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TITLE

Ship Frame/Grillage Research Program-
Investigation of Finite Element Analysis Boundary Conditions

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SUMMARY

This report described work conducted as part of a comprehensive study of the ultimate strength of ships frames. The current focus is on frames subject to intense local lateral loads, such as ice loads. The work was begun with support from Transport Canada to study single frames. The US Coast Guard then joined in the project and enabled an expansion of the experimental and numerical analysis. To date 8 single frames have been tested, with two more to be tested. The experimental program has been further expanded with the support of the Ship Structures Committee, which is funding the experimental investigation of large grillages.

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SHIP FRAME RESEARCH PROGRAM - INVESTIGATION OF FINITE ELEMENT ANALYSIS BOUNDARY CONDITIONS

TR-2005-05

Claude Daley and Greg Hermanski

March 2005

OERC Report 2005-02
IOT Report TR-2005-05

Ship Frame/Grillage Research Program- Investigation of Finite Element Analysis Boundary Conditions

March 15, 2005

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SYMBOLS AND NOTATION

IACS	International Association of Classification Societies
TC	Transport Canada (Ship Safety)
DRDC	Defense Research and Development Canada (Atlantic)
USCG	United States Coast Guard
SSC	Ship Structure Committee (US-Canada Interagency Committee)
A	Frame x-sectional area
E	Young’s modulus
Et	Post-yield tangent modulus
b	height of the ice load patch
hw	height if the web
kw	area ratio
L	length of frame
P_{3h}	pressure causing collapse for case of 2 fixed supports
S	frame spacing
tw	thickness of web
Zp	plastic section modulus
Zpns	a non-dimensional modulus
σ_y	yield stress

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Acknowledgement

The work was made possible with the support of funding from a number or organizations sponsoring this research program. These include Transport Canada, Defense Research and Development Canada (Atlantic), the US Coast Guard, and the member agencies of the Ship Structures Committee. In particular we acknowledge the input and support from Mr. Jacek Dubiel (TC), Mr. Dave Stredulinsky (DRDC) and Mr. Alfred Tunik (USCG).

1. Introduction

This report described work conducted as part of a comprehensive study of the ultimate strength of ships frames. The current focus is on frames subject to intense local lateral loads, such as ice loads. The work was begun with support from Transport Canada to study single frames. The US Coast Guard then joined in the project and enabled an expansion of the experimental and numerical analysis. To date 8 single frames have been tested, with two more to be tested. The experimental program has been further expanded with the support of the Ship Structures Committee, which is funding the experimental investigation of large grillages.

The experimental program will provide empirical evidence to support the numerical and analytical investigations. The experiments will specifically explore the influence of frame geometry (for single frames), load position (central and end) and frame boundary conditions. Any single frame is joined laterally to neighboring frames through the shell plating. At its ends, the frame continues to the next bay, through a supporting stringer (or similar). The experiments examine the full range of frame support conditions. In the single frame tests, the frame ends are held rigidly (as rigidly as possible), while the sides are free. In the small grillage the ends are held rigidly, while to the side (of the central frame) there is plating and a similar frame. Attached to the plate beside the side frames, there is a heavy bar that is designed to approximate additional frames. In the large grillage, both the side and end conditions (for the central frame) are as realistic as possible.

In this report, a series of finite element analyses of the various test conditions are presented. This is being done as part of the preparations for the grillage tests. Additionally, these analyses help to clarify the behaviour of frames and show the likely levels of reserve strength in actual frames.

Frame design and regulation is generally done singly. This means that the frame capacity in rules is determined on the basis of equations that reflect the behaviour of a single frame, treated as a beam. While most ship structural rules consider on the elastic behaviour of ship frames, the new IACS Unified Requirements for Polar Ships [2], use plastic limit state behaviour. One of the aims of this study is to determine the validity of the limit state equations employed in Polar Rules. The report builds upon the work presented in [3].

2. Background

The Ocean Engineering Research Center and the Institute for Ocean Technology are conducting a research program to study the plastic behavior and ultimate limit states of ship frames and grillages subject to lateral loads. This work is closely related to the development of the new IACS Unified Requirement for Polar Ship Construction. The Polar Rules contain limit state equations for ship frames subject to lateral loads (ice loads)[3]. The limit state equations were derived on the basis of energy methods (plastic work)[4]. This research program is aimed at validating the limit state equations for single frames, determining the range of the validity, and exploring the way frames interact in grillages. The problem under study also applies to cases of hydrodynamic impact and other types of collisions. As a result, the research applies to most ship structures and many types of offshore structures.

Ship structural design is changing. Traditionally, ship structures have been designed using 'working stress' methods. This approach considers the elastic stresses in a structure and sets limits on stresses. Consequently, the elastic properties of structures (e.g. moment of inertia, elastic section modulus) are controlled and optimized. Unfortunately, this approach does not assure that structures behave adequately in overload situations. Consider the two frames sketched in Figure 1. The two frames have the same elastic section modulus, though all other geometric measures (area, inertia) are different. The two frames would be considered equally satisfactory in any 'working stress' design. However, they have quite different plastic capacities. Figure 2 illustrates the different plastic behaviours of the two frames. The flat bar frame has greater initial capacity, followed by a greater reserve and more stable behaviour. The flat bar stays upright while the tee section folds over under high loads.

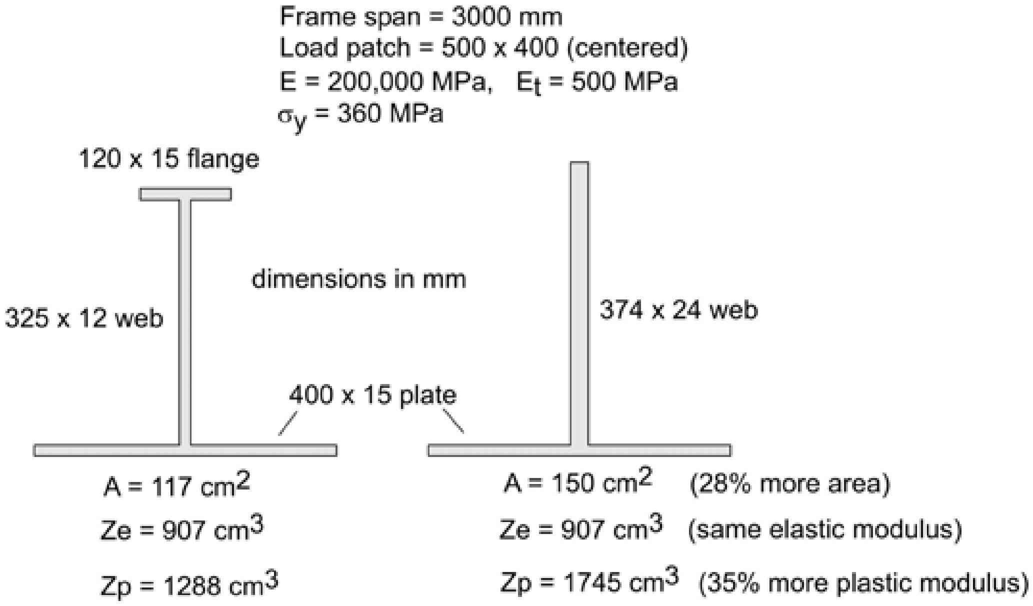


Figure 1. Two frames with equal elastic modulus.

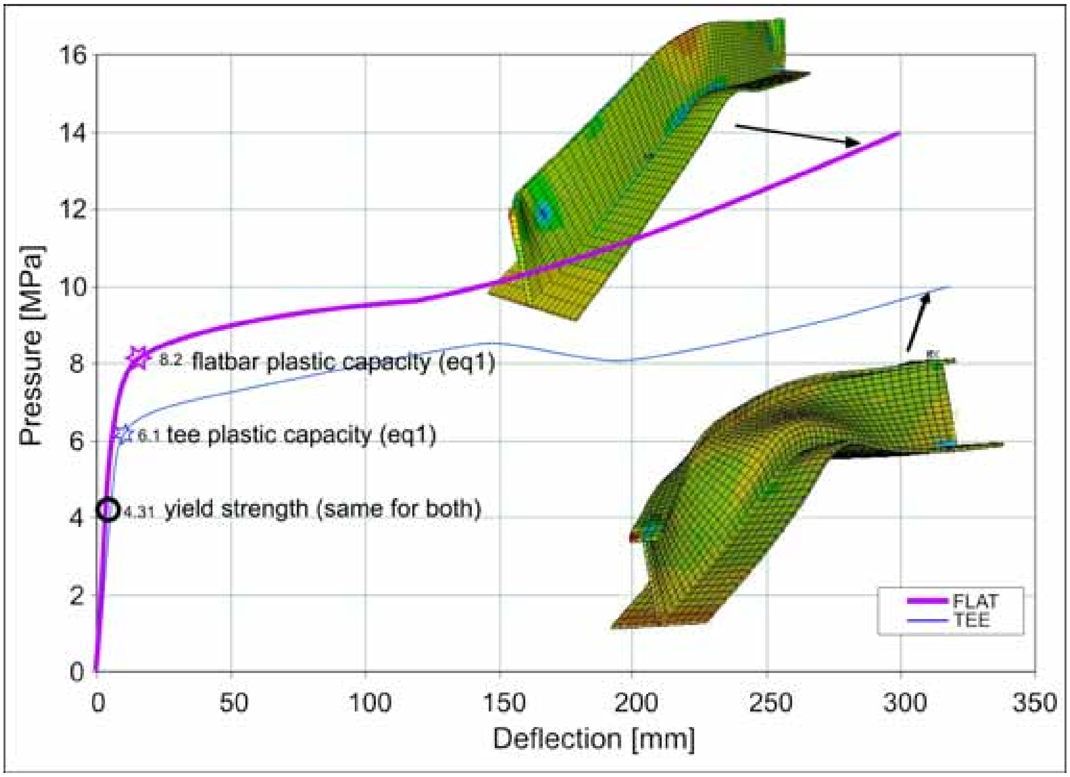


Figure 2. Comparison of load-deflection behaviour of two equal modulus frames.

Also shown in Figure 2 are the estimated plastic collapse values for the two frames. These values are found from equation (1), derived as part of the new IACS Polar Rules. The terms are defined in the nomenclature. A full explanation is given in [4,5]. The above behaviour has broad implications. Clearly the elastic section modulus is not a measure of initial strength or ultimate reserve.

$$P_{3h} = \frac{(2-kw)+kw\cdot\sqrt{1-48\cdot Zpns\cdot(1-kw)}}{12\cdot Zpns\cdot kw^{2}+1}\cdot\frac{Zp\cdot\sigma_{y}\cdot 4}{\left[S\cdot b\cdot L\cdot\left(1-\frac{b}{2\cdot L}\right)\right]} \qquad (1)$$

The frames in Figure 2 have identical modulus, but not the same weight. From one perspective the figure shows the value of considering plastic capacity, but at the cost of steel weight. In this way, conventional rules result in the flatbar being doubly penalized, once by not recognizing its superior linear and reserve performance, and secondly by adding steel weight. The next comparison shows an example where the frame weights are identical. Figure 3 shows two frames with the same weight. The tee section has a very thin web, at the limit of allowable thickness (for buckling). The flatbar actually exceeds the usual aspect ratio, but still behaves acceptably. Figure 4 compares the load-deflection curves for the two frames. Also shown are the nominal yield capacities (load which would case yield stress in simple bending) and the estimated capacity according to equation (1). Note that the flatbar has a lower plastic modulus, a lower elastic modulus, but is both initially and ultimately stronger than the slender tee. This demonstrates a number of important points. The first is that elastic properties may have little relation to structural behaviour. Second is the even the plastic section modulus may be a poor indicator of capacity, especially if plastic bending is not the dominant structural mechanism. Equation (1) inherently combines the bending and shear responses into a limit capacity, and in this case shows excellent agreement with the finite element results.

