Theory of waveguide fed array of dipoles

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THEORY OF WAVEGUIDE FED ARRAY OF DIPOLES

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OTTAWA

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NATIONAL RESEARCH COUNCIL OF CANADA
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THEORY OF WAVEGUIDE FED ARRAY OF DIPOLES

I. EQUATIONS OF TRANSFORMATION ON EQUIVALENT LINE:

The theory to be presented here was referred to in our preliminary report (PRA-64) on the design of waveguide fed arrays of dipoles. In that report was emphasized the importance of properly exciting the dipoles as regards both amplitude and phase, and that this cannot be accomplished unless due cognizance is taken of reflected waves due to the energized probes in the guide. We shall deal with the problem on the basis of the following rough picture, namely, that the guide may be treated as a transmission line periodically shunted by impedances of magnitude dependent on the probe length, and the dimensions of the dipole and its support, at each successive feed-point.

Let Z_o denote the characteristic impedance of the line

Z_r = the impedance presented by the r^{th} shunt at the point $x = d_r$

$k = 2\pi/\text{wavelength}$

$e_{r+1}(x)$ = voltage across the line between the r^{th} or $(r+1)^{th}$ shunt

$i_{r+1}(x)$ = current in the line between the r^{th} or $(r+1)^{th}$ shunt

When waves are travelling both to the right and the left:

$$\left.\begin{aligned} e_{r+1}(x) &= A_{r+1}\, e^{-jkx} + B_{r+1}\, e^{jkx} \\ i_{r+1}(x) &= \frac{1}{Z_o}\left(A_{r+1}\, e^{-jkx} - B_{r+1}\, e^{jkx}\right) \end{aligned}\right\} \quad d_r < x < d_{r+1} \qquad (1)$$

At the r^{th} shunt

$$Z_r\left(i_r(d_r) - i_{r+1}(d_r)\right) = e_{r+1}(d_r) = e_r(d_r) \qquad (2)$$

Now substitute from (1) in (2) and obtain

$$\left.\begin{aligned} A_r e^{-jkd_r} &= \left(1+\frac{Z_o}{2Z_r}\right) A_{r+1} e^{-jkd_r} + \frac{Z_o}{2Z_r} B_{r+1} e^{+jkd_r} \\ B_r e^{+jkd_r} &= -\frac{Z_o}{2Z_r} A_{r+1} e^{-jkd_r} + \left(1-\frac{Z_o}{2Z_r}\right) B_{r+1} e^{+jk_r} \end{aligned}\right\} \quad (3)$$

Now let $A_r e^{-jkd_r} = a_r$; $B_r\ e^{+jkd_r} = b_r$

$$\frac{Z_o}{2Z_r} = \gamma_r \text{ and } e^{jk(d_{r+1} - d_r)} = w_r \quad (4)$$

We then have

$$\left.\begin{aligned} a_r &= (1 + \gamma_r)\, w_r\, a_{r+1} + \frac{\gamma_r}{w_r} b_{r+1} \\ b_r &= -\gamma_r\, w_r\, a_{r+1} + \frac{1 - \gamma_r}{w_r} b_{r+1} \end{aligned}\right\} \quad (5)$$

We can therefore transform from the pair (a_{r+1}, b_{r+1}) to the pair (a_r, b_r) by means of the matrix

$$\begin{pmatrix} (1 + \gamma_r)\, w_r & \gamma_r / w_r \\ -\gamma_r\, w_r & (1 - \gamma_r)/w_r \end{pmatrix} \quad (6)$$

and the voltage across the rth shunt is given by

$$e_r\ (d_r) = a_r + b_r \quad (7)$$

II. REFLECTION, ABSORPTION AND TRANSMISSION BY A SINGLE PROBE:

In equation (3) above, put $r = 1$ and $B_2 = 0$

$$A_1 = (1 + \gamma)\ A_2 \qquad B_1 = -\gamma A_2 = -\frac{\gamma}{1 + \gamma} A_1 \quad (8)$$

Let ρ = reflection coefficient (power) = $\left| B_1^2 / A_1^2 \right|$

$$\rho = \left|\frac{\gamma}{1 + \gamma}\right|^2 = \left|\frac{Z_o}{Z_o + 2Z}\right|^2 = \frac{\frac{1}{4}}{\left(\frac{R}{R_o} + \frac{1}{2}\right)^2 + \left(\frac{X}{R_o}\right)^2} \quad (9)$$

When $Z = R + jX$

The transmission coefficient on power is given by

$$\tau = \left(\frac{A_2}{A_1}\right)^2 = \left|\frac{1}{1+\gamma}\right|^2 = \left|\frac{2Z}{Z_o + 2Z}\right|^2 = \frac{\left[\left(\frac{R}{Z_o}\right)^2 + \left(\frac{X}{Z_o}\right)^2\right]}{\left(\frac{R}{Z_o} + \frac{1}{2}\right)^2 + \left(\frac{X}{Z_o}\right)^2} \tag{10}$$

The fraction of power absorbed, $\alpha = 1 - \rho - \tau$. Hence

$$\alpha = \frac{\frac{R}{R_o}}{(R/Z_o + 1/2)^2 + (X/Z_o)^2} \tag{11}$$

Thus from (9) and (11)

$$4\,\frac{R}{R_o} = \alpha/\rho = \frac{1-\tau-\rho}{\rho} \tag{12}$$

While from (9)

$$\frac{X}{R_o} = \sqrt{\left\{\frac{1}{4p} - \left(\frac{R}{Z_o} + \frac{1}{2}\right)^2\right\}} \tag{13}$$

and

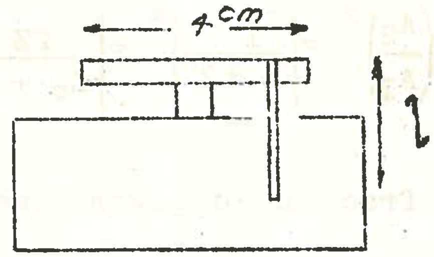
$$\frac{\tau}{\rho} = \frac{4Z^2}{Z^2}$$

The functions α, ρ, and τ of course, as given above apply only when the dipole is inserted in a matched line - any other case, however, may be calculated from the theory given. The equations (12) and (13) may be used with the appropriate measurements to calculate the shunt impedance offered by the probe connected to the dipole as a function of the probe length. In this connection, it may be noted that α of the present report is (1-p) times the fraction of power radiated of MacKinnon's report PRA-60.

We have used the above method but have found more consistent results by using ordinary standing-wave procedure. The guide was terminated with a reflector placed an odd number of quarter wavelengths from the probe, and particular care was taken to minimize the shunting effects of the detector probe. In Table I are shown the results for an end-fed dipole mounted on an 11 mm pillar above standard 3" x 1 1/2" guide.

Table I

λ cms.	$\frac{R}{Z_o}$	$-\frac{X}{Z_o}$	$-\tan^{-1}\frac{X}{R}$
5.2	0.25	0.22	41.4
5.0	.30	.45	56.3
4.8	.40	.75	61.9
4.6	.50	1.10	65.5
4.4	.70	1.60	66.4
4.2	.95	2.05	65.2
4.0	1.25	2.70	65.2
3.8	1.60	3.30	64.1



These results are shown graphically in Figure 1. In Figure 2 the energy reflected and the energy absorbed by a single probe are shown as functions of λ . It is evident that the greater the fraction of the guide energy the probe is designed to abstract, the greater will be the reflection from the probe.

