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STRENGTH OF ICE SHEETS

by

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SUMMARY

This paper describes the results of a number of large load tests performed on an Arctic Lake. The purpose of the load tests was to gain further knowledge in determining the bearing capacity of floating ice sheets.

The tests were of two types: (1) distributed loads and (2) relatively concentrated loads. In the distributed load tests a 15-ft diameter aluminum tank, with a height adjustable to 20 ft, was placed directly on the ice surface. Lake water was pumped into the tank at a more or less constant rate to load the ice. The concentrated load tests were conducted in the same manner as the distributed tests except that the tank was placed on a platform balanced on a 24 in. diameter wooden block.

The deflection of the ice sheet was measured at the load and at various distances away from the load. A deflection profile for each test is included in the report.

The time of audible cracks and their position. if observed, were recorded. The elastic theory predicts the outer circumferential crack to be at a distance greater than $1.8\,l$ for a concentrated load. The observed location was inside $1.1\,l$.

In the concentrated tests a circumferential crack did not always form. This agrees with previously reported load tests by Frankenstein (1959). There were always 13 radial type cracks present after each concentrated test.

The observed deflections for 8 of the tests were compared with Nevel's (1966) solution for a viscoelastic plate on an elastic foundation. His solution for large time can be approximated by

$$Wh^{3/2} = \left(\frac{1}{(6\,\pi\,K\,\eta_1)}\right)^{1/2} \left[\frac{1 + 3\,\eta_1}{4\,E_1\,t}\right]$$

The tests show that E_1 is a function of the ice temperature and increases when the ice tempera-

RÉSUMÉ

L'auteur décrit les résultats d'un certain nombre d'essais de chargement important réalisés sur la nappe de glace recouvrant un lac arctique. Le but visé par les essais de chargement était d'acquérir des renseignements plus précis sur la force portante des champs de glace.

Les essais étaient des deux types suivants: 1) charges réparties et 2) charges concentrées sur une aire restreinte. Pour les essais à charges réparties, un réservoir d'aluminium de 15 pi. de diamètre (4,5 m), d'une hauteur ajustable de 20 pi. (5,8 m) reposait directement sur la surface de la glace. L'eau du lac était pompée vers le réservoir à un régime plus ou moins constant pour effectuer le chargement de la nappe de glace. Les essais à charges concentrées ont été conduits de la même façon, mais le réservoir était placé sur une plateforme en équilibre sur un cylindre en bois de 24 po. (0,6 m) de diamètre.

On mesura le fléchissement de la nappe de glace à l'emplacement de la charge et à diverses distances alentour. Le présent exposé contient une reproduction du profil de fléchissement à chaque essai.

L'auteur a relevé le moment où se produisaient des craquements et leur emplacement d'origine. La théorie de l'élasticité prévoit que les fissures périphériques se produiront à une distance excédant $1{,}8\,l$ pour une charge localisée, mais l'emplacement observé se trouvait à moins de $1{,}1\,l$.

Une fissure périphérique ne s'est pas toujours formée au cours des essais de chargement concentré. Ceci est conforme aux résultats des essais de chargement réalisés antérieurement par Frankenstein (1959). L'auteur a trouvé 13 types de fissures radiales après chaque essai de chargement concentré. Il a comparé les fléchissements observés au cours de huit des essais avec les résultats du calcul de Nevel (1966) pour une

ture decreases. A curve is presented to show this relationship.

Included in the report are a number of cantilever and simply supported beam tests. The beam tests were all in situ. Comparing the strength results of the supported tests with those of the cantilever tests one cannot predict the stress concentration for the cantilever tests.

plaque de matériau viscoelastique reposant sur des fondations élastiques. La solution qu'il obtient pour un phénomène de longue durée peut être représentée approximativement par l'équation

$$Wh^{1.5} = \left(\frac{1}{6\pi K\eta_1}\right)^{0.5} \left[\frac{1 + 3\eta_1}{4E_1 t}\right]$$

Les essais montrent que E_1 est fonction de la température de la glace et qu'il croît quand la température décroît. Une courbe illustre cette relation.

Un certain nombre d'essais de lames de glace en porte-à-faux ou portant aux deux extrémités ont été menés à bien sur place. La comparaison des données fournies par les essais des lames portant aux deux extrémités et de celles se trouvant en porte-à-faux ne peut aider à prévoir la concentration des contraintes dans ces dernières.