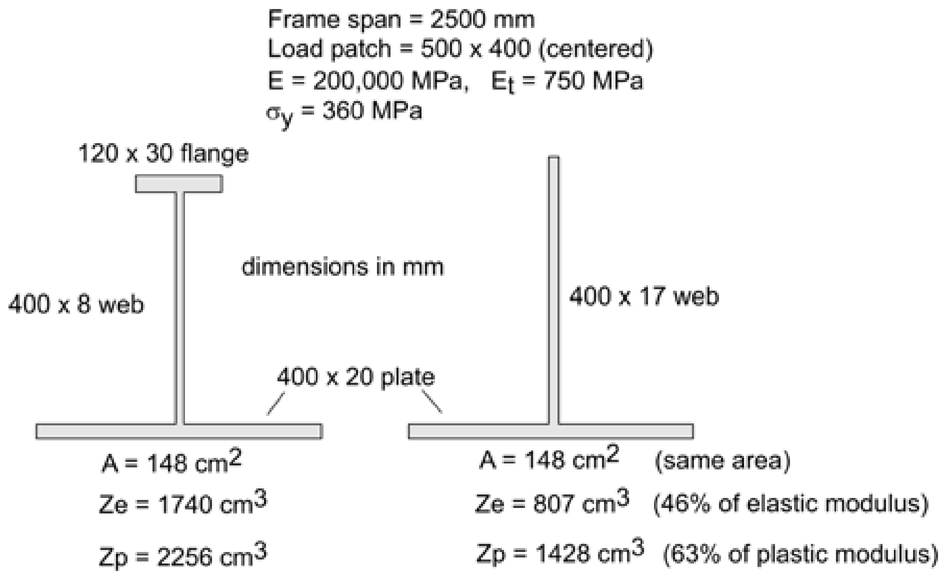


Figure 3. Comparison of load-deflection behaviour of two frames.

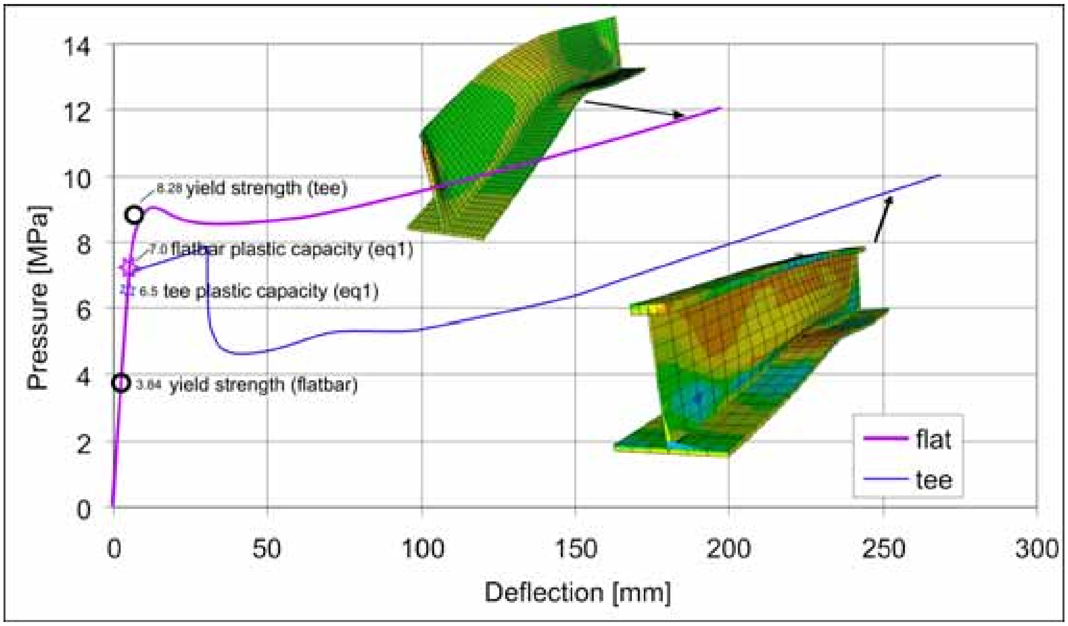


Figure 4. Comparison of load-deflection behaviour of two equal area frames.

Figure 5 sketches the typical load deflection pattern that we tend to have in laterally loaded frames. The deflection is the maximum deflection of the web under at the plate-web connection. The UR limit state (equation 1) represents a capacity comparable to that labeled “mechanism 1” in Figure 5. Prior to mechanism 1, the load-deflection curve is essentially linear and follows the slope of the original elastic trend. Yielding occurs well before mechanism 1. This is followed by the expansion of the plastic zone, during which stress redistribution takes place. Once the plastic zone fills one or more critical cross sections, a plastic mechanism forms, allowing large and permanent deformations to occur. Mechanism 1 might be called ‘collapse’, though this term is not exactly correct. Subsequent to mechanism 1, while the frame is ‘collapsing’ in bending, membrane forces tend to rise and support the growing load. Further along this curve, additional mechanisms can occur, including buckling and fracture. In the analyses in this report, some frames experience local buckling after the first mechanism. Ideally the frame would exhibit monotonically increasing capacity, even as the permanent deflections grow very large.

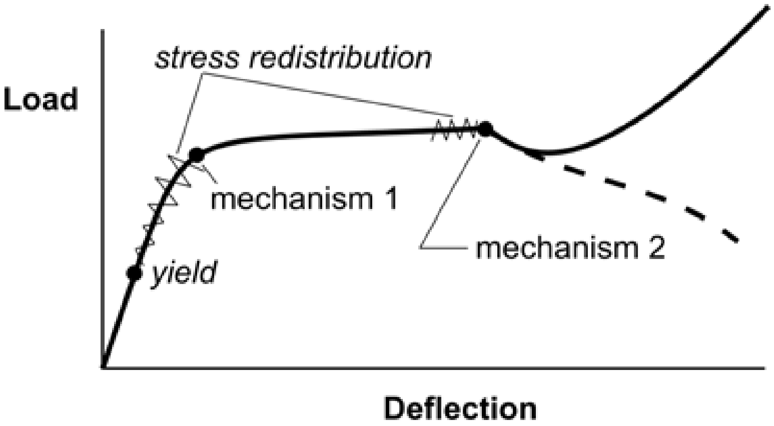
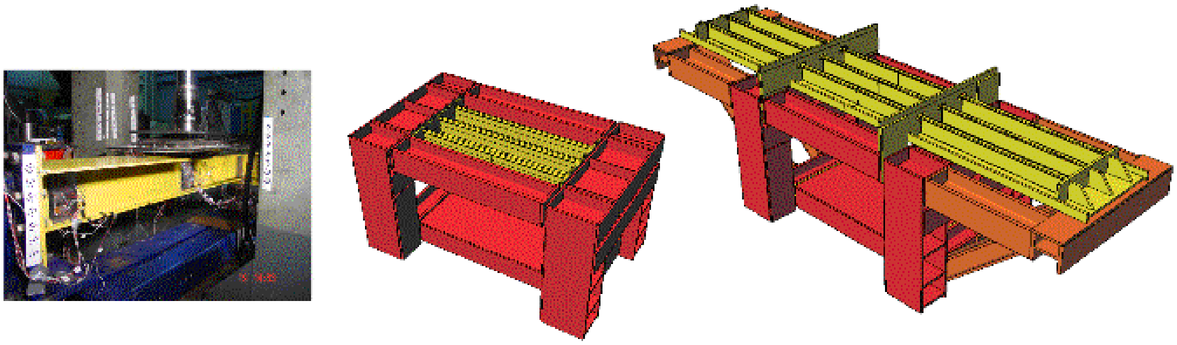


Figure 5. Idealized load-deflection curve for a frame.

3. Experimental Program

The program consists of several experimental phases (see Figure 6), each supported by numerical investigations. The first phase is the testing of a variety of single frames. The second phase is the testing of a grillage of three frames (small grillage). The third phase is the testing of a grillage of nine frames (large grillage).



Single Frame Tests

Small Grillage Tests

Large Grillage Tests

Figure 6. The three experimental phases in the ship frame research program

The three experimental phases support each other in important ways. Ships frames are always part of a structural system (a grillage). However, design and regulation normally consider single frames. The UR limit state equations were derived by considering a single frame in isolation. One of the aims of the current research is to understand the influence of boundary conditions on ship frame behaviour, especially in the plastic region. Subsequent reports will present the experimental results. In preparation for the first grillage tests, this report will examine the expected grillage behaviours and compare the effects of different boundary conditions.

Figure 7 shows the grillage support frame, with a single frame in place. The support frame will be used to test two final single frames, the small grillages and the large grillages. Figure 8 shows the first of the small grillages ready for application of the strain gauges.

Figure 7. Preparations for testing final single frames in the grillage support frame.

Figure 8. Small grillage ready for application of strain gauges.

4. Finite Element Modeling

The analysis presented here was performed using the finite element analysis program ANSYS [1]. Other than those specifically indicated, shell elements (shell-181) were used to model the frames.

The adopted shell element is suitable for analyzing thin to moderately thick shell structures. It is a 4-node element with all six degrees of freedom at each node: three translations (in the x, y, and z directions), and three rotations (about the x, y, and z-axes).

The element is well suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness is accounted for in the nonlinear analysis. In the element domain, both full and reduced integration schemes are supported. In addition, the element accounts for follower (load stiffness) effects of distributed pressures.

The geometry, node locations, and the coordinate system for this element are shown in Figure 9. The element formulation is based on logarithmic strain and true stress measures. The element kinematics allow for finite membrane strains (stretching), which makes it suitable for the present analysis. However, the curvature changes within a time increment are assumed to be small.

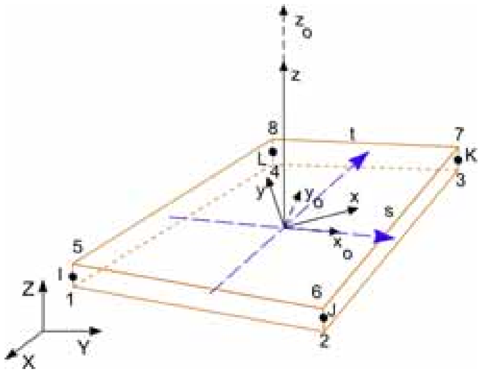


Figure 9. SHELL181 Geometry (from ANSYS help files)

Figure 10 sketches the steps involved in the finite element analysis. These steps can be performed interactively, or by creating a text input file that includes the appropriate commands and data in sequence. Appendix A includes a listing of a typical ANSYS input file. Post processing of the results was performed interactively. The load-deflection data was exported for plotting. The results were extracted and imported in a spreadsheet, and the plots were generated.

