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The Effects of Free Water on Snow Gliding

by D.M. McClung and G.K.C. Clarke

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RÉSUMÉ

Les équations classiques établissant un rapport entre l'entraînement du manteau nival et la vitesse de glissement ne tiennent pas compte des influences possibles de l'eau libre située à l'interface neige-terre ou près de celle-ci. C'est pourquoi deux des plus importantes applications de la mécanique de la neige, soit la prévision des forces maximales s'exerçant sur les structures et celle du déclenchement des avalanches de pleine hauteur, débordent le champ d'application de la théorie actuelle. Les auteurs décrivent ici deux facteurs qui peuvent réduire l'entraînement du manteau nival et accroître la vitesse de glissement : (1) la réduction de la viscosité de la neige occasionnée par la présence d'eau dans les couches inférieures, et (2) la séparation partielle entre le manteau nival et l'interface de glissement. Dans ce dernier cas, ils montrent comment la présence d'eau libre couvrant des portions de l'interface réduit la rugosité effective et accroît la vitesse de glissement; pour que cette dernière augmente de façon importante, une grande partie du lit doit être recouverte d'eau.

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The Effects of Free Water on Snow Gliding

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Conventional equations that relate snowpack drag to glide velocity take no account of the possible influences of free water at or near the snow-earth interface. For this reason two of the most important applications of snow mechanics, predicting maximum forces on structures and predicting the release of full-depth avalanches, lie beyond the scope of present theory. Here we discuss two effects that can decrease snowpack drag and increase glide velocity: (1) reduction of snow viscosity by the presence of water in the lowest layers, and (2) partial separation of the snowpack from the glide interface. For partial separation we show how the presence of free water covering portions of the interface reduces the effective roughness and increases glide speed; for significant enhancement of the glide rate a substantial fraction of the bed must be water covered.

INTRODUCTION

Field measurements show that glide rate (slip of the entire snowpack over sloping ground) depends upon the roughness at the snow-earth interface and upon snowpack properties. These observations [e.g., in *der Gand and Zupančič*, 1966; *McClung*, 1975; *Endo*, 1983] suggest the following generalizations: (1) Glide velocity may have a steady and nonsteady component, (2) short time scale velocity fluctuations (less than a day) are a persistent feature, (3) these fluctuations need not be related to snowfall, (4) processes that add water at the snow-earth interface can account for increased glide velocity (examples of such processes include rainfall, melt, and the release of stored heat in the ground), and (5) times when free water is added to the snow pack coincide with periods of high glide velocity, high forces on structures, and the release of full depth avalanches.

Present theory takes bed geometry and snow viscosity as fixed and predicts a unique relationship between glide velocity and snowpack drag [*McClung*, 1981]. The theory assumes linear viscous deformation and envisions the snowpack to be separated from the bed by an infinitesimal, continuous water film. The presence of a water film insures locally that the snowpack-glide surface contact does not support a shear stress. There is no surface-normal component of glide velocity at the contact because melt-freeze processes are neglected. The average basal shear stress, required for equilibrium, is produced by the downslope component of normal stresses on interface roughness elements. The analysis leads to a linear relation between basal shear stress and glide velocity.

The 1981 model describes the steady component of the glide process, but the most important applications, namely predicting full depth avalanche release and maximum forces on structures, cannot be described by existing theory. The missing element is the role of free water. We propose that free water causes glide velocity variations by reducing snow viscosity

near the glide interface and by inducing partial separation of the snowpack from the interface.

REVIEW OF SNOW GLIDING

Descriptive Aspects

For glide to take place at the snow-earth interface, the interface must be at 0°C [*in der Gand and Zupančič*, 1966; *McClung*, 1975]. Measurements show that over the course of an entire winter, geothermal heat flow can result in observable snow melting [*in der Gand and Zupančič*, 1966]. In the usual situation this meltwater probably exists as a thin film at the base of the snowpack and enables steady glide. Glide fluctuations occur when additional water is present. Sources of additional water include (1) rain, (2) melt at the interface resulting from stored summer heat, (3) snowpack melt by solar radiation, and (4) melt from geothermal hot spots. Figure 1 shows the influences of sources 1 and 2 on glide rate (see *McClung* [1974] for details).

The character and roughness of the glide surface also affect glide rate. The fastest rates are associated with smooth basal surfaces, such as polished rock or grassy slopes. Glide over rock surfaces appears to differ from that over vegetative surfaces. Field measurements indicate that over rock, glide does not occur until free water reaches the interface (B. Salm, personal communication, 1981). A possible interpretation is that the tiniest asperities contribute very high drag and these must be drowned before gliding can initiate. For grass- or brush-covered slopes, glide can occur without free water being present, but it is greatly enhanced when free water reaches the interface. For certain vegetative surfaces, no glide is observed, presumably because of very high roughness.

Theory of Steady Glide

The main sources of uncertainty in present theories of snow glide are in the rheological description of snow and in the actual mechanisms that contribute to glide. *McClung* [1981] proposed a linear theory in which a single mechanism, creep deformation, accounted for glide. The snowpack is taken as a linear, compressible viscous material. This rheological model

Fig. 1. Glide measurements from brushy and timbered slopes, North Cascades, United States. Increases in glide speed from summer heat stored in the ground (October–November) and midwinter rain (March) shown.

is equivalent to a compressible Newtonian viscous fluid with the static pressure term neglected [Salm, 1967].