INTRODUCTION

Many people have attempted to solve the problem of the bearing capacity of floating ice sheets by various theories. Assur (1961) and Wyman (1950) assumed an elastic failure and Meyerhof (1960) assumed a rigid-plastic failure. Nevel (1966) analyzed the problem assuming a viscoelastic or time-dependent failure. Little or no field data were available for comparison.

This paper reports the results of a number of carefully run field tests. The purpose of the tests was to determine the maximum load and deflection of an ice sheet under distributed and concentrated loading. The results of the tests are compared to some earlier theoretical statements.

PROCEDURE

The loading mechanism for the distributed tests consisted of a 15 ft diameter tank which was open at the top and sealed at the bottom with a canvas. The height of the tank could be increased in four-ft intervals. The joints were sealed to reduce leakage. The tank with its canvas bottom was placed directly on the ice surface.

A similar tank was used in the concentrated tests but the tank was placed on a platform which was then balanced on a 2.0 ft diameter block. The platform was constructed of heavy aluminum and had a wooden deck. Metal skis were attached to each corner of the frame so that the complete assembly could be easily moved to a new location. When balanced on the centre block, the skis or other parts of the frame assembly were above the ice surface.

The load was applied by pumping water from the lake into the tank. To measure the load, a level rod was placed inside the tank and read every 30 sec. until failure. All of the tests reported here were breakthrough tests. A 3 in. centrifugal pump was used to pump the water into the tank and did so at a fairly constant rate. The same pump was used for each test so the load rate was similar for all of the tests.

As soon as possible after failure the water was removed from the tank. After the water was removed, the tank and/or frame assembly was moved to a new test location.

During loading the deflection of the ice was measured at the tank and at given distances away from the tank.

TEST RESULTS

Table 1 lists each test and gives such information as ice thickness, ice temperature, maximum

TABLE I

DISTRIBUTED LOAD TEST DATA

Date	Test No.	Temperature (°C)[1] Air	Sur-face	Ice	Ice Thickness (in.)	Rod Position from Centre (ft.)	P[2] Max. Load (lbs.)	Time[3] of Failure (min.)	Max. Deflec-tion (ft.)	Crack Data Dia. of Failure Hole	Time of first Crack	No. of Circum-ferential	Dist. of Circ. frm. Edge (ft.)	No. of Radial
7 Oct 1961	2	−6.2		−1.2	6.3	1 = 11.7 2 = 24.0 3 = 49.0	23,955	11.7	1.055	17.0 ft	2.5	2	8.6 6.1	0
14 Oct 1961	3	−20.2	−5.8	−1.6	8.2	1 = 9.8 2 = 19.3 3 = 38.5 4 = 77.5	28,550	12.2	0.774	17.1 ft	2.6	2	13.0 5.4	0
16 Oct 1961	4	−10.8	−6.5	−2.2	9.5	1 = 9.6 2 = 19.3 3 = 38.7 4 = 77.5	36,980	14.6	0.870	17.9 ft	4.0	2	10.8 7.8	6
24 Oct 1961	7	−6.4		−1.5	12.8	1 = 12.0 2 = 24.0 3 = 48.0 4 = 96.0	44,330	30.5	1.557	17.8 ft	4.6	3	19.8 12.1 6.4	7
29 Oct 1961	9	−18.3	−8.5	−4.8	12.4	1 = 18.7 2 = 30.0 3 = 42.5 4 = 97.5	48,303	21.6	0.982	18.0 ft	5.6	3	25.2 12.8 10.9	5
4 Nov 1961	11	−16.2	−14.0	−8.3	15.6	1 = 11.0 2 = 21.9 3 = 43.7 4 = 100.5	77,945	30.9	1.263	18.0 ft	5.8	3	35.6 20.7 6.8	2
CONCENTRATED LOAD TEST DATA														
21 Oct 1961	5	−3.5		−1.8	11.4	1 = 10.4 2 = 20.5 3 = 41.0 4 = 82.0	30,968	9.1	0.760	26.4 in	8.1	4[4]	16.1 13.5 10.4 6.3	13
24 Oct 1961	6	−6.4		−1.5	11.0	1 = 10.0 2 = 24.0 3 = 48.0 4 = 95.5	28,970	8.6	0.446	27.9 in	4.8	0		13
28 Oct 1961	8	−16.3		−4.5	13.5	1 = 10.1 2 = 22.5 3 = 45.0 4 = 122.5	34,810	9.7	0.652	29.3 in	1.2	1	10.7	13
3 Nov 1961	10	−17.5	−14.0	−9.0	15.4	1 = 12.0 2 = 22.4 3 = 44.8 4 = 118.0	51,339	16.8	0.710	27.4 in	2.4	2[4]	26.8 13.2	13
12 Nov 1961	12	−22.2	−13.8	−9.6	17.8	1 = 12.2 2 = 26.0 3 = 52.2 4 = 119.7	61,603	21.2	0.759		3.5	1		0

