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by

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# COMPARISON OF OBSERVED AND CALCULATED GROUND TEMPERATURES WITH PERMAFROST DISTRIBUTION UNDER A NORTHERN LAKE

W. G. BROWN,* G. H. JOHNSTON,† and R. J. E. BROWN†

ABSTRACT

Making use of limited ground temperature measurements in the neighbourhood of a small shallow lake near Inuvik, N.W.T., it was possible with the help of an electronic computer to estimate the entire thermal regime under and about the lake. The results indicated a completely unfrozen zone of roughly hour-glass shape directly under the lake. Field borings under the lake and adjacent to it supported this theoretical finding.

SOMMAIRE

En partant de quelques observations faites sur la température du sol dans les environs d'un petit lac peu profond près de Inuvik, Territoire du Nord-Ouest, il a été possible, au moyen d'un calculateur électronique, de faire une estimation du régime thermique complet existant sous le lac et dans ses environs. Les résultats ont montré qu'il existait directement sous le lac une zone complètement dégelée ayant approximativement la forme d'un sablier. Des sondages effectués sous le lac et tout près de lui ont corroboré ce résultat théorique.

To determine the influence of a lake on adjacent permafrost conditions a field investigation was undertaken in April, 1961, near Inuvik, N.W.T., in the Mackenzie River delta (Johnston and Brown, 1963) by members of the Division of Building Research. Four deep borings and several shallow probings were made at a small, shallow lake, approximately 900 ft. in diameter. The

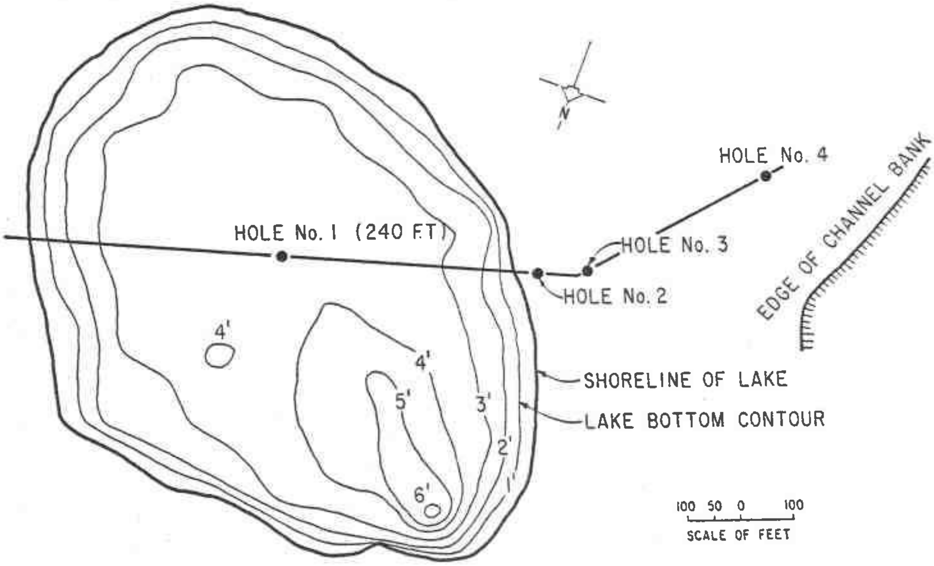


FIGURE 1. Plan of lake showing bore holes and traverse

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location of the boreholes is shown on a plan view of the lake in Figure 1. A portion of the delta with its many bodies of water which surround the lake, is shown in Figure 2. In addition to the borings and probings made to determine the distribution of permafrost, ground temperatures were observed to depths of 200 ft. Although the records obtained were limited, they warranted comparison with theoretical calculations of the temperatures in the ground.

With the help of an electronic computer program already available and the measured temperatures at several locations, it has been possible to calculate the mean annual ground surface and lake temperature and the geothermal