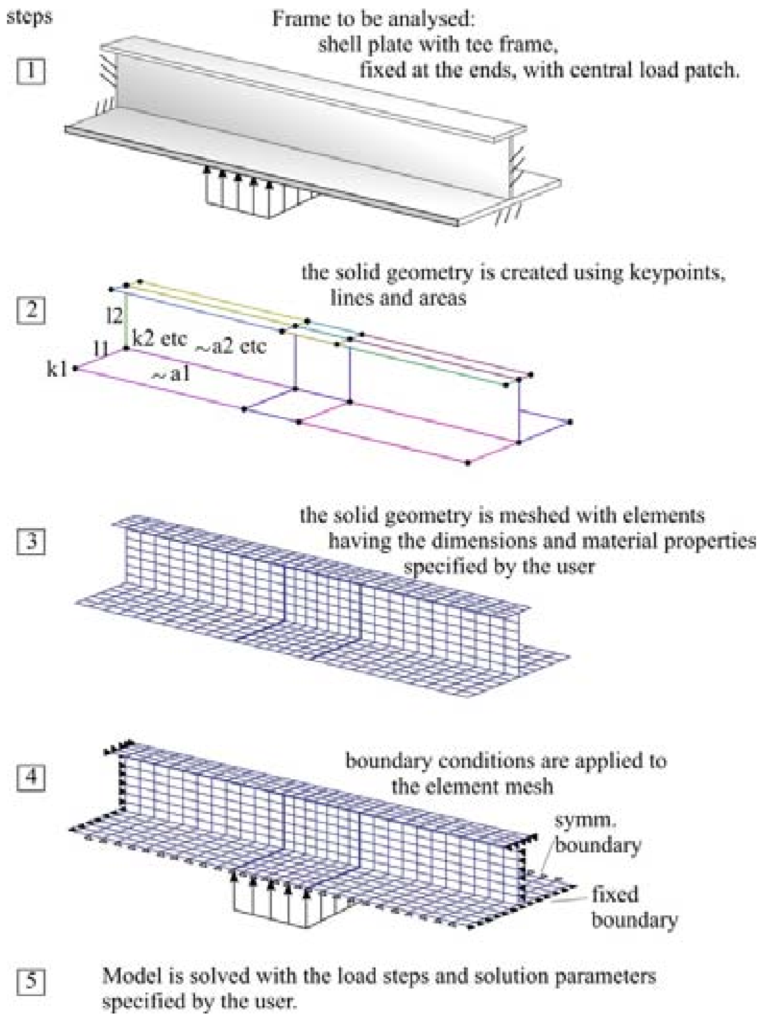


Figure 10. Outline of finite element analysis steps.

5. Frame Analysis

This section presents a variety of different finite element analyses. In all cases the frame (or frames) had the dimensions shown in Figure 11. In all cases the frames were 2m long with a 500mm load patch. Figure 12 shows the geometry of the single frame analysis (the standard case). Figure 13 shows the small grillage case. This provides the central frame with more realistic side boundary conditions. Figure 14 shows the large grillage. This provides realistic side and end boundary conditions for the central frame. Figure 15 shows a long frame, where the end boundary conditions are correct, but there are no side frames. All the configurations are shown in Figure 16.

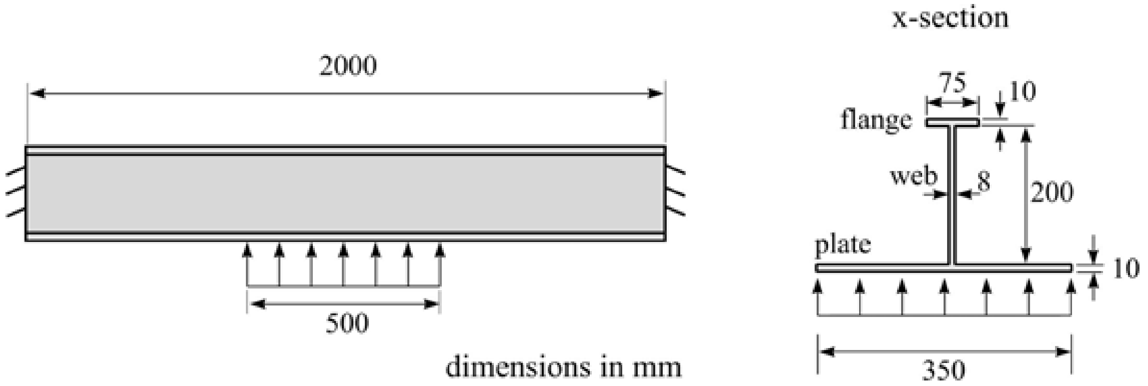


Figure 11. Basic Frame Dimensions

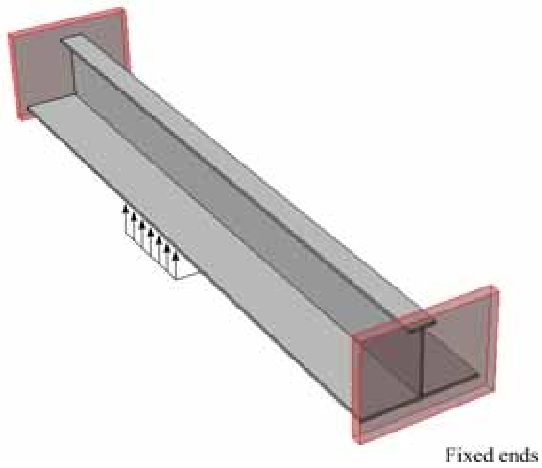


Figure 12. Single frame

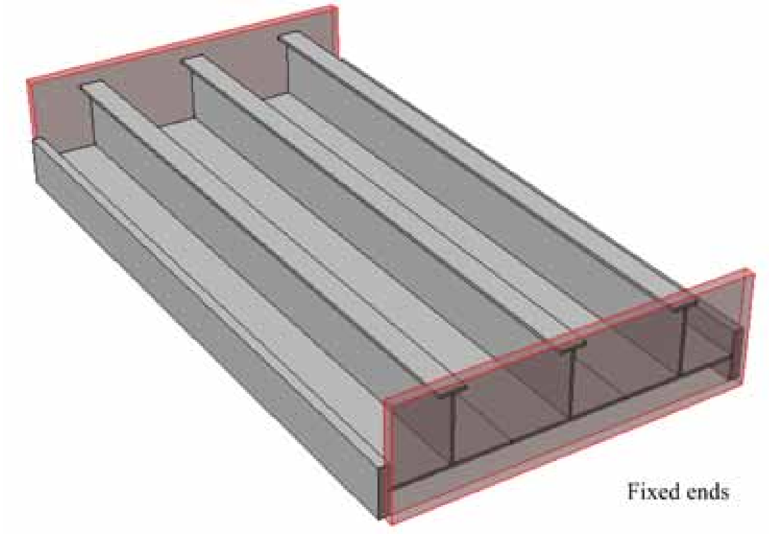


Figure 13. Small Grillage

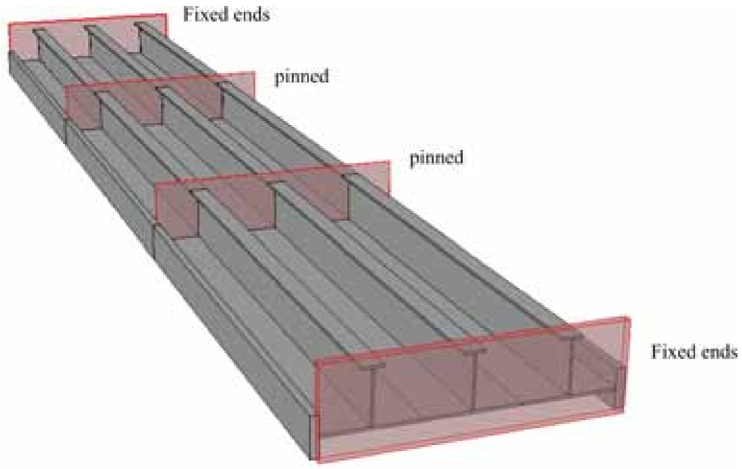


Figure 14. Large Grillage

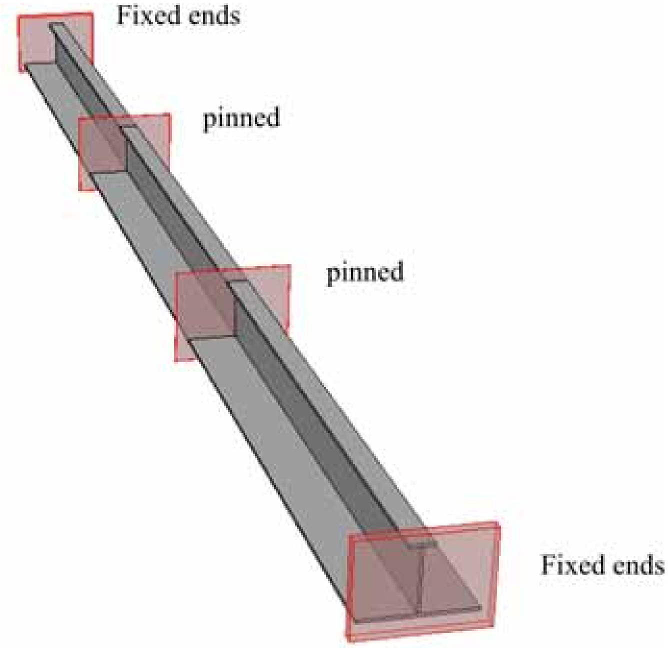


Figure 15. Long Single frame

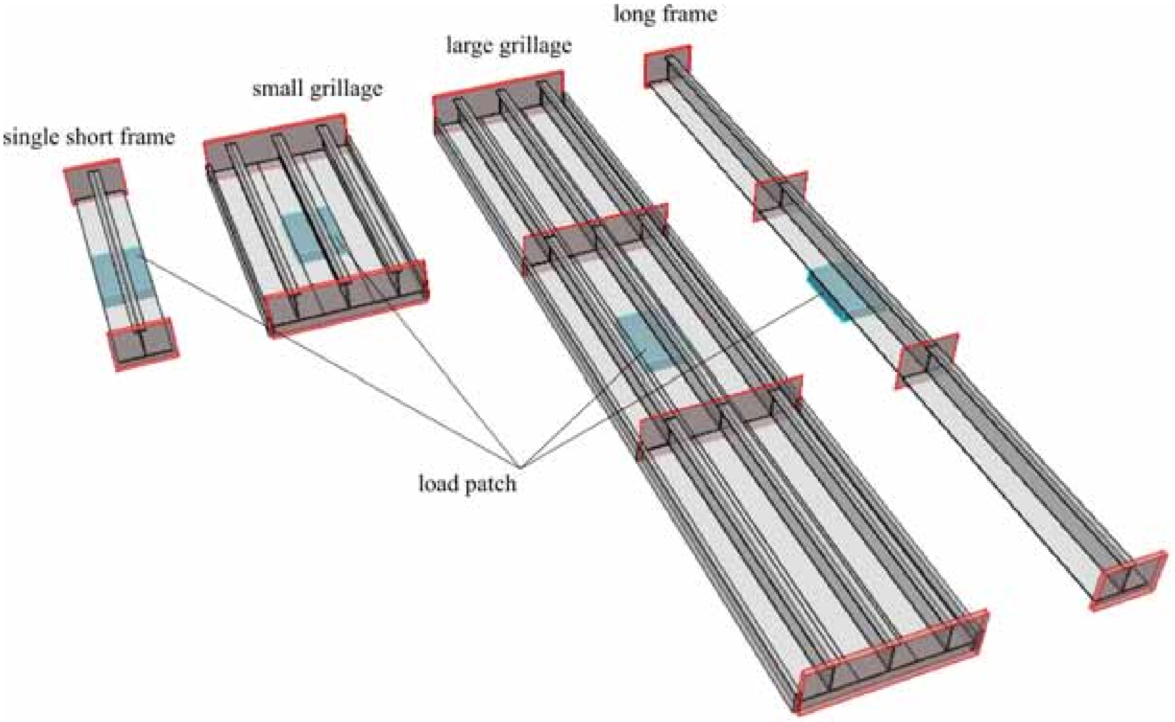


Figure 16. All four frame arrangements.