1—Ice temperatures taken 14.8 to 20.9 cm below ice surface.
2—Weight of tank and accessories are included in the maximum load. The following minor items were not figured in maximum load: observers, the water on the surface caused by flooding, and some ice that clings to the tank and accessories.
3—The time of failure is equal to the time of maximum load except in Test 7 where maximum load was reached at 17.0 minutes.
4—Circumferential cracks were not seen until after failure.
5—The outer crack is the main circumferential crack.

load, maximum deflection, number and type of cracks that formed, etc.

The deflection of the ice during the test was measured at the load and at 5 different locations away from the load. The method of measurement was to place engineer-type level rods at the above-mentioned locations and then read these rods every minute with an engineer level. The level was placed beyond the point of deflection or where the deflection was very small. The rods were placed such that the point of maximum negative deflection could be determined. Figures 1-5 are deflection profiles for each of the tests. In Test 7 the deflection at the load was approximately double that of the other tests. The increased deflection was due to creep as the maximum load remained on the ice for 13.5 min. before breakthrough.

The first cracks to form were the radial tension cracks that formed on the under surface of the ice. These radial cracks increased in length until the bending stresses equalled the flexural strength, at which time a circumferential crack formed. This process repeated itself until final breakthrough occurred. The maximum number of visible circumferential cracks was 3.

First cracking occurred very early in the distributed tests. This again proves that first cracking occurs much before final failure and that the time of the first audible crack should not be used for bearing capacity criteria. The first circumferential crack was usually visible at a time approximately equal to two-thirds of the time of maximum load and was located about $1.2l$ from

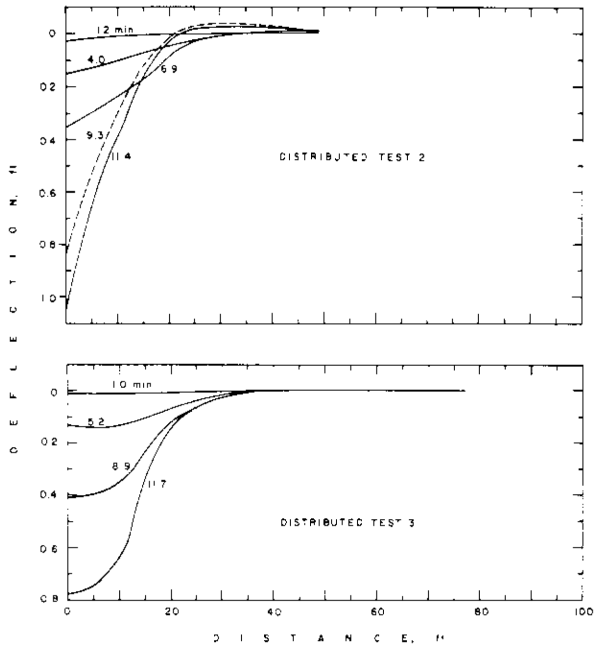

Fig. 1. Deflection of the ice at the load and at various distances away from the load. The numbers above the line indicate the time from zero loading.