When a thin water film separates the snowpack from its bed, tangential stresses vanish at the interface, and drag is calculated as the downslope component of normal stress on interface asperities. McClung [1981] gave a relation between the local basal shear stress τ and glide velocity U :

$$\tau = \mu U / 2(1 - \nu) D^0(x, y) \quad (1)$$

where x and y are downslope and cross-slope directions and μ is the shear viscosity. In this linear theory, μ and the bulk viscosity η are functions of density, temperature, and snow type. Effects of the bulk viscosity are embodied in the parameter $\nu = (3\eta - 2\mu) / (2(3\eta + \mu))$, a viscous analog of Poisson's ratio [Reiner, 1946, 1949]. Values of ν appropriate to alpine snow are discussed by Salm [1977]. The parameter $D^0(x, y)$ is called the stagnation depth [Nye, 1969] and is a function of interface geometry alone (Figure 2). For $U = 0$, due either to high roughness or subfreezing interface temperature, $D^0 = 0$.

Equation (1) is closely related to the standard equation for glacier sliding by creep over a rough bed [Nye, 1969, 1970; Kamb, 1970]. Glacier sliding theory assumes that ice is incompressible but alpine snow is better approximated as a compressible material. For the case where $\eta \rightarrow \infty$, corresponding to incompressibility, $\nu \rightarrow \frac{1}{2}$ and equation (1) reduces to the linear glacier sliding equations of Nye and Kamb. Since μ and ν are functions of density, they should increase slowly during the winter; for time scales appropriate to lubrication effects they can be taken as constant unless they are affected by water.

POSSIBLE EFFECTS OF WATER ON GLIDE MECHANICS

In our model, water has two effects on glide mechanics: it decreases snowpack viscosity and promotes separation at the base of the snowpack. Because of the high permeability of seasonal snow we believe that pore pressure effects are usually negligible. Short-term fluctuations in pore pressure should dissipate rapidly, and thick water layers, inducing flotation of the snowpack, cannot develop on alpine slopes. (The case of slush avalanches is an exception that would require special treatment.)

Effects of Water on Snow Rheology

If significant amounts of water are present in the lowest snow layers, shear and bulk viscosity may be reduced, increasing glide rate and possibly decreasing drag. Wet snow softens and becomes highly compressible [Kinosita, 1963, 1967], but quantitative estimates of the effect of water content on snow viscosity are unavailable. Estimates of the influence of water content on snow hardness are provided by Izumi and Akitaya [1985]. For water content in the range 0–22% by weight and dry densities in the range 250 kg m^{-3} to 550 kg m^{-3} , their multiple regression equations show that hardness decreases significantly as water content increases from 0 to 22%. Hardness is reduced by a factor of 2 for coarse-grained snow and by a factor of 3 for fine-grained snow. Their experiments with water-saturated snow illustrate an even greater softening effect. For snow with dry density less than 300 kg m^{-3} , hardness decreased by nearly a factor of 10 when saturated. For snow with dry density greater than 500 kg m^{-3} , hardness was nearly independent of water content. In summary, the softening effect of water is most pronounced for fine-grained snow having low dry density.

Because snowpack density progressively increases through the winter season, the water influence on shear viscosity μ should be most pronounced in early season. Figure 1 [McClung, 1975] shows that if water is available, early season glide rates can be high and later, as density increases, glide rates decrease. (In the example given, the source of meltwater is stored ground heat, not surface melting.) Other measurements show a similar progression [McClung, 1974; in der

Fig. 2. Schematic of creep and glide velocity components for the snowpack showing geometrical construction of D^0 , the stagnation depth [after Nye, 1969].

Gand and Zupančič, 1966]. These observations are consistent with the experimental results of Izumi and Akitaya [1985].

Free water may also reduce η to produce a lower Poisson's ratio in (1). We expect that ν will vary only slightly if μ and η both decrease similarly. Since ν is defined from a ratio of μ and η , the combined effect on ν would be small.

The derivation of (1) assumed constant viscosities and follows from a solution in a half-space. Extension of the theory to include reduced viscosity in the lowest layers has not been attempted. A simple replacement of the shear and bulk viscosities with their values for wet snow should predict the maximum effect. Since the viscosity of wet snow is not yet known, it does not seem worthwhile to extend the theory to a multi-layered viscosity model.

Effects of Water on Snowpack Separation

Many full-depth avalanches release in late spring when the water influence on snow hardness [Izumi and Akitaya, 1985] is expected to lose significance. Field observations show that glide cracks and full depth avalanches occur at the same locations from year to year [Lackinger, 1986], confirming the existence of a topographic influence that is independent of rheology. We think that the dominant influence of water on glide mechanics is to promote separation of the snowpack from the interface topography.

In (1) the drag arises because the normal pressure on the upstream side of bed roughness elements is greater than on the downstream side. For steep slopes and smooth glide interfaces the normal pressure on the downstream side of asperities can become low enough to allow partial separation [Nye, 1969; Kamb, 1970]; because snow is highly permeable, water infiltrates these separated regions.

When free water is present, portions of the bed may be drowned, and these drowned portions contribute negligibly to drag. For this case the drag is [McClung, 1981]

$$\tau = \mu_w U / \delta_w(x, y) \quad (2)$$

where μ_w is the viscosity of water at 0°C and $\delta_w(x, y)$ is a function of the geometry of the water layer, snowpack sole, and glide interface. Equation (2) is a standard result from lubrication theory [e.g., Batchelor, 1967]. The length scale of δ_w is closely related to mean water film thickness. In fact, δ_w is equal to water film thickness for the special case when the snowpack, modeled as a sliding block, and the glide interface have parallel sides. From field measurements [McClung, 1975], values of D^0 appear to be of the order of centimeters to meters. Using these estimates of D^0 and typical values of μ [Haefeli, 1967], equation (1) yields an estimate of drag that far exceeds that obtained from (2) for the same glide velocity. This discrepancy can only be reconciled by assuming an impossibly small value for the length scale δ_w . Thus shearing of the water film contributes negligibly to drag.