For the single frames, the edges of the section were constrained to move in the vertical direction only (symmetric boundary). This assumption reflects an actual ship frame with top plate connected at both ends of the section, thus providing some bracing to the top flange. The imposed boundary conditions are shown in Figure 17.

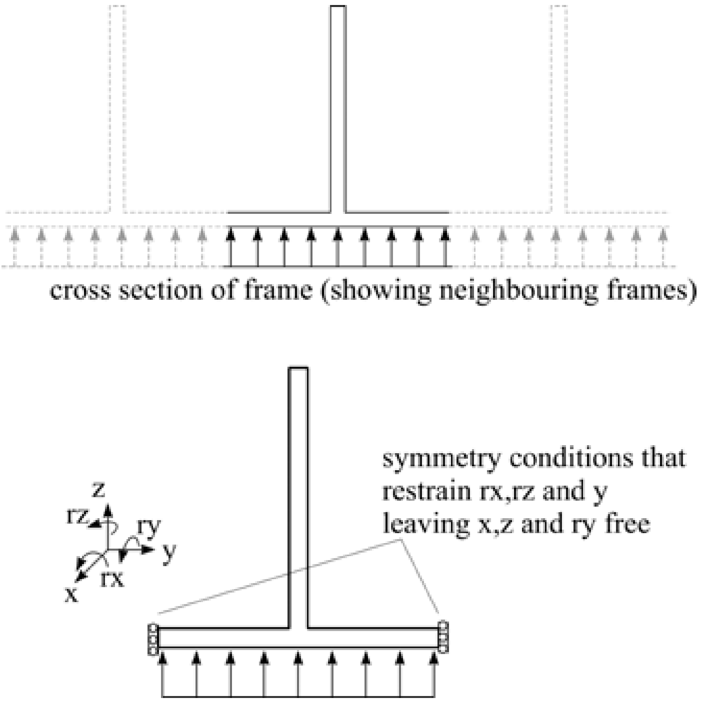


Figure 17. Sketch of the lateral boundary conditions on the frame.

The finite element meshes of the four configurations are shown in Figure 18. The lower right image shows the support condition icons as well. All models were defined with an input file (see Appendix A for an example.) All models made use of the same element and had similar mesh density. Figure 19 shows the deformation of the singe frame. Figure 20 shows the small grillage. Figure 21 shows the large grillage and Figure 22 shows the long single frame. It is interesting to note that the single and small grillage frames fall over (though only at large plastic deformation), while the long grillage and single long frames stay upright. The more compliant end conditions on the long frames presumably reduce the flange tension (ends pull in elastically) and help to stabilize the flange. This is only one possible explanation. The experiments will likely have both more a concentrated load and small imperfections that will likely result in web folding. It will be interesting to see.

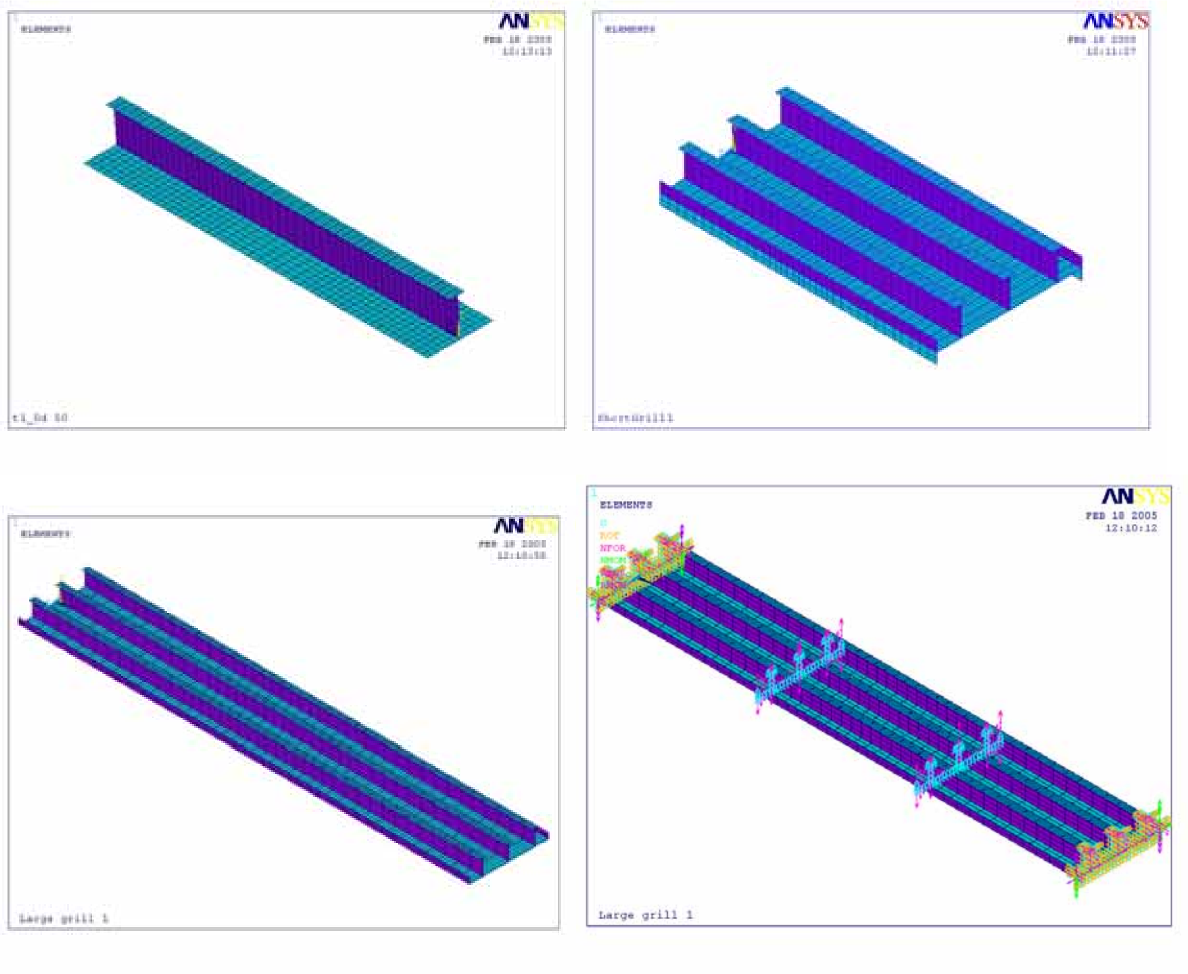


Figure 18. Finite element models of the four configurations.

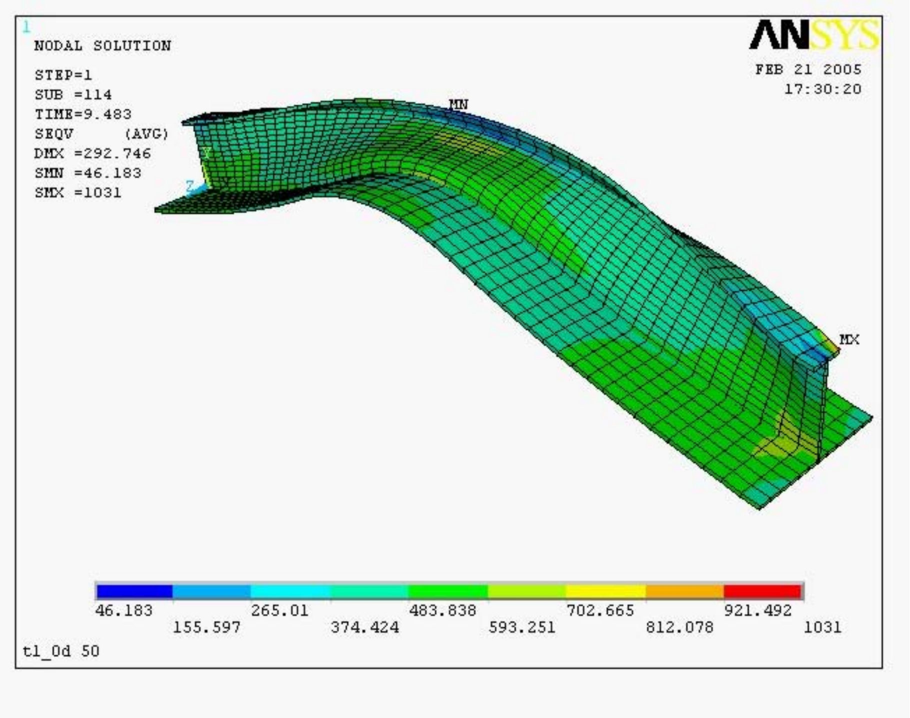


Figure 19. Single frame model with the load patch at a pressure of 9.483 MPa.

