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SHOCK STRUT DEFLECTIONS AND THE CORRESPONDING ANGLE OF THRUST-LINE TO HORIZONTAL OF TWO-SKI LANDING GEAR FOR AIRCRAFT

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BY

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Subject Shock strut deflections and the corresponding angle of thrust-line to horizontal of two-ski landing gear for aircraft

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Introduction

It was desired to analyse the behaviour of the front and rear shock absorbers of a two-ski landing gear for an aircraft passing from level ground on to a slope of $\pm 10^{\circ}$ at speeds of 20, 40 and 60 m.p.h.

Assumptions and Method

Fig. 1 is a sketch of the installation employed. A levered-suspension support, at each pedestal, acts on an oleo-spring or oleo-air shock absorber strut. The front suspension lever is free to move fore and aft in its slotted point of support. This permits the two points of attachment to the chassis to remain at a fixed distance apart despite variations in deflection between front and rear struts. To simplify the mathematical treatment of the problem, it was found necessary to assume the installation of both front and rear pedestal to be identical in every respect. Hence the free movement of the lever arm in the front pedestal had to be ignored. Furthermore, for the sake of mathematical simplicity during the analysis, the lever-suspension support and the oleo spring at each pedestal had to be replaced by an "equivalent" spring(*).

(*) Equivalent spring means the spring producing the same vertical displacement as the given installation for the same load.

If the spring constant of the oleo spring is k' then from the data given for the shock absorber strut we can deduce k, the spring constant of the "equivalent" spring (Fig. 2).

let y be the displacement of the "equivalent" spring
let y' be the displacement of the oleo spring

then

$$y' = \frac{1}{3} y$$

further, the force on the oleo spring is k'y'
the force on the "equivalent" spring is ky

hence

$$18\,ky + 6\,k\,y' = 0$$

$$18\,ky + 6\,k\,\frac{1}{3} y = 0$$

$$\therefore k = \frac{1}{9} k'$$

Another essential assumption has been made with reference to the damping action. The coefficient of damping has been taken as a constant, although it should have been a function of the displacement. No numerical value for the damping coefficient has been supplied. To make the problem more definite and considering optimum working operation of the springs, it was decided to assume critical damping which means dynamically that the vibrating system is brought to rest in the shortest time. Obviously, calculations can be performed with a specified damping constant by a slight alteration of the solution of the differential equation (1).

The centre of gravity of the aircraft has been assumed to lie half way between the attachment of the pedestals. This fact has an important bearing on the structure of the equations of motion (equations (1) and (2) shown on page 3) which then are a pair of simultaneous linear differential equations with dependent variables separated.

Under these assumptions, the problem reduces then to finding the motion of a system with a single mass on two springs mounted in parallel. This is known in dynamics as a coupled system, and will be dealt with on the following pages.

Equations of Motion of Coupled Vibrations

Fig. 3 represents a system with a single mass and two degrees of freedom. The centre of gravity G is constrained to move vertically and the mass rotate about its centre of gravity.

In general both linear and angular displacement occur at the same time, and for small amplitudes the spring forces produced by the motion are proportional to the displacements.

If the springs are alike in every respect and their mutual distance is 2d, the spring constant is k, and the damping coefficient is c, we shall be able to describe the motion at any instant t. Furthermore, let m be the total mass, neglecting the mass of the spring, and let I be the moment of inertia of the mass about G.

Suppose first the mass moves vertically with no rotation, the displacement being y downwards. Spring ① exerts a restoring force ky upwards, and this force has a moment k d.y about G in the counter-clockwise direction; similarly the spring ② exerts a restoring force ky upwards and this force has a moment-k d.y about G. The frictional force in each spring is of magnitude $c\dot{y}$ and is directed upwards. The moments of these dissipating forces about G are again equal but of opposite sign and will contribute nothing to the total moment.

Thus the net force in the y direction is 2 ky + 2 $c\dot{y}$ directed upwards.

Now suppose the mass to rotate through a small angle θ without linear motion of the centre of gravity. The rotation being about G, the displacements at the points of attachment of the springs are-dθ and dθ downwards, as θ is positive when clockwise (Fig. 4). The forces exerted by the springs are therefore kdθ + cd$\dot{\theta}$ for spring ① and -(kdθ + cd$\dot{\theta}$) for spring ② respectively. These forces have a total moment $2(kd^2\theta + cd^2\dot{\theta})$ in the counter-clockwise sense about G. The net forces in the y direction are zero.

Hence the equations of motion are

$$-m\ddot{y} = 2ky + 2c\dot{y} \qquad (1)$$

$$-I\ddot{\theta} = 2kd^2\theta + 2cd^2\dot{\theta} \qquad (2)$$

Equations (1) and (2) characterize the motion, assuming the base points of the springs to be at rest relative to the ground. If these base points simultaneously perform a prescribed motion along a given curve, equations (1) and (2) have to be referred to an absolute frame of reference [x, y] (Fig. 5).

Let $\bar{\xi}, \bar{\eta}$ be the Cartesian coordinates of the centre of gravity relative to [x y] and θ the angle of pitch when the vibrating system is in its equilibrium position at any instant t.