III. For the purpose of testing the foregoing theory we have measured the standing wave ratio from a series of similar dipole probes spaced 200° in the guide. In Figure 3 will be found the measured values calculated for a probe with λ = 3.8 cm. The agreement is good. It is striking that the S.W.R. from these loosely coupled dipoles rises as the number of probes in the series is increased, reaches a maximum of 1.5, and then decreases again. We have calculated the amplitude and phase of the voltage in the guide at each dipole in the series. The results are shown for λ = 3.8 cm probes in Figures 4 and 5, and it is clear how badly the hypothesis of loose-coupling fails. The phase of excitation is not that to be expected on the basis of ignoring the effect of the dipoles on the propagation, and the amplitude variation due to the reflected waves is considerable. However it must be pointed out that it is only the variation from the straight line in Figure 5 which leads to distortion of the beam since the phase deviation corresponding to points on this straight line result merely in a swinging of the beam.

In Figures 6, 7 and 8 corresponding data are presented for longer probes, λ = 5.0 cm. As one would expect the effects are even more strongly marked than with the short probes.

Further, we have worked out the S.W.R. to be expected from a "gabled array" in which the length of probe increases monotonically as one proceeds from the generator. These are shown in Figure 9 and support the contentions regarding the effects of reflected waves in the waveguide feed to the array.

IV. CONDITIONS UNDER WHICH A PRESCRIBED LAW OF DIPOLE EXCITATION MAY BE OBTAINED IN THE PRESENCE OF REFLECTED WAVES

Imagine that we are given a law of excitation in the form

$$\frac{e_{r+1}(d_{r+1})}{Z_{r+1}} = \mathcal{E}\frac{e_{r+1}(d_r)}{Z_r}$$

Where $\mathcal{E}$ is a given complex number yielding the proper amplitude and phase relationship of two adjacent dipoles.

From equation (7)

$$a_{r+1} + b_{r+1} = \mathcal{E}^1(a_{r+1}w_r + b_{r+1}/w_r) \qquad \mathcal{E}^1 = \frac{\mathcal{E}Z_{r+1}}{Z_r}$$

This equation must hold irrespective of the ratio b_{r+1}/a_{r+1}. Therefore

$$\mathcal{E}^1 w_r = 1 \quad \mathcal{E}^1/w_r = 1 \quad \text{i.e.} \quad (\mathcal{E}^1)^2 = 1 \text{ and } w^2 = 1$$

Thus, only if the antennae are spaced apart an integral number of half-wavelengths along the line is it possible to secure the desired distribution, and then the impedances must be in the ratio

$$\frac{Z_{r+1}}{Z_r} = \pm\frac{1}{\mathcal{E}} \tag{15}$$

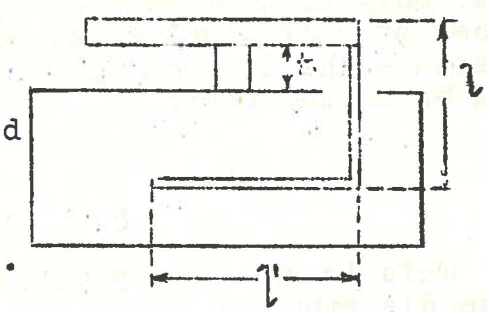
This is the law by which the desired amplitude and phase distribution among the dipoles may be secured. A given radiation pattern may be approached by the method of I. Wolff (P.I.R.E. 1937) for the spacing is regular and determined by the waveguide size.

In equation (15) the simplest choice for $\pm\frac{1}{\mathcal{E}}$ is a positive real number corresponding to a co-phased array. The proper amplitude distribution among the dipoles can be obtained only if both the resistance and reactance presented by the dipoles are correctly proportioned. From the results presented in the previous section it is quite clear that the straight probes at present used do not allow the possibility of this adjustment.

V. NON-REACTIVE SHUNT IMPEDANCE

The most hopeful way of dealing with the disturbance of phase produced by the reactance presented to the guide by the radiating elements and their coupling probes seemed to be to introduce some inductance to compensate for the capacity shunting the guide. Our first attempt to do this was to place between each dipole feeder probe and the generator a suitable "matching" probe. We no longer propose to use this method but the data may be of interest. In figure 10, corresponding to the distance of the "matching" probe from the dipole is given the lowest value of S.W.R. towards the generator obtainable by varying the length of the "matching" probe. In Figure 11 the matching probe placed at a fixed position in the guide is varied in length, the corresponding variation in the energy reflected to the generators is shown. In Figure 12 are given data on the optimum distance between dipole and matching probe for various lengths of the dipole coupling probe.

In searching for a simpler and more convenient way to achieve compensation of reactance, we have found that a simple right-angle bend at the end of the probe coupling the dipole into the guide can be made of the proper length to produce the required compensation. In Figure 13 we have plotted the family of curves $\tan^{-1} X/R$ against l^1 for a series of l. The measurements were greatly accelerated by the observation that the gradient of these curves as they cross the abscissa axis is sensibly independent of l.



On the Smith's diagram of Figure 14 are shown loci of constant l ; the value of l^1 varies around these nearly circular curves. The data are given for pillar heights h = 11 mm and h = 4 mm.

Figure 15 shows the design curves giving l^1 and the shunt resistance respectively as they depend on l.

It will be observed that for a given l the effect of the compensating L is to reduce the resistance. Secondly, where l is reduced to 2.85 cm and R is about three times the guide resistance, the slope of R against l for compensated probes is very steep, and it would be difficult to ensure gabling of the amplitude distribution along an array of such dipoles if values of R higher than the above value are required.

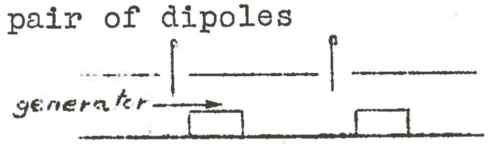
VI. METHODS FOR INCREASING SHUNT IMPEDANCE

Having succeeded in eliminating the reactance component of the shunt impedance presented to the guide by the dipole and feeder probes, we are able to excite the dipoles in phase (account being taken of alternate reversal of feed) and with an amplitude which is determined by the probe length, provided that the dipoles are spaced $\lambda_g/2$ apart. This statement may have to be qualified to allow for the disturbance produced by the waves travelling along the outside of the guide (see our Report PRA-64). Thus effect can be given to the distribution of amplitude along the array required to produce any radiation pattern symmetrical about the normal to the array. Furthermore, in principle it should be possible by suitable choice of reactance as well as resistance at 180° spacing to produce asymmetrical radiation patterns from the array. It will then be necessary to terminate the guide with a plunger suitably placed to compensate for the total reactance shunting the guide. In either case, the limiting factor in practice is the S.W.R. one is prepared to tolerate between successive feed-points.

Since we have in mind to deal with quite long arrays, it is evident that since the compensated probes can be made accurately to present only about three times the guide impedance at most, some attempt must be made to increase this figure. Even then, it may be necessary in the case of a very long array to split the feed into sections fed through matching transformers in order to match the array as a whole to the generator.

The methods which we have considered for increasing the shunt resistance follow:

(i) By means of quarter-wave transformers in the guide between each successive pair of dipoles



This method does not commend itself as practicable at the present time.

(ii) By reducing the diameter of the dipoles to 1/16", the smallest that considerations of rigidity and corona loss will allow, it is possible to raise the resistance tenfold. We have not yet investigated this possibility in detail.

(iii) By reducing the height of the pillar supporting the dipole above the guide surface, it is possible to raise the shunt resistance to at least ten times that of the guide.

As the dipole is brought closer to the guide surface (pillar height 3 mm) in order to secure a non-reactive shunt impedance on the guide, it has been found necessary to increase the length of the dipole, the point of feed being retained at the same distance from the centre. Such dipoles placed close to the guide are still capable of handling 30 KW without sparking.