Since both (1) and (2) are linear relations between τ and U , it follows that the effect of a water layer of finite thickness may be approximated as

$$\tau = \mu U / 2(1 - \nu) D^h(x, y) \quad (3)$$

In (3), $D^h(x, y)$ represents the stagnation depth modified by the presence of a basal water layer of mean thickness h . Our intention is to describe a partial drowning of the bed such that

Fig. 3. The imagined effect of a basal water film on substrate roughness.

drowned regions contribute no drag. Modifying effects of basal water on μ and ν may also coexist. Temporal changes in μ and ν (by densification and snow metamorphism) may be included in the simplest manner by increasing μ and ν .

Fluctuations in basal water can affect basal geometry to produce rapid variations in glide rate. In the present model we retain the assumption of steady glide and explore the influence of h variations on glide rate. In a more rigorous theory, time dependence, nonlinear and rate-dependent effects on rheology as well as pore pressure and transient effects would also have to be considered. Equation (1) for the infinitesimal water film and (2) for a completely drowned bed are derived from different interface boundary conditions. This makes (3) an approximation. In addition, the conditions for initiating separation and subsequent redistribution of water at the interface have not been addressed. The drag relation (3) implicitly assumes partial separation; the only drag remaining is from deformation of the sole of the snowpack over undrowned portions of the bed.

MODIFICATION OF SURFACE ROUGHNESS BY WATER LAYER

Because we have no information on how separation affects the position of excess water at the interface, a simple model will be proposed. Water will be assumed to fill the depressions as shown in Figure 3a. In this model, basal water can be viewed as a miniature sea through which islands of basal roughness protrude; the water surface is a punctured plane with a constant level z_0 of the snowpack-water interface. Since glide normally takes place only on slopes with angles exceeding 15°, this simple model is likely to be imperfect. Figure 3b, for example, may be a better model, but it is less tractable. We expect that Figure 3a is fairly realistic because the water will collect in low-pressure zones that exist in the troughs of asperities.

Topographic fluctuations of the substrate surface from its average value are described here by a homogeneous Gaussian random process $z(x, y)$. The mean and autocorrelation of z are denoted $\langle z \rangle$ and $R(x_0, y_0) = \langle z(x, y)z(x - x_0, y - y_0) \rangle$, respectively, where angle brackets indicate the statistical expectation and $x, x - x_0$ represent different positions. The Gaus-

sian assumption gives

$$\langle z \rangle = \frac{1}{(2\pi R_0)^{1/2}} \int_{-\infty}^{\infty} z \exp(-z^2/2R_0) dz = 0 \quad (4)$$

where $R_0 = \langle z^2 \rangle$ is the variance.

Consider variables z_1 and z_2 , both functions of x and y having zero mean and variance R_0 , presented by a Gaussian random process of the form $z(x, y)$. The joint probability density for these variables may be expressed as the bivariate normal probability function [Abramowitz and Stegun, 1965]

$$g(z_1, z_2, p) = \frac{1}{2\pi R_0(1-p^2)^{1/2}} \exp \left[-\frac{1}{2R_0} \left(\frac{z_1^2 + z_2^2 - 2pz_1z_2}{1-p^2} \right) \right] \quad (5)$$

where $p = R(x_0, y_0)/R_0$ is the correlation function for the two variables. It follows that [e.g., Baum, 1969]

$$R(p) = \langle z(x_1, y_1)z(x_2, y_2) \rangle = \frac{1}{2\pi R_0(1-p^2)^{1/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} z_1 z_2 \exp \left[-\frac{z_1^2 + z_2^2 - 2pz_1z_2}{2R_0(1-p^2)} \right] dz_1 dz_2 \quad (6)$$

From (6), if $p = 0$, $R(0) = 0$, whereas if $p = 1$, $z_1 = z_2 = z$ and integration yields $R(1) = \langle z^2 \rangle = R_0$. The parameter R_0 is the mean square deviation of the glide surface elevation, a measure of the strength of topographic fluctuations. The value of R_0 may also be termed the power density for the random Gaussian process z . It may be noted that $p = 1$ corresponds to $R(0, 0) = R_0$ in terms of $R(x_0, y_0)$ defined above.

We assume that the substrate is wavy only in the downslope (x) direction, thus $R = R(x)$. The power spectrum of the substrate surface is defined through the Fourier transform

$$S(k) = \int_{-\infty}^{\infty} \exp(-ikx) R(x) dx \quad (7)$$

here k is the wave number. In order to model the effects of a water layer at the bed, it is convenient to express the drag as an integral over the bed. The theory of glacier sliding presented by Nye [1969] gives calculations that are pertinent. For a linear incompressible material with viscosity η , Nye derived expressions that relate drag to the autocorrelation of the bed topography. For the one-dimensional bed the relevant expression is

$$\tau = \frac{4\eta U}{\pi} \int_0^{\infty} \frac{1}{x} \frac{d^3 R}{dx^3} dx \quad (8)$$

McClung [1981] showed that the transformation $\eta \rightarrow \mu/2(1-v)$ produces analogous expressions that apply to the linear compressible case relevant to snow gliding. Thus the linear snow glide equation becomes

$$\tau = \frac{2\mu U}{\pi(1-v)} \int_0^{\infty} \frac{1}{x} \frac{d^3 R}{dx^3} dx \quad (9)$$

or, expressed in terms of the power spectrum [Nye, 1969]

$$\tau = \frac{\mu U}{\pi(1-v)} \int_0^{\infty} S(k) k^3 dk \quad (10)$$

For the integral in (10) to converge, $S(k)$ must fall off at least

as k^{-4} . This condition is partly due to our assumption of a one-dimensional bed. For isotropic beds wavy in two dimensions, $S(k)$ must fall off at least as k^{-5} [Nye, 1970]. The physical interpretation is that roughness for asperities of the smallest scale contributes high resistance to creep.