Let Y and ϑ be the linear and angular displacements respectively measured from the instantaneous equilibrium position. Then

$$y = Y + \bar{\eta} ,$$

$$\theta = \vartheta + \bar{\theta} ,$$

and, of course, Y and ϑ satisfy the equations of motion (1) and (2)

$$m\ddot{Y} + 2c\dot{Y} + 2kY = 0 \qquad (1')$$

$$I\ddot{\vartheta} + 2cd^2\dot{\vartheta} + 2kd^2\vartheta = 0 \qquad (2')$$

or

$$m(\ddot{y} - \ddot{\bar{\eta}}) + 2c(\dot{y} - \dot{\bar{\eta}}) + 2k(y - \bar{\eta}) = 0$$

$$I(\ddot{\theta} - \ddot{\bar{\theta}}) + 2c(\dot{\theta} - \dot{\bar{\theta}}) + 2k(\theta - \bar{\theta}) = 0$$

Turning now from these general considerations to our specific problem, let us denote the uniform horizontal velocity by v, the inclination of our slope by α, and the height of the centre of gravity G, when in its equilibrium position, by h, (c.f. Fig. 6); also let $t = 0$ be the instant when the centre of gravity G reaches the point S.

$$\text{for } -\infty < t \leq -\frac{d}{v} \qquad \bar{\eta} = h, \ \ddot{\bar{\eta}} = 0$$

$$y = h + 0 = h, \ \dot{y} = \ddot{y} = 0$$

$$\text{for } \frac{d}{v} \leq t < \infty \qquad \bar{\eta} = h + vt \tan\alpha , \ \ddot{\bar{\eta}} = 0$$

Consider now the time interval $-\frac{d}{v} \leq t \leq \frac{d}{v}$, i.e. when the front spring travels on the slope while the rear spring is still on the level ground (Fig. 7, springs in equilibrium position).

$R[vt-d, 0]$, $S[vt+d, (vt+d)\tan\alpha]$, $T[vt-d, h]$, $U[vt+d, (vt+d)\tan\alpha+h]$

$\therefore G(\bar{\xi}, \bar{\eta}) = G[vt,\ h+\frac{1}{2}\tan\alpha(vt+d)]$; $\bar{\eta} = h+\frac{1}{2}\tan\alpha\,(vt+d)$

$\bar{\theta} \simeq \tan\bar{\theta} = \frac{\frac{1}{2}\tan\alpha(vt+d)}{d}$ (4) $\ddot{\bar{\theta}} = 0$

The general solution of equation (1')

i.e. $m\ddot{Y} + 2c\dot{Y} + 2kY = 0$

is obtained as follows, using operators

$$(mD^2 + 2cD + 2k)\, y = 0$$

$$D = -c/m \pm \frac{1}{m}\sqrt{c^2 - 2mk}$$

If we assume critical damping, i.e. $c^2 = 2mk$, which means physically that the system is brought to rest in the shortest time then

$Y = (A_1 t + B_1)e^{-pt}$ is the general solution of equation (1') where $p = \frac{c}{m}$ and A_1 and B_1 are arbitrary constants, later to be adjusted to the boundary conditions of the problem.

Similarly, the general solution of equation (2')

i.e. of $I\ddot{\vartheta} + 2cd^2\dot{\vartheta} + 2kd^2\vartheta = 0$

$$[ID^2 + 2cd^2D + 2kd^2]\vartheta = 0$$

$$D = -\frac{cd^2}{I} \pm \frac{d}{I}\sqrt{c^2d^2 - 2kI}$$

under the above assumption for c, (critical damping of the

vertical motion of the centre of gravity), we obtain

$$c^2d^2 < 2kI$$

hence the pitching will be damped (but not critically).

$$\therefore \quad \vartheta = A e^{-\lambda t} \cos (wt - \epsilon)$$

where $\lambda = \frac{cd^2}{I}$

$$w = \frac{d}{I}\sqrt{2kI - c^2d^2}$$

and A and ϵ are arbitrary constants.

It must be pointed out that equations (1') and (2') hold for any instant of the motion $(-\infty < t < \infty)$. The ensuing calculations will be aimed at providing a continuous solution for both coordinates and velocities, throughout the entire motion. To achieve this the discontinuity of the solutions at $t = \pm \frac{d}{v}$ will have to be smoothed out. Or in mathematical symbols,

$$y(\mp\tfrac{d}{v} - 0) = y(\mp\tfrac{d}{v} + 0) \quad (5a) \qquad \theta(\mp\tfrac{d}{v} - 0) = \theta(\mp\tfrac{d}{v} + 0) \quad (6a)$$

and

$$\dot{y}(\mp\tfrac{d}{v} - 0) = \dot{y}(\mp\tfrac{d}{v} + 0) \quad (5b) \qquad \dot{\theta}(\mp\tfrac{d}{v} - 0) = \dot{\theta}(\mp\tfrac{d}{v} + 0) \quad (6b)$$

