)(T_s - T_\infty)$
- ν = kinematic viscosity at T_f, m^2/s
- ρ = fluid density at T_f, kg/m^3
- σ = $5.670 \times 10^8\ W/m^2\, K^4$

nondimensional surface area. Note that the L/b ratio appears in Eq. (9), but it has not affect on Nu on surfaces where $\hat{n}\cdot\hat{k}$ is zero (i.e., vertical surfaces).

From the differential equation, their boundary conditions, and the equation for Nu, it is therefore concluded that Nu depends on the following nondimensional groups:

$$\mathrm{Nu}=f\left(\mathrm{Ra},\ \mathrm{Pr},\ \frac{L}{b},\ \frac{H}{b},\ \frac{t}{b},\ \frac{W}{b},\ \frac{t_{\mathrm{base}}}{b},\ \frac{R_o}{b}\right) \tag{10}$$

By introducing approximations, the dependence of Nu on some of these parameters can be eliminated. In the problems of interest, $R_o^*\rightarrow\infty$ (i.e., no enclosure effects). The changes in Nu due to changes of the base-plate thickness t_{base} can reasonably be ignored since the surface area of the edge of the base plate is small. Furthermore, when L/b is large, the dependence on L/b can usually be ignored for several reasons: z diffusion in Eqs. (3)-(5) can be ignored, the lateral pressure gradient in the u^* and v^* momentum equations can be shown to be negligible by an order-of-magnitude analysis, and the L/b that enters through Eq. (9) affects the Nusselt number only through the contribution of the top and bottom ends of the array to the heat transfer (and this area is small for large L/b). Hence, to good approximation for a vertical fin array with $R_o/b\rightarrow\infty$, large L/b, and small edge area of the base

$$\mathrm{Nu}=f\left(\mathrm{Ra},\ \mathrm{Pr},\ \frac{H}{b},\ \frac{t}{b},\ \frac{W}{b}\right) \tag{11}$$

Note that the effect of the L/b ratio is still present since it appears in the Rayleigh number, as defined by Eq. (6). That this is its *only* means of effect, for large L/b, was shown by Martin et al. (1991), for a different geometry. For an array with many fins, W/b will be large so it could also be dropped as a parameter; this is because the heat transfer from an entire array of N fins will approach N times the heat transfer from one of the interior fins, independent of W/b. In the present experiments there were as few as four fins in the array, so the W/b group is retained.

When Ra is sufficiently high for the boundary layers to be much smaller than the fin spacing, each vertical strip of the fin array will transfer the same heat as any other strip. Hence the Nusselt number over the vertical surface should become independent of the fin shape. If the heat transfer from the top and bottom ends of the array is also ignored (so that geometric parameters are not required to calculate the fraction of the surface area covered by the ends), the dependence on all length ratios in Eq. (11) disappears. Hence for the conditions on Eq. (11), plus the conditions that the end areas of the array are small compared to the vertical surface area, and that Ra is large,

$$\mathrm{Nu}=f(\mathrm{Ra},\ \mathrm{Pr}) \tag{12}$$

Experiment

Measurements were obtained for the three fin arrays shown in Fig. 2. The dimensions and aspect ratios are summarized in Table 1. The objective was to measure the convective heat transfer Q_{CONV}, from which Nu in Eq. (1) can be found. From the dimensional analysis, these Nusselt numbers should be applicable to all geometrically similar arrays, for the same Ra and Pr.

To measure the total heat loss from the fin array, a transient technique, developed for natural convection by Raithby et al. (1976) and refined by Chamberlain et al. (1985) and Hassani and Hollands (1989), was used. The fin array was hung in air from fine nylon lines, and it was heated prior to the measurements by impressing 240 volts via very fine copper leads across a heater embedded in the array. When the array reached a specified temperature above ambient, the heater was switched off and the transient cooling curve of the array (array temperature versus time) was measured. From these measurements, and a knowledge of the total heat capacitance of the array, a curve fitting procedure, described by Chamberlain (1983) and Hassani (1987), was used to find the total rate of heat loss, Q_{TOT}, at three values of ΔT during the transient decay. This heat loss consists of three components:

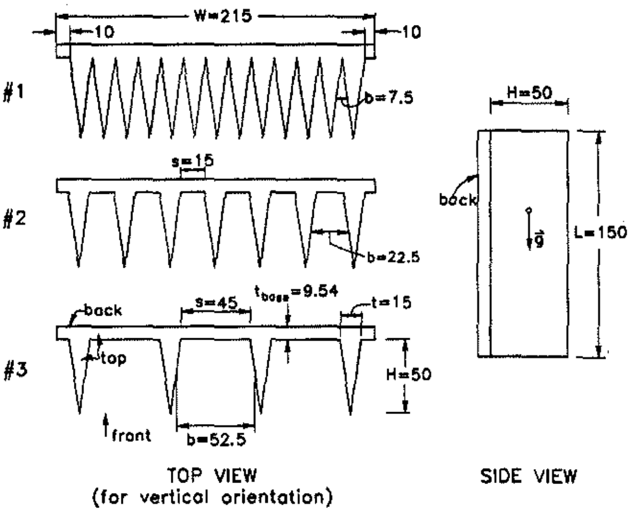

Fig. 2 Definition of the fin arrays used in the present study; all dimensions in mm

Table 1 Dimensions and aspect ratios of fin arrays (all dimensions in mm)

	L	H	W	s	b	L/b	H/b	W/b
Array #1	150	50	215	0	7.5	20	6.67	28.7
Array #2	150	50	215	15	22.5	6.67	2.22	9.56
Array #3	150	50	215	45	52.5	2.86	0.952	4.10
	t_{base} = 9.54, t = 15							

$$Q_{\mathrm{TOT}}=Q_{\mathrm{CONV}}+Q_{\mathrm{RAD}}+Q_L \tag{13}$$

where Q_{RAD} is the radiation loss, and Q_L is the loss via the leads to the array. From Eq. (13), Q_{CONV} can be found at the three ΔT values by subtracting Q_{RAD} and Q_L. The method used to find Q_{RAD} is described later; Q_L was found to be negligible.

For this method to be used, the body temperature must be uniform. This was ensured by constructing the base plate and fins from relatively good heat conductors, namely aluminum 2024-T35 and 6061-T6, respectively. The fins were bolted to the base plates, and good thermal contact was obtained by placing a strip of aluminum foil smeared with vacuum grease between the fin and base plate. Twenty 30 and 36 gage copper-constantan high precision thermocouples were imbedded throughout the model. These were used initially to check the isothermality of the model. These measurements showed that the maximum temperature variation within the array was 0.3 K, and this occurred at the largest ΔT. This high degree of isothermality was not surprising since the Biot number of the array was about 0.007. For the heat transfer measurements the thermocouples in the array were connected into a thermopile to measure the average temperature difference between the ambient air and the array.

To achieve the objective of the experiment, the heat loss from the back of the fin array had to be eliminated, as noted in the Introduction. In the present experiments, this heat loss was eliminated by creating a second identical fin array, and attaching it to the back of the first, so that the back surface was a symmetry surface. In the measurements, then, the heat transfer from the entire fin array was measured, and this was halved to get the heat transfer from the front part only.

Table 2 Measured values of C, $\mathfrak{F}$, and Nu_{COND} for the three fin arrays

Array Number (see Table 1)	Heat Capacitance, C (J/K)	$\mathfrak{F}$ (-)	Nu_{COND} (-)
1	5307	0.084	0.028
2	3630	0.132	0.160
3	2795	0.173	0.533

From the description so far, Nu values would be found at only three ΔT corresponding to three Ra values. To increase the range of Ra for a given array, measurements were carried out with the array hung near the center of a pressure vessel, and the transient cooling experiment was repeated at various pressure levels. Three data points were obtained at each pressure level, as already described. Varying the Rayleigh number by varying pressure was originally proposed by Saunders (1936) and recently discussed by Hollands (1988). The pressure tank (Shewen, 1986) was 1.5 m in diameter and 2.1 m long, with a working pressure range of 1 Pa to 1.04 MPa. The large dimensions of the vessel ensured that the outer wall was sufficiently far from the fin array to avoid interference (i.e., $R_o^* \to \infty$ in Eq. (8), so the enclosure effects were negligible). The inner wall of the pressure vessel was covered by a copper water-cooled liner to maintain isothermality. A heat exchanger at the same temperature as the liner was operated at the top of the tank to remove heat from the plume rising from the array. By these measures, the temperature variations within the ambient air around the array were held to within 1/30 of the temperature difference between the body and the air. The tank liner temperature, T_l, was measured for use in calculating the radiation loss.