To sum up, we intend to construct an array to the guide feed of which each dipole presents a high resistance of a value determined by a law of "gabling" chosen on the basis of the radiation pattern to be expected. The dipoles will be $\lambda_g/2$ apart and excited in phase. The guide will be terminated so as not to shunt the dipole feeders and hence no energy should be lost at the end of the guide.

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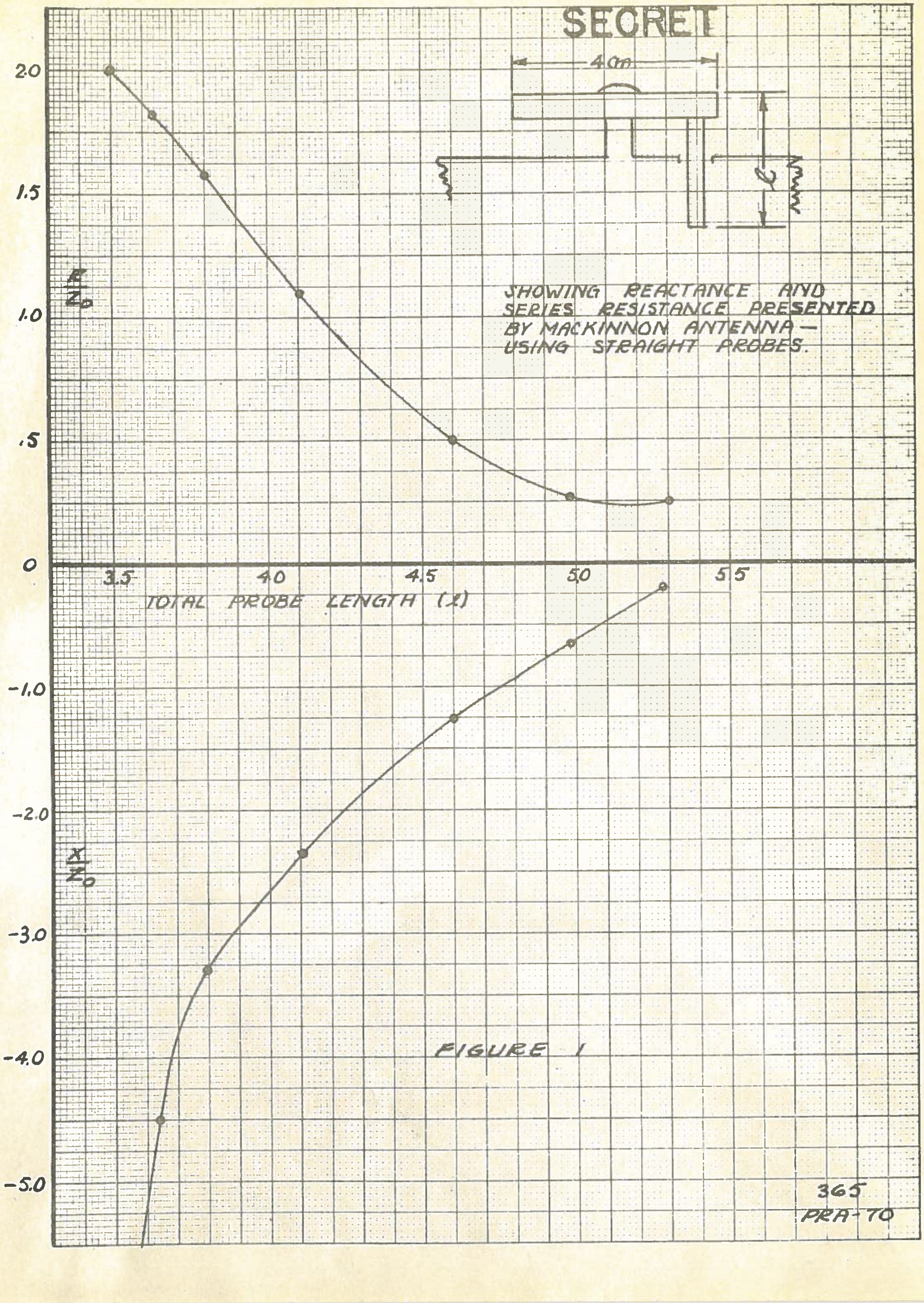
20-1-43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4 cm
ℓ
SHOWING REACTANCE AND SERIES RESISTANCE PRESENTED BY MACKINNON ANTENNA — USING STRAIGHT PROBES.
R/Z₀
X/Z₀
2.0
1.5
1.0
.5
0
-1.0
-2.0
-3.0
-4.0
-5.0
3.5
4.0
4.5
5.0
5.5
TOTAL PROBE LENGTH (λ)
365
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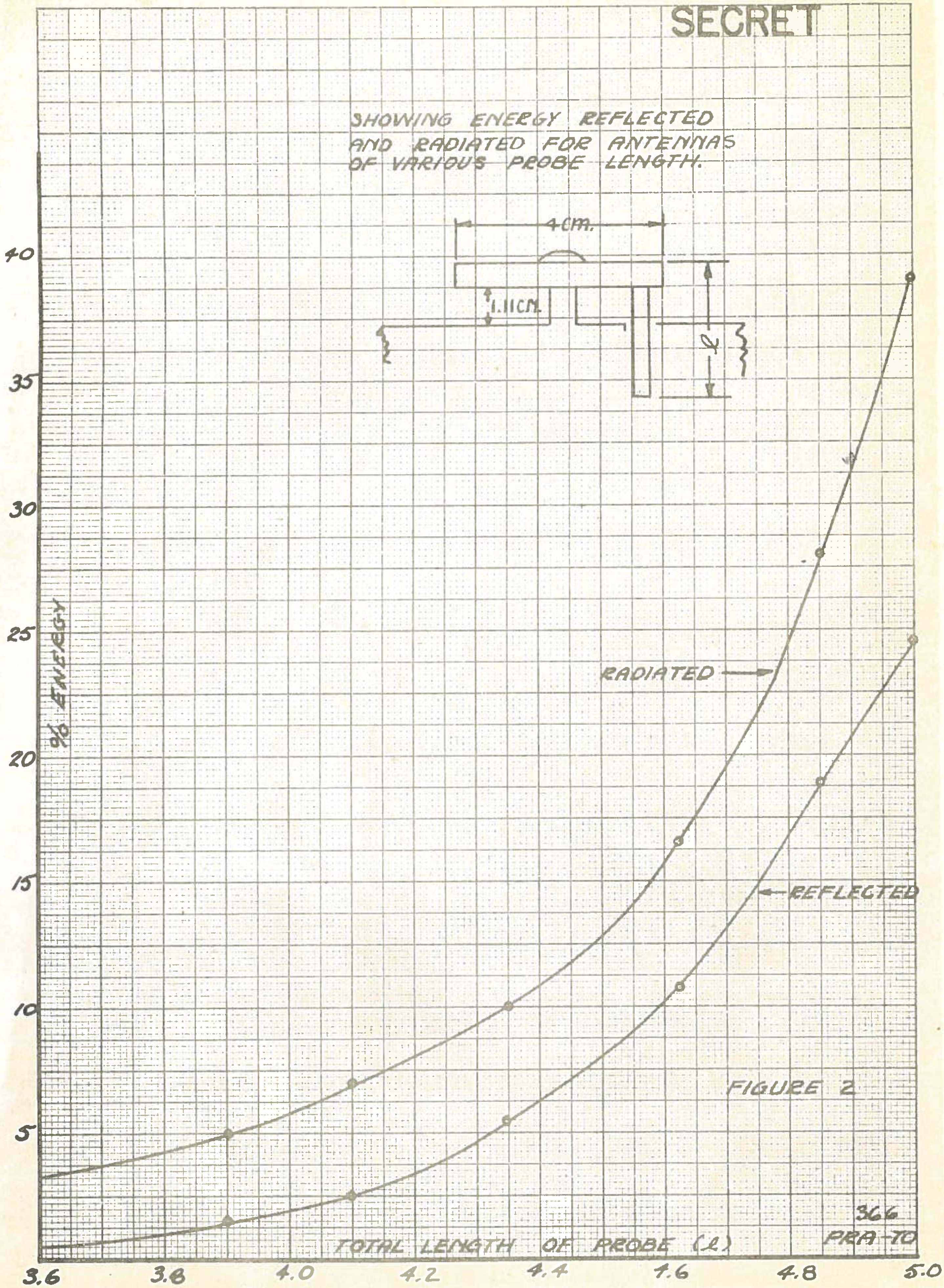
MADE IN U.S.A.
FIGURE 1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SHOWING ENERGY REFLECTED AND RADIATED FOR ANTENNAS OF VARIOUS PROBE LENGTH.
4 CM.
1.11CM.
ℓ
RADIATED
REFLECTED
FIGURE 2
% ENERGY
TOTAL LENGTH OF PROBE (ℓ)
40
35
30
25
20
15
10
5
3.6
3.8
4.0
4.2
4.4
4.6
4.8
5.0
366
PRA-70