Two simple spectra proposed by Nye [1969] meet the convergence condition implied by (10): (1) a Gaussian spectrum centered on the origin, and (2) a constant spectrum with a cutoff at high wave number. For the Gaussian spectrum the results depend only on a wave number falloff parameter k_0 , and there is no difficulty with respect to convergence of (10). The power spectrum is given by

$$S(k) = S_0 \exp(-k^2/k_0^2) \quad (11)$$

where S_0 and k_0 are constants. This spectrum may apply to vegetative surfaces where steady glide can take place throughout the entire winter.

For the constant spectrum we introduce the cutoff wave number k_m . Without this cutoff the integral (10) diverges, indicating infinite drag resistance and no gliding; this situation describes a snowpack, lacking excess water, resting on a rough rock surface. If high wave numbers (tiny asperities) are absent, finite drag results. The power spectrum is

$$S(k) = C_0 [\mathbb{I}(k_m - k) + \mathbb{I}(k + k_m)] \quad (12)$$

where $\mathbb{I}(k)$ is the Heaviside step function. The autocorrelation functions associated with (11) and (12) are

$$R(x) = \frac{S_0 k_0}{2(\pi)^{1/2}} \exp[-(k_0 x)^2/4] \quad (13)$$

and

$$R(x) = \frac{C_0}{\pi} \frac{\sin k_m x}{x} \quad (14)$$

For zero argument, (13) gives

$$R(0) = S_0 k_0 / 2(\pi)^{1/2} \quad (15)$$

and expression (14) yields

$$R(0) = C_0 k_m / \pi \quad (16)$$

as $x \rightarrow 0$, so the correlation functions are

$$p(x) = \exp[-(k_0 x)^2/4] \quad (17)$$

and

$$p(x) = \frac{\sin k_m x}{k_m x} \quad (18)$$

respectively.

The mean and autocorrelation function of the partially drowned bed are now derived. Recalling that $z(x, y)$ describes the fluctuations of the bedrock surface, $w(z)$ is introduced to describe the fluctuations of the lower snowpack surface. In the absence of basal water and cavitation, $w \equiv z$, but if a water layer is present,

$$w(z) = z\mathbb{I}(z - z_0) + z_0\mathbb{I}(z_0 - z) \quad (19)$$

where z_0 is the level of the snowpack-water interface (Figure

3a). The mean value of w is simply

$$\langle w \rangle = \int_{-\infty}^{\infty} w(z) \frac{\exp(-z^2/2R_0)}{(2\pi R_0)^{1/2}} dz = \int_{-\infty}^{\infty} w \left[a\delta(w - z_0) + \frac{1(w - z_0)}{(2\pi R_0)^{1/2}} \exp(-w^2/2R_0) \right] dw = h \quad (20)$$

In (20), h is the mean thickness of the water layer, $\delta(z)$ is the Dirac delta function and

$$a = \frac{1}{(2\pi R_0)^{1/2}} \int_{-\infty}^{z_0} \exp(-z^2/2R_0) dz \quad (21)$$

The term

$$f^z(z) = \frac{1}{(2\pi R_0)^{1/2}} \exp[-z^2/2R_0] \quad (22)$$

in (20) can be recognized as the probability density function for z and

$$f^w(w) = a\delta(w - z_0) + \frac{1(w - z_0)}{(2\pi R_0)^{1/2}} \exp[-w^2/2R_0] \quad (23)$$

as the probability density function for w .

The autocorrelation of w is somewhat more difficult to calculate, but fortunately, the effects of nonlinear systems on Gaussian random processes have been investigated in great detail by electrical engineers. Methods for computing the autocorrelation of a clipped random process w have been developed. The approach used here is described by *Laning and Battin* [1956] and *Baum* [1969]. The autocorrelation of w is given by

$$R^h(p) = \frac{1}{2\pi R_0(1-p^2)^{1/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w(z_1)w(z_2) \exp\left[-\frac{z_1^2 + z_2^2 - 2pz_1z_2}{2R_0(1-p^2)}\right] dz_1 dz_2 \quad (24)$$

and the superscript h in $R^h(p)$ is introduced to indicate the presence of a waterfilm of mean thickness h . For $p = 0$ ($R(x) = 0$), expression (24), together with (20), gives

$$R^h(0) = \frac{1}{2\pi R_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w(z_1)w(z_2) \exp[-(z_1^2 + z_2^2)/2R_0] dz_1 dz_2 = \left\{ \frac{1}{(2\pi R_0)^{1/2}} \int_{-\infty}^{\infty} w(z) \exp[-z^2/2R_0] dz \right\}^2 = h^2 \quad (25)$$

Integration of (25) with the use of (19) yields

$$R^h(0) = R_0 \left[\alpha P(\alpha) + \frac{1}{(2\pi)^{1/2}} \exp(-\alpha^2/2) \right]^2 \quad (26)$$

where $\alpha = z_0/(R_0)^{1/2}$ and

$$P(x) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^x \exp(-t^2/2) dt \quad (27)$$

is the normal probability function tabulated by *Abramowitz and Stegun* [1965, p. 972]. For computation, the function $P(x)$ can also be expressed in terms of the error function $\text{erf } x$. For $x \leq 0$, $P(x) = \frac{1}{2}[1 - \text{erf}(-x/2^{1/2})]$, and for $x \geq 0$, $P(x) =$

$\frac{1}{2}[1 + \text{erf}(+x/2^{1/2})]$, where

$$\text{erf } z = 2/\pi^{1/2} \int_0^z \exp(-t^2) dt$$

An expression for the water layer thickness is then

$$h = \alpha(R_0)^{1/2}P(\alpha) + (R_0/2\pi)^{1/2} \exp(-\alpha^2/2) \quad (28)$$

where $(R_0)^{1/2} = \langle z^2 \rangle^{1/2}$, the root mean square deviation of the glide surface topographic fluctuations, a measure of the strength of the fluctuations.