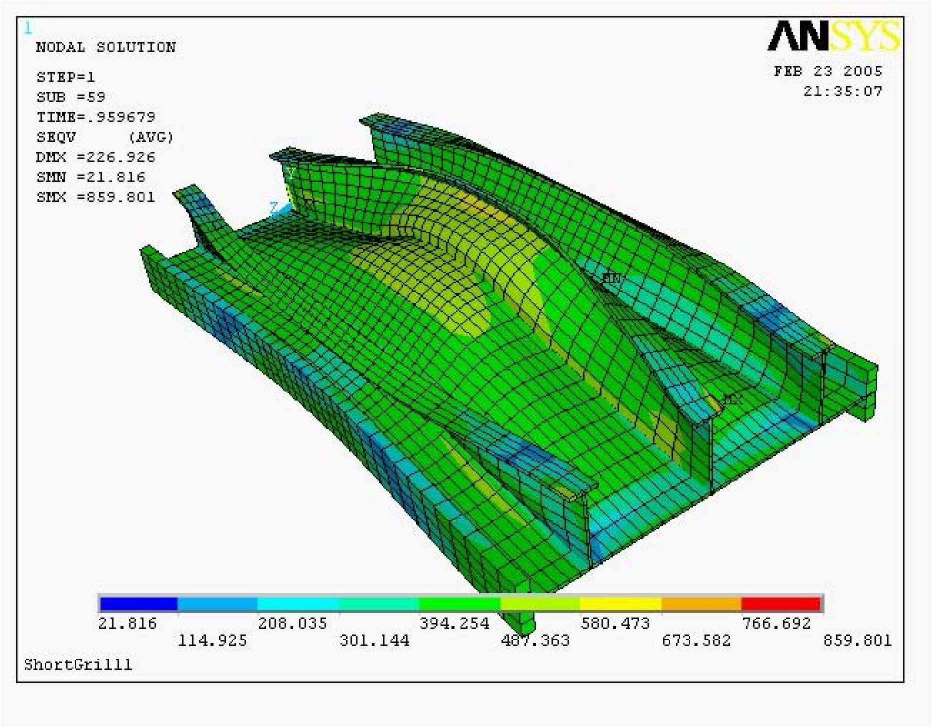


Figure 20. Small Grillage model with the load patch at a pressure of 19.2 MPa.

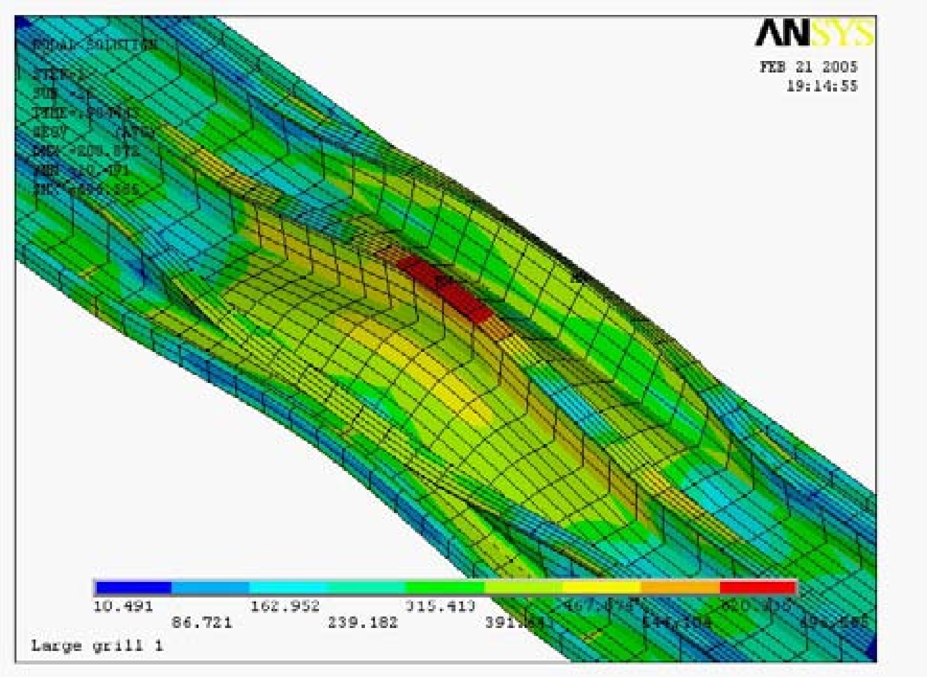


Figure 21. Large Grillage model with the load patch at a pressure of 18 MPa.

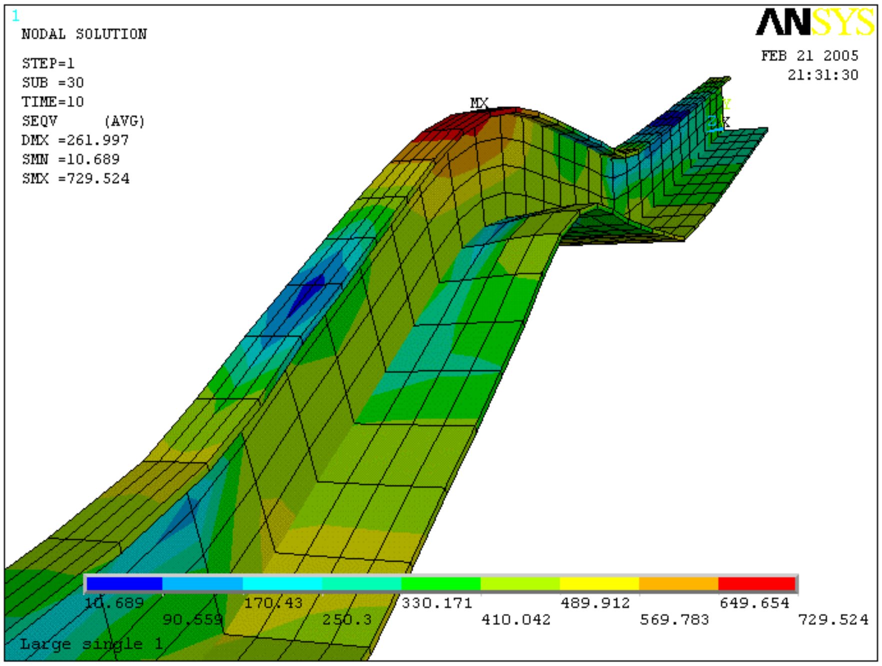


Figure 22. Long Frame model with the load patch at a pressure of 10 MPa.

For each of the four configurations Figure 23 shows the load-deflection curves. Figure 24 shows just the initial part of the curves. The load in Figure 23 and 24 is just on the middle frame. In that case the surrounding frames (if they exist) help support the frame. However, the neighboring frames may be partially of fully loaded. Figure 25 shows a sketch of neighboring frame loads. Figure 26 shows the load deflection relationship for the small grillage, with varying amounts of load on the adjacent frames. Figure 27 shows the initial part of the curves from Figure 26. Figure 28 is a check on the adequacy of the mesh. A coarser mesh model is compared to the normal mesh. This shows that the normal mesh is fine enough.

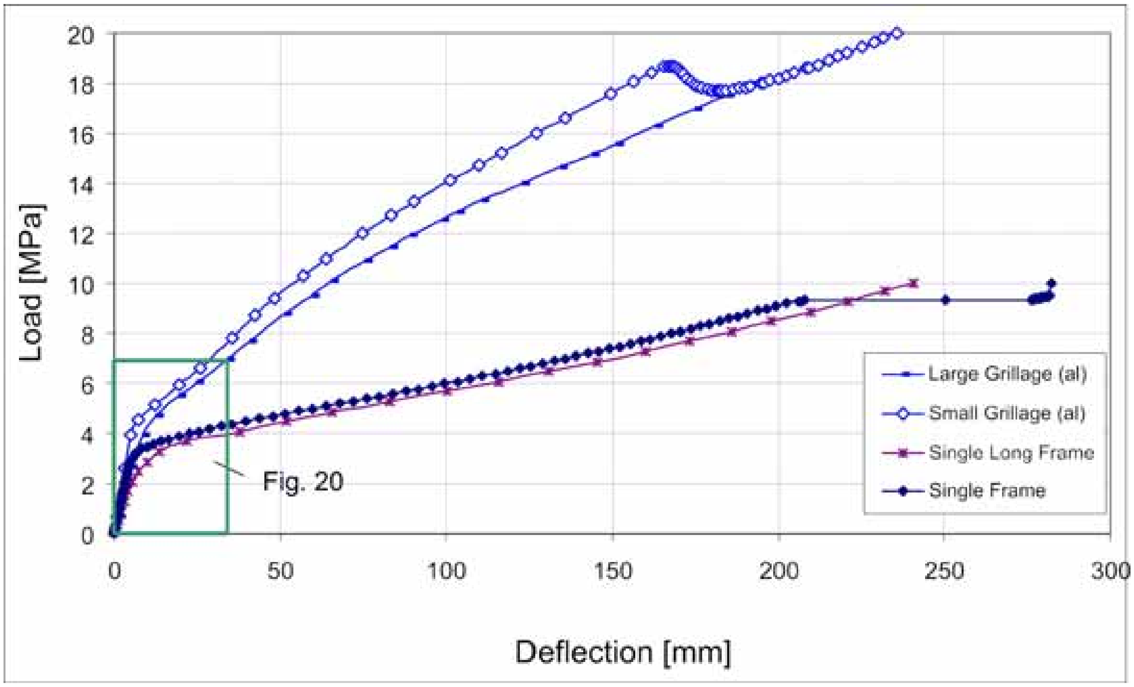


Figure 23. Comparison of the load-deflection curves for the four configurations.

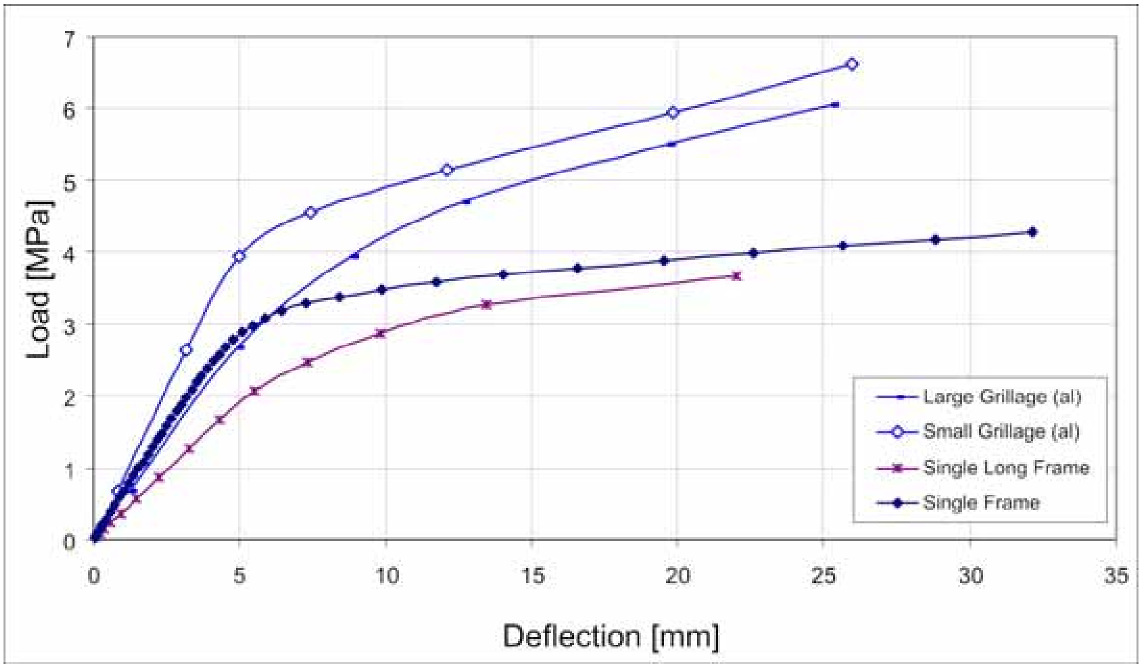


Figure 24. Initial portion of the load-deflection curves for the four configurations.