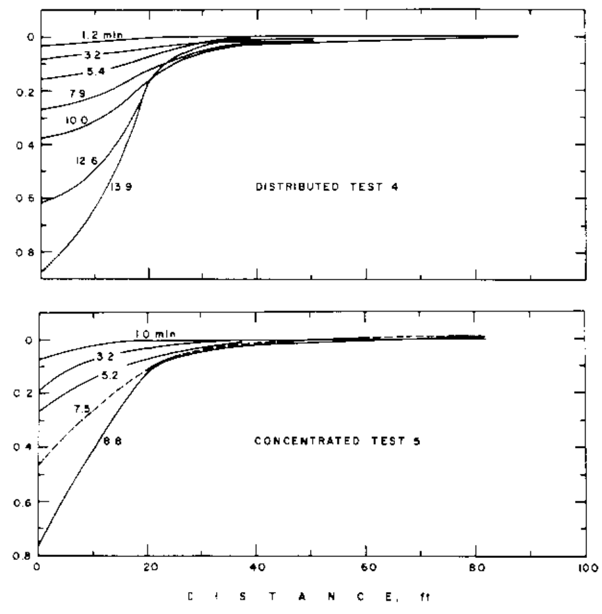

Fig. 2. Deflection of the ice at the load and at various distances away from the load. The numbers above the line indicate the time from zero loading.

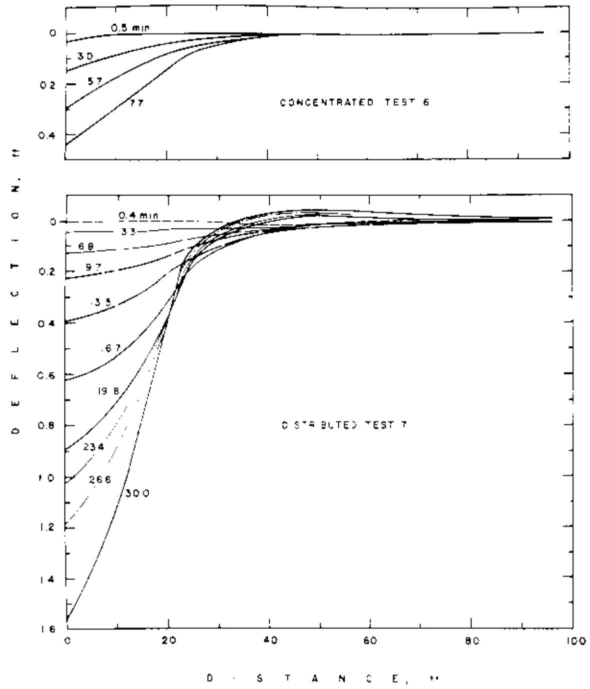

Fig. 3. Deflection of the ice at the load and at various distances away from the load. The numbers above the line indicate the time from zero loading.

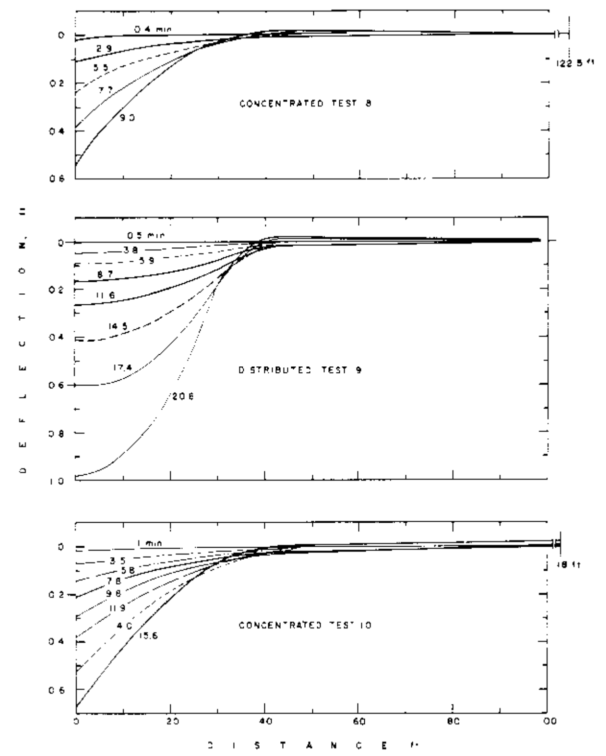

Fig. 4. Deflection of the ice at the load and at various distances away from the load. The numbers above the line indicate the time from zero loading.