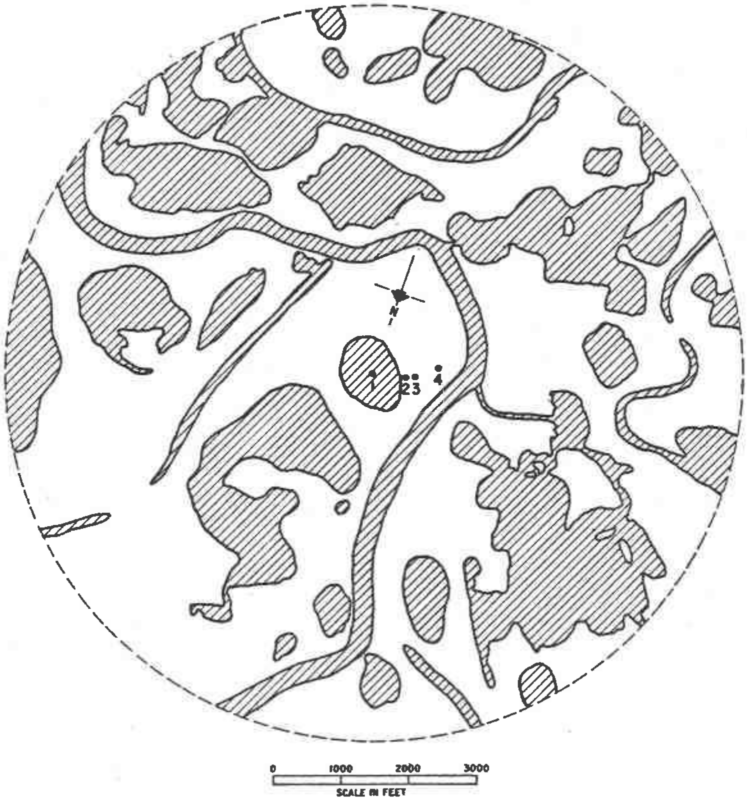


FIGURE 2. General view of delta showing bodies of water adjacent to lake investigated

gradient for the region. In addition, some of the temperature records obtained inland from the lake were sufficient to establish the approximate amplitude of the annual ground surface temperature wave.

It is the purpose of this paper to describe the procedure used in deriving the theoretical ground temperature regime and to compare this regime with the field results.

## GROUND TEMPERATURES

Ground temperatures measured in May, 1961, by means of thermocouples placed at various depths to 100 ft. in hole no. 3 and to 200 ft. in hole no. 4,

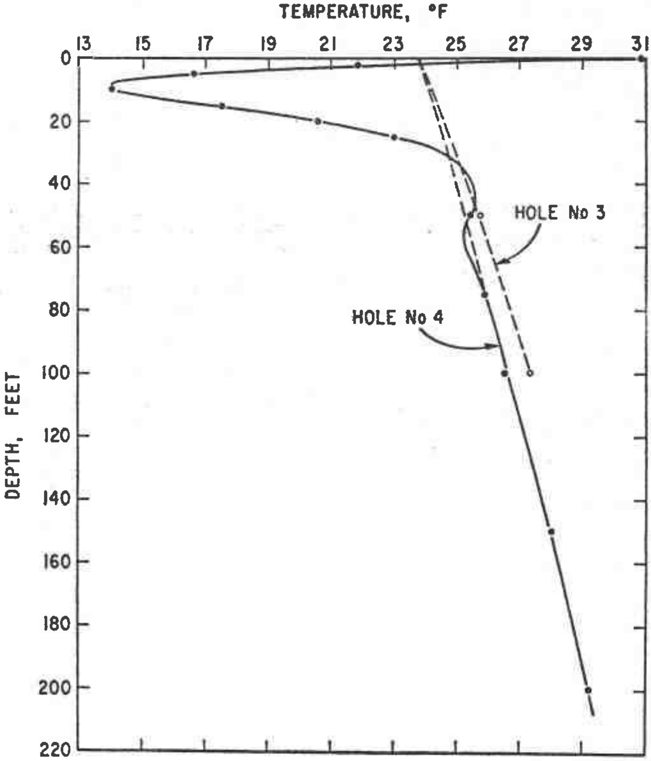


FIGURE 3. Ground temperatures in holes nos. 3 and 4. Dashed lines are calculated mean annual steady-state conditions; solid line is calculated non-steady-state for hole no. 4

are shown in Figure 3 (plotted as points). The calculated temperatures (represented by the dashed and solid curves), which will be discussed later, are also shown. Hole no. 4 is well removed from any body of water and, as a consequence, the ground temperatures to depths of about 100 ft. are practically free of distortion from the influence of surface water. Although only two temperature measurements were made in hole no. 3, these were sufficient to indicate higher temperatures than at corresponding depths in hole no. 4 because of the proximity of the lake.