SHOWING S.W.R. (VOLTAGE) IN FRONT OF NO.1, NO.2, ETC. ANTENNA. TOTAL LENGTH OF PROBE — 3.81 CMS. EACH ANTENNA RADIATES 5 % OF GUIDE ENERGY.

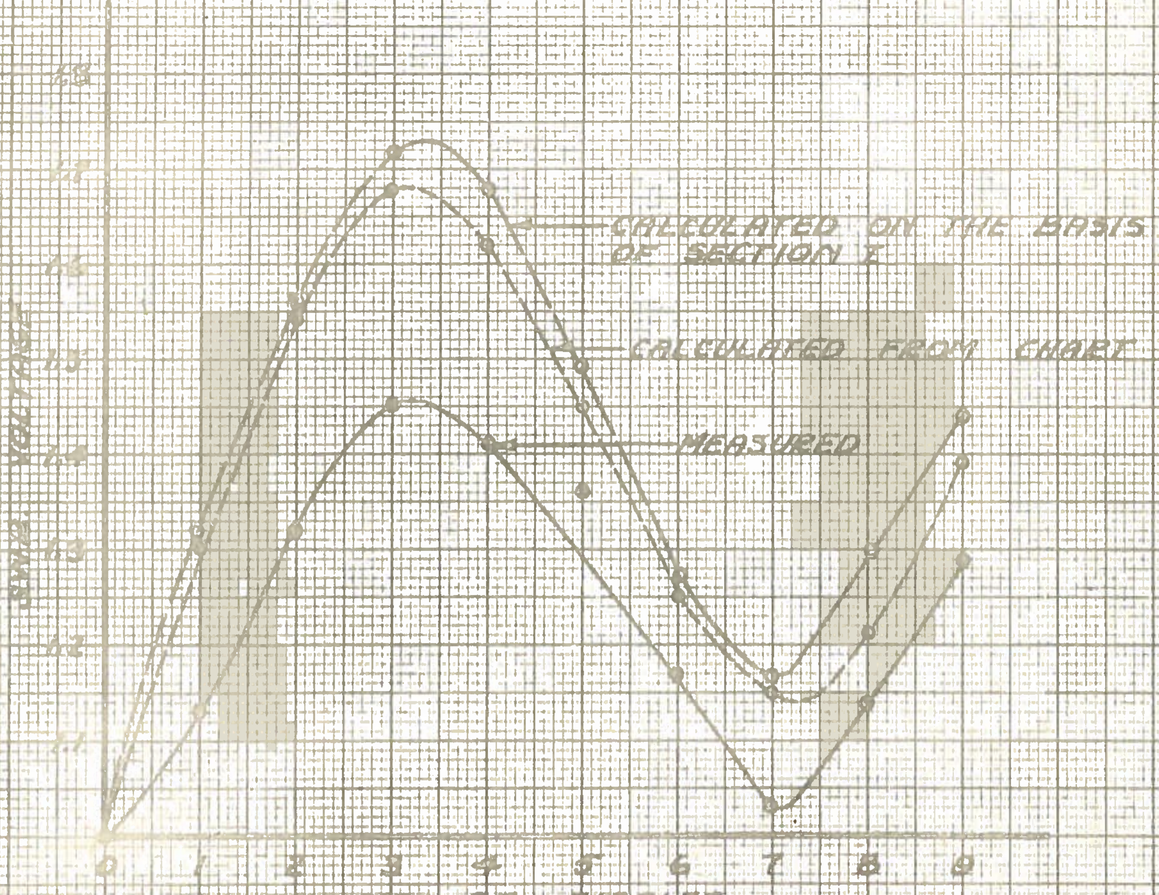

FIGURE 3

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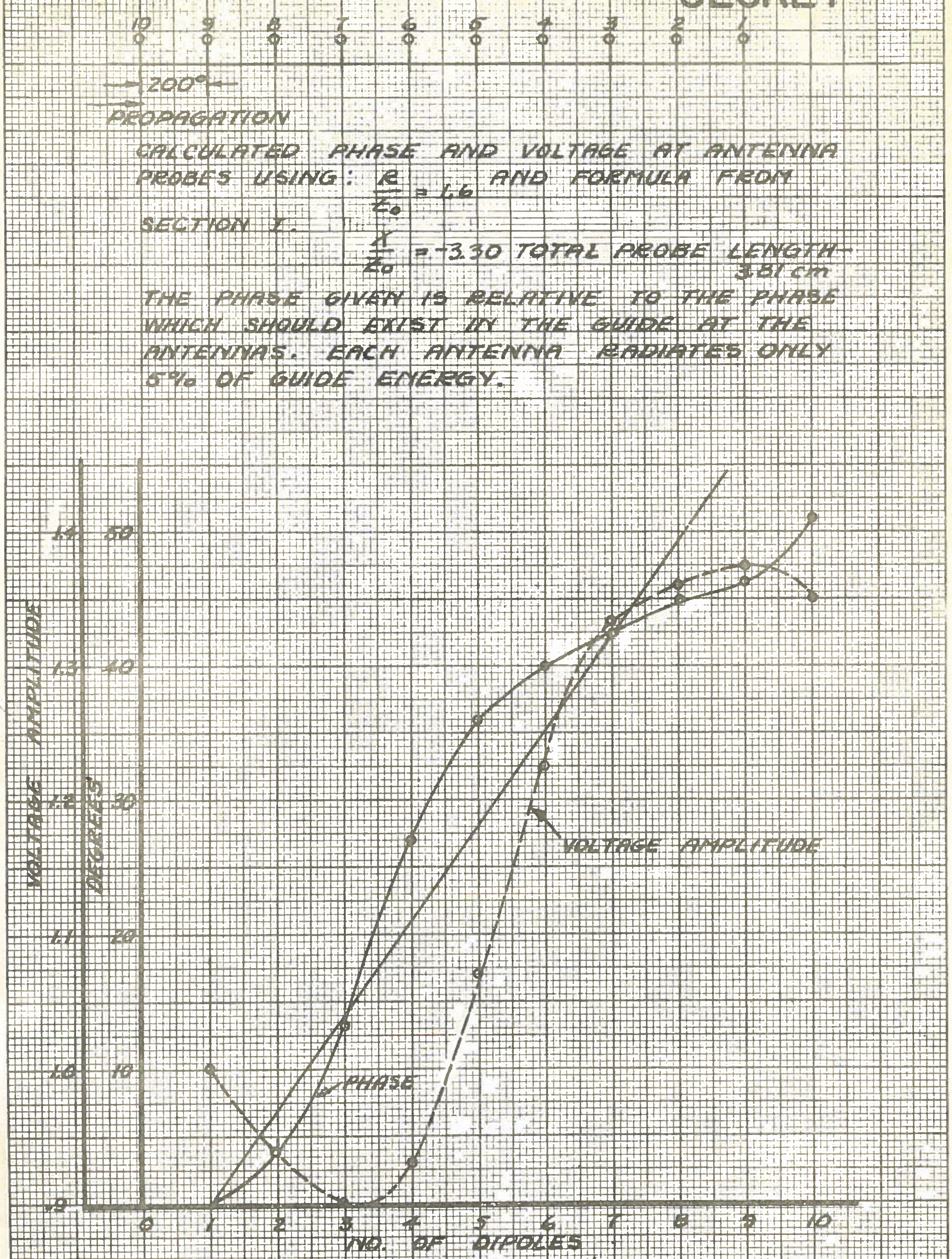