In order to express $R^h(p)$ at the limits $p \pm 1$, (24) may be rewritten as

$$R^h(p) = \frac{1}{2\pi R_0(1-p^2)^{1/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w(z_1)w(z_2) \exp\left[-\frac{(z_1 - pz_2)^2}{2R_0(1-p^2)}\right] \exp\left[-\frac{z_2^2}{2R_0}\right] dz_1 dz_2 \quad (29)$$

Using the identity

$$\lim_{p \rightarrow \pm 1} \frac{1}{[2\pi R_0(1-p^2)]^{1/2}} \exp\left[-\frac{(z_1 - pz_2)^2}{2R_0(1-p^2)}\right] = \delta(z_1 - pz_2) \quad (30)$$

gives

$$R^h(\pm 1) = \frac{1}{(2\pi R_0)^{1/2}} \int_{-\infty}^{\infty} w(\pm z)w(z) \exp(-z^2/2R_0) dz \quad (31)$$

From (31), $R^h(1) = \langle w^2 \rangle$ and $R^h(-1) = \langle w(-z)w(z) \rangle$. The integrals may be expressed in terms of the probability density for w , but it is more convenient to work with the Heaviside functions given in (19). There are two cases: For $z_0 \geq 0$ it can be shown that $w(z)w(-z) = z_0z\mathcal{L}(z - z_0) - z_0z\mathcal{L}(-z - z_0) + z_0^2\mathcal{L}(z_0 - z)\mathcal{L}(z + z_0)$, and for $z_0 \leq 0$, $w(z)w(-z) = -z^2\mathcal{L}(z - z_0)\mathcal{L}(-z - z_0) + z_0z\mathcal{L}(z - z_0) - z_0z\mathcal{L}(z_0 - z)$. Given values of $\alpha = z_0/(R_0)^{1/2} \geq 0$, equation (31) is equivalent to

$$R^h(1) = R_0[\alpha^2 P(\alpha) + \frac{1}{2}Q(\alpha^2|3)] \quad (32)$$

$$R^h(-1) = R_0 \left[\alpha^2 [2P(\alpha) - 1] + \frac{2\alpha}{(2\pi)^{1/2}} \exp(-\alpha^2/2) \right]$$

where $P(x)$ has been defined previously and $Q(\chi^2|3)$ is the chi-square integral defined as

$$Q(\chi^2|3) = [2^{3/2}\Gamma(3/2)]^{-1} \int_{\chi^2}^{\infty} t^{(3/2)-1} e^{-t/2} dt \quad (33)$$

and tabulated by *Abramowitz and Stegun* [1965, p. 978]. Similarly, for $z_0/(R_0)^{1/2} \leq 0$, the integrals give

$$R^h(1) = R_0[1 + \alpha^2 P(\alpha) - \frac{1}{2}Q(\alpha^2|3)] \quad (34)$$

$$R^h(-1) = R_0 \left[\frac{2\alpha \exp(-\alpha^2/2)}{(2\pi)^{1/2}} + Q(\alpha^2|3) - 1 \right]$$

For $\alpha = 0$, $Q(\alpha^2|3) = 1$, and it is easily verified that both (32) and (34) yield the boundary conditions $R^h(1) = R_0/2$ and $R^h(-1) = 0$. The foregoing expressions for $R^h(0)$ and $R^h(\pm 1)$ will subsequently be used as boundary conditions on a differential equation for $R^h(p)$. Equation (26) gives an additional condition for $R^h(0)$ that can be used to check the accuracy of numerical solutions of the differential equation.

No simple procedure exists for directly integrating (24), but

Fig. 4. The variation of the normalized autocorrelation function $R^h(p)/R_0$ for various values of the dimensionless quantity $hR_0^{-1/2}$. This parameter measures the degree of water cover; for a half-covered surface, $hR_0^{-1/2} \approx 1.4$.

the autocorrelation of $d^2w/dz^2 = \delta(z - z_0)$ is simply

$$R^{w''}(p) = \frac{1}{2\pi R_0(1-p^2)^{1/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(z_1 - z_0) \delta(z_2 - z_0) \cdot \exp \left[-\frac{z_1^2 + z_2^2 - 2pz_1z_2}{2R_0(1-p^2)} \right] dz_1 dz_2$$

$$= \frac{1}{2\pi R_0(1-p^2)^{1/2}} \exp \left[-\frac{z_0^2}{R_0(1+p)} \right] \quad (35)$$

With some difficulty it can be shown that

$$R_0^2 R^{w''}(p) = d^2 R^h(p)/dp^2 \quad (36)$$

[Price, 1958] so that a differential equation governing R^h is given by

$$d^2 R^h(p)/dp^2 = \frac{R_0}{2\pi(1-p^2)^{1/2}} \exp \left[-\frac{z_0^2}{R_0(1+p)} \right] \quad (37)$$

The boundary conditions on R^h are given by (32) and (34). The special case $z_0 = 0$ (a half-wave rectifier) has the analytic solution

$$R^h(p) = \frac{R_0}{2\pi} [p \cos^{-1}(-p) + (1-p^2)^{1/2}] \quad (38)$$

This solution affords a check on the accuracy of the numerical solution of (37). A second check on solution accuracy is the known value of $R^h(0)$ given by (26).