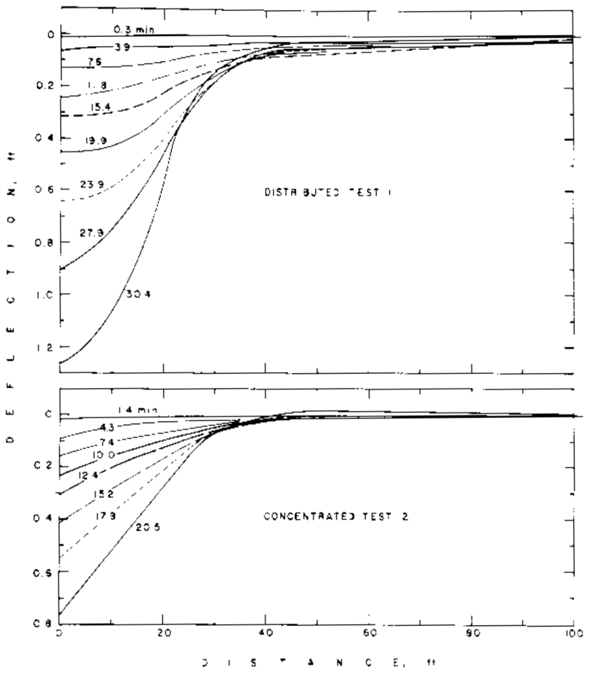

Fig. 5. Deflection of the ice at the load and at various distances away from the load. The numbers above the line indicate the time from zero loading.

the load where l is the characteristic length or action radius and is equal to

$$l^4 = \frac{Eh^3}{12k\,(1-\nu^2)} \qquad (1)$$

where E is the modulus of elasticity, h the ice thickness, k is the reaction of the foundation, and ν is Poisson's ratio and equal to $\frac{1}{3}$. There were usually 3 circumferential cracks visible during or after a distributed test but in some of the tests on thinner ice only 2 cracks were visible

The failure hole for the distributed tests contained many pie-shaped pieces which indicates that failure occurred along the radial tension cracks. These pie-shaped pieces were only visible for ice thickness less than 10 in. For ice thickness greater than this. the pie-shaped pieces would be broken up into many small pieces. The failure hole of the distributed tests increased somewhat with ice thickness but the minimum and maximum failure hole diameter was 17.0 and 18.0 ft respectively.

The crack phenomenon for the concentrated tests differed greatly from that of the distributed tests. The distributed tests always yielded a circumferential crack but in some of the concentrated tests, no circumferential cracks were visible either during loading or after failure. Earlier tests, Frankenstein (1963), also on fresh water ice and where the bearing surface measured 1.4 ft diameter, yielded the same result. The radial cracks which formed during the concentrated tests were whitish in appearance and looked as if they were caused by crushing. There were always 13 of these radial cracks visible. The failure hole diameter for the concentrated tests was very close to the diameter of the bearing block. In some of the tests the block had to be forced from the failure hole.

ANALYSIS

The elastic solution predicts that the deflection of the plate under the load is proportional to the load. Figure 6 is a typical time-deflection curve for the field tests which indicates that the data does not fit the elastic solution. For a concentrated load the elastic solution predicts a minimum value of $1.8l$ for the location of maximum negative bending moment for values of ν from 0 to $\frac{1}{2}$. The maximum observed value for this moment was at $1.1l$.

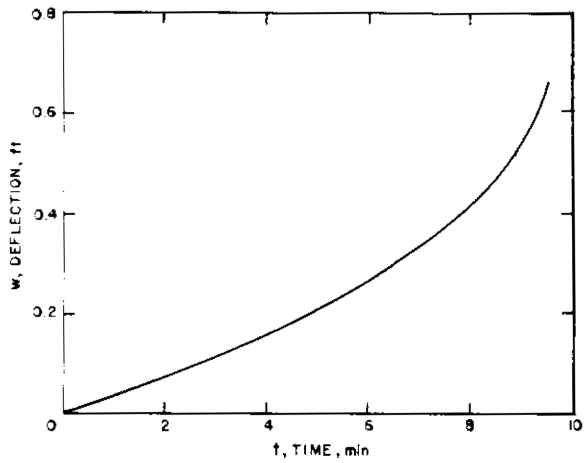

Fig. 6. A typical deflection versus time curve for the load tests. The curve indicates that the failure is not a true elastic failure.