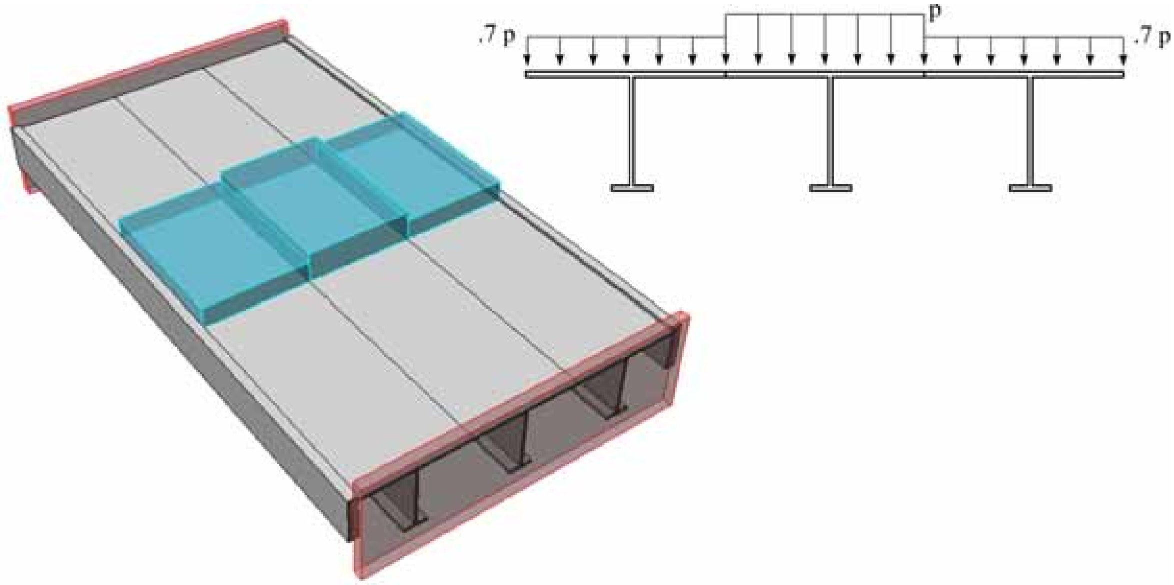


Figure 25. Load pattern on the small grillage with 70% load on neighboring frames.

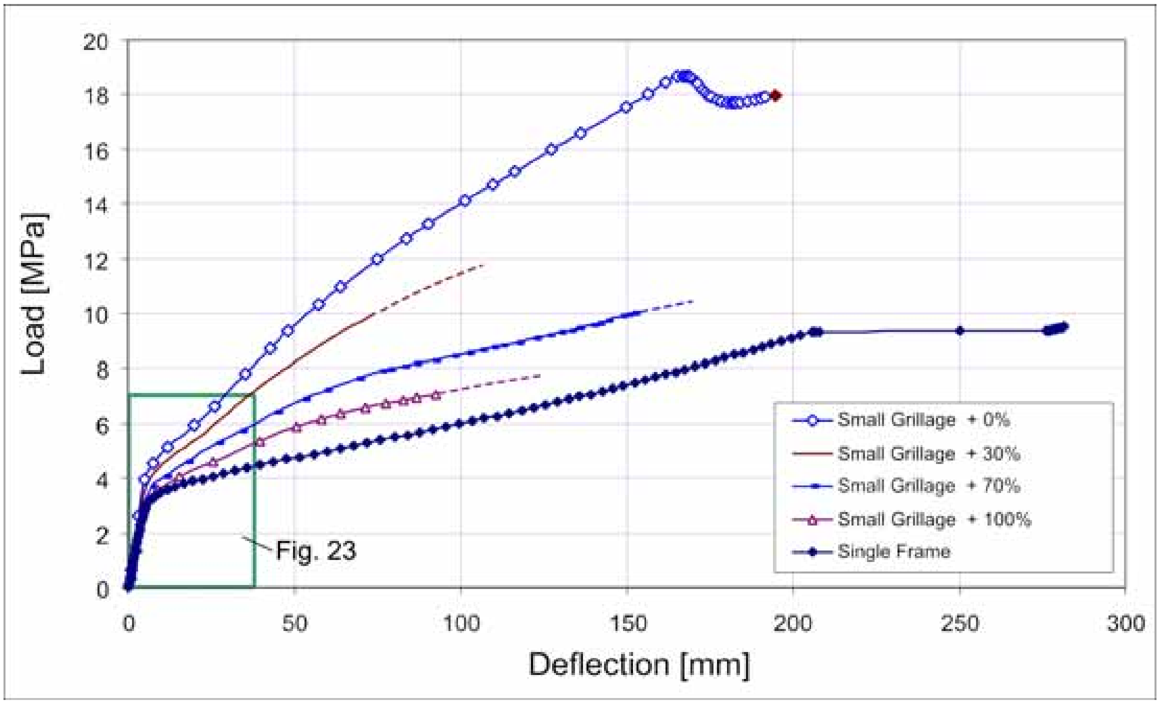


Figure 26. Load-deflection curves for the small grillage with varying load % on neighboring frames.

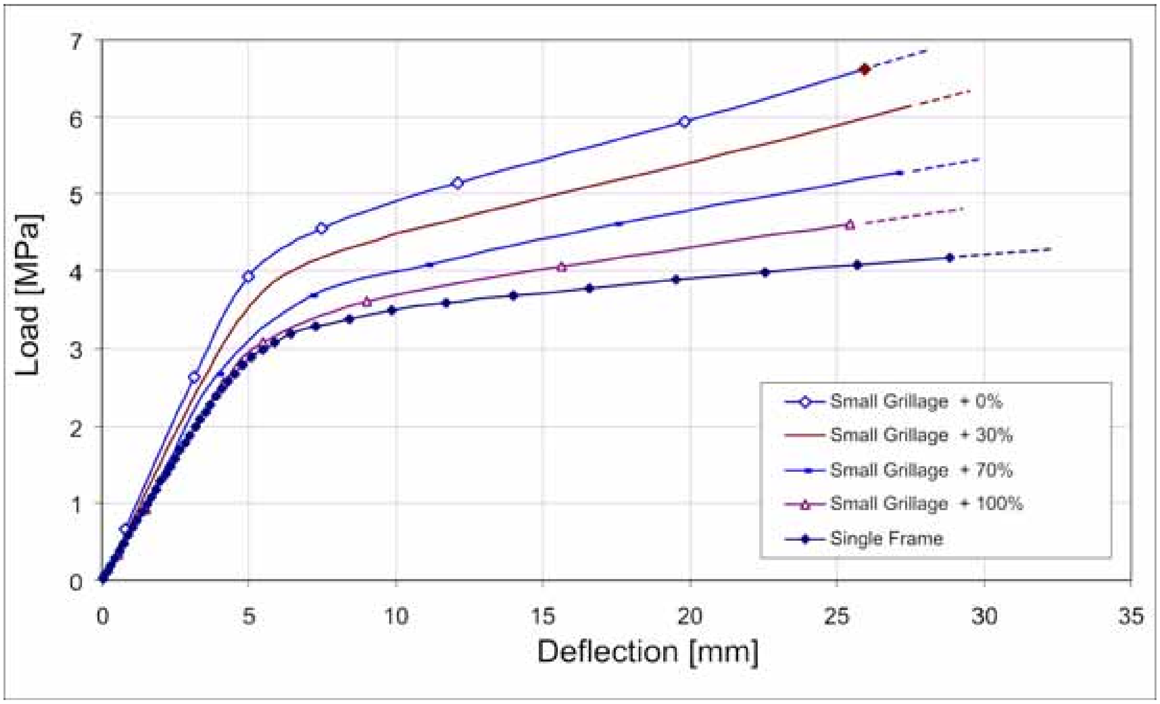


Figure 27. Initial portion of the load-deflection curves for Figure 26.

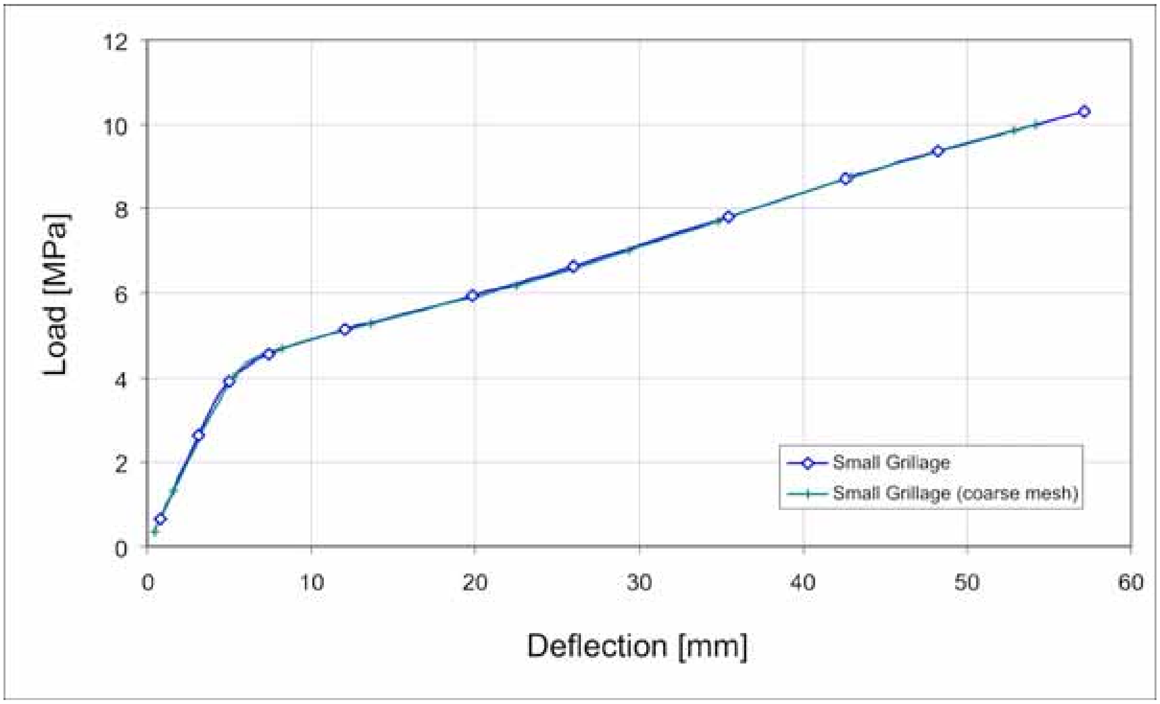


Figure 28. Influence of mesh size on small grillage results.

6. Discussion of Results

The above analysis has been done both to prepare for the grillage tests and to explore the differences between single and multiple frames. The results do show a number of very interesting results.

Figure 2 illustrates the value of examining plastic behaviour. The two frames would be considered identical by any standard based on the “working stress” concept (including most current ship rules). In this case the heavier flat bar frame would not get credit for its increased initial (linear) capacity, nor for its improved reserve. Figure 4 makes the same point with two frames of the same weight. The flat bar frame has better initial and reserve behaviour, but due to lower values of elastic and plastic section modulus, would be wrongly considered as being weaker. The case shown in Figure 4 shows what would happen if the elastic properties were optimized to obtain the maximum possible modulus from the steel. This presses the limits of the local buckling requirements, and results in a stiff but ‘brittle’ frame.