As shown on page 4

$$-\infty < t < -\frac{d}{v} \qquad y = h, \qquad \dot{y} = 0$$

$$-\frac{d}{v} < t < \frac{d}{v} \qquad y = h + \tfrac{1}{2}\tan\alpha(vt+d) + e^{-pt}(A_1 t + B_1)$$

$$\dot{y} = \tfrac{v}{2}\tan\alpha + e^{-pt}(-pA_1 t - pB_1 + A_1)$$

Applying (5a) we get, taking the upper sign,

$$y(-\tfrac{d}{v}) = h + e^{\frac{pd}{v}}(-A_1\tfrac{d}{v} + B_1) = h$$

$$\therefore \quad B_1 = A_1\frac{d}{v}$$

and from (5b)

$$\dot{y}(-\frac{d}{v}) = \frac{v}{2}\tan\alpha + e^{\frac{pd}{v}}\left(pA_1\frac{d}{v} - pA_1\frac{d}{v} + A_1\right) = 0$$

$$\therefore A_1 = -\frac{v}{2}\tan\alpha\, e^{-\frac{pd}{v}},$$

$$B_1 = -\frac{d}{2}\tan\alpha\, e^{-\frac{pd}{v}}$$

hence $y = h+\frac{1}{2}\tan\alpha(vt+d) - \frac{1}{2}\tan\alpha(vt+d)e^{-p(t+\frac{d}{v})}$

$$y = h+\tfrac{1}{2}\tan\alpha(vt+d)\left[1-e^{-p(t+\frac{d}{v})}\right]$$

$$\dot{y} = \tfrac{1}{2}v\tan\alpha - \tfrac{1}{2}\tan\alpha\, v e^{-p(t+\frac{d}{v})} + \tfrac{1}{2}\tan\alpha(vt+d)pe^{-p(t+\frac{d}{v})}$$

$$= \tfrac{1}{2}\tan\alpha\left[v\left\{1 - e^{-p(t+\frac{d}{v})}\right\} + (vt+d)\,pe^{-p(t+\frac{d}{v})}\right]$$

Applying (5a) and (5b) taking the lower sign, we can write

$$y(\frac{d}{v}) = h+\tfrac{1}{2}\tan\alpha\, 2d\left[1-e^{-\frac{2pd}{v}}\right] = h+d\tan\alpha\left[1-e^{-\frac{2pd}{v}}\right]$$

$$\dot{y}(\frac{d}{v}) = \tfrac{1}{2}\tan\alpha\left[v(1-e^{-\frac{2pd}{v}}) + 2dpe^{-\frac{2pd}{v}}\right] = \tfrac{1}{2}\tan\alpha\left[v+(2pd-v)e^{-\frac{2pd}{v}}\right]$$

now in $\frac{d}{v} < t < \infty$

$$Y = e^{-pt}(A_2t + B_2) \qquad (A_2,\ B_2 \text{ arbitrary constants})$$

$$\bar{\eta} = h + vt\tan\alpha$$

$$y = h + vt\tan\alpha + e^{-pt}(A_2t + B_2)$$

$$\dot{y} = v\tan\alpha + e^{-pt}\left[-pA_2t - pB_2 + A_2\right]$$

From (5a) and (5b) taking the lower sign:

$$y\left(\frac{d}{v}\right) = h + d\tan\alpha + e^{-\frac{pd}{v}}\left[A_2\frac{d}{v} + B_2\right] = h + d\tan\alpha\left[1 - e^{-\frac{2pd}{v}}\right]$$

$$\dot{y}\left(\frac{d}{v}\right) = v\tan\alpha + e^{-\frac{pd}{v}}\left[-pA_2\frac{d}{v} - pB_2 + A_2\right] = \frac{1}{2}\tan\alpha\left[v + (2pd - v)e^{-\frac{2pd}{v}}\right]$$

Solving for A_2 and B_2, we get

$$A_2\frac{d}{v} + B_2 = -d\tan\alpha\, e^{-\frac{pd}{v}}$$

$$v\tan\alpha + e^{-\frac{pd}{v}}\left[pd\tan\alpha\, e^{-\frac{pd}{v}} + A_2\right] = \frac{1}{2}\tan\alpha\left[v + (2pd - v)e^{-\frac{2pd}{v}}\right]$$

$$\frac{v}{2}\tan\alpha + A_2 e^{-\frac{pd}{v}} = -\frac{1}{2}v\tan\alpha\, e^{-\frac{2pd}{v}}$$

$$A_2 = -\frac{v}{2}\tan\alpha\left(1 + e^{-\frac{2pd}{v}}\right)e^{\frac{pd}{v}} = -v\tan\alpha\cosh\frac{pd}{v}$$

$$B_2 = -d\tan\alpha\, e^{-\frac{pd}{v}} + d\tan\alpha\cosh\frac{pd}{v} = d\tan\alpha\sinh\frac{pd}{v}$$

hence, summarizing:

$$\begin{cases} y = h & \left(-\infty < t \leq -\frac{d}{v}\right) \\ y = h + \frac{1}{2}\tan\alpha\,(vt + d)\left[1 - e^{-p\left(t + \frac{d}{v}\right)}\right] & \left(-\frac{d}{v} \leq t \leq \frac{d}{v}\right) \\ y = h + vt\tan\alpha + \tan\alpha\, e^{-pt}\left[d\sinh\frac{pd}{v} - vt\cosh\frac{pd}{v}\right] & \left(\frac{d}{v} \leq t < \infty\right) \end{cases}$$

A similar procedure gives θ, the pitching angle referred to the absolute coordinate system

$$\theta = \vartheta + \bar{\theta}$$

in $-\infty < t < -\frac{d}{v}$, $\vartheta = 0$, $\bar{\theta} = 0$, $\theta = \dot{\theta} = 0$ (c.f. Fig. 4)

from the differential equation (2')
we obtain the general solution

$$\vartheta = Ae^{-\lambda t} \cos (wt - \epsilon) \qquad \text{where} \quad \lambda = \frac{cd^2}{I}$$

$$w = \frac{d}{I}\sqrt{2kI - c^2d^2}$$

and A and ϵ are arbitrary constants.