Nevel (1966) predicts the deflection, at the centre for concentrated loads, by a viscoelastic analysis. He solved the following differential equation of a plate

$$l^4 \nabla^4 w + w = q/k \qquad (2)$$

where w is the deflection of the plate at the centre and

$$q = H(t)\,\dot{P}t/a^2 \quad \text{for} \quad r < a$$

and

$$q = 0 \quad \text{for} \quad r > a$$

where P is the load rate, a the radius of the uniform load, H(t) is Heavysides step function, and t is the time. Nevel assumed a Maxwell model (where E_1 is the elastic constant and η_1 is the viscosity constant) for two times the shear modulus and $\nu = \frac{1}{2}$, obtained

$$w = \frac{\dot{P}t\,H(t)}{8k\,l^2}\left\{\frac{e^{-\xi/2}}{3}\left[(3+2\xi)\,I_0\,(\xi/2) + (1+2\xi)\,I_1\,(\xi/2)\right] + \frac{\alpha^2}{16\pi}\left[(2+\xi)\left(-5 + E_1\,(\xi) + \ln\frac{\gamma^5\alpha^4\xi}{16}\right) + \frac{1-e^{-\xi}}{\xi}\,\frac{3\xi}{2} - e^{-\xi}\right] + R_1\right\}. \qquad (3)$$

where $\alpha = a/l$, $\xi = E_1\, t/n_1$, $\ln V$ is Euler's constant and equal to 0.577. The constants E_1 and η_1 were determined by applying a

$$\frac{Wh^{3/2}}{\dot{P}t\sqrt{t}} = \frac{1}{\sqrt{6\pi k\eta_1}}\left[1 + \frac{3\,\eta_1}{4\,E_1\,t}\right] \qquad (4)$$

least square fit of the data to Eq. (4). Figure 7 shows the test results plotted with the solution of Eq. (3) and Table 2 lists the values of E_1 and η_1.

Tests 4 and 11 have an unusually high value for η_1. These higher η_1 values cause the computed curve to fall below the plotted data. In Test 7 the computed curve fits the data for the first 6 minutes but then falls below the data. This test was a creep test so one should expect the curve to fall well below the data. Figure 7 shows that Eq. (4) fits the data very well for the concentrated tests but varies somewhat for the distributed tests (Tests Nos. 4, 7 and 11) because Nevel's analysis was based on a small concentrated load.

TABLE 2
Viscoelastic Properties

Test #	E x 10^6 lb/ft^2	n_1 x 10^6 lb-Min/ft^2	t(−°C)	h(ft)
4	58.23	99.90	2.2	0.79
5	62.80	46.37	1.8	0.95
6	8.886	25.27	1.5	0.92
7	51.20	33.67	1.5	1.07
8	9.517	18.04	4.5	1.12
10	34.87	32.93	9.0	1.28
11	29.03	124.2	8.3	1.30
12	11.94	51.93	9.6	1.48

Figure 8 shows E versus temperature. As expected, the value for E increases with a decrease

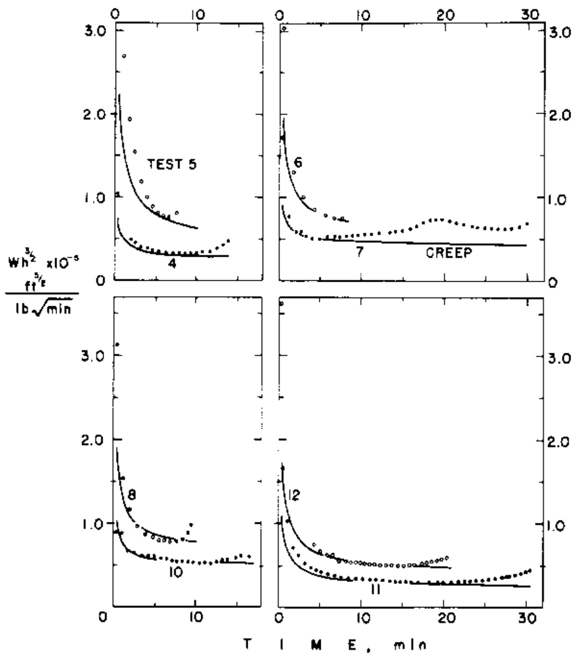

Fig. 7. Observed deflections plotted versus Eq. 3. Tests 4, 7, and 11 are distributed tests.