## THEORETICAL TEMPERATURE REGIME

Owing to access difficulties lake temperature variations during the year were not measured. As a result no attempt can be made to calculate the seasonal variation in ground temperature under the lake. On the other hand, as shown in Figure 3, the annual temperature cycle for all practical purposes penetrates no more than 80 ft. below the ground surface. Beyond this depth, temperatures are governed only by the mean annual temperatures at the ground surface both under the lake and remote from it. In the absence of evidence to the contrary it should be permissible to assume no intrinsic change in mean annual temperatures with time. The possibility that the physical margins of the lake vary with time poses a far more difficult problem, however, for in order to calculate the temperature in the ground it is necessary to know the rate, if any, at which the shape of the lake changes. There is reason to believe that the lake in question has not, in fact, varied its boundaries greatly, at least for about 200 years. In the first place the lake is unconnected with other bodies of water except during occasional spring flood periods, hence sediment accumulation is expected to be low. Field observations indicated an average rate for sediment deposition to be about ½ in.per year. Perhaps of more importance is the presence of trees about 220 years old around the lake periphery. It can be shown with the help of an equation given by Lachenbruch (1957) for the temperatures under a circular lake that if a lake with a diameter of 1,000 ft., whose mean annual temperature is 9° F. higher than the mean annual ground surface temperature, has been in existence for 200 years then the mean annual temperatures at depths of up to 200 ft. will deviate from the final or steady temperatures by less than 1° F. Under the circumstances then, it should be reasonable to assume, at least as a first approximation for calculation purposes, that the steady conditions have been reached for the real lake. It should be remembered that not only the lake itself but also all other nearby bodies of water (Figure 2) contribute to the resulting temperature in the ground. Hence calculation of the ground temperature must also take into account the effect of these other bodies of water.

## Calculation of Ground Temperature

The mean temperature in the ground was calculated assuming the mean annual water and ground surface temperatures everywhere constant. This mean annual temperature ($\bar{T}$) at any location in the ground can be expressed by the following equation due to Lachenbruch (1957):

$$\text{(1)} \qquad \bar{T} - \bar{T}_G = (\bar{T}_W - \bar{T}_G) \sum_{\sigma=0}^{\sigma=2\pi} \frac{\Delta\sigma}{2\pi}\left[1 - \frac{1}{\sqrt{[1 + (R/z)^2]}}\right] + G_G z.$$

The first term represents the sum of the temperature contributions at the common apex of circular sectors of radius $R$ and angle $\Delta\sigma$, where $R$ is the horizontal distance from the point in question to the edge of a body of water and $z$ is the depth. The second term is the temperature contribution of the geothermal gradient $G_G$. $\bar{T}_W$ is the mean annual temperature of the water of the lakes and rivers and $\bar{T}_G$ is the mean annual ground surface temperature.

Computer programs are available to evaluate the first term in Equation (1) using either the Bendix G-15 or the IBM 1620. The latter is more rapid and requires about six hours to establish completely the temperature profile along the traverse line in Figure 1. The computation procedure requires only insertion of Cartesian co-ordinates specifying the outline of the lake and assures internally that the circular sectors are of sufficiently small angle to minimize error. Since the computer program had a capacity of only 49 pairs of co-ordinates it was necessary to subdivide the lake-river areas external to the lake in question in the manner shown in Figure 4. By approximating the actual water area in this way it was possible to treat it as just two lakes

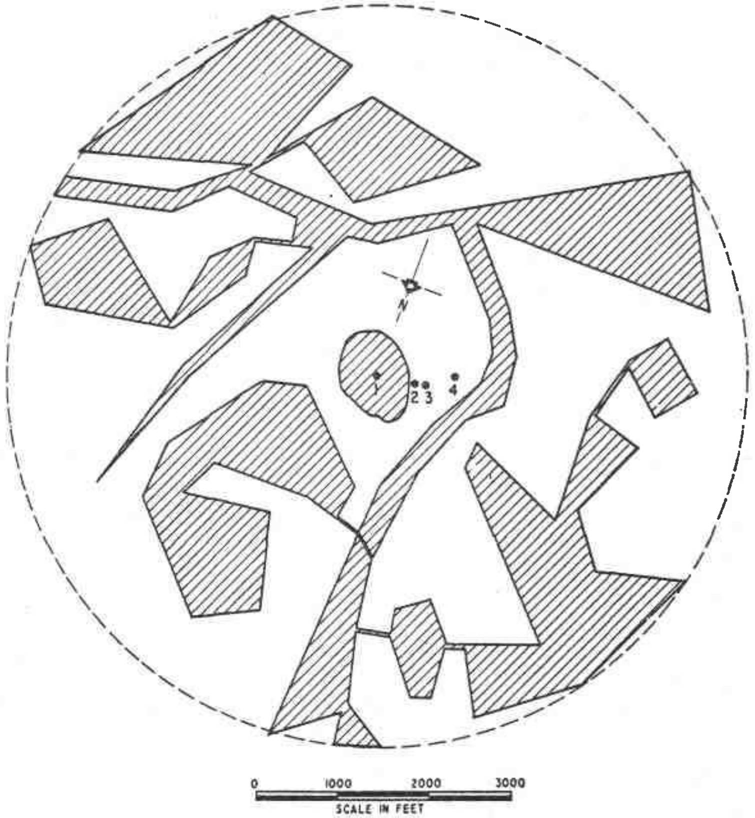


FIGURE 4. Geometrical approximation of surrounding bodies of water for computation purposes