Given the boundary conditions defined by (26) and (28), the differential equation (37) can be integrated numerically as a simple two-point boundary value problem; the apparent singularities at $p = \pm 1$ pose no difficulties. The curves in Figure 4 give $R^h(p)/R_0$ plotted for a range of the dimensionless quantity $h/(R_0)^{1/2}$ defined by (28); the mean values have not been removed so that $R^h(0) = h^2$. With the values of $R^h(p)$ defined, the values of $R^h(x)$ are obtained from the correlation functions defined by (17) and (18).

For the Gaussian bed, solutions for $R^h(x)$ are in the domain $0 \leq p \leq 1$ so it is appropriate to use the boundary conditions at $R^h(1)$, and $R^h(0)$. In Figure 5 the normalized zero-mean autocorrelations $[R^h(x) - h^2]/R_0$ for the one-dimensional

Fig. 5. Normalized autocorrelation functions for the Gaussian bed with the mean values h^2/R_0 removed. Values of $hR_0^{-1/2}$ are given next to the curves.

Gaussian bed are plotted for values of $h/(R_0)^{1/2}$ in the range 0–1.5. Similar calculations are given for the constant spectrum assumption in Figure 6, but these extend to negative values of p so that $R^h(-1)$ must be used as a boundary condition.

SNOW GLIDING FOR THE PARTIALLY DROWNED BED

For the partially drowned bed the relevant drag expressions are obtained from (9). With the replacement of $R(x)$ by $R^h(x)$, it is clear that

$$[D^h]^{-1} = \frac{4}{\pi} \int_0^\infty \frac{1}{x} \frac{d^3 R^h(x)}{dx^3} dx \quad (39)$$

For the thin film case the drag expressions are easily derived from (10) using (11) and (18). The Gaussian spectrum gives

$$\tau = \frac{\mu S_0 k_0^4 U^0}{2\pi(1-\nu)} = \frac{\mu R(0) k_0^3 U^0}{\pi^{1/2}(1-\nu)} \quad (40)$$

and the constant spectrum with a cutoff has drag given by

$$\tau = \frac{\mu C_0 k_m^4 U^0}{4\pi(1-\nu)} = \frac{\mu R(0) k_m^3 U^0}{4(1-\nu)} \quad (41)$$

Fig. 6. Normalized autocorrelation functions for a bed with a constant spectrum having a high wave number cutoff k_m similar to Figure 5.

In the case of a partially submerged bed, (3) must be numerically integrated over the bed using the appropriate autocorrelation functions obtained by solving (37). The triple derivative can be expanded to yield

$$\frac{d^3 R^h}{dx^3} = \frac{d^3 p}{dx^3} \frac{dR^h(p)}{dp} + 3 \frac{dp}{dx} \frac{d^2 p}{dx^2} \frac{d^2 R^h(p)}{dp^2} + \left(\frac{dp}{dx}\right)^3 \frac{d^3 R^h(p)}{dp^3} \quad (42)$$

From the correlation functions (17) and (18) the derivatives of $p(x)$ were obtained analytically. The values of $dR^h(p)/dp$ may be calculated by standard numerical procedures for the derivative of a tabulated function. The values of $d^2 R^h(p)/dp^2$ were derived from the differential equation (37). The third derivative was calculated as the first derivative of the tabulated values of $d^2 R^h(p)/dp^2$. This method allows calculation of the derivatives at equally spaced intervals and reduces the error that would be introduced by taking higher numerical derivatives.

A region of a snowpack free of macroscopic longitudinal stress gradients and material properties is termed a neutral zone. In a neutral zone the enhancement in glide speed caused by a water layer of mean thickness h may be expressed as the glide gain ratio

$$\frac{U^h}{U^0} = \frac{D^h}{D^0} \quad (43)$$

For the Gaussian spectrum the solution for $h = 0$ was obtained by setting $\alpha = -10.0$ as defined by (28). From equations (9) and (40), it is clear that

$$\int_0^\infty \frac{1}{x} \frac{d^3 R}{dx^3} dx = \frac{\pi^{1/2}}{2} R(0) k_0^3 \quad (44)$$

To obtain good precision in the solution (0.5% error), it was necessary to use $\sim 10^4$ increments over the range $0 \leq p \leq 1$. The same procedure was used for calculations in the range $0 < h/(R_0)^{1/2} \leq 2.5$. Since the solution for $h = 0$ converges more slowly than for $h > 0$, it is felt that the accuracy is quite good for all calculations. The numerical solutions (43) are plotted in Figure 7 as a function of $h/(R_0)^{1/2}$ for the Gaussian bed.

For the constant spectrum with a cutoff, (9) and (41) give

$$\int_0^\infty \frac{1}{x} \frac{d^3 R}{dx^3} dx = \frac{\pi}{8} R(0) k_m^3 \quad (45)$$

With this spectrum, solutions are in the range $-2/3\pi \leq p \leq 1$. Calculations showed that the values of the integration oscillated from positive to negative as the solution approached the true value. For $h = 0$ ($\alpha = -10.0$), integration over the first (positive) phase showed that an upper limit on the true value of the integral was provided. This value was 6% greater than the true value. Integration through the first positive and first negative intervals provided a lower limit on the integral that was 11% too small. A simple average of these limits yielded a value of the integral within 3% of the expected value. This procedure was followed for the remainder of the calculations. Since the solution for $h = 0$ converges more slowly than those for $h > 0$, it is felt that the solutions for $h > 0$ are more accurate. This method avoids carrying the integration beyond $k_m x = 3\pi$, and it provides solutions which are sufficiently ac-

Fig. 7. Glide gain ratio U^h/U^0 for the Gaussian bed versus dimensionless water layer thickness $h/(R_0)^{1/2}$. The approximate analytic solution from equation (47) is also shown.

curate when the physical assumptions in the model are considered. The numerical solutions for the constant spectrum with a cutoff are plotted in Figure 8.