For the horizontal orientation in Fig. 1(B), the array was simply rotated 90 deg, keeping the base plate vertical.

(The present measurement technique could not be used for a horizontal base plate, because heat loss can no longer be blocked from the back side of the base plate, because of the loss of symmetry.)

To determine accurately the total heat loss from the array from the transient cooling curve, the heat capacitance of the body, C, was needed. Furthermore, the radiant loss, Q_{RAD}, was needed to provide the convective heat transfer. Q_{RAD} was calculated from

$$Q_{RAD} = \sigma A_s \mathfrak{F} (T_s^4 - T_l^4) \qquad (14)$$

where the radiant exchange factor $\mathfrak{F}$ (Kreith, 1968) accounts for both geometric and surface emissivity effects governing radiant exchange between the fins and the tank liner. Both C and $\mathfrak{F}$ were measured using the method of Hassani and Hollands (1989). This involved measuring the transient temperature response of the array, for a constant power at the array heater, with the array in an evacuated bell jar ($p = 10^{-4}$ Pa) where convection and conduction are virtually eliminated so the heat transfer is by radiation alone. The values of C and $\mathfrak{F}$ for the three arrays are given in Table 2. The measured capacitance includes the capacitance of embedded heaters, bolts holding the fins to the base plate, the aluminum base plate and fins, etc. It is assumed that the value of $\mathfrak{F}$ is the same when the array is in the pressure vessel as it is when the array is in the bell jar, because in both cases the containing vessel emissivity was close to unity and its area was substantially greater than the fin array area.

A detailed error analysis for this experiment has been prepared by Karagiozis (1991). Following the procedure of Moffat (1988), precision errors and bias errors were combined to produce an overall measure of uncertainty based on 95 percent confidence values for the precision errors. This analysis showed that the uncertainty in the measured Ra values decreased from 3.5 percent at $Ra = 10^{-2}$ to 2.3 percent at $Ra = 10^8$; the major source of uncertainty in Ra was the measurement of the low values of pressure. The uncertainty in Nu decreased from 12 percent for $Nu = 0.08$ (at very low pressures) to 2.5 percent at $Nu = 7$. The large uncertainty for low Nu values was dominated by uncertainty in the radiation correction; heat loss via the lead wires contributed little to either the heat transfer or the uncertainty.

Validation of Experimental Method

The transient procedure was used because it permitted many data points to be obtained quickly. (In the more usual measurement procedure, power is supplied to the heater at a constant known rate, which equals the heat loss from the body when steady state is achieved; the time to reach steady state can be very large.) To ensure that the transient and steady-state procedures gave the same results, steady-state measurements were performed on one of the fin arrays, and results compared to the transient method results. Steady-state tests were repeated at seven pressure levels, the temperature difference between the array and the air varying by less than 0.07°C over each one-hour test. The tests yielded Nu values between 0.5 and 9.2, and the maximum difference in Nu measured by the two procedures was 2.0 percent.

Nusselt numbers were also measured with the fin array replaced by the base plate alone. The base plate itself constituted a 170×150×9.54 mm vertical plate. For the same Ra, the results were compared with measurements of Hassani (1987) who used a plate (81×81×8.1 mm) that was close to being geometrically similar. The two sets of measurements were found to agree to within 5.5 percent, which was within the combined experimental uncertainty of 6 percent.

Measurements for a fin array were normally obtained over the portion of the transient decay curve from $\Delta T = 20$ to 5°C. For one array, these measurements were repeated for the $\Delta T = 40$ to 30°C portion of the decay curve. The Nusselt numbers from the two independent experiments agreed to within about 1 percent, showing that Nu was independent of the range of ΔT used. In making these comparisons, it is important to evaluate property values at the temperatures indicated in the nomenclature.

As a final validation test, heat transfer from Array #1 in Fig. 2 was measured at low Ra for both the vertical and horizontal orientations (Fig. 1). It would be expected that heat transfer would become independent of orientation for sufficiently low Ra, because heat conduction from the array would dominate. For $Ra < 1$, the Nu values from the two orientations were indeed found to be virtually identical.