$-\frac{d}{v} < t < \frac{d}{v}$, $\quad \bar{\theta} = \frac{(vt+d)\tan\alpha}{2d}$ [c.f. (4) page 5]

$$\vartheta = Ae^{-\lambda t} \cos (wt - \epsilon)$$

$$\therefore \theta = \frac{(vt+d)\tan\alpha}{2d} + Ae^{-\lambda t} \cos (wt - \epsilon)$$

$$\dot{\theta} = \frac{v \tan\alpha}{2d} - Ae^{-\lambda t}\left[\lambda \cos (wt - \epsilon) + w \sin (wt - \epsilon)\right]$$

Continuity at $t = -\frac{d}{v}$ gives, using relation (6a) (6b)

$$\theta(-\tfrac{d}{v}) = Ae^{\frac{\lambda d}{v}} \cos (\tfrac{wd}{v} + \epsilon) = 0$$

$$\therefore \frac{wd}{v} + \epsilon = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

$$\therefore \epsilon = \frac{\pi}{2} - \frac{wd}{v}$$

$$\dot{\theta}(-\tfrac{d}{v}) = \frac{v \tan\alpha}{2d} - Ae^{+\frac{\lambda d}{v}}\left[\lambda 0_{(-)}^{(+)} w \sin \tfrac{\pi}{2}\right] = 0$$

$$\therefore A = -\frac{v \tan\alpha}{2dw} e^{-\frac{\lambda d}{v}}$$

$$\theta = \frac{(vt+d)\tan\alpha}{2d} - \frac{v\tan\alpha}{2wd}\, e^{-\lambda(\frac{d}{v}+t)} \cos\left(wt - \frac{\pi}{2} + \frac{wd}{v}\right)$$

$$\theta = \frac{\tan\alpha}{2d}\left[(vt+d) - \frac{v}{w}\, e^{-\lambda(\frac{d}{v}+t)} \sin w\,(t+\tfrac{d}{v})\right] \quad (-\tfrac{d}{v} < t < \tfrac{d}{v})$$

$$\dot{\theta} = \frac{\tan\alpha}{2d}\left[v - \frac{v}{w}\, e^{-\lambda(\frac{d}{v}+t)} (w\cos w\,(t+\tfrac{d}{v}) - \lambda \sin(wt + \tfrac{d}{v}))\right]$$

$$\left.\begin{aligned} \theta\,(\tfrac{d}{v}) &= \frac{\tan\alpha}{2d}\left[2d - \frac{v}{w}\, e^{-\lambda\frac{2d}{v}} \sin\frac{2wd}{v}\right] \\ \dot{\theta}\,(\tfrac{d}{v}) &= \frac{\tan\alpha}{2d}\left[v - \frac{v}{w}\, e^{-\frac{2\lambda d}{v}} (w\cos\frac{2wd}{v} - \lambda\sin\frac{2wd}{v})\right] \end{aligned}\right\}$$

Consider $(\frac{d}{v} < t < \infty)$ a possible solution will be

$$\theta = \tan\alpha + Be^{-\lambda t}\cos(wt - \epsilon_1) \qquad (7)$$

i.e. $\bar{\theta} = \tan\alpha$, and $\vartheta = Be^{-\lambda t}\cos(wt - \epsilon_1)$
where B, ϵ_1 are arbitrary constants and since α is small we replace it by $\tan\alpha$. Attention may be drawn to the fact that solution (7), in addition, gives for $t \to \infty$
$\theta \to \alpha$

Applying conditions of continuity at $t = \frac{d}{v}$

$$\theta = \tan\alpha + Be^{-\lambda t}\cos(wt - \epsilon_1)$$

$$\dot{\theta} = -Be^{-\lambda t}\left[\lambda\cos(wt - \epsilon_1) + w\sin(wt - \epsilon_1)\right]$$

$$\cancel{\tan\alpha} + Be^{-\frac{\lambda d}{v}}\cos\left(\frac{wd}{v} - \epsilon_1\right) = \cancel{\tan\alpha} - \frac{v}{2wd}\tan\alpha\, e^{\frac{2\lambda d}{v}} \sin\frac{2wd}{v}$$

$$-Be^{-\frac{\lambda d}{v}}\left[\lambda\cos\left(\frac{wd}{v}-\epsilon_1\right)+w\sin\left(\frac{wd}{v}-\epsilon_1\right)\right]=\frac{v\tan\alpha}{2d}\left[1-\frac{e^{-\frac{2\lambda d}{v}}}{w}\left(w\cos\frac{2wd}{v}-\lambda\sin\frac{2wd}{v}\right)\right]$$