FIGURE 4 & 5

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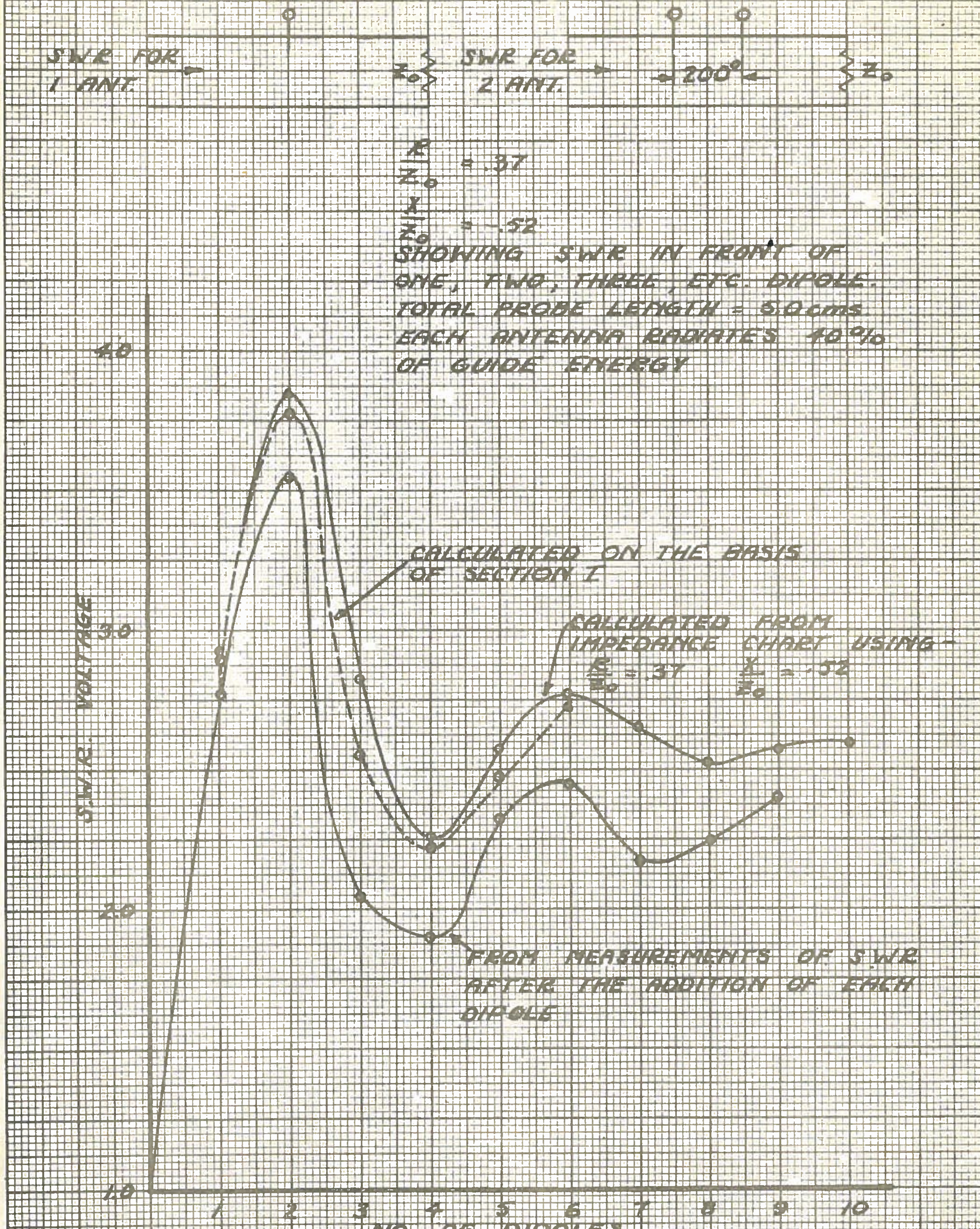

FIGURE 6

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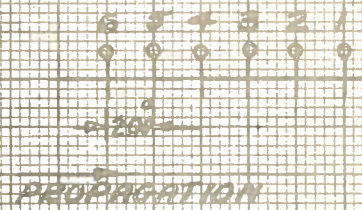

CALCULATED PHASE AND VOLTAGE AT ANTENNA PROBES USING :- $\frac{R}{Z_0} = .37$ AND FORMULA FROM

$-j\frac{X}{Z_0} = .52$ SECTION I.

THE PHASE GIVEN IS RELATIVE TO THE PHASE WHICH SHOULD EXIST AT THE POSITION OF THE ANTENNA IN A PURE TRAVELLING WAVE.

EACH ANTENNA RADIATES 40% OF GUIDE ENERGY WHEN INSERTED ALONE IN A GUIDE WITH A MATCHED TERMINATION.

FIGURE 7 & 8

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S.W.R. FOR 1, 2, — 8 DIPOLES OF AN 8-DIPOLE ARRAY CABLED SUCH THAT

#1 DIPOLE RADIATES 10% OF GUIDE ENERGY
#2 " " 9% " " "
#3 " " 8.3% " " "
#4 " " 7.8% " " "
#5 " " 7.3% " " "
#6 " " 6.9% " " "
#7 " " 6.5% " " "
#8 " " 6.0% " " "

CALCULATED FROM TRANSMISSION LINE CHART USING VALVES OF $\frac{R}{Z_o} + j\frac{X}{Z_o}$ DETERMINED BY EXPERIMENT.