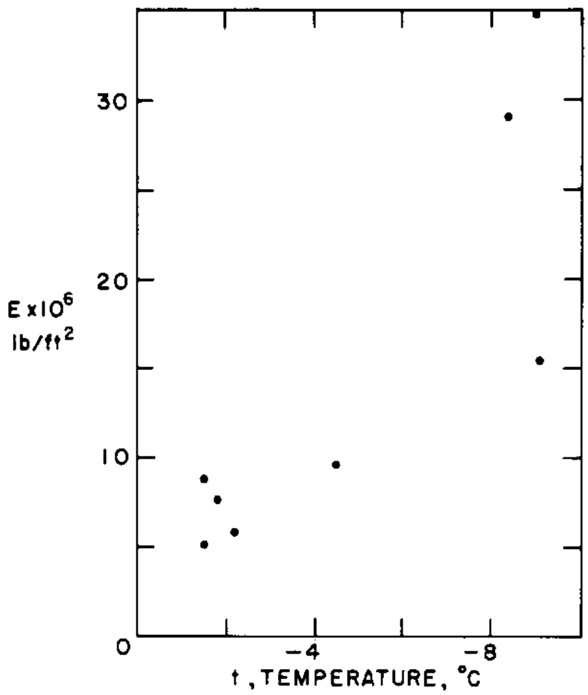

Fig. 8. Elastic constant E plotted versus the ice temperature.

in temperature. Hitch (1959) found the same from stress-strain relationships of ice beams.

Jellinek and Brill (1956) conducted a creep tension test on an ice cylinder. They analyzed their data using a 4 element model. Converting the test results from the 4 element model to the Maxwell model, his value for E_1 ranged from $20.1 \leq E_1 \leq 57.3$ lb/ft^2 compared to $6.28 \leq E_1$ 58.2×10^6 lb/ft^2 computed from the load tests. His viscosity constant η_1 had a range of from 4.1 to 21.6 x 10^9 compared to 18.04 to 124.2 x 10^6 computed from the load tests. The tests run by Jellinek and Brill were creep tests of longer duration while ours were of constant load rate of short duration which might account for the difference of the viscosity constant.

The deflection profiles (Figures 1-5) show how the deflection increases with time at the centre of the load and at various distances away from the load. The profiles also show how the point of maximum negative deflection increases and moves towards the load as the ice creeps. The amount of maximum negative deflection can be expressed by the transformed differential equation.

$$l^4 \nabla^4 \bar{w} + \bar{w} = O \qquad (5)$$

where $\lambda = S/(S + E_1/\eta_1)$. The solution of Eq. (5) (Wyman, 1950) that remains finite for $O \leq r \leq \infty$ is

$$\bar{w} = A \text{ kei } r/l\lambda \qquad (6)$$

The constant A is determined by the vertical equilibrium of the plate given by

$$\int_0^\infty kw2\pi r dr = H(t)\dot{P}t \qquad (7)$$

Taking the transform we get

$$2\pi k A \int_0^\infty r \text{ kei } r/l\lambda \, dr = \dot{P}/S^2 \qquad (8)$$

Hence

$$A = \frac{-P}{S^2 2\pi k l^2 \lambda^2} \qquad (9)$$

and

$$\bar{w} = \frac{-P}{S^2 2\pi k l^2 \lambda^2} \text{ kei}(r/l\lambda) \qquad (10)$$

For the maximum negative deflection we require $\frac{\partial w}{\partial r} = O$. The transform of this is $\frac{\partial \bar{w}}{\partial r} = O$.

This will be satisfied when $\bar{r}/l\lambda = 4.93$. Evaluating Eq. (10) at this value we obtain

$$\bar{w} = \frac{\dot{P}\,(.0112)}{S^2 2\pi k l^2 \lambda^2} \qquad (11)$$

We note from Nevel (9) that $\bar{w}$ evaluated at $r = O$ for a concentrated load is

$$\bar{w}_o = \frac{\dot{P}}{8k l^2 \lambda^2 S^2} \qquad (12)$$

Therefore the amount of maximum negative deflection is

$$\frac{w_{max}}{w} = \frac{\bar{w}}{\bar{w}_o} = \frac{4}{\pi}\,(.0112) \qquad (13)$$

Eq. (13) agrees reasonably well when applied to the results of the observed field data.