hence $$B=-\frac{v}{2wd}\tan\alpha\, e^{-\frac{\lambda d}{v}}\sin\frac{2wd}{v}\sec\left(\frac{wd}{v}-\epsilon_1\right)$$

$$\lambda\cos\left(\frac{wd}{v}-\epsilon_1\right)+w\sin\left(\frac{wd}{v}-\epsilon_1\right)=\frac{we^{\frac{\lambda d}{v}}-e^{-\frac{\lambda d}{v}}\left(w\cos\frac{2wd}{v}-\lambda\sin\frac{2wd}{v}\right)}{e^{-\frac{\lambda d}{v}}\sin\frac{2dw}{v}\sec\left(\frac{wd}{v}-\epsilon_1\right)}$$

$$\lambda+w\tan\left(\frac{wd}{v}-\epsilon_1\right)=\frac{we^{\frac{2\lambda d}{v}}-w\cos\frac{2wd}{v}}{\sin\frac{2wd}{v}}+\lambda$$

$$\tan\left(\frac{wd}{v}-\epsilon_1\right)=\frac{e^{\frac{2\lambda d}{v}}-\cos\frac{2wd}{v}}{\sin\frac{2wd}{v}}$$

$$\sec\left(\frac{wd}{v}-\epsilon_1\right)=\frac{\sqrt{1+e^{\frac{4\lambda d}{v}}-2e^{\frac{2\lambda d}{v}}\cos\frac{2wd}{v}}}{\sin\frac{2wd}{v}}$$

$$B=-\frac{v}{2wd}\tan\alpha\sqrt{e^{-\frac{2\lambda d}{v}}+e^{\frac{2\lambda d}{v}}-2\cos\frac{2wd}{v}}=-\frac{v}{2wd}\tan\alpha\sqrt{2\left(\cosh^2\frac{2\lambda d}{v}-\cos\frac{2wd}{v}\right)}$$

further, to get ϵ_1

$$\cos(wt-\epsilon_1)=\cos\left(\overline{wt-\frac{wd}{v}}+\overline{\frac{wd}{v}-\epsilon_1}\right)=$$

$$=\cos\left(wt-\frac{wd}{v}\right)\cos\left(\frac{wd}{v}-\epsilon_1\right)-\sin\left(wt-\frac{wd}{v}\right)\sin\left(\frac{wd}{v}-\epsilon_1\right)$$

$$\cos\left(\frac{wd}{v} - \epsilon_1\right) = \frac{\sin\frac{2wd}{v}}{\sqrt{\quad}}, \quad \sin\left(\frac{wd}{v} - \epsilon_1\right) = \frac{e^{\frac{2\lambda d}{v}} - \cos\frac{2wd}{v}}{\sqrt{\quad}}$$

$$\cos(wt - \epsilon_1) = \cos\left(wt - \frac{wd}{v}\right)\sin\frac{2wd}{v} - \sin\left(wt - \frac{wd}{v}\right)\left(e^{\frac{2\lambda d}{v}} - \cos\frac{2wd}{v}\right)\frac{1}{\sqrt{\quad}}$$

and hence

$$\theta = \tan\alpha - \frac{v}{2wd}\tan\alpha\, e^{-\lambda\left(t+\frac{d}{v}\right)}\left[\cos w\left(t-\frac{d}{v}\right)\sin\frac{2wd}{v} - \sin w\left(t-\frac{d}{v}\right)\left(e^{\frac{2\lambda d}{v}} - \cos\frac{2wd}{v}\right)\right]$$

$$= \tan\alpha\left\{1 - \frac{v}{2wd}e^{-\lambda t}\left[e^{-\frac{\lambda d}{v}}\sin w\left(t+\frac{d}{v}\right) - e^{\frac{\lambda d}{v}}\sin w\left(t-\frac{d}{v}\right)\right]\right\}$$

Summarizing, we have

$$\theta = 0 \qquad \left(-\infty < t < -\frac{d}{v}\right)$$

$$\theta = \frac{\tan\alpha}{2d}\left[(vt+d) - \frac{v}{w}e^{-\lambda\left(\frac{d}{v}+t\right)}\sin w\left(t+\frac{d}{v}\right)\right] \qquad \left(-\frac{d}{v} < t < \frac{d}{v}\right)$$

$$\theta = \tan\alpha\left\{1 - \frac{v}{2wd}e^{-\lambda t}\left[e^{-\frac{\lambda d}{v}}\sin w\left(t+\frac{d}{v}\right) - e^{\frac{\lambda d}{v}}\sin w\left(t-\frac{d}{v}\right)\right]\right\} \qquad \left(\frac{d}{v} < t < \infty\right)$$

Hitherto the theoretical analysis furnishes results for the independent motion of the centre of gravity and the angle of pitch in terms of time. Without further difficulty, the calculations will be extended on the following pages to obtain the deflections of both front and rear springs as functions of the time.