neighbouring on the central lake. The temperature at any location is then the sum of the contributions of all three lakes. Bodies of water more remote from the lake than those shown in Figures 2 and 4 were ignored as their temperature contribution would be negligible. Equation (1) can be written in the form

$$\bar{T} - \bar{T}_G = (\bar{T}_W - \bar{T}_G).f(x, y, z) + G_G z, \quad (2)$$

where $f$ is the calculated factor obtained with the computer program. Values of the computed function $f$ for holes nos. 3 and 4 are given in Table I.

With the help of Equation (2) and using these values for $f$ and the measured data from Figure 3, values of 33° F., 23.8° F., and 1.88° F. per 100 feet were obtained for $\bar{T}_W$, $\bar{T}_G$, and $G_G$ respectively. The dashed curves of Figure 3 representing the mean annual temperature were thus obtained using these calculated values.

It will be seen that these curves represent the experimental records quite well for depths greater than about 80 ft. For shallower depths the annual temperature cycle at the ground surface causes the ground temperature to oscillate about the mean annual temperature and is the reason for the large

TABLE I

Dimensionless Temperature Function $f(x, y, z)$ for boreholes no. 3 and 4

| $z$, ft. | Hole no. 3 | Hole no. 4 |
|---|---|---|
| 0 | 0 | 0 |
| 20 | 0.0408 | 0.0218 |
| 40 | 0.0797 | 0.0436 |
| 60 | 0.1163 | 0.0653 |
| 80 | 0.1489 | 0.0840 |
| 100 | 0.1762 | 0.1035 |
| 140 | 0.2196 | 0.1392 |
| 180 | 0.2484 | 0.1683 |
| 220 | 0.2696 | 0.1908 |
| 260 | 0.2833 | 0.2104 |
| 300 | 0.2940 | 0.2261 |

disparity between the measured temperatures and the dashed (mean annual) curves.

As pointed out previously, it is not possible to determine the cyclic variation of ground temperature near the lake because the annual lake temperature cycle was not recorded. In spite of this it is possible to show that the measured temperatures in hole no. 4 can be accounted for by theory. The reason for this is that hole no. 4 is sufficiently remote from bodies of water to be unaffected by the cycling of their temperatures. It is known that the ground surface temperature variation can be approximated by a sine wave of the form $A \sin wt$, where $A$ is the amplitude, $w = 2\pi/P$, $P$ being the period (365 × 24 hr.), and $t$ is the time. Under the influence of this sine wave the temperature deviation from the mean annual at depth $z$ is:

$$Ae^{-z\sqrt{(w/2\alpha)}}\sin[wt - z\sqrt{(w/2\alpha)}]$$

where $\alpha$ = thermal diffusivity. By curve fitting to the measured temperatures for hole no. 4 in Figure 3, it was found that with $A = 38°$ F., $wt = 0.185$, and $\alpha = 0.030$ sq. ft./hr. the complete temperature profile at hole no. 4 could be represented by the expression

$$T - 23.8 = 9.2f + \frac{1.88z}{100} + 38e^{-0.11z}\sin(0.185 - 0.11z). \quad (3)$$

It will be seen from Figure 3 that the agreement between measured temperatures (points) and the curve represented by Equation (3) (solid line) is excellent for all depths. This indicates that for the general case if the temperature conditions at the surface are well known then the temperatures at various depths can be predicted with good accuracy. Should it be desirable to know the ground temperatures in hole no. 4 at some other time of year it is only necessary to insert the appropriate value of $wt$ into Equation (3) in place of the value 0.185.