FLEXURAL STRENGTH

The flexural strength was determined from loading in-situ cantilever and supported beams. The tests were conducted on Fresh Water Lake and Elson Lagoon, both located at Barrow, Alaska. The above-described tests were also conducted on Fresh Water Lake.

A cantilever beam test is probably the easiest and most convenient method for determining in-situ flexural strength. The beams were prepared by cutting the three sides of the cantilever such that the sides were completely free of the ice sheet. The load was applied by pushing down or pulling up the free end of the beam. The flexural strength is determined by

$$\sigma = \frac{6\,P\,L}{b\,h^2} \qquad (15)$$

where P is the applied load, L the length, b the width, and h the ice thickness.

The applied load was measured by various methods, depending on the ice thickness. When the ice was thin, say 10 cm or less, a spring scale was used for the pull-up tests and a proving ring for the push-down tests. When the ice thickness was greater than 10 cm, the drum and lever arm method Frankenstein (1959) was used.

Brown (1962) states that the stress concentration factor for a cantilever test is 2.80. His assumption was based on results of tests using Plexiglas. Frankenstein (1961), comparing cantilever tests with supported tests on fresh water ice, could not detect the effect of stress concentrations. In some tests the cantilever tests yielded higher strength values than the supported tests.

The supported beam test resembles a simple beam test. The beam is cut from the ice sheet and floated to and under the two supports (one at each end of the beam). The beam has vertical movement while under the supports, Frankenstein (1961) and the method for applying the load was the drum and lever arm method. The flexural strength was determined by

$$\sigma = \frac{3 P l}{2 b h^2} \tag{16}$$

Table 3 is a summary of the flexural tests. The results show that the strength of the cantilever beams was greater when the bottom of the ice was put in tension. Earlier tests, Frankenstein (1959, 1961), reported the same. The supported tests, fresh water ice or sea ice, give approximately the same strength value regardless of which surface was put in tension. The results also indicate the bottom cantilever tests yielded approximately the same strength value as the supported tests.

The sea ice supported tests had a much lower strength value than the fresh water tests. The results compare with the sea ice tests reported by Brown (1962). Salinity profiles were taken of the sea ice tested. The salinity was determined by the hydrometer method, corrected to 15°C. Table 4 lists the salinity and temperature profiles of the sea ice.

CONCLUSION

The results of the load tests indicate that a viscoelastic analysis using the Maxwell model can be used to analyze the test results. This is especially true for the concentrated tests.

The strength value for a cantilever is higher when the lower surface is put in tension. The beam test results show, as earlier tests also indicated (Frankenstein, 1961), that if the cantilever tests had a stress concentration the factor was very small.

ACKNOWLEDGEMENT

I wish to express my thanks to Mr. D. E. Nevel for his assistance, suggestions and for his critical review of the paper.

I would also like to express my appreciation to Mr. R. L. Garner and Mr. M. Church for their assistance in reducing the data.

TABLE 3
Summary of Tests
Fresh Water Lake

Date	Ice Thick. (cm)	Test Type	Flexural Strength (Kg/cm²) Max.	Min.	Ave.	No. of Tests	Ave. Ice Temp. (−°C)	Standard Deviation
4 Oct 61	11.8	BC	8.80	4.05	7.45	6	1.2	.89
4 Oct 61	11.8	TC	5.00	1.24	3.61	6	1.2	1.26
9 Oct 61	17.5	BC	9.00	4.74	6.82	5	1.2	2.05
9 Oct 61	17.5	TC	6.76	2.05	4.54	8	1.2	1.54
13 Oct 61	21.0	BC	8.11	6.39	7.25	6	2.3	.64
13 Oct 61	21.0	TC	6.50	3.38	4.71	12	2.3	.91
26 Oct 61	35.0	BS	9.27	5.23	6.68	6	2.7	1.55
26 Oct 61	35.0	TS	9.39	6.88	8.19	9	2.7	.98
			Elson Lagoon					
8 Nov 61	33.8	BS	4.84	2.58	3.42	6		.91
8 Nov 61	33.8	TS	3.70	2.23	2.94	6		.49
9 Nov 61	34.3	BS	3.12	2.36	2.84	3		.41
9 Nov 61	34.3	TS	3.37	3.02	3.15	3		.50