Results

Measured values of Nu are shown in Fig. 3 for the three fin arrays in their vertical orientation. For high Ra the curves coalesce along a single curve, as anticipated in Eq. (12). Plotted on the same figure is the result of the classical solution to the laminar thin layer boundary layer equations for a single vertical, thin isothermal, flat plate for $Pr = 0.71$ (Incropera and De Witt, 1990), namely $Nu_L = 0.515\, Ra_L^{1/4}$, which can be recast into the form $Nu = 0.515\, Ra^{1/4}$ The data lie within 3.3 percent of this curve for $Ra > 4000$. At low Ra the curves appear to approach a constant value, suggesting that the convective motion contributes little to the heat transfer. This constant value is the conductive limit, denoted here by Nu_{COND}.

To determine Nu_{COND}, additional Nu measurements were taken at very low pressures. These data were fit well to an equation of the form:

$$Nu = Nu_{COND} + \xi\, Ra^{\eta} \qquad (15)$$

where ξ and η are constants for a particular array. A nonlinear regression software package was used to find the three con-

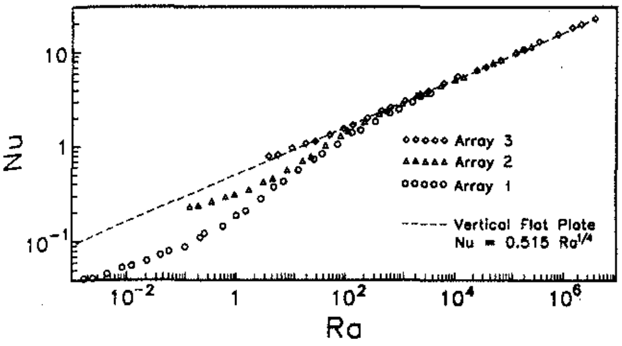

Fig. 3 Measured Nusselt numbers for the three fin arrays in Fig. 2 in the vertical orientation (Fig. 1A)

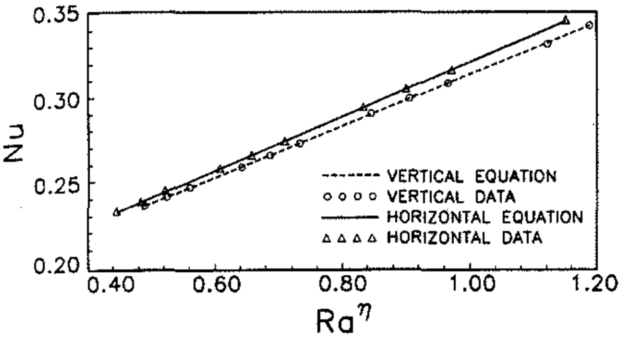

Fig. 4 Data fit at low Ra to determine the conductive limit, Nu_{COND}

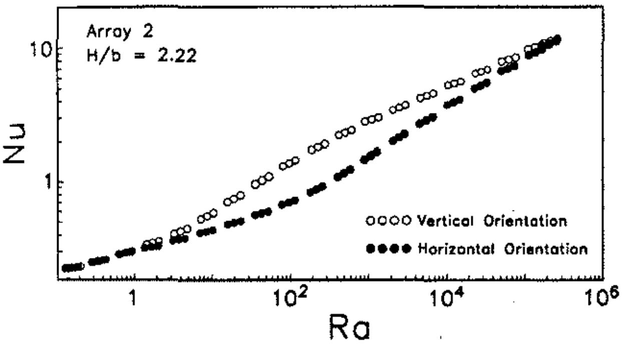

Fig. 5 Comparison of convective heat transfer from Array #2 in a vertical and horizontal orientation

stants, Nu_{COND}, ξ, and η for each body in each orientation. A plot of Eq. (15) and the data is shown in Fig. 4 for Array #2 in both the horizontal and vertical orientations. While ξ and η in Eq. (15) depend strongly on orientation, in the limit as $Ra \rightarrow 0$, $Nu \rightarrow Nu_{COND}$, independent of orientation. Nu_{COND} values obtained from the vertical and horizontal orientations were found to agree within 2.0 percent for all three arrays. The values of Nu_{COND} obtained are listed in Table 2.