Most ship rule systems treat frame design in isolation, in that single frames are designed with simple assumptions about boundary conditions. Section 5 explores a number of aspects of the actual and modeled boundary conditions. Figure 23 shows the affect of modeling the surrounding frames. Adjacent frames can support the loaded frame and make it stronger and stiffer. Modeling longer sections results in a softer response, but no significant difference in strength. The small and large grillage experiments will show how accurate this analysis has been.

Figure 28 shows that the mesh used in the analyses was sufficiently fine. A single run with a coarser mesh showed not appreciable difference.

As a final point, it should be noted that large deflection plastic analyses are not always easy to obtain. When the web is quite thin and local buckling is probably, it is likely that ANSYS will not be able to give good results up to large deformations. On the other hand, when the sections are relatively thick and robust, ANSYS tends to give good and stable results up to very large deformations.

7. Conclusions

The work presented here focuses on influence of boundary conditions. The various types of tests (single frames, small and large grillages) explore one aspect of the modeling boundary conditions, by physically incorporating more realistic support conditions. To date we have reasonably good agreement with the single frame experimental results. This report has examined the effect of moving to grillage tests and shows us what we can expect.

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Appendix A

ANSYS Input Files

Appendix A - ANSYS Input Files

The input file for the long grillage is listed below.

```
/title, Large grill 1
/prep7
!define variables
!units:mm, MPa
L=2000
hw=200
hs=75
bf=75
s=350
a=500
tw=8
tp=10
tf=10
tedg=30
E=200000
Et=2000
fy=400

n1=8
n2=2
n3=8
n4=24

!element types
et,1,shell181
ex,1,E
nuxy,1,0.3
tb,bkin,1
tbdata,1,fy,Et

r,1,tp
et,2,shell181
r,2,tw

et,3,shell181
r,3,tf

et,4,shell181
r,4,tedg

!keypoints

n=0
lx=0
k,n+1,lx,-tp/2,-s*3/2
k,n+2,lx,-tp/2,-s
k,n+3,lx,-tp/2,-s/2
k,n+4,lx,-tp/2,0
k,n+5,lx,-tp/2,s/2
k,n+6,lx,-tp/2,s
k,n+7,lx,-tp/2,s*3/2
k,n+8,lx,hw+tf/2,-s-bf/2
k,n+9,lx,hw+tf/2,-s
k,n+10,lx,hw+tf/2,-s+bf/2
k,n+11,lx,hw+tf/2,-bf/2
k,n+12,lx,hw+tf/2,0
k,n+13,lx,hw+tf/2,bf/2
k,n+14,lx,hw+tf/2,s-bf/2
k,n+15,lx,hw+tf/2,s
k,n+16,lx,hw+tf/2,s+bf/2
k,n+17,lx,hs-tp/2,-s*3/2
k,n+18,lx,hs-tp/2,s*3/2

n=18
lx=L
k,n+1,lx,-tp/2,-s*3/2
k,n+2,lx,-tp/2,-s
k,n+3,lx,-tp/2,-s/2
k,n+4,lx,-tp/2,0
k,n+5,lx,-tp/2,s/2
k,n+6,lx,-tp/2,s
k,n+7,lx,-tp/2,s*3/2
k,n+8,lx,hw+tf/2,-s-bf/2
k,n+9,lx,hw+tf/2,-s
k,n+10,lx,hw+tf/2,-s+bf/2
k,n+11,lx,hw+tf/2,-bf/2
k,n+12,lx,hw+tf/2,0
k,n+13,lx,hw+tf/2,bf/2
k,n+14,lx,hw+tf/2,s-bf/2
k,n+15,lx,hw+tf/2,s
k,n+16,lx,hw+tf/2,s+bf/2
k,n+17,lx,hs-tp/2,-s*3/2
k,n+18,lx,hs-tp/2,s*3/2

n=36
lx=1.5*L-a/2
k,n+1,lx,-tp/2,-s*3/2
k,n+2,lx,-tp/2,-s
k,n+3,lx,-tp/2,-s/2
k,n+4,lx,-tp/2,0
k,n+5,lx,-tp/2,s/2
k,n+6,lx,-tp/2,s
k,n+7,lx,-tp/2,s*3/2
k,n+8,lx,hw+tf/2,-s-bf/2
k,n+9,lx,hw+tf/2,-s
k,n+10,lx,hw+tf/2,-s+bf/2
k,n+11,lx,hw+tf/2,-bf/2
k,n+12,lx,hw+tf/2,0
k,n+13,lx,hw+tf/2,bf/2
k,n+14,lx,hw+tf/2,s-bf/2
k,n+15,lx,hw+tf/2,s
k,n+16,lx,hw+tf/2,s+bf/2
k,n+17,lx,hs-tp/2,-s*3/2
k,n+18,lx,hs-tp/2,s*3/2

n=54
lx=1.5*L+a/2
k,n+1,lx,-tp/2,-s*3/2
k,n+2,lx,-tp/2,-s
k,n+3,lx,-tp/2,-s/2
k,n+4,lx,-tp/2,0
k,n+5,lx,-tp/2,s/2
k,n+6,lx,-tp/2,s
k,n+7,lx,-tp/2,s*3/2
k,n+8,lx,hw+tf/2,-s-bf/2
k,n+9,lx,hw+tf/2,-s
k,n+10,lx,hw+tf/2,-s+bf/2
k,n+11,lx,hw+tf/2,-bf/2
k,n+12,lx,hw+tf/2,0
k,n+13,lx,hw+tf/2,bf/2
k,n+14,lx,hw+tf/2,s-bf/2
k,n+15,lx,hw+tf/2,s
k,n+16,lx,hw+tf/2,s+bf/2
k,n+17,lx,hs-tp/2,-s*3/2
k,n+18,lx,hs-tp/2,s*3/2

n=72
lx=2*L
k,n+1,lx,-tp/2,-s*3/2
k,n+2,lx,-tp/2,-s
k,n+3,lx,-tp/2,-s/2
k,n+4,lx,-tp/2,0
k,n+5,lx,-tp/2,s/2
k,n+6,lx,-tp/2,s
k,n+7,lx,-tp/2,s*3/2
k,n+8,lx,hw+tf/2,-s-bf/2
k,n+9,lx,hw+tf/2,-s
k,n+10,lx,hw+tf/2,-s+bf/2
k,n+11,lx,hw+tf/2,-bf/2
k,n+12,lx,hw+tf/2,0
k,n+13,lx,hw+tf/2,bf/2
k,n+14,lx,hw+tf/2,s-bf/2
k,n+15,lx,hw+tf/2,s
k,n+16,lx,hw+tf/2,s+bf/2
k,n+17,lx,hs-tp/2,-s*3/2
k,n+18,lx,hs-tp/2,s*3/2

n=90
lx=3*L
k,n+1,lx,-tp/2,-s*3/2
k,n+2,lx,-tp/2,-s
k,n+3,lx,-tp/2,-s/2
k,n+4,lx,-tp/2,0
k,n+5,lx,-tp/2,s/2
k,n+6,lx,-tp/2,s
k,n+7,lx,-tp/2,s*3/2
```

```
k,n+8,lx,hw+tf/2,-s-bf/2
k,n+9,lx,hw+tf/2,-s
k,n+10,lx,hw+tf/2,-s+bf/2
k,n+11,lx,hw+tf/2,-bf/2
k,n+12,lx,hw+tf/2,0
k,n+13,lx,hw+tf/2,bf/2
k,n+14,lx,hw+tf/2,s-bf/2
k,n+15,lx,hw+tf/2,s
k,n+16,lx,hw+tf/2,s+bf/2
k,n+17,lx,hs-tp/2,-s*3/2
k,n+18,lx,hs-tp/2,s*3/2


!defining lines conecting keypoints
!sizing lines

        END FRAMES
n=0
m=0
l,n+1,n+2,n1
l,n+2,n+3,n1
l,n+3,n+4,n1
l,n+4,n+5,n1
l,n+5,n+6,n1
l,n+6,n+7,n1
l,n+2,n+9,n1
l,n+8,n+9,n2
l,n+9,n+10,n2
l,n+4,n+12,n1
l,n+11,n+12,n2
l,n+12,n+13,n2
l,n+6,n+15,n1
l,n+14,n+15,n2
l,n+15,n+16,n2

l,n+1,n+1+18,n4
l,n+2,n+2+18,n4
l,n+3,n+3+18,n4
l,n+4,n+4+18,n4
l,n+5,n+5+18,n4
l,n+6,n+6+18,n4
l,n+7,n+7+18,n4
l,n+8,n+8+18,n4
l,n+9,n+9+18,n4
l,n+10,n+10+18,n4
l,n+11,n+11+18,n4
l,n+12,n+12+18,n4
l,n+13,n+13+18,n4
l,n+14,n+14+18,n4
l,n+15,n+15+18,n4
l,n+16,n+16+18,n4

l,n+1,n+17,n2
l,n+17,n+17+18,n4

l,n+7,n+18,n2
```

```
l,n+18,n+18+18,n4


 PART OF LOADED

n=18
l,n+1,n+2,n1
l,n+2,n+3,n1
l,n+3,n+4,n1
l,n+4,n+5,n1
l,n+5,n+6,n1
l,n+6,n+7,n1
l,n+2,n+9,n1
l,n+8,n+9,n2
l,n+9,n+10,n2
l,n+4,n+12,n1
l,n+11,n+12,n2
l,n+12,n+13,n2
l,n+6,n+15,n1
l,n+14,n+15,n2
l,n+15,n+16,n2


l,n+1,n+1+18,n3
l,n+2,n+2+18,n3
l,n+3,n+3+18,n3
l,n+4,n+4+18,n3
l,n+5,n+5+18,n3
l,n+6,n+6+18,n3
l,n+7,n+7+18,n3
l,n+8,n+8+18,n3
l,n+9,n+9+18,n3
l,n+10,n+10+18,n3
l,n+11,n+11+18,n3
l,n+12,n+12+18,n3
l,n+13,n+13+18,n3
l,n+14,n+14+18,n3
l,n+15,n+15+18,n3
l,n+16,n+16+18,n3

l,n+1,n+17,n2
l,n+17,n+17+18,n3

l,n+7,n+18,n2
l,n+18,n+18+18,n3


             LOADED

n=36
l,n+1,n+2,n1
l,n+2,n+3,n1
l,n+3,n+4,n1
l,n+4,n+5,n1
l,n+5,n+6,n1
l,n+6,n+7,n1
```

```
l,n+2,n+9,n1
l,n+8,n+9,n2
l,n+9,n+10,n2