The Deflections of the Springs

Let $y^{(1)}$ be the absolute displacement of the front spring

$y^{(2)}$ be the absolute displacement of the rear spring.

then $y^{(1)} = y + d\theta$

$y^{(2)} = y - d\theta$

then the deflections of the springs relative to the ground are

for the front spring $Y^{(1)} = y^{(1)} - \bar{\eta} - h$ (*)

for the rear spring $Y^{(2)} = y^{(2)} - \bar{\eta} - h$

for $-\frac{d}{v} \leq t \leq \frac{d}{v}$ we obtain

$$y^{(1)} = h + \tfrac{1}{2}\tan\alpha(vt+d)\left[1-e^{-p(t+\frac{d}{v})}\right] + \frac{\tan\alpha}{2}\left[(vt+d) - \frac{v}{w}e^{-\lambda(\frac{d}{v}+t)}\sin w(t+\tfrac{d}{v})\right]$$

$$= h + \tan\alpha\left[(vt+d) - \tfrac{1}{2}\left\{(vt+d)e^{-p(t+\frac{d}{v})} + \frac{v}{w}\sin w(t+\tfrac{d}{v})e^{-\lambda(\frac{d}{v}+t)}\right]\right.$$

and $\bar{\eta}\,(t+\frac{d}{v}) = (vt+d)\tan\alpha$

then $Y^{(1)} = -\tfrac{1}{2}\tan\alpha\left[(vt+d)e^{-p(t+\frac{d}{v})} + \frac{v}{w}\sin w(t+\frac{d}{v})e^{-\lambda(\frac{d}{v}+t)}\right]$

$$y^{(2)} = \tfrac{1}{2}\tan\alpha\,(vt+d)\left[1-e^{-p(t+\frac{d}{v})}\right] - \frac{\tan\alpha}{2}\left[(vt+d) - \frac{v}{w}e^{-\lambda(\frac{d}{v}+t)}\sin w(t+\tfrac{d}{v})\right]$$

$$\bar{\eta}(t-\tfrac{d}{v}) = h$$

$$Y^{(2)} = -\tfrac{1}{2}\tan\alpha\left[(vt+d)e^{-p(t+\frac{d}{v})} - \frac{v}{w}e^{-\lambda(\frac{d}{v}+t)}\sin w(t+\tfrac{d}{v})\right]$$

Considering $\frac{d}{v} < t < \infty$

for the front spring $\eta(vt+d) = (vt+d)\tan\alpha$

for the rear spring $\eta(vt-d) = (vt-d)\tan\alpha$

$$Y^{(1)} = +vt\tan\alpha + \tan\alpha\, e^{-pt}\left[d\sinh\frac{pd}{v} - vt\cosh\frac{pd}{v}\right] - (vt+d)\tan\alpha$$

$$+d\tan\alpha\left\{1 - \frac{v}{2wd}e^{-\lambda t}\left[e^{-\frac{\lambda d}{v}}\sin w(t+\tfrac{d}{v}) - e^{\frac{\lambda d}{v}}\sin w(t-\tfrac{d}{v})\right]\right\}$$

(*) for the front spring $\bar{\eta}$ must be taken at $\bar{\eta}\,(t+\frac{d}{v})$

for the back spring $\bar{\eta}$ must be taken at $\bar{\eta}\,(t-\frac{d}{v})$

$$\therefore Y^{(1)} = \tan\alpha\left[e^{-pt}\left(d\sinh\frac{pd}{v} - vt\cosh\frac{pd}{v}\right) - \frac{v}{2w}e^{-\lambda t}\left(e^{-\frac{\lambda d}{v}}\sin w\left(t+\frac{d}{v}\right) - e^{\frac{\lambda d}{v}}\sin w\left(t-\frac{d}{v}\right)\right)\right]$$

$$Y^{(2)} = vt\tan\alpha + \tan\alpha\, e^{-pt}\left[d\sinh\frac{pd}{v} - vt\cosh\frac{pd}{v}\right] - (vt-d)\tan\alpha$$

$$-d\tan\alpha\left\{1 - \frac{v}{2wd}e^{-\lambda t}\left[e^{-\frac{\lambda d}{v}}\sin w\left(t+\frac{d}{v}\right) - e^{\frac{\lambda d}{v}}\sin w\left(t-\frac{d}{v}\right)\right]\right\}$$

$$\therefore Y^{(2)} = \tan\alpha\left\{e^{-pt}\left[d\sinh\frac{pd}{v} - vt\cosh\frac{pd}{v}\right] + \frac{v}{2w}e^{-\lambda t}\left[e^{-\frac{\lambda d}{v}}\sin w\left(t+\frac{d}{v}\right) - e^{\frac{\lambda d}{v}}\sin w\left(t-\frac{d}{v}\right)\right]\right\}$$

and, of course, for $-\infty < t < -\frac{d}{v}$

$$Y^{(1)} = Y^{(2)} = 0.$$

Conclusion and Summary of Results with Nomenclature

All the formulae required for a continuous solution of the problem follow. They involve the horizontal speed of the plane and the angle of elevation (assumed to be small) as independent parameters. It must be borne in mind, however, that this is the solution for the "equivalent" spring. It is an easy matter to obtain the results for the given oleo springs using the equivalent spring constant.