FIGURE 9

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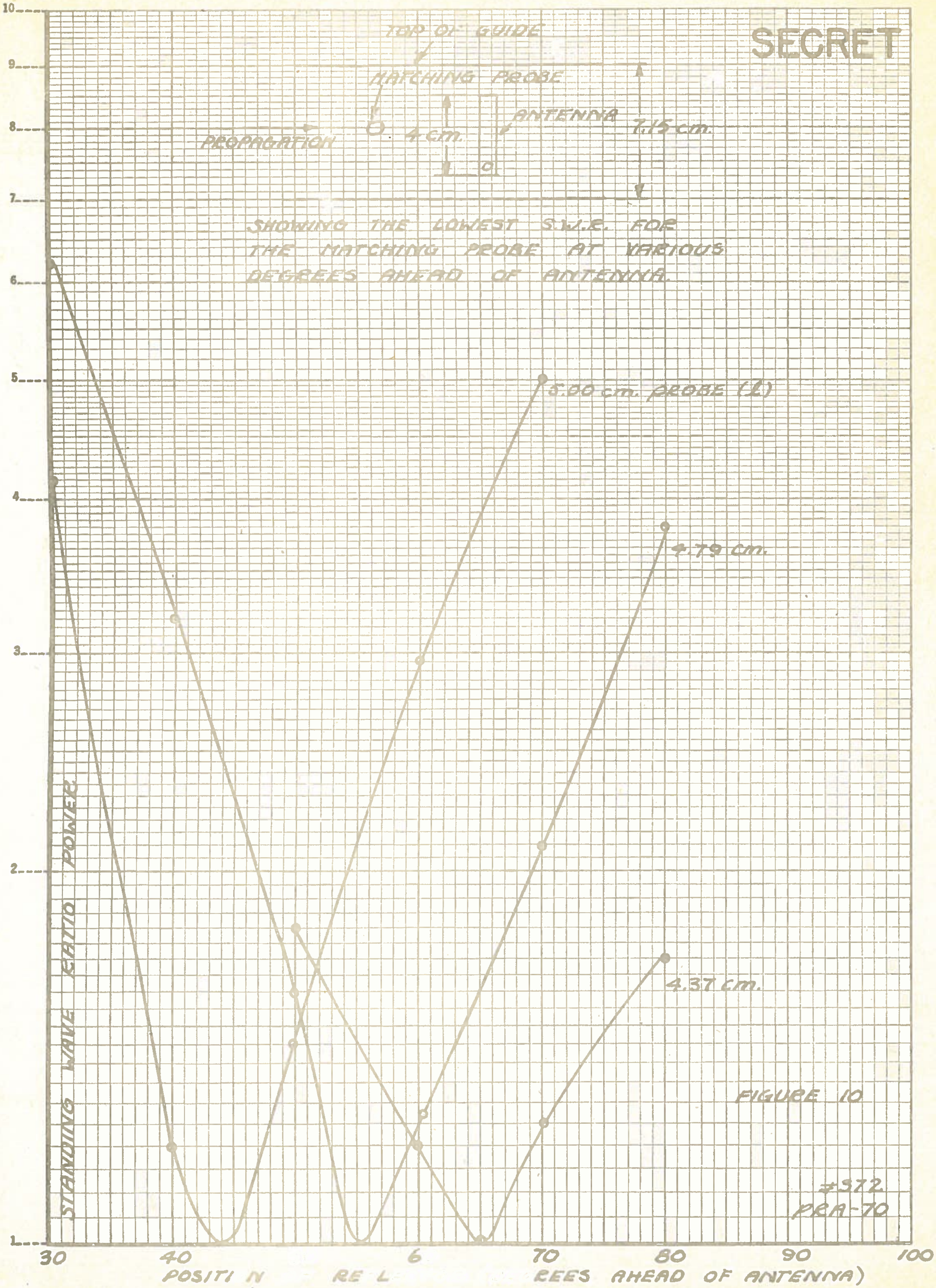
SECRET
TOP OF GUIDE
MATCHING PROBE
ANTENNA
PROPAGATION
4 cm.
7.15 cm.
SHOWING THE LOWEST S.W.R. FOR THE MATCHING PROBE AT VARIOUS DEGREES AHEAD OF ANTENNA.
5.00 cm. PROBE (L)
4.79 cm.
4.37 cm.
FIGURE 10
#572
PRA-70
STANDING WAVE RATIO POWER
POSITI N RE L REES AHEAD OF ANTENNA)
10
9
8
7
6
5
4
3
2
1
30
40
6
70
80
90
100

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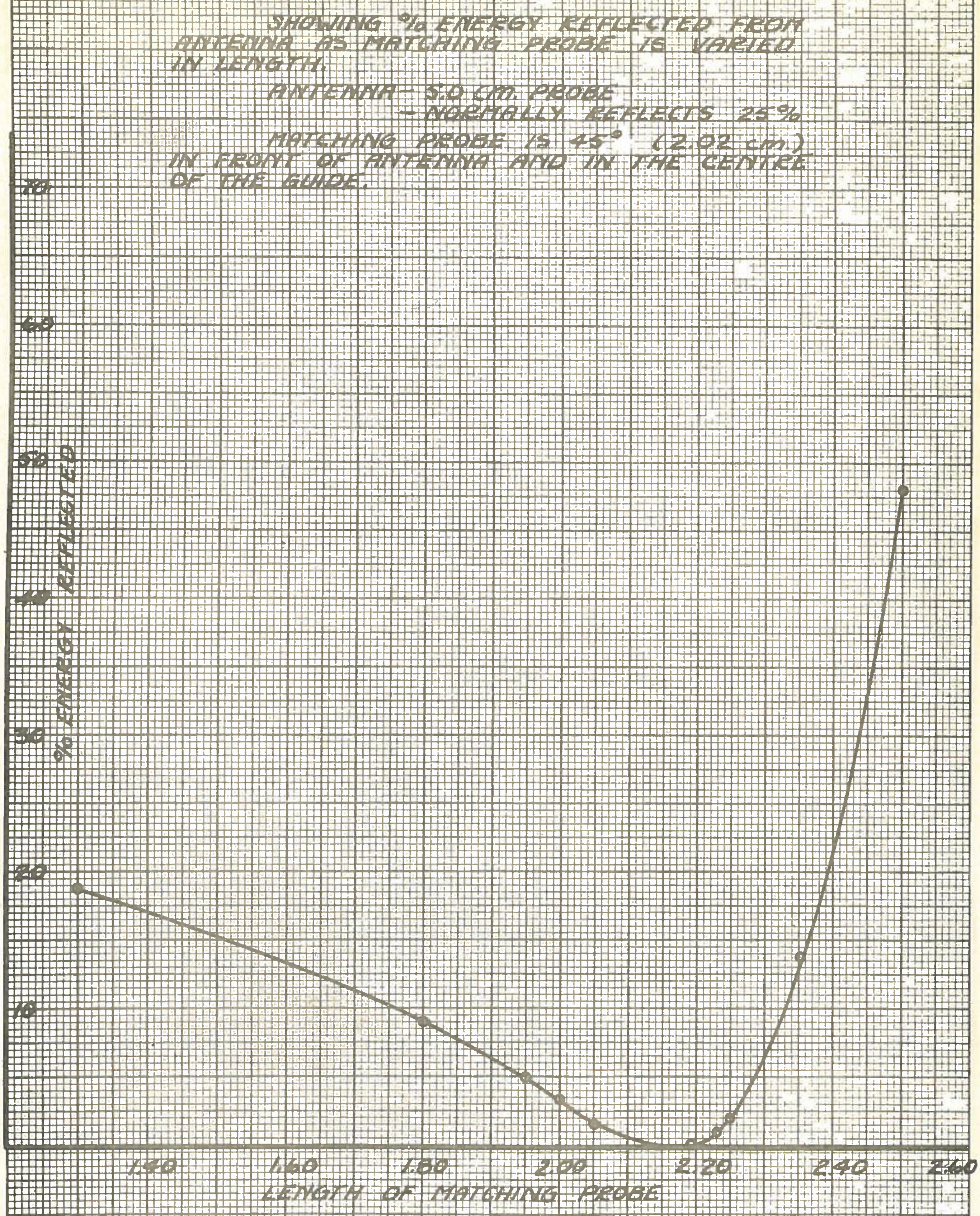