The convective heat transfer from Array #2 is plotted in Fig. 5 for both the horizontal and vertical orientations. For $Ra \rightarrow 0$, the heat transfer becomes independent of Ra, as already discussed. At intermediate Ra, which is the usual range of interest to designers, the Nusselt number for the vertical orientation is up to 87 percent higher than for the horizontal orientation. Based on these results, it is difficult to justify the practice of choosing the horizontal orientation to cool electronic components. At high Ra, $Ra > 2 \times 10^5$, Nu becomes virtually the same for both orientations, and the trends suggest that the horizontal orientation might even give slightly higher Nu outside the experimental range of Ra.

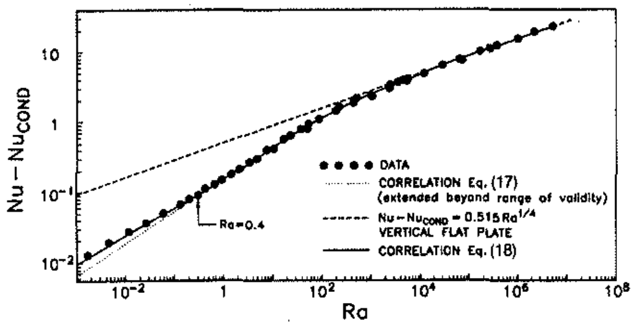

Fig. 6 Heat transfer ($Nu - Nu_{COND}$) from vertical fin arrays, together with correlation equations

Correlation of Data, Vertical Orientation

When replotted in the form $Nu - Nu_{COND}$ versus Ra, the data for all three arrays in the vertical orientation were found very nearly to collapse onto a single curve, as shown in Fig. 6. This was somewhat surprising for low Ra, since a dependence on geometric parameters was expected from Eq. (11). For fin geometries outside the range used in the present experiments, the expected dependence could still arise.

For $Ra > 4 \times 10^3$, the data fall near the classical equation for a vertical flat plate, discussed previously:

$$Nu - Nu_{COND} = 0.515\, Ra^{1/4}; \quad Ra > 4 \times 10^3 \qquad (16)$$

Equation (16) is plotted on Fig. 6. The rms deviation from this equation is 3.3 percent and the maximum deviation is 8.4 percent.

At lower Ra, thermal interference between the fins reduce the heat transfer. For $Ra \geq 0.4$ the following correlation equation has been derived using the Churchill-Usagi (1972) procedure

$$Nu - Nu_{COND} = 0.515\, Ra^{1/4}\left[1 + \left(\frac{3.26}{Ra^{0.21}}\right)^3\right]^{-1/3}; \quad Ra > 0.4 \qquad (17)$$

This equation is also plotted in Fig. 6. The measured Nu values are fit by this equation with an rms error of 4.8 percent and a maximum error of 11 percent.

For $Ra < 0.4$, Eq. (17) can normally be used with little overall error because conduction and radiation dominate the heat transfer. For completeness, however, the convection data in Fig. 6 can be fitted by adding a quantity denoted δNu to Eq. (17) as follows:

$$Nu - Nu_{COND} = 0.515\, Ra^{1/4}\left[1 + \left(\frac{3.26}{Ra^{0.21}}\right)^3\right]^{-1/3} + \delta Nu \qquad (18a)$$

where

$$\delta Nu = \{0.147\, Ra^{0.39} - 0.158\, Ra^{0.46},\, 0\}_{MAX}; \quad Ra > 10^{-3} \qquad (18b)$$

In Eq. (18*b*) the maximum of the two quantities in brackets is used. Equation (18) fits all the data with an rms error of 4.8 percent and a maximum error of 11 percent.