The Deflection of the Front Springs

$Y^{(1)} = 0$ ..... in $-\infty < t \leq -\frac{d}{v}$

$$Y^{(1)} = -\tfrac{1}{2}\tan\alpha\left[(vt+d)e^{-p\left(t+\frac{d}{v}\right)} + \frac{v}{w}\sin w\left(t+\frac{d}{v}\right)e^{-\lambda\left(\frac{d}{v}+t\right)}\right] \cdots -\frac{d}{v} \leq t \leq \frac{d}{v}$$

$$Y^{(1)} = \tan\alpha\left[e^{-pt}\left(d\sinh\frac{pd}{v} - vt\cosh\frac{pd}{v}\right) - \frac{v}{2w}e^{-\lambda t}\left(e^{-\frac{\lambda d}{v}}\sin w\left(t+\frac{d}{v}\right) - e^{\frac{\lambda d}{v}}\sin w\left(t-\frac{d}{v}\right)\right)\right]$$

in $\frac{d}{v} \leq t < \infty$.

The Deflection of the Rear Springs

$$Y^{(2)} = 0 \quad \ldots\ldots\ldots \quad \text{in} \quad -\infty < t \leq -\frac{d}{v}$$

$$Y^{(2)} = -\tfrac{1}{2}\tan\alpha\left[(vt+d)e^{-p(t+\frac{d}{v})} - \frac{v}{w}e^{-\lambda(\frac{d}{v}+t)}\sin w(t+\frac{d}{v})\right] \ldots \text{in} \ -\frac{d}{v} \leq t \leq \frac{d}{v}$$

$$Y^{(2)} = \tan\alpha\left\{e^{-pt}\left[d\sinh\frac{pd}{v} - vt\cosh\frac{pd}{v}\right] + \frac{v}{2w}e^{-\lambda t}\left[e^{-\frac{\lambda d}{v}}\sin w(t+\frac{d}{v}) - \right.\right.$$

$$\left.\left. - e^{\frac{\lambda d}{v}}\sin w(t-\frac{d}{v})\right]\right\} \quad \ldots\ldots \quad \text{in} \quad \frac{d}{v} \leq t < \infty$$

Angle of Pitch

$$\theta = 0 \quad \ldots\ldots\ldots \quad -\infty < t \leq -\frac{d}{v}$$

$$\theta = \frac{\tan\alpha}{2d}\left[(vt+d) - \frac{v}{w}e^{-\lambda(\frac{d}{v}+t)}\sin w\,(t+\frac{d}{v})\right] \quad \ldots\ldots \quad -\frac{d}{v} \leq t \leq \frac{d}{v}$$

$$\theta = \tan\alpha\left\{1 - \frac{v}{2wd}e^{-\lambda t}\left[e^{-\frac{\lambda d}{v}}\sin w(t+\frac{d}{v}) - e^{\frac{\lambda d}{v}}\sin w(t-\frac{d}{v})\right]\right\} \ldots \frac{d}{v} \leq t < \infty$$

---

where $p = \frac{c}{m}$

$\lambda = \frac{cd^2}{I}$

$w = \frac{d}{I}\sqrt{2kI - c^2d^2}$

$k = \frac{1}{9}k'$

The following data must be known:

m - total mass of aircraft and cargo less mass of skis.

I - pitching moment of inertia of the aeroplane with respect to the centre of gravity

$2d$ - distance between shock struts

k' - spring constant of oleo-spring

($k = \frac{1}{9} k'$ - spring constant of equivalent spring)

c - damping constant (i.e. damping force per unit vertical velocity) In this mathematical analysis critical damping is assumed, and is thus known $c = \sqrt{2mk}$.