FIGURE 11

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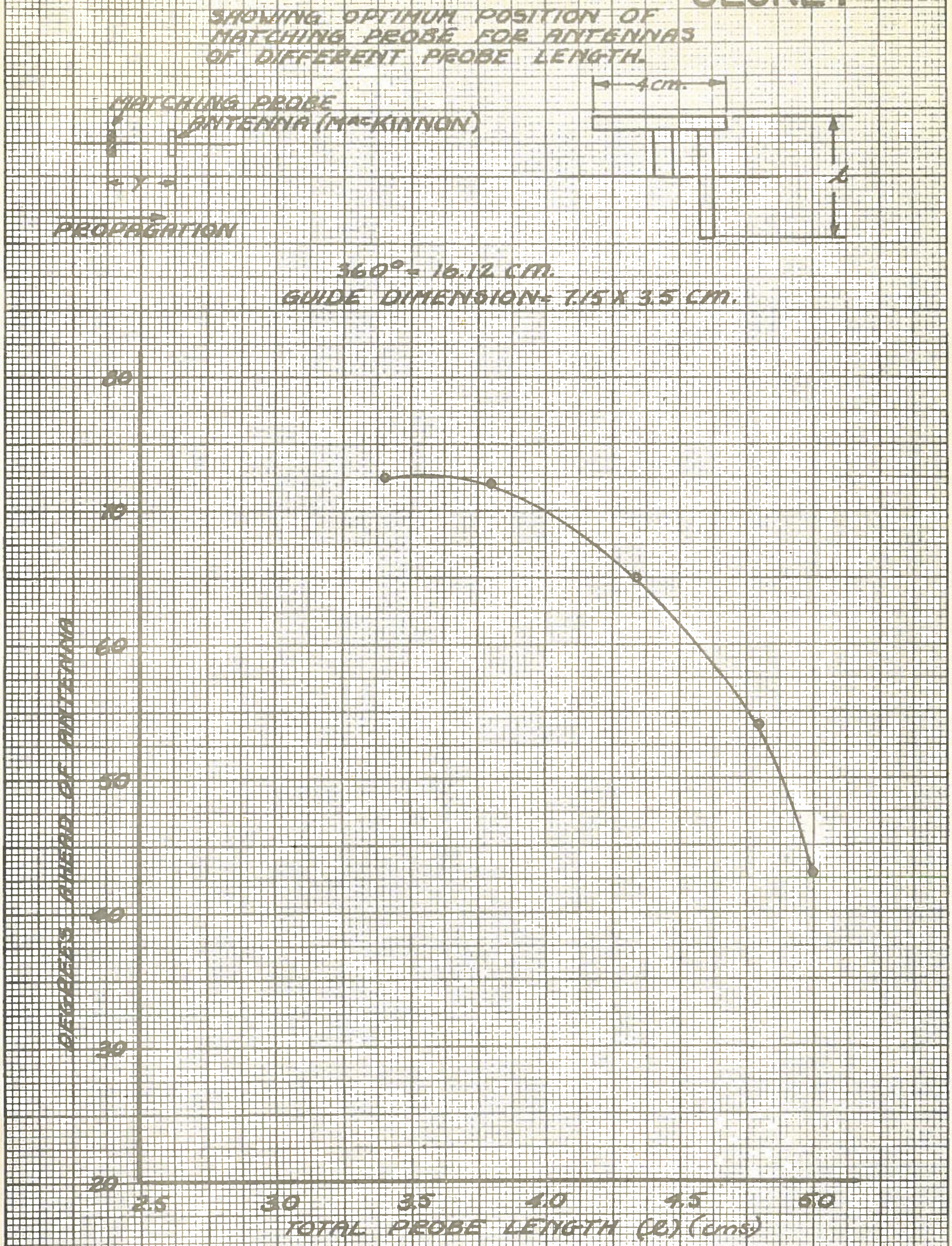

FIGURE 12

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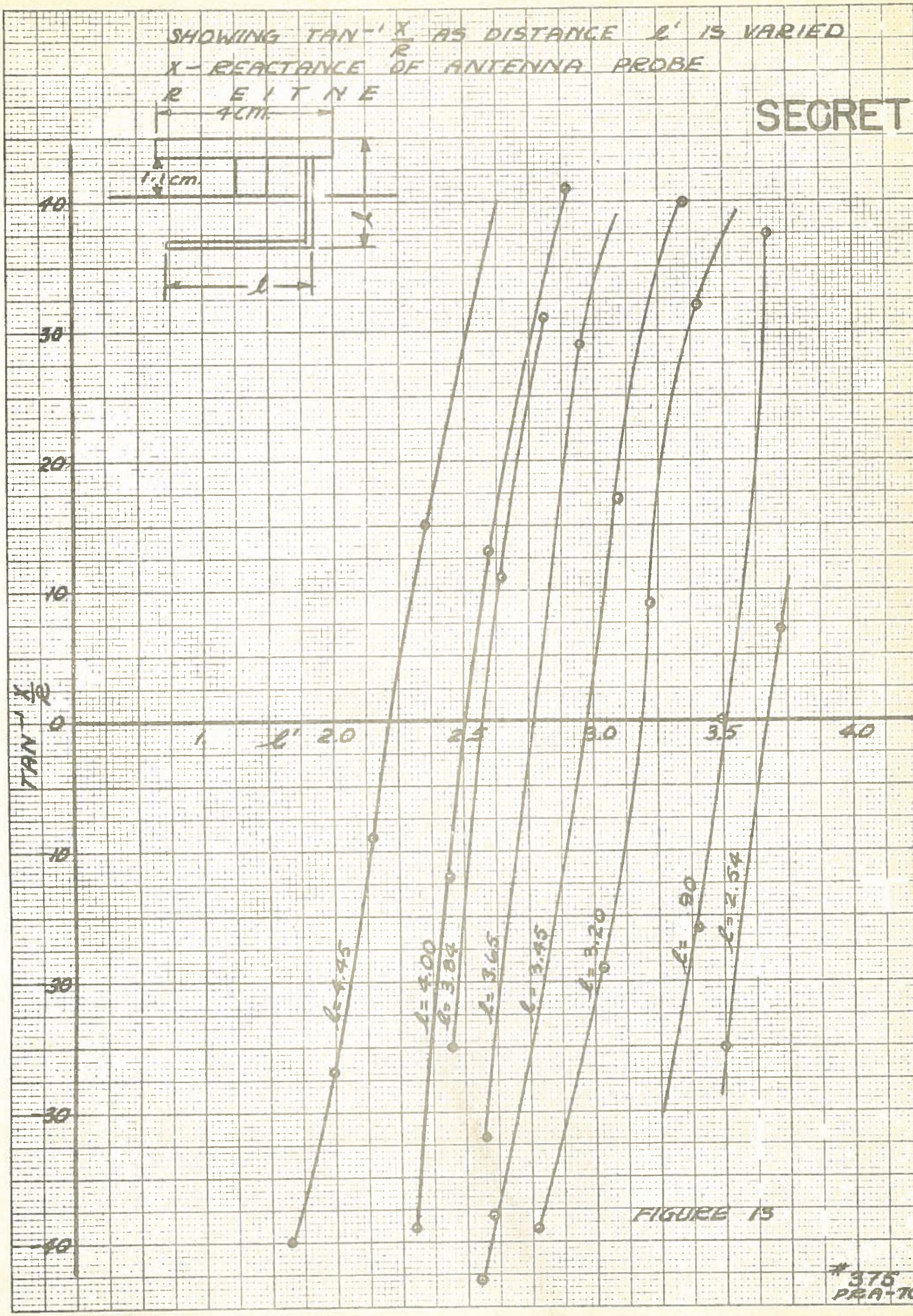

FIGURE 13

#375
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PLOT OF REACTANCE AND RESISTANCE ON SMITH DIAGRAM, AS THE RIGHT ANGLE LENGTH ℓ' IS CHANGED