l,n+4,n+12,n1
l,n+11,n+12,n2
l,n+12,n+13,n2
l,n+6,n+15,n1
l,n+14,n+15,n2
l,n+15,n+16,n2

l,n+1,n+1+18,n1
l,n+2,n+2+18,n1
l,n+3,n+3+18,n1
l,n+4,n+4+18,n1
l,n+5,n+5+18,n1
l,n+6,n+6+18,n1
l,n+7,n+7+18,n1
l,n+8,n+8+18,n1
l,n+9,n+9+18,n1
l,n+10,n+10+18,n1
l,n+11,n+11+18,n1
l,n+12,n+12+18,n1
l,n+13,n+13+18,n1
l,n+14,n+14+18,n1
l,n+15,n+15+18,n1
l,n+16,n+16+18,n1

l,n+1,n+17,n2
l,n+17,n+17+18,n1

l,n+7,n+18,n2
l,n+18,n+18+18,n1


               PART OF LOADED

n=54
l,n+1,n+2,n1
l,n+2,n+3,n1
l,n+3,n+4,n1
l,n+4,n+5,n1
l,n+5,n+6,n1
l,n+6,n+7,n1
l,n+2,n+9,n1
l,n+8,n+9,n2
l,n+9,n+10,n2
l,n+4,n+12,n1
l,n+11,n+12,n2
l,n+12,n+13,n2
l,n+6,n+15,n1
l,n+14,n+15,n2
l,n+15,n+16,n2


l,n+1,n+1+18,n3
l,n+2,n+2+18,n3
```

```
l,n+3,n+3+18,n3
l,n+4,n+4+18,n3
l,n+5,n+5+18,n3
l,n+6,n+6+18,n3
l,n+7,n+7+18,n3
l,n+8,n+8+18,n3
l,n+9,n+9+18,n3
l,n+10,n+10+18,n3
l,n+11,n+11+18,n3
l,n+12,n+12+18,n3
l,n+13,n+13+18,n3
l,n+14,n+14+18,n3
l,n+15,n+15+18,n3
l,n+16,n+16+18,n3

l,n+1,n+17,n2
l,n+17,n+17+18,n3

l,n+7,n+18,n2
l,n+18,n+18+18,n3


n=72
l,n+1,n+2,n1
l,n+2,n+3,n1
l,n+3,n+4,n1
l,n+4,n+5,n1
l,n+5,n+6,n1
l,n+6,n+7,n1
l,n+2,n+9,n1
l,n+8,n+9,n2
l,n+9,n+10,n2
l,n+4,n+12,n1
l,n+11,n+12,n2
l,n+12,n+13,n2
l,n+6,n+15,n1
l,n+14,n+15,n2
l,n+15,n+16,n2


l,n+1,n+1+18,n4
l,n+2,n+2+18,n4
l,n+3,n+3+18,n4
l,n+4,n+4+18,n4
l,n+5,n+5+18,n4
l,n+6,n+6+18,n4
l,n+7,n+7+18,n4
l,n+8,n+8+18,n4
l,n+9,n+9+18,n4
l,n+10,n+10+18,n4
l,n+11,n+11+18,n4
l,n+12,n+12+18,n4
l,n+13,n+13+18,n4
l,n+14,n+14+18,n4
l,n+15,n+15+18,n4
l,n+16,n+16+18,n4
```

```
l,n+1,n+17,n2
l,n+17,n+17+18,n4

l,n+7,n+18,n2
l,n+18,n+18+18,n4


n=90
m=175


l,n+1,n+2,n1
l,n+2,n+3,n1
l,n+3,n+4,n1
l,n+4,n+5,n1
l,n+5,n+6,n1
l,n+6,n+7,n1
l,n+2,n+9,n1
l,n+8,n+9,n2
l,n+9,n+10,n2
l,n+4,n+12,n1
l,n+11,n+12,n2
l,n+12,n+13,n2
l,n+6,n+15,n1
l,n+14,n+15,n2
l,n+15,n+16,n2

l,n+1,n+17,n2

l,n+7,n+18,n2


!create area

n=0
a,n+1,n+2,n+2+18,n+1+18 !1
a,n+2,n+3,n+3+18,n+2+18 !2
a,n+3,n+4,n+4+18,n+3+18 !3
a,n+4,n+5,n+5+18,n+4+18 !4
a,n+5,n+6,n+6+18,n+5+18 !5
a,n+6,n+7,n+7+18,n+6+18 !6
a,n+2,n+9,n+9+18,n+2+18       !7
a,n+8,n+9,n+9+18,n+8+18 !8
a,n+9,n+10,n+10+18,n+9+18 !9
a,n+4,n+12,n+12+18,n+4+18
!10
a,n+11,n+12,n+12+18,n+11+18 !11
a,n+12,n+13,n+13+18,n+12+18 !12
a,n+6,n+15,n+15+18,n+6+18
!13
a,n+14,n+15,n+15+18,n+14+18 !14
a,n+15,n+16,n+16+18,n+15+18 !15
a,n+1,n+17,n+17+18,n+1+18 !16
a,n+7,n+18,n+18+18,n+7+18 !17
```

```
n=n+18

a,n+1,n+2,n+2+18,n+1+18 !18
a,n+2,n+3,n+3+18,n+2+18 !19
a,n+3,n+4,n+4+18,n+3+18 !20
a,n+4,n+5,n+5+18,n+4+18 !21
a,n+5,n+6,n+6+18,n+5+18 !22
a,n+6,n+7,n+7+18,n+6+18 !23
a,n+2,n+9,n+9+18,n+2+18       !24
a,n+8,n+9,n+9+18,n+8+18 !25
a,n+9,n+10,n+10+18,n+9+18 !26
a,n+4,n+12,n+12+18,n+4+18
!27
a,n+11,n+12,n+12+18,n+11+18 !28
a,n+12,n+13,n+13+18,n+12+18 !29
a,n+6,n+15,n+15+18,n+6+18
!30
a,n+14,n+15,n+15+18,n+14+18 !31
a,n+15,n+16,n+16+18,n+15+18 !32
a,n+1,n+17,n+17+18,n+1+18 !33
a,n+7,n+18,n+18+18,n+7+18 !34
n=n+18

a,n+1,n+2,n+2+18,n+1+18 !35
a,n+2,n+3,n+3+18,n+2+18 !36
a,n+3,n+4,n+4+18,n+3+18 !37
a,n+4,n+5,n+5+18,n+4+18 !38
a,n+5,n+6,n+6+18,n+5+18 !39
a,n+6,n+7,n+7+18,n+6+18 !40
a,n+2,n+9,n+9+18,n+2+18 !41
a,n+8,n+9,n+9+18,n+8+18 !42
a,n+9,n+10,n+10+18,n+9+18 !43
a,n+4,n+12,n+12+18,n+4+18 !44
a,n+11,n+12,n+12+18,n+11+18 !45
a,n+12,n+13,n+13+18,n+12+18 !46
a,n+6,n+15,n+15+18,n+6+18 !47
a,n+14,n+15,n+15+18,n+14+18 !48
a,n+15,n+16,n+16+18,n+15+18 !49
a,n+1,n+17,n+17+18,n+1+18 !50
a,n+7,n+18,n+18+18,n+7+18 !51
n=n+18

a,n+1,n+2,n+2+18,n+1+18 !52
a,n+2,n+3,n+3+18,n+2+18 !53
a,n+3,n+4,n+4+18,n+3+18 !54
a,n+4,n+5,n+5+18,n+4+18 !55
a,n+5,n+6,n+6+18,n+5+18 !56
a,n+6,n+7,n+7+18,n+6+18 !57
a,n+2,n+9,n+9+18,n+2+18 !58
a,n+8,n+9,n+9+18,n+8+18 !59
a,n+9,n+10,n+10+18,n+9+18 !60
a,n+4,n+12,n+12+18,n+4+18 !61
a,n+11,n+12,n+12+18,n+11+18 !62
a,n+12,n+13,n+13+18,n+12+18 !63
a,n+6,n+15,n+15+18,n+6+18 !64
a,n+14,n+15,n+15+18,n+14+18 !65
```

```
a,n+15,n+16,n+16+18,n+15+18 !66
a,n+1,n+17,n+17+18,n+1+18 !67
a,n+7,n+18,n+18+18,n+7+18 !68
n=n+18


a,n+1,n+2,n+2+18,n+1+18 !69
a,n+2,n+3,n+3+18,n+2+18 !70
a,n+3,n+4,n+4+18,n+3+18 !71
a,n+4,n+5,n+5+18,n+4+18 !72
a,n+5,n+6,n+6+18,n+5+18 !73
a,n+6,n+7,n+7+18,n+6+18 !74
a,n+2,n+9,n+9+18,n+2+18 !75
a,n+8,n+9,n+9+18,n+8+18 !76
a,n+9,n+10,n+10+18,n+9+18 !77
a,n+4,n+12,n+12+18,n+4+18 !78
a,n+11,n+12,n+12+18,n+11+18 !79
a,n+12,n+13,n+13+18,n+12+18 !80
a,n+6,n+15,n+15+18,n+6+18   !81
a,n+14,n+15,n+15+18,n+14+18 !82
a,n+15,n+16,n+16+18,n+15+18 !83
a,n+1,n+17,n+17+18,n+1+18   !84
a,n+7,n+18,n+18+18,n+7+18   !85


asel,s,area,,1,6
asel,a,area,,18,23
asel,a,area,,35,40
asel,a,area,,52,57
asel,a,area,,69,74
AATT,1,1,,

amesh,all

asel,all
asel,s,area,,7
asel,a,area,,24
asel,a,area,,41
asel,a,area,,58
asel,a,area,,75

asel,a,area,,10
asel,a,area,,27
asel,a,area,,44
asel,a,area,,61
asel,a,area,,78

asel,a,area,,13
asel,a,area,,30
asel,a,area,,47
asel,a,area,,64
asel,a,area,,81
```

```
AATT,1,2,,

amesh,all

asel,all
asel,s,area,,8,9
asel,a,area,,25,26
asel,a,area,,42,43
asel,a,area,,59,60
asel,a,area,,76,77

asel,a,area,,11,12
asel,a,area,,28,29
asel,a,area,,45,46
asel,a,area,,62,63
asel,a,area,,79,80

asel,a,area,,14,15
asel,a,area,,31,32
asel,a,area,,48,49
asel,a,area,,65,66
asel,a,area,,82,83

AATT,1,3,,

amesh,all

asel,all

asel,s,area,,16,17
asel,a,area,,33,34
asel,a,area,,50,51
asel,a,area,,67,68
asel,a,area,,84,85

AATT,1,4,,

amesh,all
save

/soln
antype,0

nsel,s,loc,x,0
d,all,all,0
nsel,all

nsel,s,loc,x,L
d,all,uy,0
d,all,uz,0
nsel,all
```

```
nsel,s,loc,x,2*L
d,all,uy,0
d,all,uz,0
nsel,all

nsel,s,loc,x,3*L
d,all,all,0
nsel,all


save
/soln
antype,static
nlgeom,on
sstif,on
neqit,30
nropt,full,,off

!lnsrch,on  !!

arclen,on  !!
pred,on,,on  !!


cnvtol,F,,0.05,,1
cnvtol,M,,0.05,,1
ncnv
outres,all,all

!ksel,s,kp,,44
!fk,all,fz,100
!ftran
!ksel,all

py=20

nsubst,30  !

!autots,on  !!
!time,py   !!
!nsubst,1   !!
!deltim,0.04,0.01,0.4  !!

asel,s,area,,37,38
sfa,all,,pres,py
asel,all
sftran

asel,all

save
solve
save
```