Comparison of the calculated gains for the two spectra show that the differences are less than 10% for the cases considered. This suggests that the precise details of the bed spectra are not very crucial, at least for the one-dimensional models considered here. Similar calculations (unpublished) for bed spectra with constant roughness in one and two dimensions also show the same trend. Thus it is likely that the effect of introducing waviness in the transverse direction will not have a strong effect on calculated gains. Nye [1969] showed that an isotropic two-dimensional Gaussian spectrum increases drag for the thin film case by 18% over the one-dimensional Gaussian spectrum. Calculation of gains for the two-dimensional Gaussian spectrum have not yet been undertaken.

From the results of Figures 5 and 6 a simple approximation to the gliding gain functions is available. Inspection of Figure 5 shows that the autocorrelation functions are nearly Gaussian, and this was confirmed by numerical curve fitting. The

Fig. 8. Glide gain ratio U^h/U^0 for the constant spectrum having a high wave number cutoff plotted against $h/(R_0)^{1/2}$. The predictions of equation (47) are also shown.

curves of Figure 5 may be approximated by

$$R^h(x) \approx \left\{ \left[\frac{R^h(x)}{R_0} - \frac{h^2}{R_0} \right] \right\}_{\lim x \rightarrow 0} R_0 \exp \left[-\frac{1}{2} \left(\frac{k_0 x}{\sigma^h} \right)^2 \right] \quad (46)$$

where σ^h represents the standard deviation of the function with a water layer thickness $h/(R_0)^{1/2}$ and the quantity in braces represents the normalized zero-mean autocorrelation function evaluated at $x = 0$. This latter quantity can be given an analytic expression by noting that $x \rightarrow 0$ corresponds to $p = 1$. The bracketed expression becomes $[(R^h(1)/R_0) - (R^h(0)/R_0)]$. In order to calculate the gains the integral in (39) must be evaluated. With the stated approximations this integral has the value $(2\pi)^{1/2} R_0 k_0^3 / (\sigma^h)^3$ times the expression in braces, and for $h = 0$ it is clear from (17) that $\sigma^0 = 2^{1/2}$.

The gliding gain is then given by

$$\frac{U^h}{U^0} \approx \left[\frac{R^h(1)}{R_0} - \frac{R^h(0)}{R_0} \right]^{-1} \left(\frac{\sigma^h}{\sigma^0} \right)^3 \quad (47)$$

The curves of Figure 5 show that σ^h decreases as $h/(R_0)^{1/2}$ increases. From Figure 5 a simple approximate empirical relation for the standard deviation ratio is $\sigma^h/\sigma^0 = [1 + 0.6h/(R_0)^{1/2}]^{-1}$. This ratio together with (47) gives an approximation that is plotted on Figures 7 and 8. Important features of the model become evident. Obstacle drowning reduces both the amplitude and standard deviation of the autocorrelation functions. Thus in (47), $\sigma^h \leq \sigma^0$ and the term $(\sigma^h/\sigma^0)^3$ tends to counteract the gain enhancement contributed by the term in brackets. Comparison of the approximate analytic solutions with the numerical results shows quite good agreement; in view of the great uncertainties in the model assumptions the approximate solutions are entirely adequate.

The calculations represented in Figures 7 and 8 show that gliding is enhanced by about 2.5 if $h/(R_0)^{1/2} \rightarrow 1$ and about 15 if $h/(R_0)^{1/2} \rightarrow 2$. These results indicate an exponential increase in glide speed with linear increase in mean water layer thickness. Our intention, based upon concepts introduced by *Lliboutry* [1968], is to ignore any drag or the details of such over the drowned portions of the bed. Therefore the area of the bed over which partial separation occurs is a more important parameter than h for which to gauge our results. For reference, if a simple sine wave is considered for a bed geometry, it is easy to show that $h/\langle z^2 \rangle^{1/2} = 1.4$ implies that the surface area is half covered with water. Thus a major feature of the model is that a substantial fraction of the bed must be covered with water to precipitate rapid gliding.

When partial separation occurs the stress near undrowned asperities is intensified. For nonlinear snow rheology a further enhancement of the glide rate would result. A quantitative description of such conditions is beyond present knowledge of the properties of alpine snow.

PARTIAL SEPARATION

In the foregoing sections it is presupposed that separation takes place. We shall now relate, in an approximate fashion, our calculations of glide rate to the conditions for separation. Separation becomes possible when the fluctuations of normal stress become comparable to the overburden pressure. The normal pressure at the interface fluctuates about a mean value Π_0 due to the overburden pressure, and if negative fluctuations on the downstream portions of obstacles become great

enough to reduce the pressure to zero, the threshold for separation is attained.