Discussion

Since the paper provides information for design, it seems appropriate to stress that radiation and conduction can play significant roles. Figure 7 shows, in the bottom-most curve, a plot of $Nu - Nu_{COND}$. Adding Nu_{COND} from Table 1 to obtain the convective Nusselt number Nu (plotted in the next curve up) causes a very significant difference at low Ra. This Nu_{COND} value, in the present experiments, is based on heat transfer only to the surrounding air. If other cool surfaces were located in the vicinity of the fin array, Nu_{COND} will be higher due to direct conduction to those surfaces. On the other hand, if the

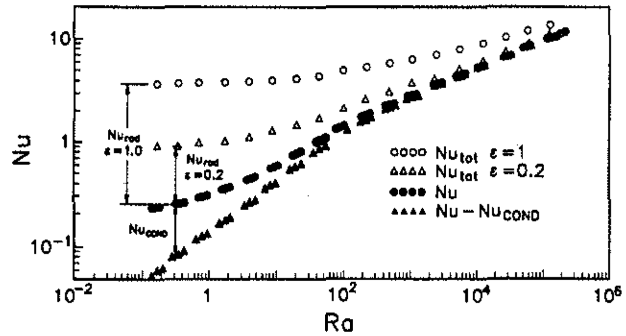

Fig. 7 The effect of conduction and radiation on the heat transfer from Array #2

fin array were mounted on a heated surface that extends beyond the array, Nu_{COND} could be decreased. Each application will have its own Nu_{COND}, and the determination of its value will often be important.

To show the magnitude of radiation on the heat loss from Array #2 in the present experiments, the radiative heat transfer, Q_{RAD}, is calculated for $T_s = 34.8°C$ and $T_l = 14.7°C$ using Eq. (14), and this is used in a "radiation Nusselt number" Nu_{RAD} where Q_{CONV} in Eq. (1) is replaced by Q_{RAD}. The total Nusselt number, $Nu_{TOT} = Nu + Nu_{RAD}$, is also plotted in Fig. 7 for every third data point. The emissivity of the present arrays was quite low ($\epsilon \approx 0.2$); this value was calculated from the measured $\mathfrak{F}$ in Eq. (14), by roughly estimating the angle factor from the array to its environment. (This value of emissivity was confirmed using direct measurements using a Gier–Dunkle infrared reflectometer.) For a black surface, $\epsilon = 1.0$, the Nu_{RAD} value would be larger by five times; the Nu_{TOT} for a black surface is shown as the top curve in Fig. 7. It is clear from this figure that failure to account for radiation will result in a severe underestimation of the heat loss from the surface.

The results obtained in the present study strictly apply only to fin arrays that are geometrically similar to the tested ones. For Array #1, Fig. 2, the heat transfer is obtained for many fins so that application of the results to wider arrays (i.e., larger W/b) would seem to be valid, as discussed in relation to Eq. (11). For Array #3, however, there are only four fins of which the two outside fins "see" quite different conditions than the interior fins; applying Eqs. (17) or (18) to wider arrays of such fins therefore seems quite risky. This caution is somewhat balanced by two facts: The fins should behave as vertical flat plates in the high Ra limit, and the data for all Ra did collapse to a single curve for the three arrays that would not be expected if the edge fins in Array #3 had significantly different heat transfer than the interior fins.

No attempt has been made to correlate the results for the horizontal orientation because the heat transfer would depend on the number of fins. The air heated by the lower fins is swept up, and blankets the higher fins, and the effects of this will depend on the number of fins present. A correlation for the specific arrays in this study would therefore by of very limited interest. Besides, the present results show that the horizontal orientation should be avoided.

The present results also supply strictly to isothermal fins. The error in applying the correlation for the convective heat transfer should be quite small for fin efficiencies near unity. Dusinberre (1958) claims that the heat transfer coefficients can be adequately corrected to account for nonisothermally for fin efficiencies as low as 75 percent.

Summary

The present paper reports measurements of convective heat transfer from three triangular, isothermal, fin arrays to air. The main points related to heat transfer from a vertical fin array are summarized as follows:

1 The measured values of Nu approach very closely the theoretical equation for a vertical flat plate for $Ra \geq 4000$.
2 As $Ra \to 0$, $Nu \to Nu_{COND}$, where Nu_{COND} is the limiting value for heat loss by conduction to the ambient air.
3 For $Ra < 4000$, the appropriate Nu value depends on Nu_{COND}, which will change with geometry.
4 All results collapsed on a single curve when plotted as $Nu - Nu_{COND}$ versus Ra. A correlation equation was provided.
5 It was noted that radiation and Nu_{COND} will depend strongly on the application of interest.

For fins running horizontally on a vertical base plate, the Nusselt number was found to be significantly lower than when the fins were vertical.

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