:EM

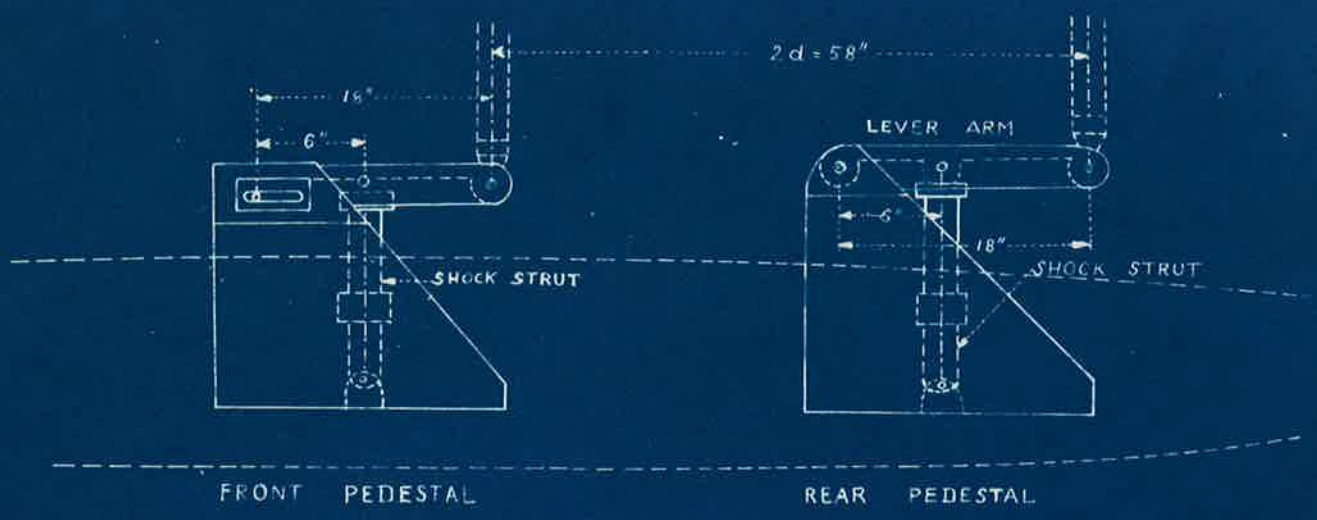

FIG. 1.

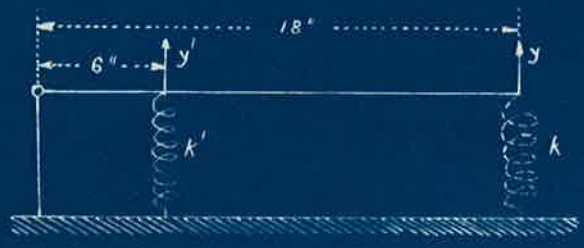

FIG. 2

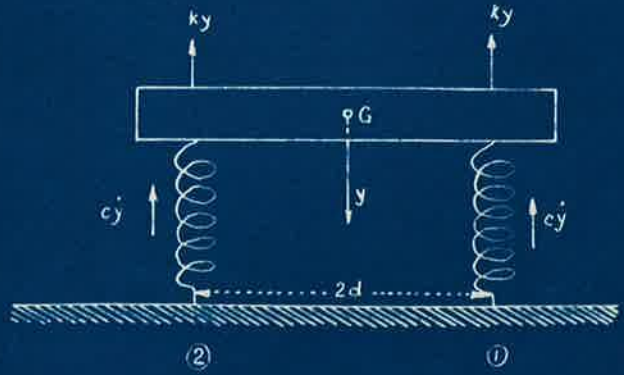

FIG. 3.

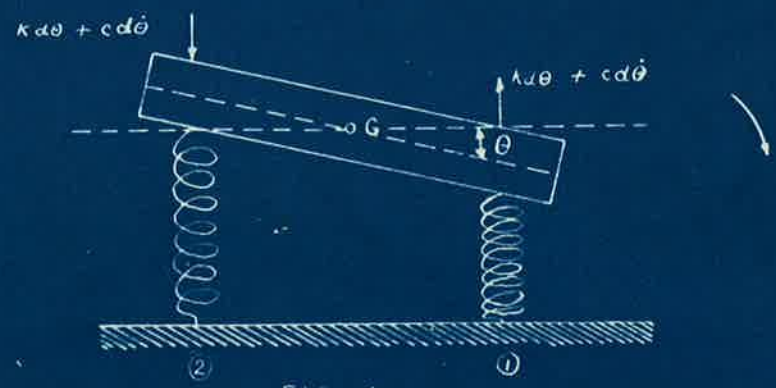

FIG. 4.

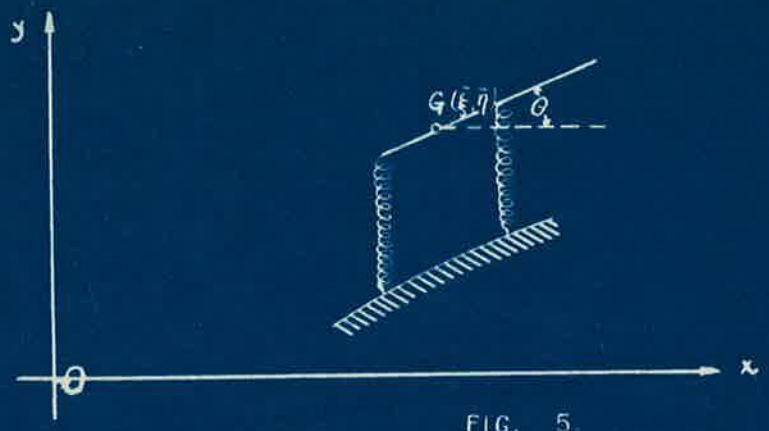

FIG. 5.

FIG. 6.

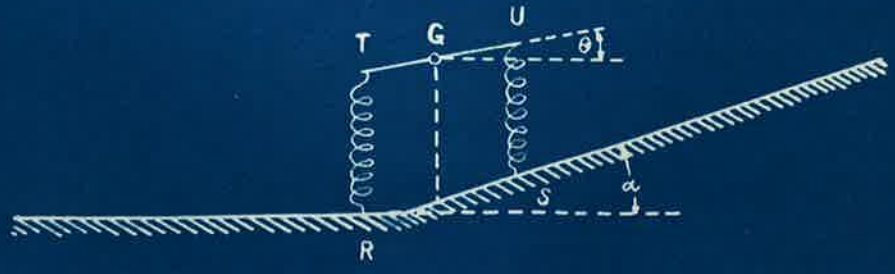

FIG. 7.