The mean-square value of the pressure fluctuations $\Pi(x)$ may be extracted from *Nye* [1970] for the one-dimensional bed

$$\langle \Pi^2 \rangle = \frac{1}{\pi} \int_0^\infty \left[\frac{\mu}{1-\nu} \right]^2 U^2 S(k) k^4 dk \quad (48)$$

With the spectrum of equation (11), integration gives

$$\langle \Pi^2 \rangle^{1/2} = \frac{3^{1/2}}{2} \left(\frac{\mu}{1-\nu} \right) [R(0)]^{1/2} k_0^2 U \quad (49)$$

and equation (12) yields a similar calculation

$$\langle \Pi^2 \rangle^{1/2} = \frac{1}{5^{1/2}} \left(\frac{\mu}{1-\nu} \right) [R(0)]^{1/2} k_m^2 U \quad (50)$$

For a neutral zone the ratio of τ to the overburden stress Π_0 is given by $\tan \psi$, where ψ is the slope angle. Use of (40) and (41) yields the ratios

$$\frac{\langle \Pi^2 \rangle^{1/2}}{\Pi_0} = \frac{\tan \psi}{[R(0)]^{1/2}} \left(\frac{3\pi}{2^{1/2} k_0} \right) \quad (51)$$

for the Gaussian bed and

$$\frac{\langle \Pi^2 \rangle^{1/2}}{\Pi_0} = \frac{\tan \psi}{[R(0)]^{1/2}} \left(\frac{4}{5^{1/2} k_m} \right) \quad (52)$$

for a constant spectrum with a cutoff.

From *Kamb* [1970] and *Nye* [1970] an approximate condition for extensive separation is $\langle \Pi^2 \rangle^{1/2} / \Pi_0 \geq 1$. For a constant spectrum and $\lambda_m = 2\pi/k_m$ this condition gives

$$\lambda_m \geq \frac{2\pi(5)^{1/2}}{4 \tan \psi} [R(0)]^{1/2} \quad (53)$$

For typical values $h \sim [R(0)]^{1/2}$ and expression (53) implies $\lambda_m \geq 3.5h$ for $\psi = 45^\circ$ and $\lambda_m \geq 13h$ for $\psi = 15^\circ$. Thus extensive separation implies that the cutoff wavelength must be several times the mean water layer thickness. This is actually a condition on bed surface roughness. If wavelengths shorter than the above estimates are present, the surface would be too rough for extensive cavitation to develop. If roughness r is defined as $r \equiv \langle z^2 \rangle^{1/2} / \lambda$, then r cannot be greater than about 0.3 for extensive cavitation to develop on a 45° slope or about 0.07 on a 15° slope. This seems a reasonable condition to fulfill, and it gives quantitative support to the observation that only smooth surfaces allow rapid glide. For rapid glide to be possible, r values are expected to lie in the approximate range 0.1–0.01.

The physical interpretation of the cutoff wavelength is that small-amplitude fluctuations $\langle z^2 \rangle^{1/2}$ are drowned by the presence of excess free water. If r is in the range 0.1–0.01 and h is near 1 mm, obstacles less than millimeter size would be removed from the drag process, and obstacles larger than 1 mm would be passed by creep over them. Similar calculations for the Gaussian bed using (51) yield a condition on falloff wavelength λ_0 ; a high wave number cutoff is unnecessary.

DISCUSSION AND CONCLUSIONS

If creep over asperities is taken as the only glide mechanism, then to achieve finite drag, the spectrum must fall at least as k^{-4} . This condition should be checked against actual field or

possibly laboratory measurements. Field measurements show that some beds will not allow gliding; whether this is because of the form of the roughness spectrum or some other cause is unclear. Laboratory experiments might answer this question quickly, and they may be essential to determine how realistic our basal water distribution is.

We are unsure if additional mechanisms should be incorporated in the gliding theory. In the case of glacier sliding, the pressure melting mechanism is also active, and the problem of high wave number asperities leading to infinite drag does not arise. Unpublished calculations that include the conventional pressure melting (but ignore several possible complicating factors) show that as $h/(R_0)^{1/2}$ increases, glide gain is significantly enhanced over the estimates plotted in Figures 7 and 8. We found that if σ^h/σ^0 is set equal to 1.0, expression (47) provides an upper limit on the glide gain ratio provided by these calculations. The results predict an increase in glide gain over the present model of up to a factor of 5 as $h/(R_0)^{1/2} \rightarrow 1.0$ and up to 15 as $h/(R_0)^{1/2} \rightarrow 2.5$. It is expected that pressure melting effects on snow glide would be small, if not negligible, compared to the effect on glacier sliding. Nevertheless until a proper analysis is undertaken and the range of possible glide is mechanisms better understood, the calculations in this paper should be taken as a lower limit on glide gain.

A major feature of the model is that a substantial fraction of the bed must be covered with water for significant drag reduction. For the model that we have chosen, the Gaussian assumption implies that enhancement in glide speed depends only on h for a given bed geometry. It is expected that the effects of separation on glide velocity will depend only on the location of separation and contact points [Lliboutry, 1968]. This makes the area of separation more important than mean water layer thickness for assessing separation effects. For the present model, no account of the drag is taken for drowned portions of the bed, and it is implied that water pressure is hydrostatic there. If a different geometry was chosen for the snowpack sole over the portions separated, use of h as the only parameter would imply a dependence of drag on the details of the snowpack sole geometry in the areas where separation takes place. The discussion following equation (2) indicates that once separation takes place, the drag will be essentially independent of h . Therefore the physical principles of our model make it possible that even thin zones of separation can yield high glide rates.

Our model assumes steady glide, but variations in the supply and drainage of water can lead to unsteady glide. The simplest approach to time dependence would be to introduce a time-variable water thickness into our model. A more realistic treatment would also allow nonlinearity and rate dependence of the deformation of the material, including any pore pressure effects. In spite of gaps in knowledge about how precisely to model such effects, there is little doubt that the flux and distribution of water in the basal regions are crucial to the explanation of tensile glide crack formation and full-depth avalanche release.

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