## Mean Annual Temperatures for the Entire Traverse

With the appropriate values of mean annual lake and ground surface temperatures and the geothermal gradient established, the entire mean annual

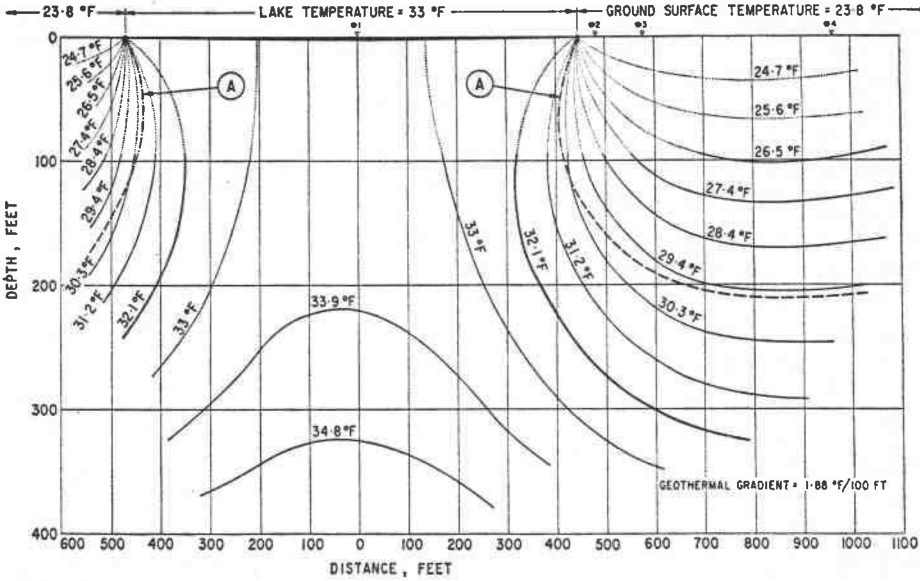


FIGURE 5. Computed temperature regime under traverse line. Mean lake water temperature of 33° F., ground surface temperature of 23.8° F., and geothermal gradient of 1.88° F./100 ft. calculated from measured temperatures at holes nos. 3 and 4. Dashed curve *A* is the position of the 32° F. isotherm for a lake water temperature of 35° F. (Note: For depths less than about 80 ft., the temperature varies under the influence of the annual climate cycle.)

temperature regime in the ground can now be calculated with the help of Equation (2). Ground temperature profiles were thus computed for several points on the traverse line shown in Figure 1. Mean annual temperature isotherms were then drawn as shown in Figure 5.

## Discussion and Conclusion

The computed results in Figure 5 show that the entire region under the lake, as indicated by the position of the 32° F. isotherms, is unfrozen, whereas under surface areas not covered by water the ground remains frozen to depths of up to several hundred feet. Of interest is the relative constancy of mean annual temperature under the lake centre indicating a temperature change of only 1.8° F. in moving from the bottom of the lake to a depth of over 300 ft. The positions of the 33° F. isotherms, which intersect the lake at 90° angles, appear strange, but are the result of the combined effect of geothermal gradient, ground surface temperature, and lake temperature.

The field probings in the bore holes are all consistent with the results of Figure 5. At hole no. 1 no permafrost was encountered to a depth of at least 229 ft., where bedrock was reached. Permafrost was encountered for the full depth (115 ft.) of hole no. 2 and at hole no. 3 the ground was completely frozen to a depth of at least 196 ft. No unfrozen ground was encountered in hole no. 4 where bedrock was reached at 260 ft.

The value of mean annual ground surface temperature of 23.8° F. obtained from Figure 3 is obviously fairly accurate. The value for mean annual water temperature of 33° F. is subject to an error of perhaps 2 or 3 degrees F. It is known that the lake does not everywhere freeze to the bottom in winter indicating a wintertime temperature of 32° F. for a period of at least six months. During the summer the water temperature will probably be quite high. In a similar lake at Point Barrow, Alaska, Brewer (1958) reports a mean annual lake bottom temperature of close to 35° F. Because of possible error in the ground temperature measurements and the necessity of assuming constant ground thermal properties and a steady-state condition, the true mean annual water temperature at the bottom of the delta lake could easily be 35° F. instead of 33° F. The positions of the 32° F. isotherms for this possibility are also given in Figure 5. By comparing these isotherms with the 32° F. isotherms resulting from the water being at 33° F. it can be seen that small deviations in water temperature can markedly alter the position of the permafrost. In any event, however, the theory supports the general field findings of no permafrost under the central region of the lake.

To the best of the authors' knowledge the analysis undertaken here is the first attempt of its kind to use an electronic computer for estimating temperatures in the ground under lakes. The results have been encouraging in spite of the limited field records available for comparison and indicate that future work of this kind could be expected to yield useful results.

The analysis has brought out the importance of obtaining precise knowledge of temperatures and temperature variations at the ground surface. For regions near the ground surface the annual cycling of temperature at the surface is of great importance whereas at greater depths only the mean annual ground (and water) surface temperatures are important. Furthermore, at great depths any slow variation with time of the boundaries of surface areas must be considered. Nearer the surface this factor is not critical because temperature regimes are established fairly rapidly.

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