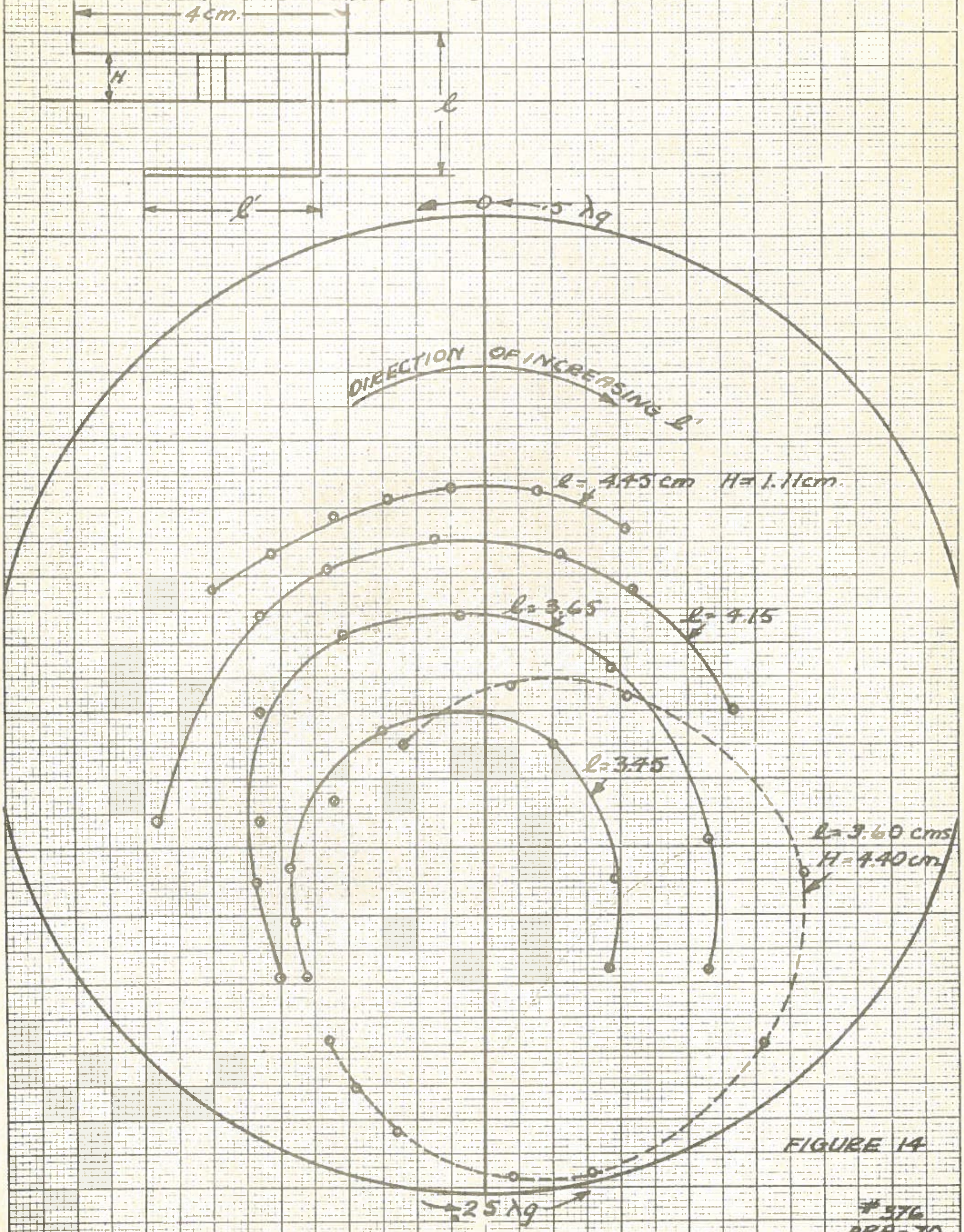

FIGURE 14

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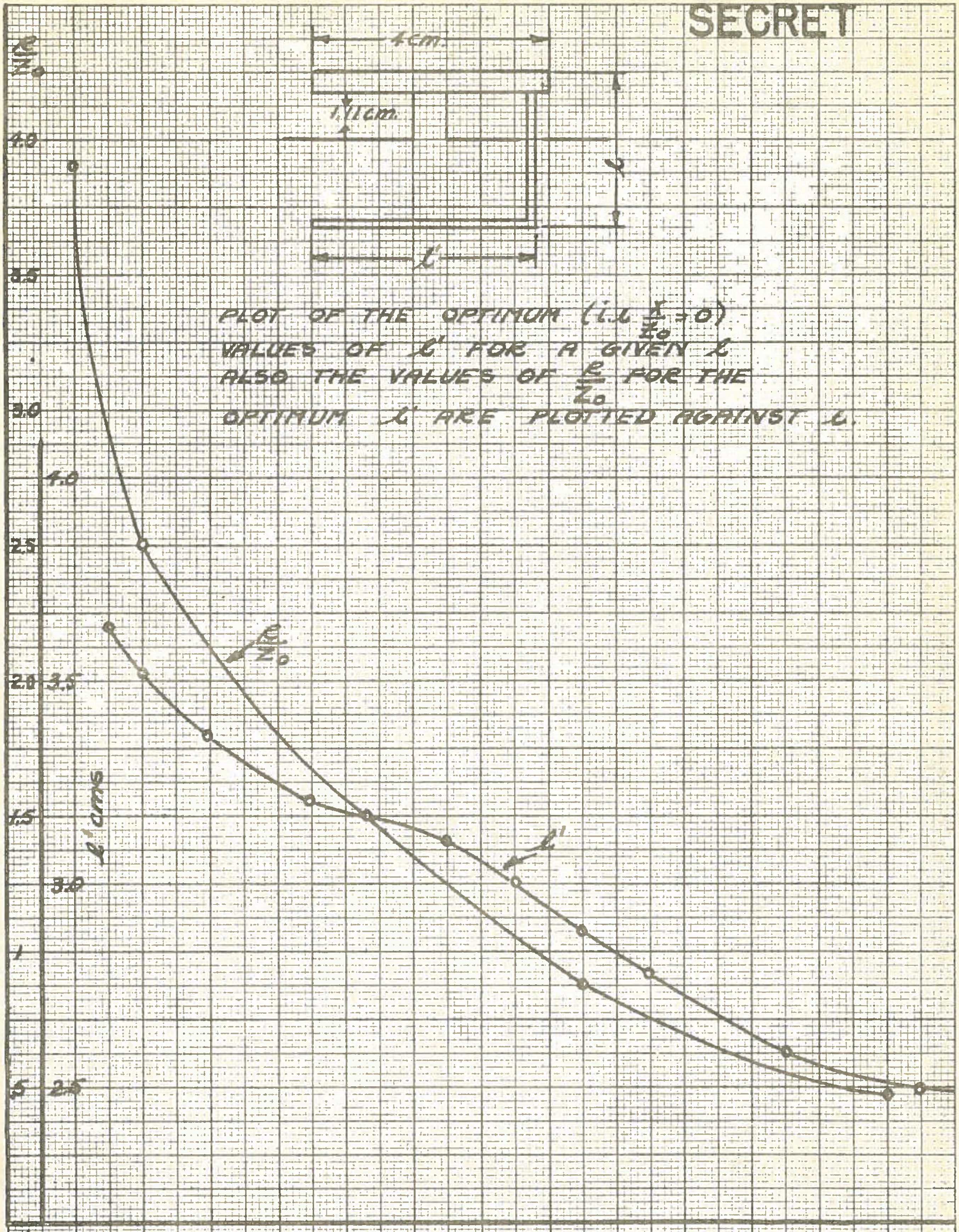

FIGURES 15 & 16

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